3D Geometric Transformations

Graphics Systems /

Computer Graphics and Interfaces

3D Geometric Transformations

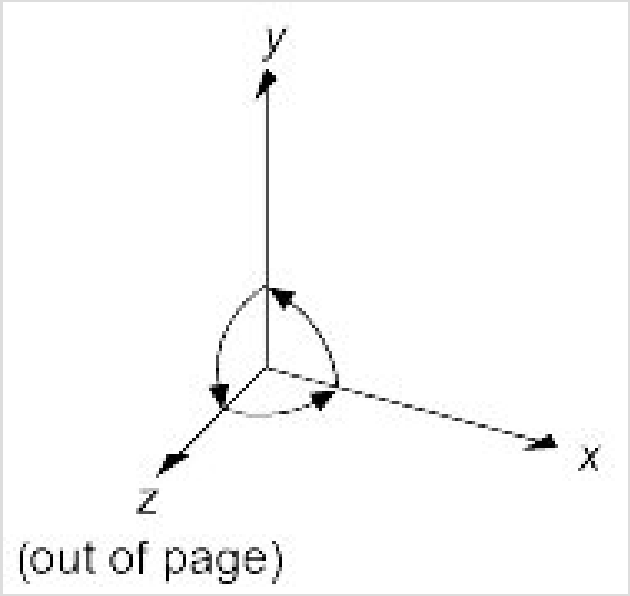
Extension of 2D methods now including the Z coordinate

Transformations:

- Translation
- Scaling
- Rotation

3D coordinate system:

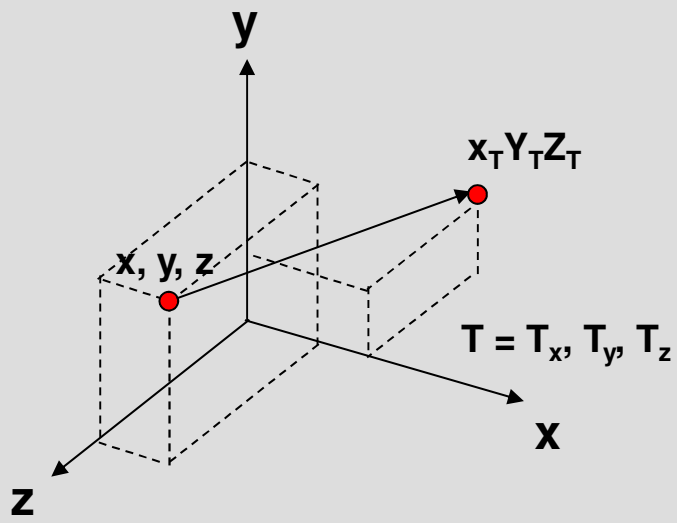
Right-Hand Rule



Axis rotation	Positive direction of rotation
x	y to z
y	z to x
z	x to y

Translation

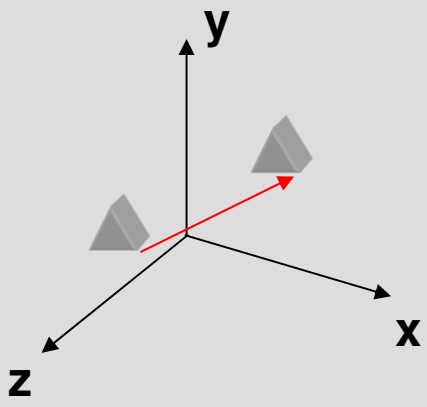
Translation of a Point



$$\begin{cases} x_t = x + T_x \\ y_t = y + T_y \\ z_t = z + T_z \end{cases}$$

$$\begin{bmatrix} x_T \\ y_T \\ z_T \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The translation of an object is carried out by applying the operation to each of its vertices.

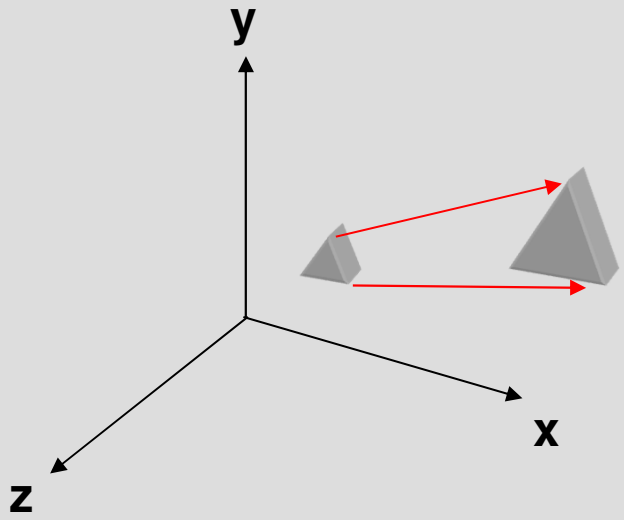


Scaling

Regarding the origin:

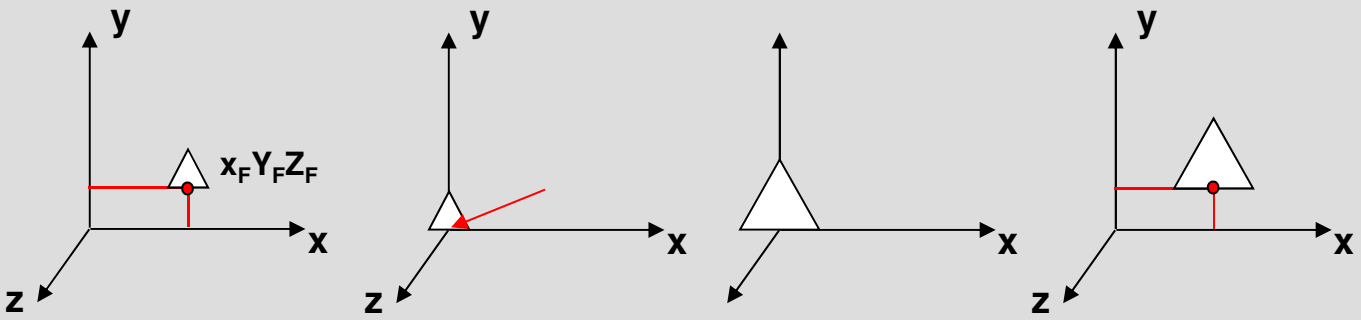
$$\begin{cases} x_s = x.s_x \\ y_s = y.s_y \\ z_s = z.s_z \end{cases}$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Scaling

In relation to an arbitrary point:

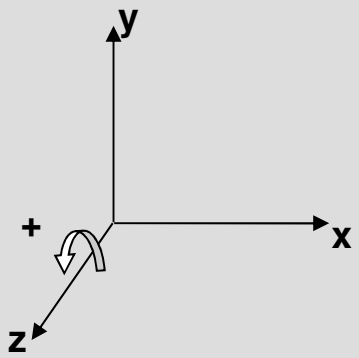


$$\boldsymbol{T}(X_F Y_F Z_F).\boldsymbol{S}(S_x, S_y, S_z).\boldsymbol{T}(-X_F, -Y_F, -Z_F) = \begin{bmatrix} S_x & 0 & 0 & (1-S_x).x_F \\ 0 & S_y & 0 & (1-S_y).y_F \\ 0 & 0 & S_z & (1-S_z).z_F \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation

- In 2D the axis of rotation is perpendicular to the XY plane
- 3D rotation axis may be
  - x, y or z
  - An axis arbitrarily placed in the space

Around z-axis → constant z

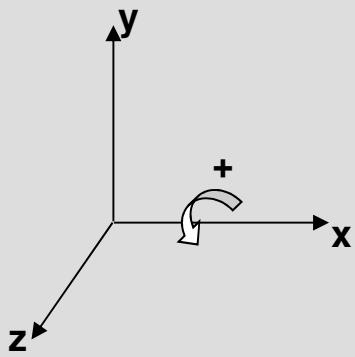


$$\begin{cases} x_{Rz} = x\cos(a) - y\sin(a) \\ y_{Rz} = x\sin(a) + y\cos(a) \\ z_{Rz} = z \end{cases}$$

$$\begin{bmatrix} x_{Rz} \\ y_{Rz} \\ z_{Rz} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(a) & -\sin(a) & 0 & 0 \\ \sin(a) & \cos(a) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation

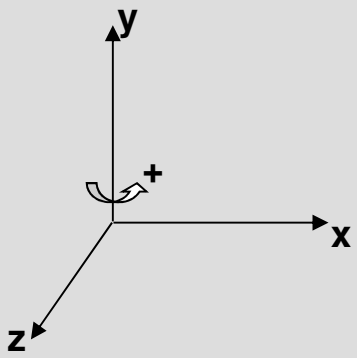
Around x-axis → x constant



$$\begin{cases} x_{Rx} = x \\ y_{Rx} = y\cos(a) - z\sin(a) \\ z_{Rx} = y\sin(a) + z\cos(a) \end{cases}$$

$$\begin{bmatrix} x_{Rx} \\ y_{Rx} \\ z_{Rx} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) & 0 \\ 0 & \sin(a) & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Around y-axis → constant y

