Performance Analysis of Real-Time Extended and Unscented Kalman Filters

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***Abstract*—This paper performs a deep Performance Analysis of the most utilised filters for prediction estimation on tracking systems which are the *Extended Kalman Filter* (*EKF*) and the *Unscented Kalman filter* (*UKF*). A real-time simulation is used to compare both filters and praise the advantages of each approach, where both are compared with their respective *MATLAB* function. All the performed scenarios aim to test the capabilities of the filters in a real-life scenario and, in the end, discover what are the best conditions to use each method.**

***Keywords*—** Extended Kalman filter; Unscented Kalman filter.

II. Introduction

Rudolf Emil Kálmán (1930-2016) was an electrical engineer distinguished with the IEEE Medal of Honour in 1974. At that time his filtering ideas met less scepticism in the mechanical engineering department where he published the first results of his co-invention: Kalman-Bucy Filter (commonly know only as Kalman Filter). This filter was firstly used during the Apollo program (1961-1972) and later on in many types of vehicles ranging from submarines to missiles or even space shuttles [1]. Widely used in many systems, highlighting the radar world where the *Kalman Filter* is used extensively, according to the author of [2], the filter had tremendous impact in the modernisation of these systems in the middle 20^{th} century . As he says "systems may be dynamic with measurements made from moving platforms at consistent time intervals such as in autonomous navigation.". Usually it is present in models incorporating electronic and gyroscopic systems, that the "new method" as R.E.Kálmán called in his famous research "A new approach to linear filtering and predicting problems" [3]. Moreover, it is a recursive algorithm that works under linear and gaussian conditions. According to the last aspect, improvements were made resulting on filters for non-linear systems and also adaptations to work with approximate Gaussian's.

The normal *EKF* is widely used in the field of non-linear estimation, with applications in both *state-estimation* and *machine learning* for parameter estimation. However, as it is explained in the next sections, the EKF is a variation of the normal *Kalman Filter* that can be used for non-linear models, by approximating the optimal terms as:

$$\hat{x}_k^- \approx F(\hat{x}_{k-1}, \overline{v}); \quad \hat{y}_k^- \approx H(\hat{x}_k^-, \overline{n}) \tag{1}$$

$$\hat{x}_k \approx \hat{x}_k^- + \kappa \cdot [y_k - \hat{y}_k] \tag{2}$$

$\hat{x}_k$- Update estimate; $\hat{x}_k^-$ - Prediction estimate;
$\hat{y}_k$ - Measurement prediction; $\kappa \in [0, 1]$ - Optimal Gain.

This meaning, that in the *EKF* the state distribution is approximated by taking into account the first-order terms of the *Taylor* expansion applied to the equation that describes the state transition. This approximation can introduce large errors, for example in the covariance estimation which may lead to sub-optimal performances and even the divergence of the filter[4][5].

In contrast, the *UKF* addresses this approximation issues, since now the state distribution is specified using a minimal set of carefully chosen sample points (*sigma points*). These *sigma points*, when propagated through the non-linear system can actually capture the posterior mean of a covariance accurately to the 3^{rd} order of the Taylor Series expansion for any non-linearity which is far better than the 1^{st} order approximation of the *Extended Kalman Filter*.

In theory, the *UKF* provides an alternative to the *EKF* for especially more nonlinear models, that have difficult jacobians both related to the state and measurement model, that would have more errors associated with the Taylor Series approximation.

III. Background Concepts - Kalman Filter

The *Kalman filter* has two different steps per iteration: prediction and update, as it is shown in figure 1. This is a recursive method since the output of a step is used cyclically in future iterations. Moreover, this method is composed by five equations that were also present in his paper [3].

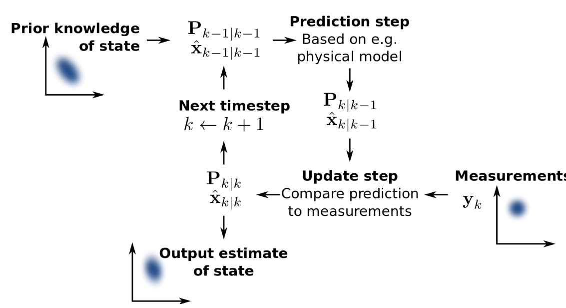

Fig. 1. Workflow of the Kalman filter

a) Prediction Step: It combines the prior knowledge of the state and the instance to predict the next state. It is composed by two equations: state (15) and covariance (24) extrapolation present in [3].

b) Update Step: Combining a periodic measurement and the prediction, in this step it is computed the Optimal gain (25) and the update equations: state (21) and covariance (26) according to [3].

