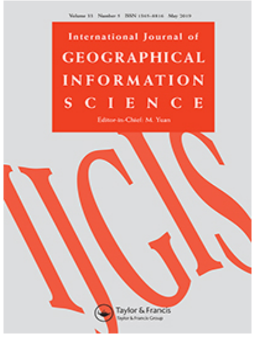

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Commercial GIS location analytics: capabilities and performance

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ABSTRACT

An essential location analytic method is GIS, able to support map-based display along with geographic data creation, management, manipulation in addition to functions for suitability evaluation. Location models have proven to be critical as well, with many prominent approaches found in popular GIS software. This paper reviews a class of location models and provides an overview of the methods used to solve these models. This is significant because of the broad use and application of location models in GIS to address important problems and issues facing society. Spatial analytical insights are essential. The implications of models and methods available through GIS are explored, particularly notions of solution quality. Case studies are offered to highlight ease of access to location models in GIS along with observed computational performance.

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Introduction

The importance and significance of GIS (geographic information systems), and more broadly GIScience, is undeniable. It represents a major focus of basic research but also is widely applied to address a range of substantive issues, problems and concerns. GIS is arguably the first location analytic tool used in any study, providing the capacity to create, manage, manipulate and display spatial information of all sorts (Clarke 2011). User-friendly, point-and-click access through GIS to a range of spatial analytical methods is also common, including spatial statistics and location modeling. This has removed past limitations and overhead of advanced technical knowledge and training necessary in order to apply GIS and associated analytics, making it accessible to even the most novice of users.

While there are a range of location analytics supported in GIS, of increasing interest is the use of location models as a component of broader spatial analysis capabilities. Combined with GIS, much has been written on the significance of location modeling (Church 1999, 2002, Murray 2010, Bruno and Giannikos 2015), and more generally spatial optimization (Tong and Murray 2012, Li *et al*. 2016). Further, commercial GIS packages like ArcGIS and TransCAD explicitly structure and solve location models. What is particularly interesting about this is that available model options within GIS software are solved using heuristic methods. By definition, a heuristic is an ad hoc strategy for solving

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an optimization problem (Church 2001, Church and Murray 2009, Tong and Murray 2012). A major benefit of using a heuristic is that it may identify a solution more efficiently, requiring less computing time and resources. This may be more economical too as there is no need to acquire software or develop an exact solution method, if one exists for the model of interest. Thus, solution via a heuristic within GIS bypasses the need for a solver that would enable exact, optimal solutions to be found for a structured problem. However, the likelihood that sub-optimal results will be obtained in any application is high as there is no guarantee for (most) heuristics that they will derive good or optimal solutions (see Church and Sorensen 1996, Murray and Church 1996, Tong and Murray 2012). Further, observed performance characteristics for a heuristic utilized for one optimization problem generally do not carry over when the heuristic is applied to another optimization problem. It may or may not perform equally as well, or as poorly, because it depends on the heuristic, problem properties, constraining conditions, etc.

Because of advances in GIS technology and easy to apply, user-friendly point-and-click interfaces, the application of location models is now common and widespread. Application studies relying on location models through GIS are readily found in reports, theses, dissertations, books, chapters, journal articles and conference proceedings. Studies using GIS for location modeling have increased in recent years (see Table 1 summary). Yet, with this increase in the application, particularly the work appearing in the academic literature, one must confront the reality of whether reported findings and results are of sufficient quality in all (or any) cases. This is relevant because many heuristics used in commercial GIS packages for solving location models do not provide quality assessment or assurance in reported results. Thus, questions related to application assessment are salient when efficiencies and service provision are involved. Ultimately this raises issues of significance because non-optimal results may not be significant in the classic sense of optimization.

This paper seeks to evaluate location coverage models accessible in GIS software packages in terms of their solution characteristics, including computational time and the quality of obtained results. The next section provides background for this research. This is followed by a review of associated coverage models that can be structured and solved in GIS. Details about system functionality and operation along with a description of the heuristics utilized are provided. Model application results are demonstrated for a number of planning and analysis situations. The paper ends with discussion and concluding comments.

Background

The breadth of location models to support different analyses, planning, management and decision-making contexts is remarkable and extensive. Reviews can be found in Francis *et al*. (1992), Church and Murray (2009) and Daskin (2013), among others, but it is possible to view location models in terms of the categories for which they can be applied, such as continuous space, discrete space, cover, median, center, dispersion, disruption, single facility, multiple facilities, etc. Two categories of location models, in particular, have seen broad adoption, use and/or application, cover and median, and as a result figure prominently in contemporary GIS software. Of particular interest in this

Table 1. Applications of location cover models relying on commercial GIS software for direct solution.

Reference	Application	Publication
Fraser *et al.* (2018)	Public cooling center	Journal
Khan *et al.* (2018)	Solid waste facilities	Journal
Mustain (2018)	Pedestrian network improvements	Thesis
Kwon *et al.* (2017)	Automated external defibrillators	Journal
Dodson *et al.* (2017)	Antiretroviral therapy clinics	Journal
Maghfiroh *et al.* (2018)	Pre-positioning ambulances	Journal
Gwak *et al.* (2017)	Green roofs	Journal
Adesina *et al.* (2017)	Fire stations	Journal
McNamara (2017)	Emergency care facilities	Thesis
Tali *et al.* (2017)	Fire stations	Journal
Loraamm and Downs (2016)	Wildlife crossing structures	Journal
Ahmad (2016)	Domestic solid waste sites	Report
Basudan (2016)	Food outlets and delivery	Thesis
Seger (2016)	Medical imaging centers	Thesis
Polo *et al.* (2015)	Veterinary clinics	Journal
Tomintz *et al.* (2015)	Police stations	Book chapter
Burciu *et al.* (2015)	Hubs for port activities	Proceedings
Jackson (2015)	On-farm biomass processing facilities	Dissertation
Suwan (2015)	College/university campus siting	Thesis
Oviasu (2014)	Kidney disease treatment centers	Journal
Hohn *et al.* (2014)	Biogas plants	Journal
Jankowski and Brown (2014)	Health care centers	Journal
Huang and Liu (2014)	Bus stops	Journal
Daubee (2014)	Aged residential care facilities	Thesis
Robbins (2013)	Fire stations	Thesis
García-Palomares *et al.* (2012)	Bike-share stations	Journal
Algharib (2011)	Fire stations	Dissertation
Zang (2011)	Community access centers	Thesis
Brown (2010)	Primary health care facilities	Thesis
Woodhouse *et al.* (2000)	Conservation of bird species	Journal
Gerrard *et al.* (1997)	Nature reserve sites	Journal

paper are coverage location models, though we return to median approaches in some of the discussion that follows. Beginning with the work by Toregas *et al.* (1971) and Toregas and ReVelle (1972), there has been substantial interest in cover location models for siting service facilities of some sort. A recurring application context has been emergency facilities, such as fire and ambulance response (ReVelle 1991, Murray 2013, Church and Li 2016). The goal is to site an efficient, least-cost configuration of facilities, where there is a coverage feature to service provided. For example, when facilities are fire stations, a municipality desires the fewest number of stations capable of responding to any emergency call for service within a 6-min standard. The drive time of 6 min or less, therefore, represents the ability to cover a residence or neighborhood by personnel at a particular fire station site in this case. Of course, there are many other situations that reflect cover standards, including cellular communication, access to transit, species presence in nature reserve design, mail delivery, sensor detection, search and rescue, and lighting, among others (Church and Murray 2018).

Toregas *et al.* (1971) detailed the location set covering problem (LSCP) to represent the planning context where the fewest facilities are sought such that all demand is covered within the stipulated maximum service response standard. Subsequent work by Church and ReVelle (1974) introduced the maximum covering location problem (MCLP) to account for a budget, seeking a configuration of facilities that can serve the greatest amount of potential demand within the stipulated service response standard. The LSCP and MCLP have been evaluated, extended and further developed by many researchers. Noteworthy is the more recent work of Murray (2005), Tong and Church (2012), Wei and Murray (2015), Church and Li (2016), Murray (2016), Tong and Wei (2017) and Murray (2018). A text focused exclusively on coverage models, including the LSCP and MCLP, is that of Church and Murray (2018).

