FIRST EVALUATION OF FUZZY MIMO CONTROL LAWS

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Abstract–This paper presents two fuzzy multivariable control structures based on a qualitative modelling of a process. We use a benchmark to compar the results obtained with the fuzzy strategies with those of an LQ control law.

LQ = linear quadratic

Keywords: **Fuzzy sets, multivariable fuzzy controllers**

I. INTRODUCTION

Fuzzy control has seemed very attractive since the publication of efficient results in various applications, mainly in Japan, USA and Europe. However, these applications are more often single input–single output (SISO) control problems. Little attention has been paid to multivariable problems for multiple input–multiple output systems, despite some pioneering theoretical work [3].

In our approach, presented in this paper, we want to tackle multivariable problems, but taking profits of our numerous studies on SISO fuzzy controllers [4]. For this, starting from an example, we begin by developing a decentralized control structure including two robust fuzzy controllers after an analysis of the couplings in the multivariable system under consideration. The decentralized structure is, in a second step, improved taking into account secondary cross couplings between the main loops. Our approach is compared with a classical LQ controller, and the results appear very promising with a need, of course, of generalization.

II. EXAMPLE OF A MULTIVARIABLE PROCESS

In order to be pragmatic, we will develop progressively the design of a multivariable control using a physical example. The process, proposed by Jaume and al.[1], consists of mixing streams of two flow liquids in a continuously stirred tank. The

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production objectives is to maintain the tank level (H) and the tank concentration (C) at their set point, by manipulating the flow rates of the two inputs. A schematic of the process is shown in Fig.1 .

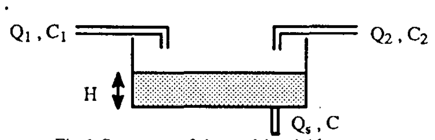


Fig.1 Structure of the multivariable process

The inputs are :
– liquid 1 : flow rate Q_1 and constant concentration C_1
– liquid 2 : flow rate Q_2 and constant concentration C_2

The output liquid has a flow rate Q_s and a concentration C.

We begin our study by a qualitative modelling of the process behavior. So, we apply different input couples (q_1, q_2) to the process, and we analyze the behavior of the outputs (see Fig.2).

2.

	Inputs		Outputs	
	q_1	q_2	h	c
	+	0	++	–
2	++	+	+++	0
	–	0	– –	+
	– –	–	– – –	0
	0	+	++	++
1	–	+	0	+++
	0	–	– –	– –
	+	–	0	– – –

Keys of the symbols
+ increase of the variable
– decrease of the variable
0 variation of the variable null or quite null

The number of signs represents the variation magnitude of the variable

Fig.2 Qualitative analysis results

Then, we have to analyze the results of the qualitative analysis and to deduce from it fuzzy control structures.

III. SYNTHESIS OF THE FUZZY MIMO CONTROLLERS

Two kinds of structures have been designed. The first one, the simplest, is based on the association of two local controllers. The second one is more complex, taking into account the interactions of the process.

A) Design of the decentralized structure

From the analysis of the shaded rows of the table Fig.2 , we can detect srong coupling effects in the system, and draw a causal graph (see Fig.3).

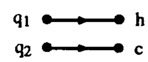
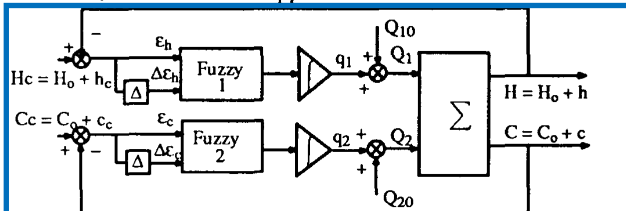


Fig.3 Simple causal graph

From this graph, we can design the decentralized control structure (FS1) of the Fig.4 , including two simple SISO fuzzy controllers. The rule tables are derived from the Mac Vicar Whelan one [2]. We consider the system as a black box, and we tune the controllers with a heuristic approach.

3.