The demonstration of the equations is studied in [6], including the predict equations (9) and (10), gain (11) and update equations (12) and (13).

IV. Literature Review

An application of the *Extended Kalman Filter* to a "vehicle tracking based on the road surface vibration measurements" [7] was tested on a real experiment, and the results indicate that this method is feasible under certain circumstances. However the results were not optimal, since the measurements are considerably different from the true values often in the first half of the test. Furthermore *Ashok et al.* applied a common and a modified gain EKF (*MGEKF*) in an underwater active target tracking [8]. Analysing the two scenarios obtained, and the conclusions of the authors claiming that the *MGEKF* is more suitable for the research purpose than the *EKF*, which is highlighted the orientation error obtained with the *EKF* as the main responsible for the poor results.

Lastly, taking these two experiments into account, the purpose of this research is to create a non-linear model that estimates a pre-defined path (under certain conditions) and measure the error of the estimated 2D position and their true values as in [7]. In order to avoid notorious differences in the orientation as [8], the integral of the orientation error is a system feedback.

The *Unscented Kalman Filter* was initially proposed by *Julier and Uhlmann* [9] as an initial implementation of the already popular *Unscented Transform* for the prediction of random variables for a nonlinear transformation.

This work was based on previous papers of the same group regarding the study of nonlinear filtering systems [10], which in it's early stages still lacked some optimisation and fine tuning of the important variables such as β and λ which will be addressed further on the article.

Nevertheless, the main objective of the *Julier et al.* research group was to implement a new approach to the normal *EKF*, that had been in use for over 30 years, that would be far easier to implement, easier to tune and more reliable for more nonlinear systems.

Trough the years more and more research groups have developed improvements to the normal *Unscented Kalman Filter*, in order deploy a more optimisation-based strategy to the parameters of the *UKF* or even to the sampling of the *sigma points*, by finding better ways to spread them to achieve a better overall estimation [11].

V. Filtering methods

The model used during the research is related to a GPS and an orientation sensor and is used to follow a pre-defined trajectory. Therefore the non-linear model is written by:

$$h(\hat{x}_k^-) = \begin{bmatrix} \hat{\rho}_k \\ \hat{\theta}_k \end{bmatrix} = \begin{bmatrix} \hat{\rho}_k({}_x\hat{\rho}_k, {}_y\hat{\rho}_k) \\ \arctan_2(\frac{{}_y\hat{\rho}_k - {}_y\hat{\rho}_{k-1}}{{}_x\hat{\rho}_k - {}_x\hat{\rho}_{k-1}}) \end{bmatrix} \tag{3}$$

Therefore the Jacobian Matrix $\left(\dfrac{\partial h(\hat{x}_k^-)}{\partial({}_x\hat{\rho}, {}_y\hat{\rho}, \hat{\theta})}\right)$ is given by:

$$H(\hat{x}_k^-) = \begin{bmatrix} \frac{{}_x\hat{\rho}_k}{\sqrt{{}_x\hat{\rho}_k + {}_y\hat{\rho}_k}} & \frac{{}_y\hat{\rho}_k}{\sqrt{{}_x\hat{\rho}_k + {}_y\hat{\rho}_k}} & 0 \\ \frac{-({}_y\hat{\rho}_k - {}_y\hat{\rho}_{k-1})}{{}_x\hat{\rho}_k{}^2 + {}_y\hat{\rho}_k{}^2} & \frac{{}_x\hat{\rho}_k - {}_x\hat{\rho}_{k-1}}{{}_x\hat{\rho}_k{}^2 + {}_y\hat{\rho}_k{}^2} & 0 \end{bmatrix} \tag{4}$$

The $\hat{x}_k^-$ or $\hat{x}_{k|k-1}$ is computed in the prediction step from status transition equation at the prior moment (k-1), $({}_x\hat{\rho}_k, {}_y\hat{\rho}_k)$ are the 2D position of the object carrying the sensors.

