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Lista por partes

1- $\int x \operatorname{sen} x \, dx$

$u = x \qquad dv = \operatorname{sen} x \, dx$
$du = dx \qquad v = -\cos x$

$-x \cdot \cos x + \int \cos x \, dx$

$\int x \operatorname{sen} x \, dx = -x \cdot \cos x + \operatorname{sen} x + C$

2- $\int x e^{2x} \, dx$

$u = x \qquad dv = e^{2x} \, dx$
$du = dx \qquad v = \frac{1}{2} e^{2x}$

$\cancel{e^{2x} dx}$ $\qquad \int e^{2x} dx \qquad u = 2x$
$du = 2 \quad dx = \frac{du}{2}$

$\cancel{u = 2x}$
$\cancel{du = 2 \quad dx = \frac{du}{2}}$ $\qquad \int e^{u} \frac{du}{2} = \frac{1}{2} \int e^{u} du$

$= \frac{1}{2} e^{u} = \frac{1}{2} e^{2x}$

$x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx$

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$\int xe^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) + C$

3- $\int \ln x \, dx$

$u = \ln x \qquad dv = dx$
$du = \frac{1}{x}dx \qquad v = x$

$\ln|x|.x - \int x \cdot \frac{1}{x}\, dx = \ln|x|.x - \int \frac{\cancel{x}}{\cancel{x}_1} dx$

$x\ln x - x + C$

4- $\int x^2 e^x \, dx$

$u = x^2 \qquad dv = e^x dx$
$du = 2x\,dx \qquad v = e^x$

$\cancel{x^2 e^x - \int e^x 2x\,dx = x^2 e^x - e^x x^2 + C}$

$x^2 e^x - \int e^x 2x\,dx = x^2 e^x - 2\int xe^x dx$

$\int xe^x dx$

$u = x \qquad dv = e^x$
$du = dx \qquad v = e^x$

$x.e^x - \int e^x dx = xe^x - e^x$

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$$\int x^2 e^x dx = x^2 e^x - 2(xe^x - e^x) + C$$

5. $\int x \arctan x \, dx$

$u = \arctan(x)$ $\quad dv = x\,dx$

$du = \frac{1}{1+x^2} dx$ $\quad v = \frac{x^2}{2}$

$$\arctan(x) \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2}$$

$$\int \frac{x^2}{2} \cdot \frac{1}{1+x^2} = \frac{1}{2} \int \frac{x^2}{1+x^2}$$

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6 - $\int e^x \cos x \, dx$

$u = \cos x$ $\quad dv = e^x dx$
$du = -\text{sen}\, x$ $\quad v = e^x$

$\cos x \cdot e^x - \int e^x - \text{sen}\, x = \cos x \cdot e^x + \int e^x \,\text{sen}\, x \, dx$

~~$\cos x \cdot e^x + e^x - \cos x + e$~~

$\int e^x \,\text{sen}\, x \, dx$

$u = \text{sen}\, x$ $\quad dv = e^x dx$
$du = \cos x \, dx$ $\quad v = e^x$

$\text{sen}\, x \cdot e^x - \int e^x \cos x \, dx$

$\int e^x \cos x \, dx = \cos x \cdot e^x + \text{sen}\, x \cdot e^x - \int e^x \cos x \, dx$

7 - $\int x^3 \,\text{sen}\, x \, dx$

$u = \text{sen}\, x$ $\quad dv = x^3 dx$
$du = \cos x \, dx$ $\quad v = \frac{x^4}{4}$

$\text{sen}\, x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cos x \, dx$ $\quad u = \cos x$ $\quad dv = \frac{x^4}{4} dx$
$du = -\text{sen}\, x \, dx$ $\quad v = \frac{x^5}{5}$

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$\int x^2 \operatorname{sen} x$

$u = \operatorname{sen} x \qquad dv = x^2 dx$

$du = \cos x \, dx \qquad v = \frac{x^3}{3}$

$\operatorname{sen} x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cos x \, dx$

$\int \frac{x^3}{3} \cos x \, dx \qquad u = \cos x \qquad dv = \frac{x^3}{3} dx$

$du = -\operatorname{sen} x \, dx \qquad v = \frac{x^4}{4}$

$\cos x \cdot \frac{x^4}{4} + \int \frac{x^4}{4} \operatorname{sen} x \, dx$

$\int x \cdot 2^x dx$

$u = x \qquad dv = 2^x dx$

$du = dx \qquad v =$

$\int 2^x dx \qquad \int 2^u du$

$u = x$

$du = dx$

a) $\int \frac{dx}{(x-1)(x+2)} = \int \frac{A}{x-1}\,dx + \int \frac{B}{x+2}\,dx$