ε_h : tank level error $\Delta\varepsilon_h$: derivative of the tank level error
ε_c : concentration error $\Delta\varepsilon_c$: derivative of the concentration error

Fig.4 Fuzzy structure 1 scheme

B) Design of a global controller

The control structure discussed in the previous section is quite simple and can be applied to processes which have small coupling effects. The results given by this structure are quite good, but they can be improved by designing a global controller which takes into account the interactions of the process. We will discuss this controller in this section. The controller structure will be deduced from the analysis of the qualitative modelling.

We take, as example, the lines 1 and 2 of the table (see Fig.2).
Line 1 : If the input q_1 decreases and the input q_2 increases, then the output variation h is null or below zero and the output variation c increases. So we deduce the following control rule :
" For a positive concentration reference, if we want to keep the tank level around zero and increase the concentration then we have to decrease q_1 and increase q_2 "
Line 2 : If the input q_1 increases a "lot" and the input q_2 increases, then the output variation h increases and the output variation c stays below zero. So we write the following rule :
" For a positive tank level reference, if we want to keep the concentration below zero and increase the tank level then we have to increase significantly q_1 and increase q_2 ".
In addition, we have to translate these qualitative rules on a multivariable control structure. The measures of the errors ε_h ,ε_c and the derivative of these errors $\Delta\varepsilon_h$,$\Delta\varepsilon_c$ are available.
The analysis of the process shows that the interactions are slow dynamic phenomena. So, the interactions only act upon the tank level error and concentration error. They are taken into account in the controllers by rules, deduced from the table in Fig.2 , and written as follow :
Line 1 : " IF ε_c is Positive Big AND ε_h is Zero THEN q_1 is Negative AND q_2 is Positive ".
Line 2 :" IF ε_c is Zero AND ε_h is Positive Big THEN q_1 is Positive Medium AND q_2 is Positive ".
So, we have control a control action $q_{1\_errors} = f_1(\varepsilon_h,\varepsilon_c)$ and a control action $q_{2\_errors} = g_1(\varepsilon_h,\varepsilon_c)$.
We only use the error derivatives $\Delta\varepsilon_h$ and $\Delta\varepsilon_c$ to control the transient phases. So we generate two control actions $q_{1\_derivative} = f_2(\Delta\varepsilon_h)$ et $q_{2\_derivative} = g_2(\Delta\varepsilon_c)$.
The resulting control actions applied to the process are :

$$q_1 = F(q_{1\_errors} + q_{1\_derivative})$$
$$q_2 = G(q_{2\_errors} + q_{2\_derivative})$$

The new control structure scheme (FS2) is shown in Fig.5 .

4.

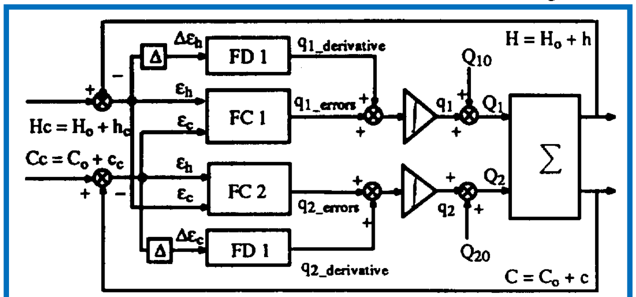


Fig.5 Fuzzy structure 2 scheme

An example of rule table for the fuzzy controller FC2 is given in Fig.6 .

ε_c \ ε_h	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NB	NB	NB
NM	NM	NM	NM	NM	NM	NM	NM
NS	NS	NS	NS	NS	NS	NS	NS
NVS	NB	NS	ZE	ZE	NS	PS	PB
ZE	NB	ZE	ZE	ZE	ZE	ZE	PB
PVS	NB	NS	PS	ZE	ZE	PS	PB
PS	PS	PS	PS	PS	PS	PS	PS
PM	PM	PM	PM	PM	PM	PM	PM
PB	PB	PB	PB	PB	PB	PB	PB

The area in black corresponds to the area where it is important to take into account the interactions.