Throughout the next section, it is used the following notation

TABLE I
EKF/UKF matrix notation

Nomenclature	Connotation
f	System Dynamics
F	Jacobian of the System Dynamics
h	Observation Dynamics,
H	Jacobian of the Observation Dynamics
P	Estimate Uncertainty
R	Measurement Uncertainty
Q	Noise Variance

Finally the 2D movement of the object (e.g. Car) is also non-linear (5a) and therefore it is used its Jacobian (5b) in the recursive method.

$$f({}_x\rho, y_\rho, \theta_k) = \begin{bmatrix} {}_x\rho_{k-1} + N\cos(\theta_{k-1}) \\ {}_y\rho_{k-1} + N\sin(\theta_{k-1}) \\ \theta_{k-1} + \sum_{n=1}^{k-1}(\theta_n^{GPS} - \theta_n) \end{bmatrix} \tag{5a}$$

$$F_k = \begin{bmatrix} 1 & 0 & -N \cdot s \\ 0 & 1 & N \cdot c \\ 0 & 0 & 1 \end{bmatrix} \tag{5b}$$

$$N = ||x_k^{GPS} - x_{k-1}^{GPS}||_2; \quad c = \cos(\hat{\theta}_{k-1}); \quad s = \sin(\hat{\theta}_{k-1}),$$

and N is the *a priori* distance between two measurements.

A. Extended Kalman Filter

The assumptions made for the *Kalman Filter* to always work is to have a gaussian distribution and a linear observation matrix. The *Extended Kalman filter* does not require the linearity since it uses the 1^{st} order-Taylor expansion around the estimate state, to transform the non-linear system into linear equations.

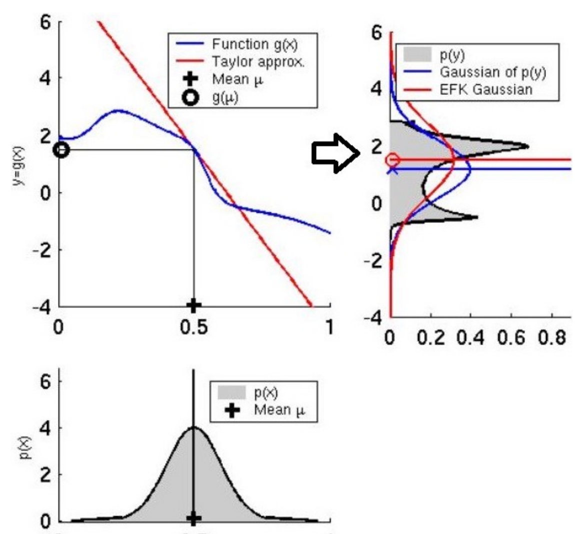

Fig. 2. 1^{st} order-Taylor expansion ($F(a) = f(a) + f'(a) \cdot \frac{(x-a)}{1!}$) used to linearise a non-linear function of one variable.

The results of the *EKF* are reasonably close to the true value gaussian when the non-linear behaviour of the functions is close to linear and continuous, since the Taylor expansion used belongs to a linear process.

a) Prediction Step: Remains unchangeable compared to the Kalman filter, and is described in the table II.

b) Update Step: Varies depending on the non-linear model. Regarding the model used in the research (GPS and sensor orientation) the non-linear equations are (3) and the jacobian (4). The equations related to this step are written in the table II.