Early research on solving the LSCP and MCLP entailed specifically designed heuristics or the use of integer-linear programming software (see Toregas and ReVelle 1973, Church 1974, Church and ReVelle 1974). There continues to be interest in developing new heuristics to directly solve these and related covering problems, especially large application instances involving continuous space. However, there is another tack that can be taken in solving problems like the LSCP and MCLP, an approach that can be characterized as indirect. Church (1974) first recognized that the MCLP could be transformed into an equivalent p-median problem, a location model introduced in Hakimi (1964) and formalized in ReVelle and Swain (1970) that seeks to site *p* facilities so that average (or total weighted) response time/distance is as low as possible. This transformation involves a simple manipulation of the facility-to-demand distance matrix. Thus, a given MCLP application instance can be represented as an equivalent p-median problem, and then any one of the techniques developed to solve a p-median problem can be used to solve this transformed problem (Church and ReVelle 1976), in theory.

Following the theoretical foundations established by Church and ReVelle (1976) as well as that of Hillsman (1979, 1984) and Church and Weaver (1986), heuristic solution strategies emerged for solving large instances of the p-median problem (e.g., Densham and Rushton 1991, 1992a, 1992b), called global regional interchange algorithm (GRIA). The GRIA heuristic was more computationally efficient than the exchange approach of Teitz and Bart (1968), limiting substitutions to be within the neighborhood of each facility coupled with a regional greedy drop and add approach. They also employed a data structure called distance strings suggested by Hillsman (1979). Essentially, a distance string is comprised of two components, a list of potential facility sites in order of increasing distances from a given demand along with the listed distances. Hillsman (1979) indicated that the length of the strings could be cut off at some point because assignment is unlikely beyond a distance range. By limiting the length of the distance strings, larger problems could be handled than what had previously been possible by reducing needed computer memory.

The work of Densham and Rushton (1991, 1992a, 1992b) became the basis for the Location-Allocation function/module integrated in ARC/INFO (the precursor ArcGIS). This allowed users to solve a p-median problem using either a GRIA or a Teitz and Bart heuristic (Church and Sorensen 1996, Church 2002). TransCAD too employed an interchange heuristic along these lines to solve the p-median problem (Church 2002). Even though the research community in Location Science has for the most part ignored the equivalence property between the p-median problem and many (single level) covering problems, this property served to sell the idea that a p-median heuristic would provide the capability to solve a wide range of problems within a GIS package.

Empirical studies in the literature have shown that the interchange heuristic of Teitz and Bart (1968) has generally been quite effective for solving the p-median problem (Church and Sorensen 1996, Murray and Church 1996, Sorensen and Church 1996). The success of the GRIA heuristic is therefore not surprising given that it attempts to retain properties of the original interchange approach, but seeks out computational efficiencies, and in particular, employs the distance matrix editing of Hillsman (1979). Sorensen and Church (1996) questioned the use of distance string cutoffs used in the GRIA heuristic, finding that they needed to be fairly long in order to find optimal solutions in evaluated p-median problem instances. In related work, Church and Sorensen (1996) tested both the GRIA and Teitz and Bart heuristics and found results to be quite comparable as long as the heuristics were restarted many times. Unfortunately, the implementation of GRIA and Teitz and Bart in ARC/INFO were fixed at no restarts (Church 2002). This meant that there was a reasonable probability that an optimal solution would not be found. Church and Sorensen (1996) also discussed the possibility of using other heuristic approaches in solving large p-median problems. One of those strategies was GRASP. ArcGIS now provides upgraded solution functionality using a GRASP with path relinking metaheuristic in Network Analyst Tools (Sandhu 2016, Esri 2017). As detailed in the next section, this approach uses a randomized greedy heuristic (based upon the design of the general GRASP heuristic) to generate starting solutions, which are then improved by employing the Teitz and Bart interchange. The result from a given implementation of GRASP (and Teitz and Bart) is then used as a candidate in a path relinking approach. Path relinking is a process that seeks to diversify the search for the best solution by combining components of two different solutions where such a combination may outperform those solutions that have already been found. This new heuristic is based upon starting the GRASP process 128 times (Sandhu 2016).

While improvements in ArcGIS (Network Analyst) likely address some of problems identified by Church and Sorensen (1996), it remains unknown how this or other p-median heuristics perform when used to indirectly solve cover models. Gerrard *et al.* (1997) compared ARC/INFO heuristics for solving the MCLP. They demonstrated that the solutions from the two heuristic approaches (GRIA and Teitz and Bart) differed for some of the problems, but did not assess whether either actually found optimal solutions. Woodhouse *et al.* (2000) indicate that ARC/INFO did not find the optimum for the LSCP problem instance they considered, though they called it an 'MCLP'. There has never been a formal assessment of solution quality when covering problems are solved using an indirect approach, and certainly, nothing examining distance editing combined with a p-median heuristic.

With the growing body of work reporting on use and application of location models supported in commercial GIS, this becomes increasingly important. These studies simply report findings, using them to make plans, devise management schemes or support decisions/conclusions, generally without consideration of solution quality/reliability issues. Table 1 provides a summary of reported work, indicating the reference, the application context, and the publication outlet. This summary is limited to publicly available journal articles, reports, theses and dissertations where the LSCP and/or MCLP is structured and solved using commercial GIS. Note in particular the range of contexts. For example, García-Palomares *et al.* (2012) evaluated bike-share station location. School siting is addressed in Menezes et al. (2014). Biogas plants were examined in Hohn *et al.* (2014). Oviasu (2014) reported on the planning of kidney disease treatment centers. Polo *et al.* (2015) examined siting of veterinarian clinics that perform sterilization of dogs and cats. Loraamm and Downs (2016) identified wildlife crossing structures for endangered panthers. Locating green roofs in urban areas to support honeybee foraging was detailed in Gwak *et al*. (2017). Ambulance pre-positioning strategies were derived in Maghfiroh *et al*. (2018). The siting of antiretroviral therapy clinics was the focus of Dodson *et al*. (2017). Finally, Khan *et al*. (2018) looked at the siting of solid waste facilities. Table 1 does suggest a need for better understanding and interpreting findings in a qualitative sense.

Methods

As noted above, there are many different location models designed to address a range of concerns and issues. Of interest in this paper are cover location models, with the two prominent approaches being LSCP and MCLP. These two are in fact explicitly available in commercial GIS software and have been applied to address various substantive planning and management situations. Mathematical formulations are detailed in this section to highlight what the specific analytical method is, how they differ and potential for heuristic solution.

Consider the following notation:

$i =$ index of demand areas (total n)

$j =$ index of potential facility sites (total m)

$N_i =$ set of potential facility sites that can suitably serve/cover demand i

$$X_j = \begin{cases} 1 \text{if facility } j \text{ is sited} \\ 0 \text{ otherwise} \end{cases}$$

With this notation, it is possible to define the LSCP (location set covering problem) originally presented in Toregas *et al*. (1971). The LSCP is as follows:

$$\textit{Minimize} \qquad \sum\nolimits_j X_j \tag{1}$$

$$\textit{Subject to} \qquad \sum\nolimits_{j \in N_i} X_j \geq 1 \forall i \tag{2}$$

$$X_j = \{0, 1\} \forall j \tag{3}$$

The LSCP objective, (1), is to minimize the total number of necessary facilities. Constraints (2) ensure that at least one facility is sited capable of serving, or covering, each demand area. Constraints (3) impose binary restrictions on decision variables.

The spatial nature of the LSCP is encapsulated in the decision variables that correspond to the selection of a location for siting a facility, X_j, but also in the set N_i because it indicates those facilities that are able to access area i within the service response standard. As mentioned previously, the major requirement of the LSCP is that each demand is covered within the service response standard.