Fig.6 An example of fuzzy control table

IV. Results and comparison

In this section, we will compare the results of our fuzzy control structures with the results obtained with an optimal controller, proposed by Jaume and al. (see Appendix). The comparison goals are not to define the best control structure, but we want to bring to the fore the advantages and the disavantages of the proposed fuzzy structures. We notice that all the simulations have been done with the non linear model of the process.
We define the following criteria to compare the controllers :

– a quadratic tank level error index

$$J(\varepsilon_h) = \int_0^t q_{11}.\varepsilon_h^2 dt$$

– a quadratic concentration error index

$$J(\varepsilon_c) = \int_0^t q_{22}.\varepsilon_c^2 dt$$

– a quadratic input cost index

$$J(q_1,q_2) = \int_0^t (r_{11}.q_1^2 + r_{22}.q_2^2) dt$$

– the global performance index, used to calculate the LQR regulator

$$J = \int_0^t (q_{11}.\varepsilon_h^2 + q_{22}.\varepsilon_c^2 + r_{11}.q_1^2 + r_{22}.q_2^2) dt$$

A) Definition of a tank level reference

In this case, the tank level set point is initialized at $h_c = 0.1$m, and the concentration set point is $c_c = 0$ mole/l.
The evolutions of the output h (c) are given Fig.7 (Fig.8).

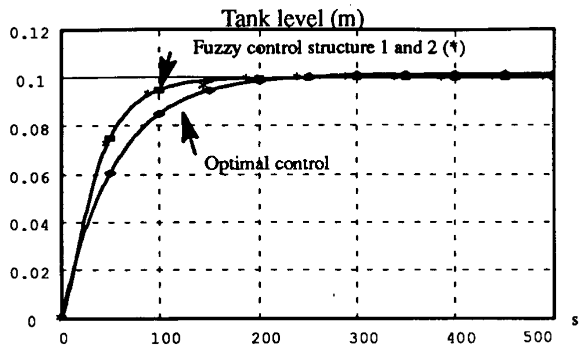


Fig.7 Tank level evolutions

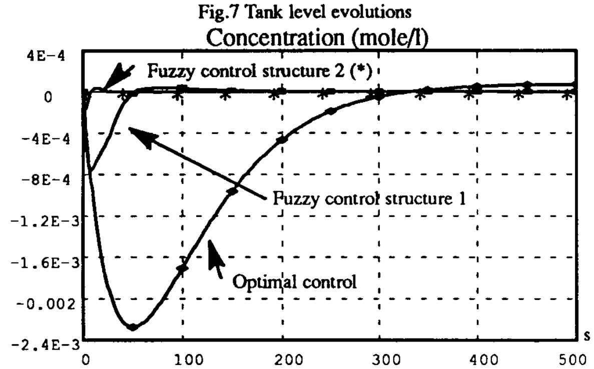


Fig.8 Concentration evolutions

The control actions q_1 (q_2) are given Fig.9 (Fig.10).

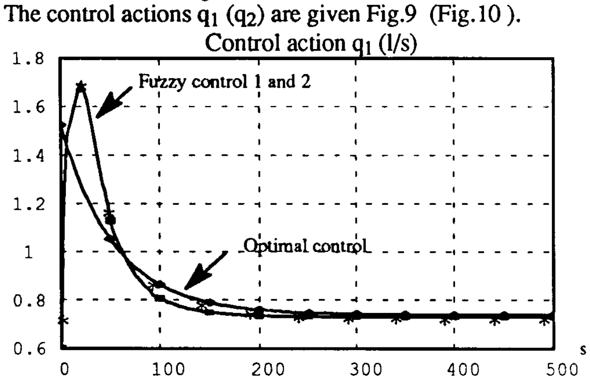


Fig.9 Control actions q_1

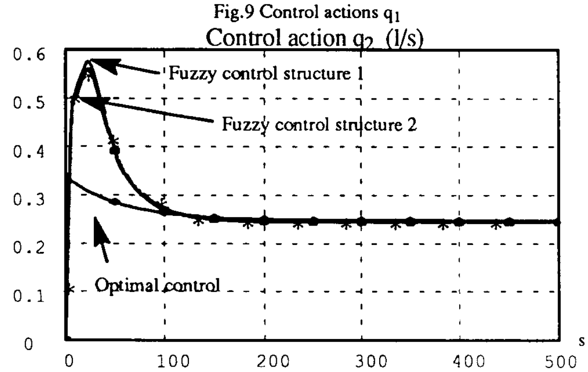


Fig.10 Control actions q_2

The different indexes are given Fig.11 .