TABLE II
Extended Kalman Filter

	Initialisation: *a priori* values
1.	Prediction:
	$\hat{x}_{k\|k-1} = f(\hat{x}_{k\|k-1})$
	$P_{k\|k-1} = FP_{k-1}F^T + FQF^T$
2.	Gain:
	$K_k = P_{k\|k-1}H^T \cdot [H_k P_{k\|k-1} H^T + H_k R H^T]^{-1}$
3.	Update:
	$P_k = (I - K_k H_k) \cdot P_{k\|k-1}$
	$\hat{x}_k = \hat{x}_{k\|k-1} + K_k \cdot [x_k^{GPS} - h(\hat{x}_{k\|k-1})]$

To conclude, the *Extended Kalman Filter* evaluates how feasible the data is compared to its knowledge about the system. This method computes the state estimate that minimises the mean-square covariance error. Moreover, uncertainties (P and R) define the Kalman gain that in collaboration with the prediction state and the measurements, contributes to the updated state. For example taking into account the case when the estimate uncertainty goes to 0, the Kalman gain is 0 and therefore the next state relies on the prediction of the model, and vice verse for the case when the measurements uncertainty is small the Kalman gain matrix tends to the inverse of the observation's jacobian and so the next state relies on the measurements.

B. Unscented Kalman Filter

As it was previously said, the UKF addresses much of the approximation issues of the *EKF*, and it does this using by applying an *unscented transformation*(UT), which is a method for determining the probability of a random variable that undergoes a specific nonlinear transformation, such as the dynamics model of the a Car that will be addressed on the experimental stage.

a) Prediction Step: In order to explain this transformation, let us first consider that *x* has a mean $\tilde{x}$ and covariance of P_x. To calculate the propagation of *x* with a nonlinear transformation (assumed y), the UT creates a matrix χ of *2L + 1 sigma vectors* χ_i (L is the number of state variables), each with a corresponding weight W_i that can be personalised by the user to give more emphasis on, for example, the initial value of *x* instead of the sigma points. Next, to all this points, the nonlinear transformation is applied following the weight criteria defined for each in order to have a final median estimate of the next prediction.

$$
\begin{aligned}
\chi &= \tilde{x}; \\
\chi_i &= \tilde{x} \pm (\sqrt{(L+\lambda)P_x}),\ i = 1, ..., L; \\
W_0 &= \frac{\lambda}{L+\lambda}; \\
W_i &= \frac{1}{2(L+\lambda)},\ i = 1, ...2L;
\end{aligned} \tag{6}
$$

TABLE III
UNSCENTED TRANSFORMATION

Nomenclature	Connotation
β	Defines the amount of prior knowledge of the Gaussian distribution of *x*(set to 2)
λ	Scaling Parameter

As it was previously said, the *unscented transform* applied the nonlinear dynamics model to each *sigma point*, following the expressions.

TABLE IV
UNSCENTED KALMAN FILTER - DYNAMICS STAGE

Dynamics Model Processing
$\chi_{[i,k\|k-1]} = F[\chi_{[k-1]}]$
$\hat{x_k}^- = \sum_{i=0}^{2L} W_i \chi_{[i,k\|k-1]}$
$P_k = \sum_{i=0}^{2L} W_i[\chi_{[i,k\|k-1]} - \hat{x_k^-}][\chi_{[i,k\|k-1]} - \hat{x_k 0}^-]^T$

A visual version of what was previously said, can be seen from figure 3, where it is possible to see how the *sigma points* influence the estimate in contrast with the normal linearisation of the system by the *EKF* and with the total sampling analysis relative with the Monte Carlo Estimate.

TABLE V
UKF TABLE OF NOTATIONS

Nomenclature	Connotation
$F[\chi_{[k-1]}]$	Defines the Non-Linear transformation in this case related with the Dynamics model
$\hat{x_k}^-$	State Prediction from the Non-Linear Dynamics Model
P_k	Defines the covariance matrix regarding the Estimate
$\chi_{[i,k\|k-1]}$	Defines the expected state for a given point