Given budget realities and the need for agencies and organizations to be fiscally responsible, it has been recognized that coverage to all demand may not be reasonable. For example, if a somewhat isolated demand area has infrequent service needs, it might be impractical to build and maintain a facility dedicated to serving that area only. Further, it may be more reasonable to provide service in such a case that is a little beyond the standard. However, we likely would want to make sure that the most demand possible is served by the limited number of facilities afforded. This is the essence of the MCLP, effectively relaxing the more rigid cover requirements of the LSCP that all demand be served.

Consider the following additional notation:

$p =$ number of facilities to site

$a_i =$ expected service demand in area i

$$Y_i = \begin{cases} 1 \text{if demand } i \text{ is suitably served (or covered)} \\ 0 \text{ otherwise} \end{cases}$$

With this notation, the MCLP (maximum covering location problem) introduced in Church and ReVelle (1974) is as follows:

$$\text{Maximize} \quad \sum\nolimits_i a_i Y_i \tag{4}$$

$$\text{Subject to} \quad \sum\nolimits_{j \in N_i} X_j \geq Y_i \forall i \tag{5}$$

$$\sum\nolimits_j X_j = p \tag{6}$$

$$X_j = \{0, 1\} \forall j \tag{7}$$

$$Y_i = \{0, 1\} \forall i \tag{8}$$

The MCLP objective, (4), is to maximize the total demand covered by sited facilities. Constraints (5) stipulate that at least one facility be sited capable of serving each demand area within the coverage standard before it can be counted as covered. Constraint (6) indicates that exactly p facilities are to be sited. Constraints (7) and (8) impose binary restrictions on decision variables.

In addition to the spatial location variables corresponding to siting a facility, X_j, the MCLP also accounts for whether a geographical demand area is provided service using the variable Y_i. There are multiple ways that location and proximity are therefore incorporated into this mathematical formulation, making it an interesting spatial optimization problem.

As with any spatial optimization problem, among which includes the LSCP and MCLP as well as other location models, the mathematical specification is one step in addressing analysis, planning, management and decision-making processes. There is a need for supporting spatial and aspatial information, namely N_i and a_i as well as demand and potential facility locations in the cases of the LSCP and MCLP. With this, there is also an issue of solving the associated problem instance. There have in fact been many developed exact and heuristic approaches for solving the LSCP and MCLP directly (Church and Murray 2018). A GIS environment makes tremendous sense for structuring and solving these models, if possible, because spatial location and proximity can be readily derived. Further, visualization and subsequent evaluation are possible.

Access and solution in GIS

The primary approach to date in GIS software to solve cover location models is through the use of heuristic methods. As noted previously, this has been done using an indirect approach in ArcGIS, where the models are transformed to an equivalent p-median problem (Sandhu 2016, Esri 2017). Church and ReVelle (1976) provided a theoretical basis for doing this, demonstrating mathematically that the MCLP can be structured as an equivalent p-median problem. Church and Murray (2018) also detail how the LSCP can be structured and solved using this indirect approach. This being the case, a solution approach developed for the p-median problem can then be applied to problem variants capable of being structured as an equivalent p-median problem.

To illustrate how this is theoretically possible, additional notation is needed:

$d_{ij} =$ shortest distance/travel time from demand in area i to facility j

$$Z_{ij} = \begin{cases} 1 \text{if demand } i \text{ is allocated to facility } j \\ 0 \text{ otherwise} \end{cases}$$

With this and previously defined notation, the p-median problem structured in ReVelle and Swain (1970) follows:

$$\textit{Minimize} \quad \sum_i \sum_j a_i d_{ij} Z_{ij} \tag{9}$$

$$\textit{Subjected to} \quad \sum_j Z_{ij} = 1 \forall i \tag{10}$$

$$\sum_j X_j = p \tag{11}$$

$$Z_{ij} \leq X_j \forall i, j \tag{12}$$

$$X_j = \{0, 1\} \forall j \tag{13}$$

$$Z_{ij} = \{0, 1\} \forall i, j \tag{14}$$

The p-median problem objective, (9), is to minimize the total demand weighted assignment distance. Constraints (10) stipulate that each demand area is to be served by one facility. Constraint (11) indicates that p facilities are to be sited. Constraints (12) require that a facility is sited if allocation for service is made. Constraints (13) and (14) impose binary restrictions on decision variables.

Church and ReVelle (1976) proved that the p-median problem could be used to structure a corresponding MCLP. This can be demonstrated using additional notation:

$S =$ service response standard (distance or travel time)

$$d'_{ij} = \begin{cases} 0 \text{ if } d_{ij} \leq S \\ 1 \text{ otherwise} \end{cases}$$

Objective (9) can then be modified as follows:

$$\textit{Minimize} \quad \sum_i \sum_j a_i d'_{ij} Z_{ij} \tag{15}$$

This means that using the p-median structure, where objective (15) replaces (9), the interpretation of the p-median problem becomes minimizing the total demand not serviced within the desired response standard. This is equivalent to maximizing the total demand covered, the very objective of the MCLP, (4).

Church and Murray (2018) detail how the p-median problem structure can also be used to solve the LSCP. Perhaps the most direct approach is as an MCLP where objective (15) replaces (9) in the p-median formulation. As indicated above, the objective then represents minimizing the total demand not serviced within the desired response standard. If the number of facilities, p, is increased to the smallest value where no demand is not served, this represents an optimal solution to the LSCP. Church and Murray (2018) also detail an approach based on Hillsman (1984) involving the introduction of pseudo nodes.

Theoretically, these transformations make it possible to structure and indirectly solve an LSCP, MCLP or other variants under an umbrella model framework. With this in mind, the description of location cover models and solution in ArcGIS and TransCAD are given. ArcGIS indicates the following (Esri 2017):

> *Location-allocation is a solver … to minimize weighted impedance, maximize coverage, or achieve a target market share. Heuristics are used to solve the location-allocation problems. The location-allocation solver starts by generating an origin-destination matrix of shortest-path costs between all the facilities and demand point locations along the network. It then constructs an edited version of the cost matrix by a process known as Hillsman editing. This editing process enables the same overall solver heuristic to solve a variety of different problem types. The location-allocation solver then generates a set of semirandomized solutions and applies a vertex substitution heuristic (Teitz and Bart) to refine these solutions creating a group of good solutions. A metaheuristic then combines this group of good solutions to create better solutions. When no additional improvement is possible, the metaheuristic returns the best solution found. The combination of an edited matrix, semirandomized initial solutions, a vertex substitution heuristic, and a refining metaheuristic quickly yields near-optimal results.*

Coupled with the above models and transformation details this makes some sense. However, this is it. There is no technical description provided to the user beyond this. But the same is true for TransCAD, with the following offered associated with location modeling capabilities and solution heuristic(s) (Caliper 2017):

> *When you use the Facility Location Model to add a fixed number of facilities, the algorithm works in two stages:*
>
> - It identifies a set of initial facility locations using a greedy heuristic
> - It attempts to improve on the initial set of locations by swapping candidates with chosen facilities on a pair-wise basis until no improvement can be made
>
> The greedy heuristic chooses the next best location by evaluating all the candidates and selecting the one that best achieves the desired objective.
> When you use the [F]acility [L]ocation [M]odel to determine the number of facilities to add, the algorithm works in two stages:
>
> - It determines whether to add an additional facility location using a greedy heuristic. If no facility is added, the algorithm stops.
> - It attempts to improve on the current set of locations by swapping candidates with chosen facilities on a pair-wise basis until no improvement can be made.
> - It returns to the first stage and attempts to add another facility.

ArcGIS provides access to both the LSCP and MCLP. This can be accomplished in a number of ways, depending on how a user wishes to interact with the software.