$1 = A(x+2) + B(x-1) = Ax + 2A + Bx - B = (A+B)x +$

$A+B=0$

$2A-B=1$

$3A=1 \quad A=\frac{1}{3} \quad \frac{1}{3}+B=0 \quad B=-\frac{1}{3}$

$$\int \frac{\frac{1}{3}}{x-1}\,dx - \int \frac{\frac{1}{3}}{x+2}\,dx = \frac{1}{3}\int \frac{1}{x-1}\,dx - \frac{1}{3}\int \frac{1}{x+2}\,dx$$

$$= \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+2)$$

b) $\int \frac{x-1}{x^2+2x+10}\,dx$

$x^2+2x+10=0$

$\frac{-2 \pm \sqrt{4-40}}{2}$ $\Delta = -36$

$x^2+2x+10 = (x^2+2x+1)-1+10 = (x+1)^2+9$

$\int \frac{x-1}{(x+1)^2+9}\,dx$ $\quad v = x+1$ $\quad x-1 = (v-1)-1 = v-2$

$dv = dx$

$\int \frac{v-2}{v^2+9}\,dv = \int \frac{v}{v^2+9}\,dv - \int \frac{2}{v^2+9}$ $\quad w = v^2+9$

$dw = 2v\,dv \quad v\,dv = dw/2$

$\int \frac{1}{w}\frac{dw}{2} = \frac{1}{2}\int \frac{dw}{w} = \frac{1}{2}\ln(w) = \frac{1}{2}\ln(v^2+9)$

$-2\int \frac{1}{v^2+3^2}\,dv = \frac{1}{a}\arctan\left(\frac{v}{a}\right)$ $\quad a=3 = -\frac{2}{3}\arctan\left(\frac{v}{3}\right)$

$\frac{1}{2}\ln(v^2+9) - \frac{2}{3}\arctan\left(\frac{v}{3}\right)+C = \frac{1}{2}\ln((x+1)^2+9) - \frac{2}{3}\arctan\left(\frac{x+1}{3}\right)+C$

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c) $\int \frac{x^4+48x-1}{(x+7)(x^2+x+1)} = \int \frac{A}{x+7}\,dx + \int \frac{Bx+C}{x^2+x+1}\,dx$

$x^4+48x-1 = A(x^2+x+1) + Bx + C((x+7)$

$x^4+48x-1 = Ax^2 + Ax + A + Bx^2 + 7Bx + Cx + 7C$

$0x^3 + x^4 + 48x - 1 = (A+B)x^2 + (A+7B+C)x + A + 7C$

$A + 7C = -1$

$A = -1 - 7c$

$A+B=0 \quad A=-B$

$A + 7B + C = 48$

d) $\int \frac{1}{x(x+1)}\,dx = \int \frac{A}{x}\,dx + \int \frac{B}{x+1}\,dx$

$1 = A(x+1) + B(x) = Ax + A + Bx = (A+B)x + A$

$A+B=0$

$A=1$

$1+B=0 \quad B=-1$

$\int \frac{1}{x}\,dx + \int \frac{1}{x+1}\,dx = \ln|x| - \ln(x+1)\,dx$

e) $\int \frac{dx}{x^2+4} = \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$

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f) $\int \frac{x}{(x+2)(x-3)^2}\,dx = \int \frac{A}{x+2}\,dx + \int \frac{B}{x-3}\,dx + \int \frac{C}{(x-3)^2}\,dx$

$\frac{x}{(x+2)(x-3)^2} = A(x-3)^2 + B(x+2)(x-3) + C(x+2)$

$x = A(x-3)(x-3) + B(x+2)(x-3) + C(x+2)$

$x = Ax - 3A(x-3) + Bx + 2B(x-3) + Cx + 2C$

$x = Ax^2 - 3Ax - 3Ax + 9A + Bx^2 - 3Bx + 2Bx - 6B + Cx + 2C$

$x = Ax^2 + Bx^2 - 6Ax - 1Bx + Cx + 9A - 6B + 2C$

$x = (A+B)x^2 - (6A - B + C)x + 9A - 6B + 2C$

$x = -2 \quad A(2-3)^2 + B(0) + C(0)$

$-2 = A(-5)^2 \quad -2 = 25A \quad A = \frac{-2}{25}$

$x = 3 \quad A(0) + B(0) + C(5) \quad 3 = 5C \quad C = \frac{3}{5}$

$(A+B)x^2 = 0 \quad A+B = 0 \quad \frac{-2}{25} + B = 0 \quad B = \frac{2}{25}$

$\frac{-2}{25}\int \frac{1}{x+2}\,dx + \frac{2}{25}\int \frac{1}{x-3} + \frac{3}{5}\int \frac{1}{(x-3)^2}\,dx \qquad \frac{(x-3)^{-1}}{-1}$