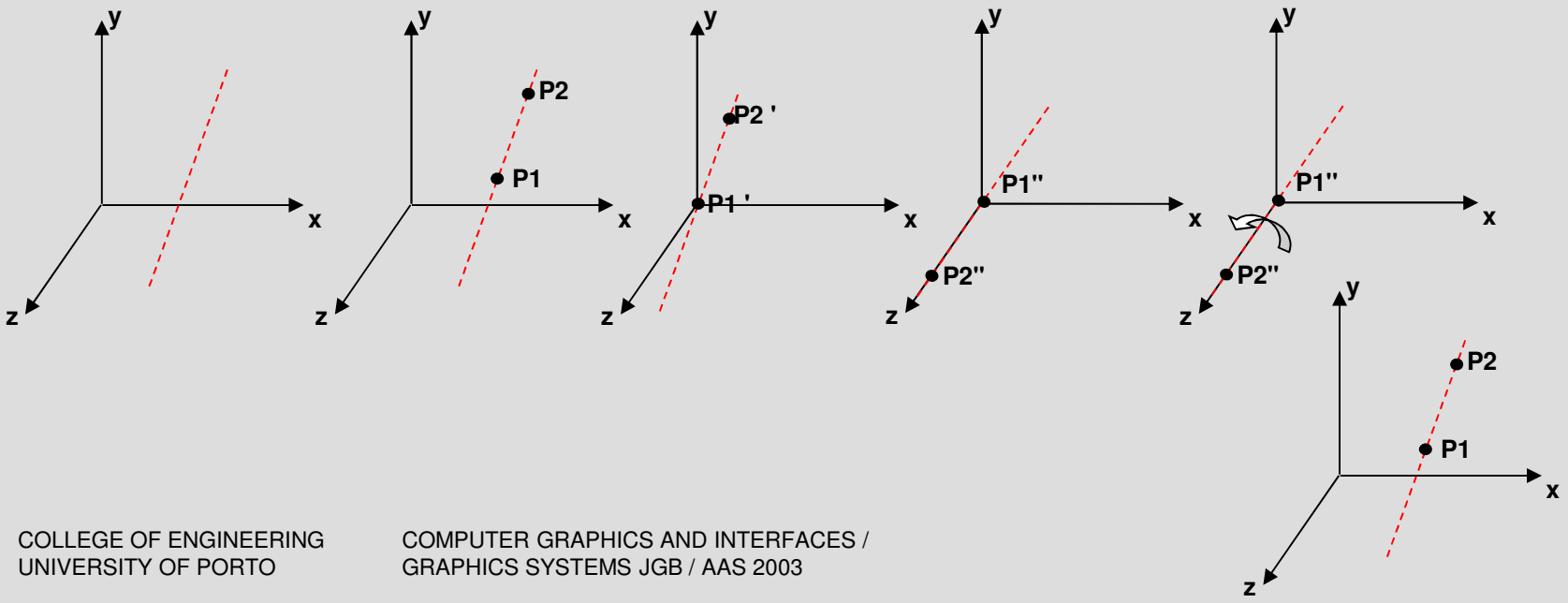


$$\begin{bmatrix} x_{Ry} \\ y_{Ry} \\ z_{Ry} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(a) & 0 & \sin(a) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(a) & 0 & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation

Rotation about an axis placed arbitrarily in 3D space:

1. Applying the translation to place the axis of rotation passing through the origin of the coordinate system.
2. Rotating the object so that the axis of rotation coincides with one of the coordinate axes.
3. Apply the desired rotation about this axis.
4. Apply the inverse rotation of point 2.
5. Apply the inverse translation of 1.



Exercises

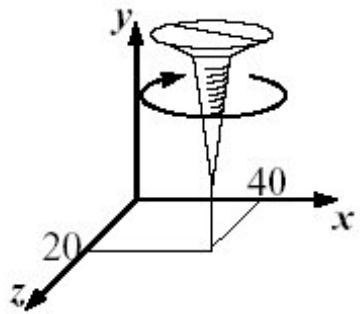
4. Pretende-se realizar a seguinte sequência de transformações geométricas 3D:

1. "Espelho" no plano $y=k$;
2. Ampliação de S vezes, na dimensão y.
3. Rotação de $|\mathbf{a}|$ no sentido dos ponteiros do relógio, para quem observa de $y=\infty$ para $y=0$.

a)- Determine a matriz de transformação equivalente.

b)- Será possível obter o mesmo resultado, com as mesmas operações por outra ordem?

5. Um parafuso encontra-se, no espaço 3D, de tal forma que o seu eixo coincide com a recta $x=40$, $z=20$. A rosca do parafuso é direita e faz o parafuso avançar **2** unidades por volta.



a)- Calcule a matriz de transformação geométrica 3D que traduz o movimento do parafuso quando este roda de **10º** no sentido indicado na figura

b)- Diga se seriam suficientes os dados fornecidos se, além dos movimentos enunciados, o parafuso fosse também alvo de um escalamento $S(1, 1.2, 1)$.