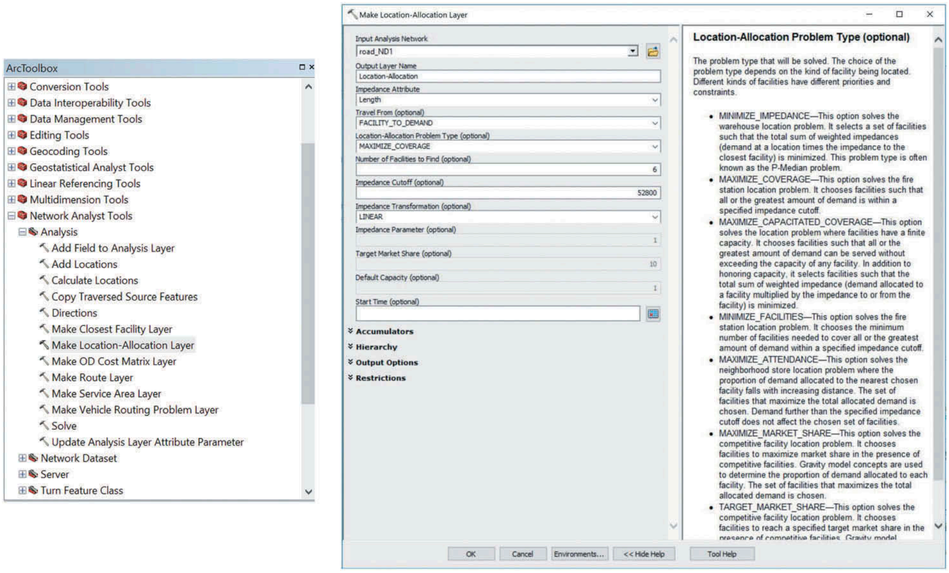
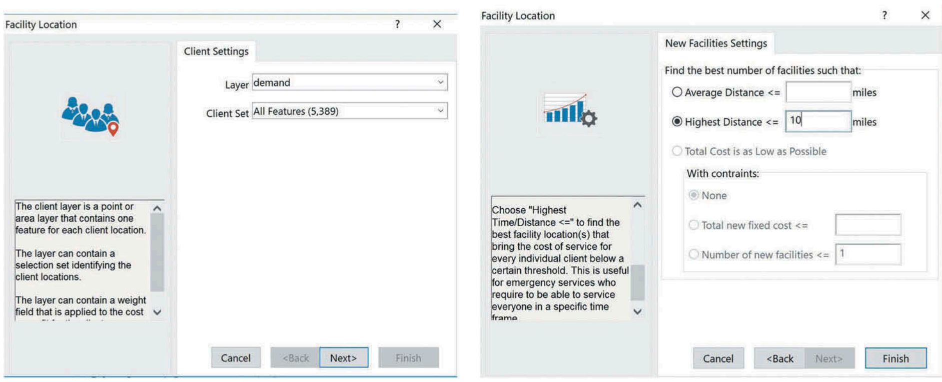


Figure 1. User interface windows in ArcGIS (Network Analyst Tools) for accessing location models.

One approach is through the use of the 'toolbox', accessible through the ArcToolbox window shown in Figure 1 (left window). Through a series of point-and-click selections (ArcToolbox → Network Analyst Tools → Analysis → Make Location-Allocation Layer), the location model interface window can be accessed, shown in Figure 1 (right window).[1] This window enables model specification through 'Location-Allocation Problem Type' selection, with available options shown in the right panel. For example, the LSCP could be selected, which is referred to in ArcGIS as 'MINIMIZE_FACILITIES'. Specification of the MCLP in ArcGIS is done through, 'MAXIMIZE_COVERAGE' as the Location-Allocation Problem Type, requiring as well the number of facilities to select ('Number of Facilities to Find').

In TransCAD it is only possible to readily solve the LSCP. Solution of the MCLP would require additional programming and customization, where the user would need to manipulate the distance matrix in order to structure an equivalent p-median problem as discussed for objective (15). The LSCP is accessed in TransCAD through the Facility Location Model module (Figure 2).[2] Through point-and-click selections, the 'Client Settings' (demand) and 'Facility Settings' (potential facilities) frames are encountered, Figure 2(a, b), along with others, such as the 'Cost Matrix' frame. In order to solve LSCP in TransCAD, the 'Number of new facilities' selection item must be set to 'Automatic' and the maximal service response standard should be filled in the blank after 'Highest Distance ≤' under the 'Find the best number of facilities such that' item (Figure 2(b)).

This constitutes what is known about the utilized heuristic solution techniques available in these packages to solve location cover problems. They are expressly noted to be heuristics, and therefore may not identify an optimal solution when applied. Obtaining any qualitative characterization of any application results presumably remains up to the user to figure out. In what follows, we attempt to provide some insight on solution performance and solution quality.



(a) Client Settings frame (b) New Facility Settings frame

Figure 2. User interface windows in TransCAD (Facility Location Models) for accessing location models.

Application results

A number of case studies are presented to provide a comparison of performance characteristics when heuristic approaches relied upon in GIS are applied. Again, solution quality is not understood when indirect heuristic techniques are relied upon. The LSCP and MCLP are structured and solved in ArcGIS. The LSCP is structured and solved in TransCAD. The heuristic results are then compared to exact solutions derived using Xpress (FICO 2015), a commercial optimization solver. Exact solution involves exporting demand and potential facility information along with travel distance matrices from GIS for use in Xpress. Road networks (TIGER/Line shapefiles) for each area were downloaded from the US Census Bureau, unless otherwise noted. All processing and computation are done on a desktop personal computer (Intel Xeon E5 CPU, 2.30 GHz with 96 GB RAM).

Fire response in Northwest Boston

Analysis and planning to support improved access and response to potential structure fires were explored for the eight suburbs of Northwest Boston (Acton, Bedford, Carlisle, Concord, Lincoln, Maynard, Sudbury, and Wayland). Fire protection is a priority for public service, saving lives and property. Examined here are 511 structure fires between 1990 and 2004 that were reached within 4 min of a call for service. The analysis that follows considers each location both a demand and a potential facility site. The response standard was assumed to be 2.46 miles. The goal then is to site fire stations in the most efficient manner possible. Given the nature of service response and articulated needs for efficiency, the LSCP and MCLP reflect important planning approaches for fire station siting in this region.

Reported in Table 2 (first row) are the findings when applying the LSCP to derive the minimum number of facilities necessary to provide coverage for all demand in Northwest Boston (potential structure fires) using ArcGIS, TransCAD and Xpress. The use of ArcGIS identifies 20 stations as being necessary for coverage of all demand within the response standard. TransCAD suggests that the minimum is 22 stations. The

Table 2. LSCP results (number of facilities needed) by case study.

Case Study (*n*, *m*)	ArcGIS	TransCAD	Xpress
Northwest Boston (511, 511)	20 (12.86 s)	22 (40.78 s)	20 (0.13 s)
Santa Barbara Co. (5,389, 6,000)	30 (1,263.00 s)	a	28 (17.22 s)
San Jose (59, 500)	42 (0.54 s)	41 (41.30 s)	41 (0.06 s)

[a]Solution was terminated after 5 days (432,000.0 sec.) of processing.

optimum, indicated in the Xpress column, is actually 20 fire stations. Deviation above the optimum of 20 stations is significant because fixed and annual costs are not trivial associated with fire stations. The optimal configuration is depicted in Figure 3. Solution time does differ as well, with the ArcGIS heuristic requiring 12.86 s and the TransCAD heuristic taking 40.78. The optimal solution obtained using Xpress needed 0.13 s. Note that solving this problem using Xpress required the origin-destination distance matrix to be derived and exported from GIS, with ArcGIS taking 5.20 s and TransCAD 0.32 s for this operation.

To assess the impacts of siting fewer facilities, the MCLP was also applied. The possibility of siting 1 up to 20 fire stations ($p = 1$–20), with $p = 20$ being the maximum identified by ArcGIS using the LSCP was considered. A summary of the findings are given in Figure 4(a). Many of the problem instances were solved optimally by ArcGIS. However, Figure 4(a) shows that the ArcGIS heuristic used to solve the MCLP when p equals 5, 13 and 18 was not able to identify the optimal solution. In the case of $p = 5$ this corresponds to 98.85% of the optimum. That is, the heuristic solution configuration of fire stations can cover 345 potential structure fires, but the optimal configuration of eight stations can actually cover 349 structure fires. Solution time for the MCLP is summarized in Figure 4(b). Xpress required about 0.19 s on average to solve across values of p, whereas the ArcGIS shows a trend of increasing as p increases, with a high of 5.61 s when p equal to 19.