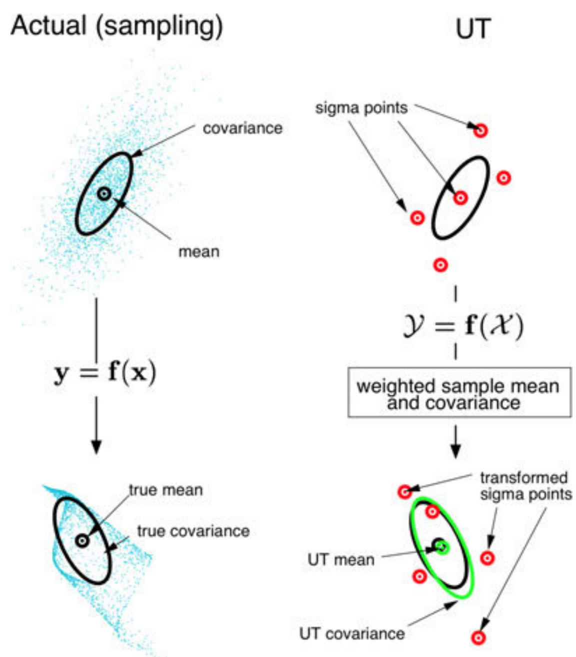

Fig. 3. Unscented Transformation and normal Monte Carlo sampling

TABLE VI
UNSCENTED KALMAN FILTER - MEASUREMENT STAGE

Measurement Model Processing
$\hat{y}_{[k\|k-1]} = H[\hat{y}_{[k\|k-1]}, y_{[k-1]}]$
$\hat{y}_k^- = \sum_{i=0}^{2L} W_i \hat{y}_{[i,k\|k-1]}$
$P_{\hat{y}} = \sum_{i=0}^{2L} W_i[\hat{y}_{[i,k\|k-1]} - \hat{y}_k^-][\hat{y}_{[i,k\|k-1]} - \hat{y}_k^-]^T$

TABLE VII
UKF TABLE OF NOTATIONS

Nomenclature	Connotation
$H[y_{[k\|k-1]}, y_{[k-1]}[]$	Defines the model of the Measurements
$\hat{y_k}^-$	Defines the expected measurements
P_k	Defines the covariance matrix regarding the Estimate
$\hat{y}_{[i,k\|k-1]}$	Defines the expected measurements for a given point
$P_{\hat{y}}$	Defines the Covariance matrix for the Measurements

b) Update Step: Next the *Unscented Kalman Filter* takes advantage of the Measurement Model by applying the same *Unscented Transform* applied to the Dynamics model, as it can be seen in equations VI.

After obtaining the expected measurements, the next steps are to determine the *Kalman Gain*, update the estimate based on the measurements and, in the end, update the Covariance matrix regarding the estimate, which can be seen in equations VIII.

An important remark is that the *UKF* does not require any type of Jacobian determination which gives it an upper-hand in certain scenarios in contrast with the *EKF*.

TABLE VIII
UNSCENTED KALMAN FILTER - UPDATE STAGE

Update Processing
$P_{\hat{x}_k \hat{y}_k} = \sum_{i=0}^{2L} W_i [\chi_{[i,k\|k-1]} - \hat{x_k}^-][\hat{y}_{[i,k\|k-1]} - \hat{y_k}^-]^T$
$\kappa = P_{\hat{x}_k \hat{y}_k} P_{\hat{y}}^- 1$
$x_k = \hat{x}_k^- + \kappa(y_k - \hat{y}_k)$
$P_k = P_k^- - \kappa P_{\hat{y}} \kappa^T$

TABLE IX
UKF TABLE OF NOTATIONS

Nomenclature	Connotation
$P_{\hat{x}_k \hat{y}_k}$	Defines the covariance associated with the State Model
κ	Defines the expected measurements
P_k	Defines the Kalman Gain
x_k	Defines the prediction after the Update Step

VI. EXPERIMENTAL RESULTS

In order to prove the theoretical component and analyse the performance for multiple scenarios it was developed both the *Extended Kalman Filter* and the *Unscented Kalman Filter*, which are then compared, on every simulation, with the respective version from *MATLAB*, in this case, the functions *trackingEKF* [12] and *trackingUKF* [13], respectively.

In order to analyse the performance of each method, both filters were applied to a 2D moving object with GPS and an Orientation Sensor. Moreover, in order to have a better understanding of the benefits of each filter, both scenarios were tested with normal conditions, high error applied to the Measurements, difficult paths and also the kidnapping situation, where the filter starts with an estimated initial position that is not the correct one, to see the convergence ability of the filter to the true path.

First it is important to understand the conditions of the experiments. In this case, it was found through the research made that the normal GPS Error is about 10^{-3}% of the measurement and since it was not found much information regarding the Orientation sensor it was considered to have the same error percentage.

