Linear Algebra 101 - Systems of equations

What is it?

A branch of mathematics that:

Provides the tools to represent, analyze, and solve multiple linear equations simultaneously using matrices, and matrix operations.

Solving Ax = b is no more than <u>finding where these objects intersect</u>.

Key concepts (systems of linear equations):

- Matrix[A] Grid of numbers to represent systems or transformations;
- Linear dependence/Independence Dependent if one row/column = combo of others;
- Singularity/non-singularity If the determinant of a matrix is 0, then it is said to be singular;
- Determinant (det) Scalar that tells if a matrix has a unique solution;
- Rank Number of independent rows/columns of a matrix. Rank = # rows Solution space dimension;
- Gaussian Elimination Step-by-step method to solve equations;
- Row echelon form Matrix simplified to a staircase pattern;
- Pivots First nonzero entries in each row.

Matrices as the translation of linear equations

$$a + b = 0$$

$$a + 2b = 0$$

$$1$$

$$1$$

$$1$$

$$2$$

$$a + b = 0$$
 $2a + 2b = 0$
 a
 b
 1
 2
 2

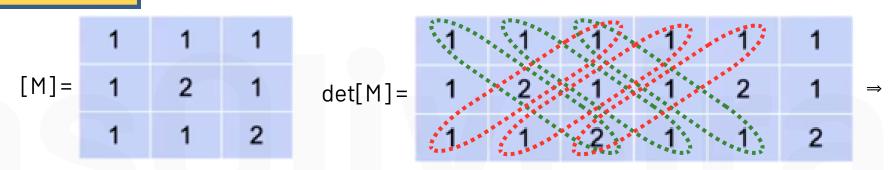
2 possible cases

Independent columns \Rightarrow Non-singular A Non-singular A \Rightarrow det(A) \neq 0 det(A) \neq 0 \Rightarrow **Unique solution** to Ax = b

Dependent columns \Rightarrow Singular A Singular A \Rightarrow det(A) = 0 det(A) = 0 \Rightarrow **No or infinite solution** to Ax = b

Redundant systems $(0x + 0y + 0z = 0) \Rightarrow$ infinite solutions. \bigcirc Contradictory systems $(0x + 0y + 0z != 0) \Rightarrow$ no solutions. \bigcirc

Determinants



$$\Rightarrow \det[M] = 1x2x2 + 1x1x1 + 1x1x1 - 1x2x1 - 1x1x1 - 2x1x1$$
main diagonals product (positive)
anti-diagonals product (negative)

$det[M] = 1 \Rightarrow Unique solution$

Properties

$$\det(AB) = \det(A)\det(B)$$

$$\det(A^T) = \det(A) \quad \det(A^{-1}) = \frac{1}{\det(A)}$$

Effect of Row Operations

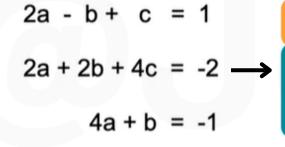
Solution:

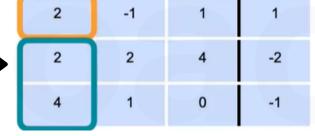
- Swapping 2 rows: **det()*(-1)**.
- Multiplying a row by a scalar k: **det()*k**.
- Adding a multiple of one row to another: No change.

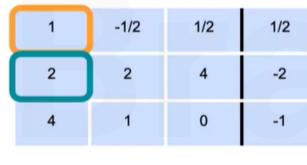
Gaussian Elimination

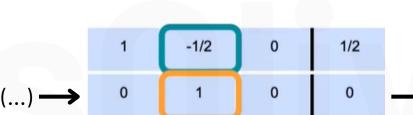
Augmented matrix [A|b]

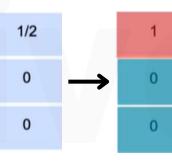
Reduce matrices to row echelon form to solve systems through row operations.

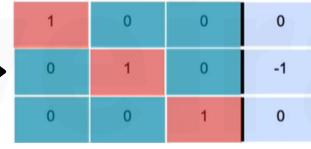












System of linear equations

Augmented matrix form

Row 1 \rightarrow 1/2Row 1

Row 1 \rightarrow Row 1 – 1/2 Row 2

Row echelon form/System Solution