Supplemental nutrition program in Santa Barbara County

Analysis and planning to support the Special Supplemental Nutrition Program for Women, Infants, and Children in Santa Barbara County was also carried out. This is a federally funded program administered through the State of California, providing aid for buying healthy foods, nutrition and breastfeeding education and support, and referrals to health care and community services. This is done through offices sited throughout the county. Demand for service is considered using 5,389 census blocks in the county with the total population (2010) of 424,220 people. The block centroid is used as the demand point. There were 6,000 locations identified as potential facility sites. The maximum service distance to access a facility is assumed to be 10 miles. The goal then is to site offices for this program in the most efficient manner possible, with supporting analysis carried out using the LSCP and MCLP approaches.

Table 2 (second row) summarizes the findings for applying the LSCP to derive the minimum number of facilities necessary to provide coverage to all demand in Santa Barbara County (census blocks) using ArcGIS, TransCAD, and Xpress. In this case, the ArcGIS heuristic suggests that 30 facilities are the minimum for coverage of all demand.

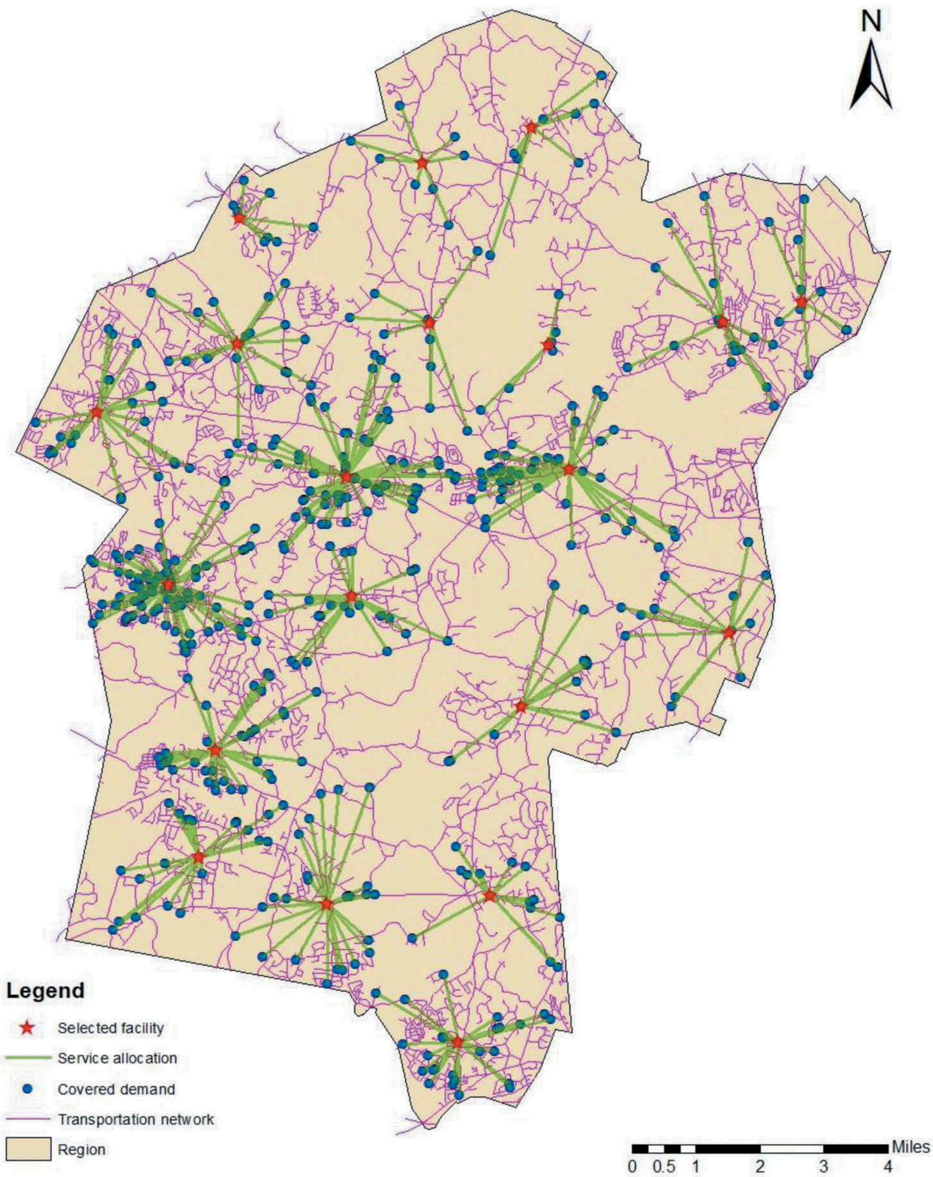
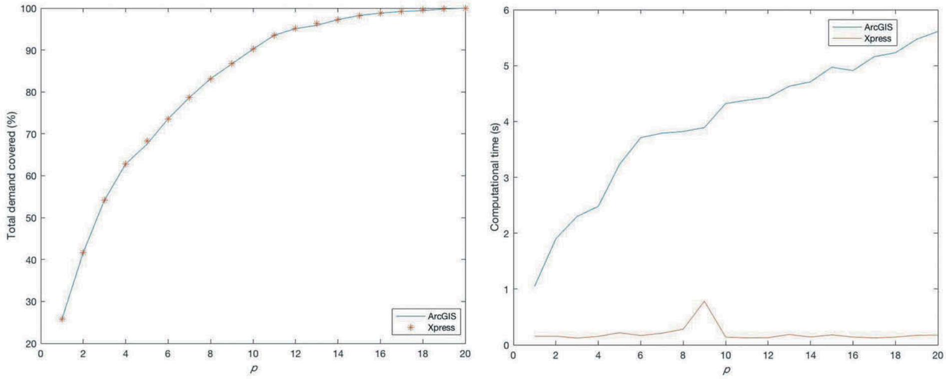


Figure 3. LSCP derived configuration of 20 facilities for serving the Northwest Boston area.

TransCAD was terminated after five days of computing without identifying a solution. The optimum, using Xpress, is 28 facilities. This is fairly significant considering the fixed and annual recurring costs associated with siting a system of service facilities, even one or two too many could be devastating to a regional authority. Computationally, solution times again varied, with the ArcGIS heuristic taking 1263.00 s. Xpress found the optimum in 17.22 s. The extraction of the origin-destination distance information took ArcGIS 173.56 s and TransCAD 2.21 s.

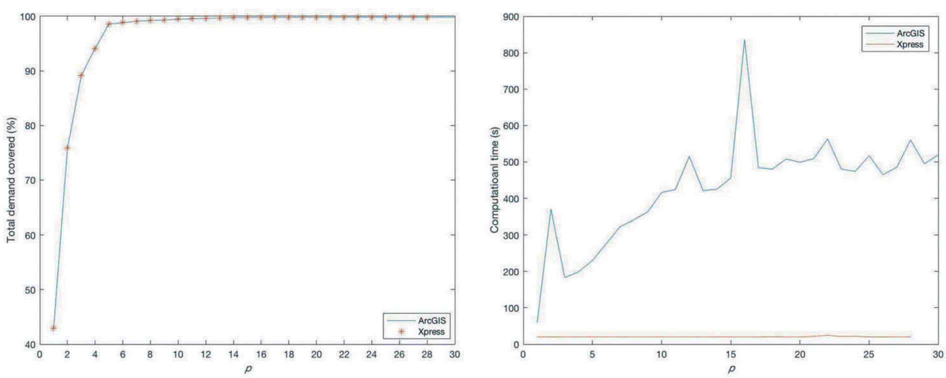


(a) Demand coverage by number of facilities (p)

(b) Solution time by number of facilities (p)

Figure 4. Summary of MCLP solution details for the Northwest Boston case study.