TABLE X
EXPERIMENT TABLE OF INFORMATION

Connotation	Value
Normal Measurement Error	0.001% of y_n
High Measurement Error	0.01% of y_n
Super High Measurement Error	0.1% of y_n
Kidnapped Situation	Defines the Situation where the Prediction Model is initially in a different Position than the Object.

Moreover, to test all the different scenarios it is also important to take note that the initial covariance regarding the matrices Q and R varied depending on the scenario, for example the kidnapped scenario requires a higher covariance regarding the measurements since the measurements do not correspond to the actual position in the initial state.

A. Software Interface

The software developed allows the user to draw a new path or test the filtering in two different default scenarios. The first one contains a simple curve and the second is more complex with an hand-made draw of the "kalman" word. The simulation is performed in real-time periodically with 0.2 seconds between measurements. The user interface is composed by a plot of the all test on the left, the area of interest zoomed on the top involving the actual instance and the legend of the colours, respectively for each filter on the top right corner. Bellow the area of interest is the euclidean distance between the estimate location and the true location for each group of filters (*EKF* and *UKF*).

B. Simulation with Normal Measurement Error

The first battery of tests was under a normal measurement error, firstly with a smooth curve and the second with abrupt change of directions. The figure 4 represents the first scenario (curve) where the performance of the filter is almost perfect during the first 90% of the test, then it presents an error order of approximately 1‰on both developed filters and in the *UKF* of the matlab [13].

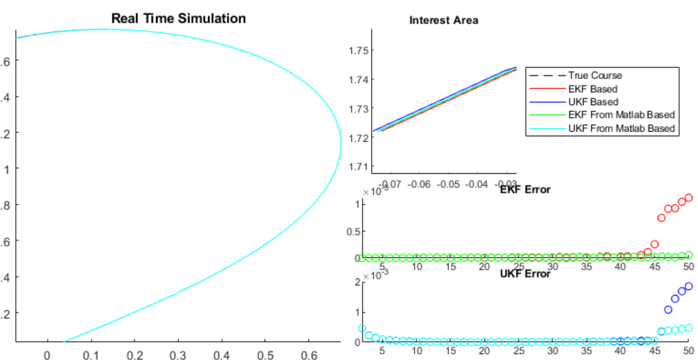

Fig. 4. Curved Simulation with Normal Measurement Error

Bearing these results in mind and taking into account the results of the second scenario 5, where both *EKFs* performed better that the *UKFs*, it is correct to state that under normal circumstances / low measurement errors the *Extended Kalman filters* are more appropriated. This is a result of the number of estimations this group of filter does (1) compared to the *Unscented Kalman filters* and therefore they converge quicker.

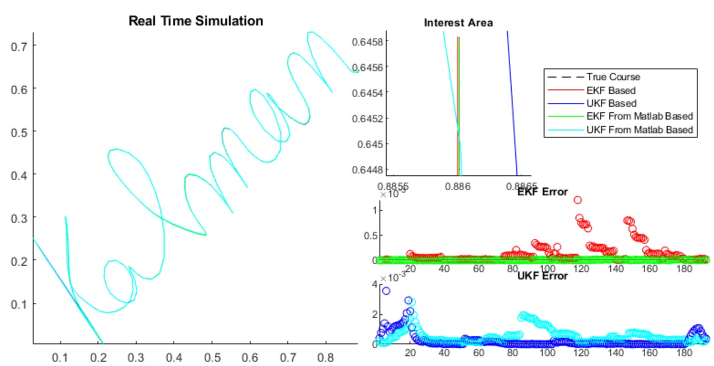

Fig. 5. Kalman Word Simulation with Normal Measurement Error

Regarding the second scenario (fig. 5) it is evident that once the error has reached a relevant increase it can be considered a peak, since the accumulated error of the angle does not let the system forget about past deviation and corrects the future orientation until it converges to the true values. This approach is not biased since the correction is made on the angle and the error is observed in the euclidean distance compared to the state variables.

C. Simulation with High Measurement Error

From table X it is possible to see the measurement values used, since in these tests it is mainly valued the capacity of the filters to coup with noisy measurements. For this reason it is tested with two different scenarios and one of them with a much higher noise percentage.