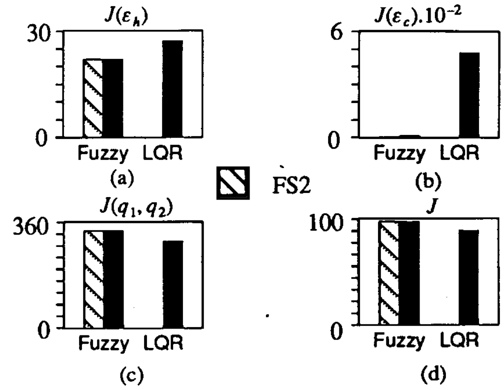


Fig.11 Performance indexes

The results obtained with the fuzzy structures are very suitable. The output h has a good dynamic behavior (see Fig.7 and Fig.11 (a)). The coupling effects are lower for the fuzzy structures than the optimal one (see Fig.8). We can notice the efficiency of the proposed fuzzy structure 2 for the compensation of the interactions. Indeed, the index $J(\varepsilon_c)$ is equal to 10^{-3} for FS1 and 10^{-5} for FS2 (see Fig.11 (b)). But, the control action cost is bigger for the fuzzy structures (13% more Fig.11 (c)). The global index J is 10% more for the fuzzy structures (see Fig.11 (d)).

B) Definition of a concentration reference

In this case, the concentration set point is initialized at c_c = 0.01mole/l , and the tank level set point is h_c = 0 m.

The evolutions of the output h (c) are given Fig.12 (Fig.13).

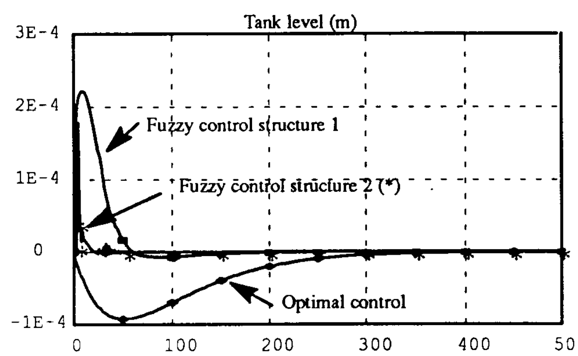


Fig.12 Tank level evolution

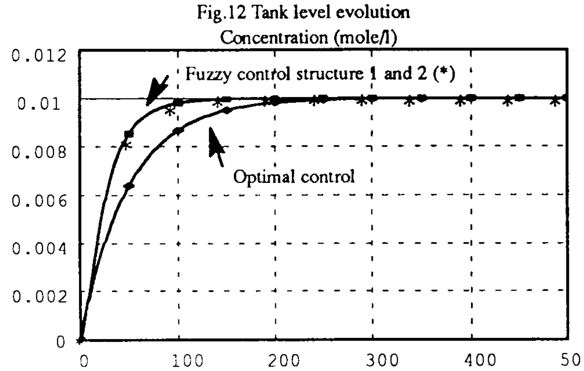


Fig.13 Concentration evolution

The different indexes are given Fig.14 .

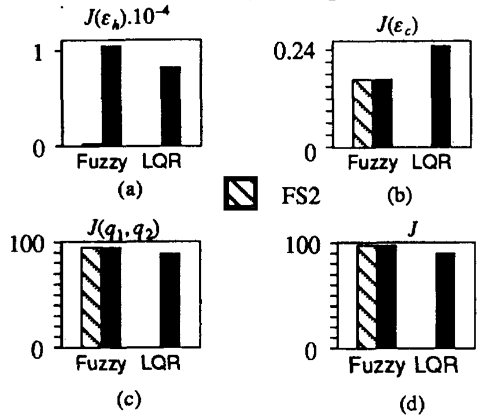


Fig.14 Performance indexes

The dynamic behavior of the concentration is good (see Fig.13 and Fig.14 (b)). If we focus our attention on the output h (Fig.12), we could make few remarks. First, the coupling effects are bigger with FS1 than the LQR control (see Fig.14 (a)) and the control action cost is 6% bigger (see Fig.14 (c)). Second, when we use FS2, we bring down the coupling effects for the same control action cost (see Fig.14 (a)). These results show the quality of the fuzzy control structure 2.