To assess the impacts of siting fewer facilities, the MCLP was also applied. The possibility of siting 1 up to 30 offices ($p = 1$–30), with $p = 30$ being the maximum identified by ArcGIS using the LSCP was considered. A summary of the findings is given in Figure 5(a) as well as in Table 3. Out of the 30 problems solved using ArcGIS, only 10 were optimal. Figure 6 shows the optimal spatial configuration of facilities for $p = 6$ (Xpress) with covered demand indicated. From Table 3, this configuration serves 419,570 people in the region, whereas the identified configuration using ArcGIS covers only 419,148. While only a decrease of less than 1% for the ArcGIS solution, the optimal configuration is able to serve some 422 more people. That is, enhanced service access is provided to an additional 422 people within the stipulated coverage standard. The computational time for the ArcGIS heuristic averaged approximately 429 s in Figure 5(b). Xpress needed 20.40 s on average to find the optimal solution.



(a) Demand coverage by number of facilities (p)

(b) Solution time by number of facilities (p)

Figure 5. Summary of MCLP solution details for the Santa Barbara County case study.

Table 3. MCLP results (total demand covered) for the Santa Barbara County case study.

p	ArcGIS	Xpress
1	182,139	182,139
2	321,754	321,754
3	378,265	378,265
4	399,313	399,313
5	418,248	418,248
6	419,148	419,570
7	420,470	420,470
8	420,902	421,050
9	421,256	421,614
10	421,920	422,052
11	422,305	422,387
12	422,623	422,705
13	422,882	422,939
14	423,116	423,116
15	423,229	423,229
16	423,272	423,279
17	423,316	423,323
18	423,332	423,351
19	423,360	423,370
20	423,379	423,383
21	423,392	423,394
22	423,403	423,403
23	423,409	423,411
24	423,410	423,416
25	423,415	423,420
26	423,421	423,423
27	423,423	423,424
28	423,424	423,425
29	423,424	
30	423,425	

Beverage container recycling in San Jose

The final case study involves analysis and planning to support beverage container recycling in the City of San Jose, California. The State of California, like many others across the United States, requires beverage sales to include a charge for the container, the California Redemption Value (CRV). Accordingly, recycling centers where redemption is possible for the consumer are mandated to be sited within a half-mile, at least for large volume points of sales like supermarkets. There are 59 supermarkets in the San Jose region, representing the demand for service in this case. There were 500 potential facility sites identified as potential recycling facilities for container redemption. Given the nature of access and proximity, in this case, a complete network reflecting Euclidean distance was structured for use in ArcGIS and TransCAD. The maximum service distance to a facility is assumed to be 0.5 miles given the mandated response standard. The goal then is to site recycling facilities for this program in the most efficient manner possible, with supporting analysis carried out using the LSCP and MCLP approaches.

Table 2 (third row) summarizes the findings for applying the LSCP to derive the minimum number of facilities necessary to provide coverage to all demand in San Jose (supermarkets) using ArcGIS, TransCAD, and Xpress. In this case, the ArcGIS heuristic suggests that 42 facilities are the minimum for coverage of all demand. The TransCAD heuristic finds that 41

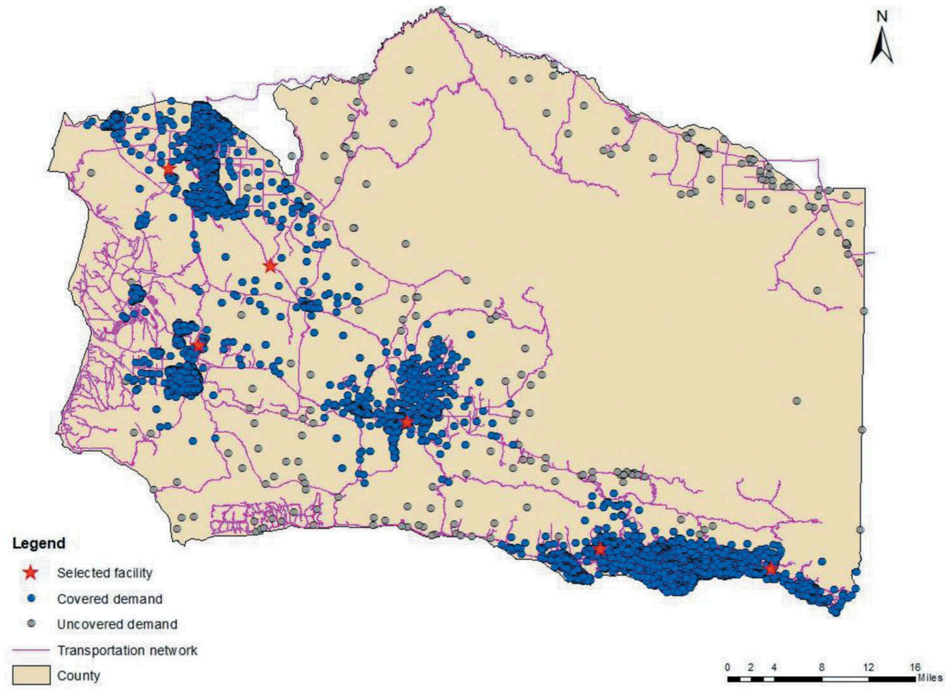


Figure 6. MCLP derived facility configuration (p = 6) and coverage for serving Santa Barbara County.

facilities are needed. The optimum using Xpress shows the minimum to be 41 facilities. This spatial configuration is shown in Figure 7. Again, fixed and annual recurring costs associated with siting a system of service facilities make any findings above the actual minimum an important difference economically. Computationally, solution times varied with the ArcGIS heuristic taking 0.54 s and the TransCAD heuristic needing 41.30 s. Solution time using Xpress required 0.06 s. The derivation of the origin-destination distance matrix took ArcGIS 4.27 s and TransCAD 0.56 s in this case.

To assess the impacts of siting fewer facilities, the MCLP was also applied. The possibility of siting 1 up to 42 recycling facilities ($p = 1–42$), with $p = 42$ being the maximum identified by ArcGIS using the LSCP was considered. A summary of the findings is given in Figure 8(a). Of the 42 problem instances solved using the ArcGIS heuristic in this case, only 12 were optimal. That means over 70% of the problem instances were not optimal. The spatial configuration for $p = 18$ is shown in Figure 9. This configuration serves 36 stores, whereas the identified configuration using ArcGIS only serves 34 within the coverage standard. This is over 5% less than the optimal configuration. Interestingly, such a tradeoff solution covers 61% of the supermarkets, but does so with only 44% of the investment of resources compared to serving all supermarkets by the 41 recycling facilities identified using the LSCP. The solution times for each value of p are given in Figure 8(b), with the ArcGIS heuristic requiring approximately 0.69 s on average and the exact approach using Xpress needing about 0.03 s on average.