The first example is the figure 6, where in contrast with the normal noise values, the *UKF* has much better errors than the *EKF*, which is also to be expected since the *Unscented Kalman Filter* takes into account a bigger sample(*sigma points*) than the only one point sample on the prediction phase of the *Extended Kalman Filter*. Thus the *UKF* has a better ability to coup with noisy data than the extended version.

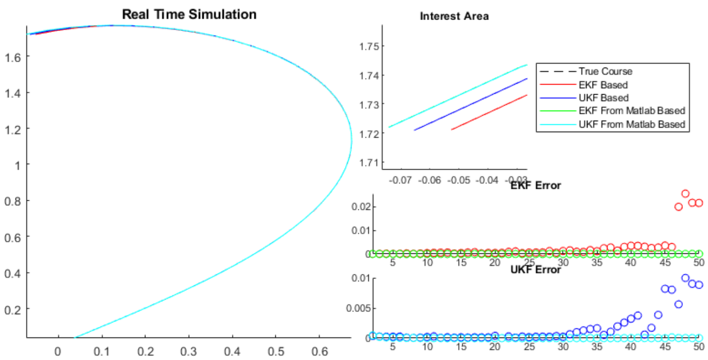

Fig. 6. Curved Simulation with High Measurement Error

On the next scenario, figure 7, the authors wanted to go a step further and test in a much more difficult scenario which is describing the "kalman" word. Here again the *UKF* performed much better than the standard *EKF*, which actually tended to have high errors (seen in red near the last 'a'), but managed to correct them with ease.

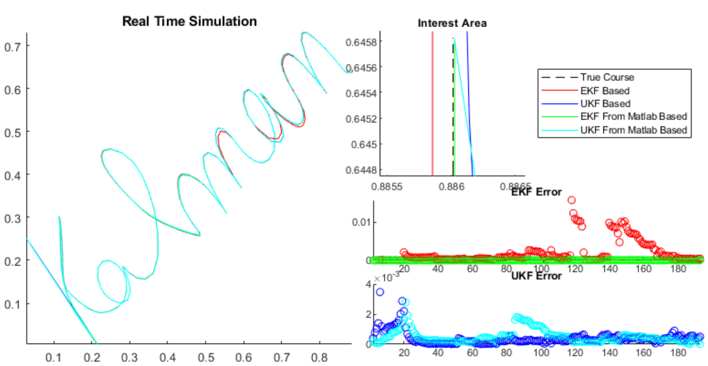

Fig. 7. Kalman Word with High Measurement Error

The last scenario, figure 8 is still the word kalman, but now the percentage of error applied to measurements was increased, now describing the 'Super High Measurement Error' connotation. Here it was immediately evident that the *UKF* has much better results, which can be explained by the same line of thinking as the first example, but now the *EKF* actually diverged in some instances (letter 'm' and last 'a') but managed to correct itself, which will be better explained in the Kidnapped scenario where the convergence ability is tested.

D. Kidnapped

As it has been explained previously the Kidnapped Situation refers to the scenario where the Prediction Model starts with an different initial position than the actual object. It is an important scenario since it tests the convergence of the model. In this case it can clearly be seen by the left image of figure 9 that the initial position of the prediction model is 0.05 units (e.g. meters) away from the actual position.

From what is possible to see from the error showcased in the bottom right corners of the figure 9 it can be clearly seen that the *EKF* converged much faster than the *UKF*, which resulted in lower errors. However, if the analysis was based on the closest neighbours and not the error at every instant, which is showcased on the top right image, it is seen that the *UKF*(in blue) had a much better convergence to the original signal.