$= \frac{-2}{25}\ln(x+2) + \frac{2}{25}\ln(x-3) + \frac{3}{5}\int (x-3)^{-2}\,dx = -\frac{3}{5(x-3)} + C$

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g) $\int \frac{1}{x^2+x+2}\,dx$

$x^2+x+2=0$

$\frac{-1 \pm \sqrt{1^2-8}}{2}$ $\Delta = -9$

h) $\int \frac{x^3+1}{x(x+4)}\,dx = \int \frac{A}{x}\,dx + \int \frac{B}{x+4}\,dx$

$x^3+1 = A(x+4) + B(x) = Ax + 4A + Bx = (A+B)x + 4A$

$A+B=0$

$4A=1 \quad A=\frac{1}{4} \quad B=-\frac{1}{4}$

$\frac{1}{4}\int \frac{1}{x}\,dx - \frac{1}{4}\int \frac{1}{x+4}\,dx = \frac{1}{4}\ln(x) - \frac{1}{4}\ln(x+4)$

1- $\int_1^{\infty} \frac{1}{x^3} dx = \lim_{t \to \infty} \int_1^{t} \frac{1}{x^3} dx$

$\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2} \Big|_1^t = \frac{t^{-2}}{-2} - \frac{1}{-2} = \frac{-1}{2t^2} - \frac{1}{-2}$

$\lim_{t \to \infty} \left[\underbrace{\frac{-1}{2t^2}}_{0} - \frac{1}{-2} \right] = \frac{-1}{2}$

t	$\frac{-1}{2t^2}$
10	-0,00005
100	-0,000005
	$\downarrow$ 0

2- $\int_0^{\infty} e^{-x} dx = \lim_{t \to \infty} \int_0^{t} -e^{x} dx$

$\int -e^{-x} dx = -e^{-x} \Big|_0^t = -e^{t} + \cancel{e^0}\,1 = 1 - e^{t}$

$\lim_{t \to \infty} \left[1 - \underbrace{e^{t}}_{\infty} \right] = 1 - \infty = \infty$ diverge

t	e^t
10	1095
	$\downarrow$ ∞

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3 - $\int_1^\infty \frac{1}{x} dx = \lim_{t \to \infty} \int_1^t \frac{1}{x} dx$

$\int \frac{1}{x} dx = \ln(x) \Big|_1^t = \ln(t) - \underbrace{\cancel{\ln(1)}}_{0}$

$\lim_{t \to \infty} \left[\ln(t) \right] = \infty$

4 - $\int_0^1 \frac{1}{x^p} dx = \lim_{t \to 0^+} \int_t^1 \frac{1}{x^p} dx$

Se $p = 1$

$\int \frac{1}{x} = \ln(x) \Big|_t^1 \; \underbrace{\cancel{\ln(1)}}_{0} - \ln(t) = -\ln(t) = \infty$

A integral diverge

6) $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = \underbrace{\int_0^\infty \frac{1}{1+x^2} dx}_{\text{I}} + \underbrace{\int_{-\infty}^0 \frac{1}{1+x^2}}_{\text{II}} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

I $\lim_{t \to \infty} \int_0^t \frac{1}{1+x^2} dx = \frac{\pi}{2}$

$\int \frac{1}{1+x^2} dx = \arctan(x) \Big|_0^t = \arctan(t) - \arctan(0)$

$\lim_{t \to \infty} \left[\underbrace{\cancel{\arctan(t)}}_{\frac{\pi}{2}} - \underbrace{\cancel{\arctan(0)}}_{0} \right]$

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(1) $\lim_{t \to -\infty} \left[\underbrace{\arctan(t)}_{+\frac{\pi}{2}} - \underbrace{\arctan(0)}_{0} \right] = +\frac{\pi}{2}$

2) $\int_0^{\infty} \frac{x}{(1+x^2)^2} = \lim_{t \to \infty} \int_0^t \frac{x}{(1+x^2)^2}$

$\int \frac{x}{(1+x^2)^2} = \int x \cdot \frac{1}{(1+x^2)^2} \, dx$ $\quad v = 1+x^2$

$dv = 2x\,dx \quad x\,dx = \frac{dv}{2}$

$\int \frac{1}{v^2} \frac{dv}{2} = \frac{1}{2} \int \frac{1}{v^2} dv = \frac{1}{2} \frac{v^{-1}}{-1} = -\frac{1}{2} (1+x^2)^{-1} \Big|_0^t = -\frac{1}{2}(1$

$+ \frac{1}{2} \cdot (-1)$

$\lim_{t \to \infty} \left[-\frac{1}{2} (1+t^2)^{-1} - \frac{1}{2} \right) = -\frac{1}{2} \lim_{t \to \infty} \left[\frac{1}{1+t^2} \right] = 0$

t	$\frac{1}{1+t^2}$
10	0,0099
100	0,00099
1000	0,000099

$\downarrow$

0