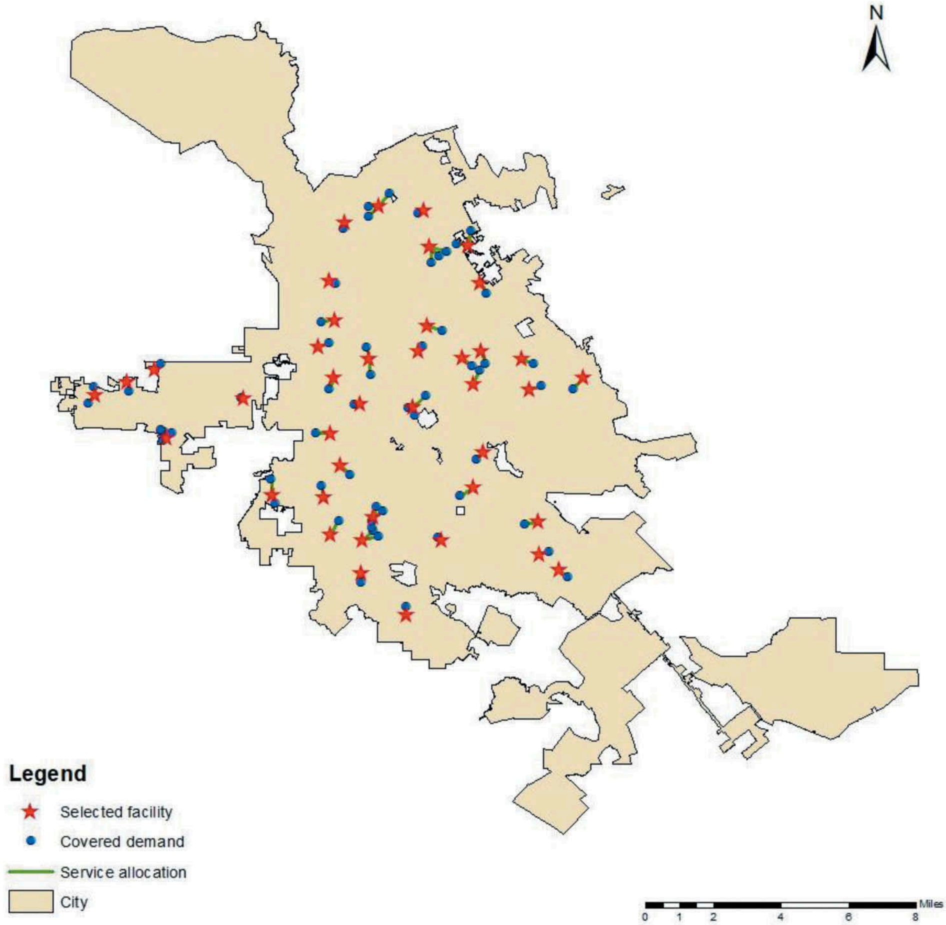


Figure 7. LSCP derived configuration of 41 facilities for serving the San Jose area.

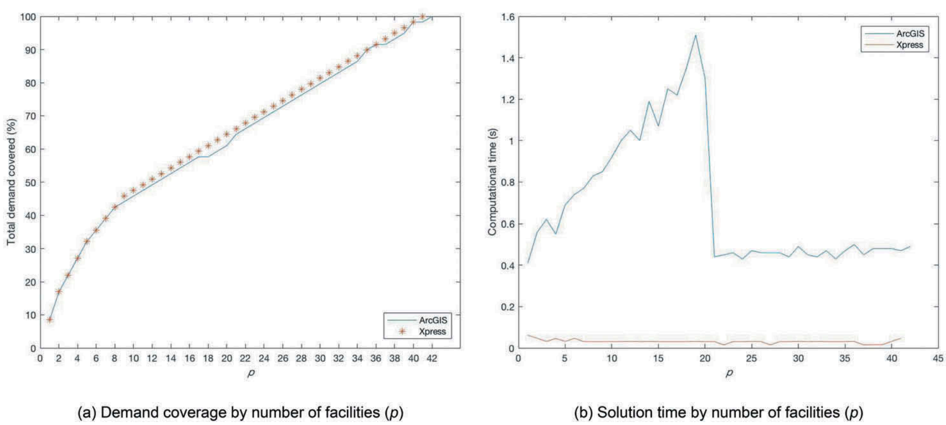


Figure 8. Summary of MCLP solution details for the San Jose case study.

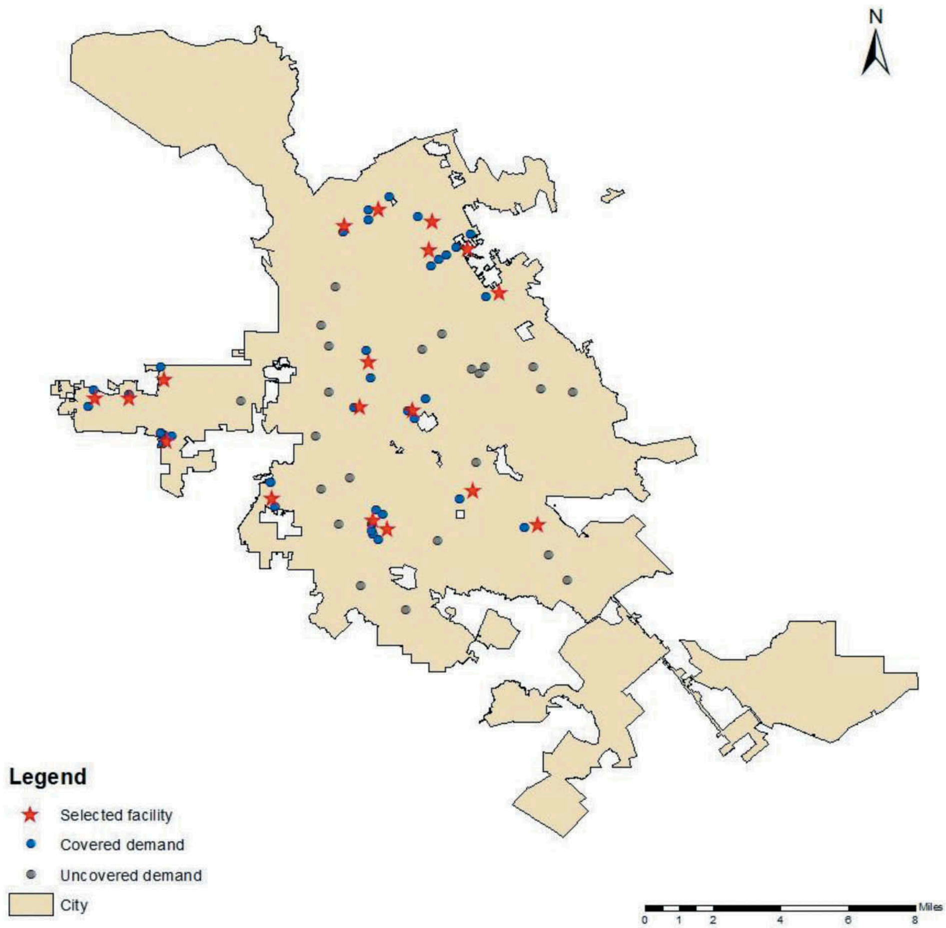


Figure 9. MCLP derived facility configuration (p = 18) and coverage for serving San Jose area supermarkets.

Discussion

The reported case studies suggest a number of issues that are worth further discussion. The intention of presenting results was to highlight that across the 1,059 different covering problem instances solved, significant variability in performance can be observed. The number of potential facility sites considered in the Santa Barbara County case study was varied from 500 up to 16,500. The purpose of this is to assess the impact of problem structure change. These 33 additional planning instances are reflected in Table 4 for a coverage standard of 10 miles, with the first column in each row indicating the number of potential facility sites. Evaluation using the MCLP along the lines reported in Figure 5 and Table 3 was repeated in each case, with the number of facilities allowed ranging from 1 to complete coverage. This means that 927 MCLP problems were subsequently solved and assessed with respect to solution quality. The MCLP results obtained using

Table 4. Heuristic optimality deviation percentage for MCLP (S= 10 miles) application (Santa Barbara County).