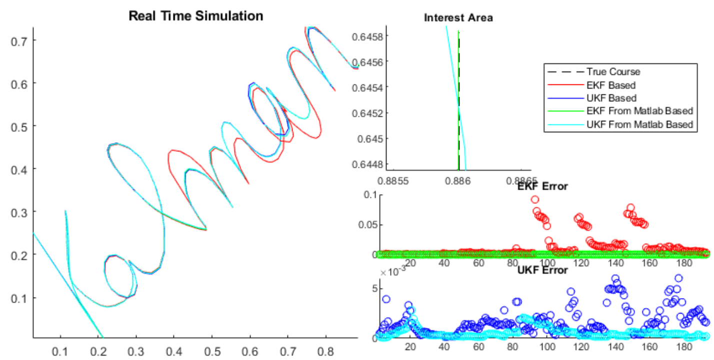

Fig. 8. Kalman Word with Super High Measurement Error

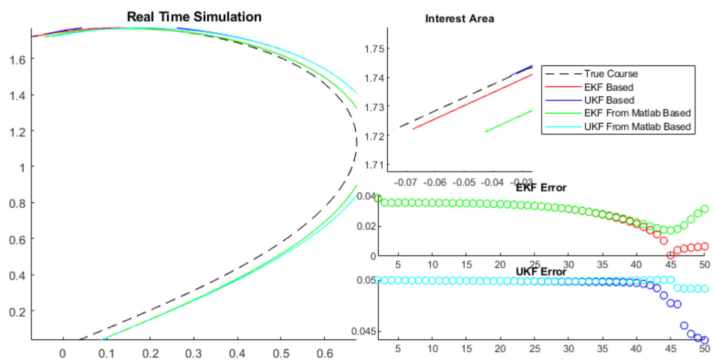

Fig. 9. Curved Simulation in Kidnapped Scenario

However since it took much longer to converge, and the error is valued by iteration, it resulted in a much larger error than the *EKF*.

These results were expected since, in short, the *EKF* considers only one sample whereas the *UKF* considers a much bigger group of samples(*sigma points*), which will result in a much slower convergence for the unscented version once it always takes into account the mean of a larger sample. In contrast, since it uses a larger sample, the *UKF* can deal much better with noisy data, which also explains the more perfect convergence even-though it was a late one.

Finally, it is also important to take notice that in this case, both the implemented version of the *EKF* and the *UKF* performed coherently with the *MATLAB* version throughout most of the process, but in a latter stage they actually performed better, obtaining better overall errors than their counterpart.

VII. Conclusion

In conclusion, it was possible to understand that both the *UKF* and the *EKF* have very similar implementations, varying mostly on the sampling strategy.

On the one hand, the *Unscented Kalman Filter* corrects the approximation problem of the *Extended Kalman Filter* for nonlinear systems, since the *UKF* takes into account more than just the 1^{st} term of the Taylor Expansion by using the *Unscented Transformation*, which contrasts with the *EKF* that has a bigger sample for the prediction process. In addition, this is also an advantage to use with systems where the observation matrix has some uncertainties. On the other hand, the *EKF* tends to have a more standardised implementation and a much better convergence speed since it only considers one point for the sampling.

From the simulations performed, it was possible to understand that the *Unscented Kalman Filter* tends to perform much better than the normal *Extended Kalman Filter* in high error scenarios applied to the sensors, mostly due to considering more than one point on the sampling phase(*sigma points*), whereas the *EKF* uses always one point only. However, the *EKF* will tend to perform better when the sensors are better and provides a much faster convergence, that was clearly showed in the Kidnapped Scenario.

In the end, both methods provide their benefits, but if the final objective is to have an implementation on a real-life scenario, the *Unscented Kalman Filter* is clearly the better option, since in these scenarios the sensors tend to have more errors and, moreover, since it's a real-life situation, process speed is a crucial factor and the *UKF* also takes an advantage since it does not need to determine the jacobians at each instant. Furthermore it actually works better with more nonlinear systems since it can accurately capture the posterior mean of a covariance up to the 3^{rd} order of the Taylor Series. However, if the main objective is based on the ease of implementation for any system or if the sensors used have a very high accuracy and low environment variance, the *Extended Kalman Filter* can be a very viable option in contrast to the *UKF*.

The software of the project is available here.

VIII. Future Work

As further studies on the Performance Analysis of both the *Unscented Kalman Filter* and the *Extended Kalman Filter* we lacked sufficient thorough research on the velocity influence on the simulation and how they may impact the scenario with a higher degree of error.

In addition, the simulations could be performed based on the *Monte Carlo method* in order to better understand the limitations of the filters and whether or not the convergence could be guaranteed.

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