C) Conclusions

We have tested two fuzzy multivariable control structures. The structure 1 gives good results but the correction of the interactions is low. The structure 2 enables us to control effectively the coupling effects and the results are very promising. In both cases, the control action cost is bigger than the optimal one. But, several goals could be defined depending on the process : minimizing the interactions without any cost constraints, or relativizing the interactions with a minimum cost.

To follow the study, we have tested the robustness of FS2. We changed the working point, we defined perturbations on the nominal flow rates, on the output signals. The robustness of the fuzzy control structure is better than the optimal law one. These results could be improved by an extension of our structure. Indeed, we correct the interactions with the slow dynamic variables and in the case of a perturbation, the fast dynamic variables are very important to calculate the control actions. So we can extend the controller FD1 and FD2 by writing rules depending on the error derivatives $\Delta\varepsilon_h$ and $\Delta\varepsilon_c$.

V. Conclusion

After the success of fuzzy control, mainly on SISO control problems, it is time to consider MIMO control problems, from a practical point of view. In our pragmatic approach to this kind of problem, we are integrating our experience on the simplicity and robustness of basic fuzzy controllers (working mainly with the error and the change of error in classical regulation problems). After a qualitative analysis of a multivariable process, detecting the main couplings within the system, a decentralized structure is proposed. This structure includes two basic fuzzy controllers as used in our example. Satisfactory results obtained in this way are however improved, taking into account, in a second step of the design, cross couplings in the system. The performances of both the structures are positively compared with those of a LQ controller. Our goal is now to give to this approach a more systematic character.

Acknowledgement

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Appendix [1]

Non linear model of the process 1.

The output flow rate of the liquid is given by $Q_s = k.\sqrt{H}$.
From the mass balance relationship, we can write a non linear model of the process :

$$\frac{dH}{dt} = \frac{Q_1}{S} + \frac{Q_2}{S} - k.\sqrt{H} \qquad (1)$$

$$\frac{dC}{dt} = \frac{1}{H}\left(\frac{Q_1}{S}.(C_1-C) + \frac{Q_2}{S}.(C_2-C)\right) \qquad (2)$$

Linear state space model

A linear space state model is calculated around the nominal values of the tank level (H_0) and of the concentration (C_0). The linearization is necessary for the calculation of the optimal regulator. The linear state space model is given Fig.15 where q_1, q_2, c, h are the relative variations of Q_1, Q_2, C and H around their nominal value Q_{10}, Q_{20}, C_0 and H_0 . The matrix of observability is the identity one. For the numerical application, we take Q_{10} = 15 l/s, Q_{20} = 5 l/s, C_1 = 1 mole/l, C_2 = 2 moles/l, H_0 = 1 m, C_0 = 1.25 moles/l, T = 50 ms . The plant model is both observable and controllable. Moreover, the plant is stable at this working point.

$$\frac{dx}{dt} = \begin{bmatrix} \frac{-1}{2.T} & 0 \\ 0 & \frac{-1}{T} \end{bmatrix} .x + \begin{bmatrix} \frac{1}{S} & \frac{1}{S} \\ \frac{C_1 - C_0}{S.H_0} & \frac{C_2 - C_0}{S.H_0} \end{bmatrix} .e$$

$$x = \begin{bmatrix} h \\ c \end{bmatrix} \qquad e = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad T = \frac{S.H_0}{Q_{10} + Q_{20}}$$

Fig.15 Linear state space model

Design of the optimal controller

The dynamic of the outputs for the closed loop system is twice as fast as the open loop one, and the coupling terms must be as low as possible. The quadratic performance index to be minimized is given by (3).

$$J = \int_0^t (x^T.Q.x + e^T.R.e).dt \qquad Q = \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \qquad R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \quad (3)$$

The control law is given solving the Riccati equation (see [1]).