# Potential facility sites (m)	# Problems solved (# non-optimal)	Percent solved optimally (%)	Median optimality deviation (%)	Maximum optimality deviation (%)
500	19 (4)	21.05	0.0000	0.0660
1,000	26 (16)	61.54	0.0019	0.1903
1,500	24 (8)	33.33	0.0000	0.0945
2,000	26 (15)	57.69	0.0002	0.0939
2,500	25 (11)	44.00	0.0000	0.2308
3,000	24 (10)	41.67	0.0000	0.1118
3,500	28 (8)	28.57	0.0000	0.2291
4,000	29 (12)	41.38	0.0000	0.2041
4,500	26 (16)	61.54	0.0009	0.1182
5,000	26 (11)	42.31	0.0000	0.0799
5,500	27 (14)	51.85	0.0005	0.2053
6,000	28 (19)	67.86	0.0007	0.1006
6,500	30 (13)	43.33	0.0000	0.2186
7,000	30 (21)	70.00	0.0008	0.1113
7,500	27 (15)	55.56	0.0017	0.2212
8,000	28 (13)	46.43	0.0000	0.0984
8,500	29 (15)	51.72	0.0009	0.0989
9,000	31 (21)	67.74	0.0005	0.1015
9,500	30 (11)	36.67	0.0000	0.1876
10,000	28 (13)	46.43	0.0000	0.1070
10,500	30 (16)	53.33	0.0013	0.1053
11,000	30 (12)	40.00	0.0000	0.0991
11,500	30 (18)	60.00	0.0013	0.1670
12,000	30 (18)	60.00	0.0005	0.1068
12,500	29 (15)	51.72	0.0005	0.0982
13,000	29 (16)	55.17	0.0009	0.1037
13,500	28 (13)	46.423	0.0000	0.0972
14,000	30 (13)	43.33	0.0000	0.0968
14,500	30 (14)	46.67	0.0000	0.0982
15,000	30 (17)	56.67	0.0006	0.0984
15,500	30 (15)	50.00	0.0002	0.0982
16,000	30 (15)	50.00	0.0002	0.1003
16,500	30 (15)	50.00	0.0002	0.0982

ArcGIS are less than 1% from the optimal in all cases, but computational effort increased significantly when problem size increased with the number of potential facility sites. Specifically, when the number of potential facility sites (m) was 16,500, solution of the MCLP increased to an average of 1349.10 s (22.48 min) across values of p using ArcGIS. In contrast, Xpress required only 51.06 s on average. In terms of solution quality, approximately 50% of the problem instances were not solved optimally using the ArcGIS heuristic reported in Table 4. The most instances of identifying an optimal solution occurred for the case where the number of potential facilities (m) was 500, with 15 of the 19 instances (79%) found to be the optimum. Median and maximum optimality deviation among the problem instances is reported in the last two columns, and relatively small. The fewest instances of identifying an optimal solution occurred for m equal to 7,000, with 9 of the 30 instances (30%) found to be the optimum.

Assessment for solving the LSCP using ArcGIS is summarized in Table 5, where the number of potential facility sites (m) is varied from 500 to 9,500 (summarized by row). In addition, the coverage standard is varied from 5 miles to 30 miles (summarized by column). This enables problem structure to be evaluated as more stringent coverage standards mean fewer facilities capable of providing coverage. It is clear in Table 5 that

Table 5. Heuristic optimality deviation percentage when varying service standard in LSCP application (Santa Barbara County).

# Potential facility sites (*m*)	Service response standard (*S*) 5 mi	10 mi	15 mi	20 mi	25 mi	30 mi
500	0.00	0.00	0.00	0.00	0.00	0.00
1,000	0.00	0.00	0.00	9.09	0.00	0.00
1,500	0.00	0.00	0.00	0.00	0.00	0.00
2,000	3.28	3.85	0.00	0.00	0.00	0.00
2,500	1.69	0.00	0.00	0.00	0.00	0.00
3,000	1.61	4.17	0.00	0.00	0.00	0.00
3,500	3.17	0.00	0.00	0.00	11.11	0.00
4,000	2.94	0.00	0.00	0.00	0.00	0.00
4,500	3.13	3.85	5.26	0.00	0.00	0.00
5,000	4.48	0.00	0.00	0.00	0.00	0.00
5,500	0.00	0.00	5.56	0.00	0.00	0.00
6,000	2.99	7.14	0.00	0.00	0.00	0.00
6,500	2.94	0.00	0.00	0.00	0.00	0.00
7,000	1.43	3.33	0.00	0.00	0.00	0.00
7,500	4.23	0.00	0.00	0.00	0.00	0.00
8,000	4.35	0.00	5.26	0.00	0.00	0.00
8,500	4.55	0.00	0.00	0.00	0.00	0.00
9,000	1.37	0.00	0.00	0.00	0.00	0.00
9,500	2.74	0.00	5.00	0.00	0.00	0.00

problems are more difficult for the heuristic to find optimal solutions when the coverage matrix is more sparse, so there are more instances of non-optimal solutions when S= 5 miles where the average optimality deviation is over 2%. When S= 30 miles, the coverage matrix is more dense and the heuristic is able to find optimal solutions. Solving the LSCP using TransCAD also proved problematic for the largest case investigated, with no solution found after five days of computing.

The intention of paper was not to be critical of any commercial GIS software package, but rather to highlight challenges in the use and application of spatial analytical tools readily available in user-friendly systems. In particular, the location modeling capabilities available in GIS rely on heuristic solution methods. The reality is that heuristics cannot guarantee optimal results. This was demonstrated across the reported findings, where optimal results are found in some cases but not in all. It was encouraging to observe that the results were generally of high quality, never worse than 7.14% from optimal in the problem instances examined in this research. Nevertheless, this can be an issue because most users tend to characterize results obtained using GIS as 'optimal'. As an example, most of the applications reported in Table 1, in fact, state that solutions are 'optimal', yet it is clear from the results here that this may well not be the case. A challenge, therefore, remains regarding how heuristic results can be presented and better communicated. Such issues in location modeling have been touched upon in the previous research (e.g., Rosing *et al*. 1979, Church 1999) and remain relevant today. Beyond this, even close to optimal may not be sufficient in many cases. When siting service facilities that cost millions of dollars to build and millions of dollars to staff and maintain, which is the case with fire services (see Murray 2013, Church and Li 2016), then any facilities above the minimum represents the needless expenditure of millions of dollars.

The case studies also highlight issues associated with access to spatial analytical methods. In terms of heuristic use and functionality in GIS software, there is a notable lack of user control regarding performance. The detailed heuristic techniques described by ArcGIS and

TransCAD generally have a range of parameters that impact speed and quality of solution(s) obtained (e.g., number of re-starts, number of iterations, convergence tolerance, etc.), yet there is no mechanism for interacting with these parameters in either GIS software package. Such issues have been noted in Church and Sorensen (1996), Church (1999) and Murray (2010), among others, and remain worth further consideration.

A final discussion point is associated with model assumptions. Both ArcGIS and TransCAD require a transportation network in order to apply and solve location cover models. While many problems are in fact network based, this need not be the case in general. In fact, the requirement of a network could be particularly limiting in some cases, such as the CRV case study in San Jose where a network had to be created beforehand to reflect continuous space travel. There are many application contexts where no network need be assumed.

Conclusions

This paper has evaluated a class of location coverage models accessible in GIS software packages in terms of their solution characteristics, including computational time and the quality of obtained results. Specifically, the LSCP (location set covering problem) and the MCLP (maximal covering location problem) were considered. The LSCP is a readily available cover model in ArcGIS and TransCAD. The MCLP is readily available in ArcGIS, but not in TransCAD. Three case studies involving 1,059 different planning problem instances were solved. The results indicated that the heuristics available in GIS generally perform well, but do not identify optimal solutions in the majority of the problem instances examined. Some 50% were sub-optimal in this study. Depending on the application, planning, management and/or decision-making context, such results in practice may be problematic. Given the broad access to location modeling and spatial optimization offered through contemporary GIS, especially user-friendly, point-and-click technology, the ability to provide reliable and defensible results is essential. Public sector contexts also bring with them important fiduciary responsibilities, making it essential to minimize costs and maximize benefit to constituents. This means that mechanisms for communication of findings and what results mean are all too important. These are challenges that remain for GIS-based location cover model approaches. Beyond this, there is evidence that computational limitations remain through the use of GIS to address location planning problems. Interestingly, the empirical results suggest that commercial solver software is very capable, significantly faster than the heuristics used in GIS software. This is a curious finding when heuristics are generally relied upon to find solutions faster.

Notes

1. The Network Analyst module of ArcGIS requires a network corresponding to travel distance or time. The network layer must be cleaned and built in order for a location model to be run. Network properties, including travel directions, connectivity, restrictions, etc., can be specified when creating the network. Further, when solving location models, the specification of potential facilities sites, demand locations, attribute weights, number of facilities, etc., must be loaded and/or specified.

2. The Facility Location Model module of TransCAD requires a network corresponding to travel distance or time. The network must be cleaned and created. In addition, inputs are necessary, including a client layer (demand) and facility layer.

Disclosure statement

No potential conflict of interest was reported by the authors.

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