

Mapping

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*Also, a very special Thank You to: **Prof. Nuno Lau, IEETA, U. Aveiro***



Background

- **Localization** – Where am I?
- **Mapping** – My (dynamic?) surroundings
- **Navigation** – How do I get where I want to go?
- TREND: **SLAM** –
 - Simultaneous Localization and Mapping

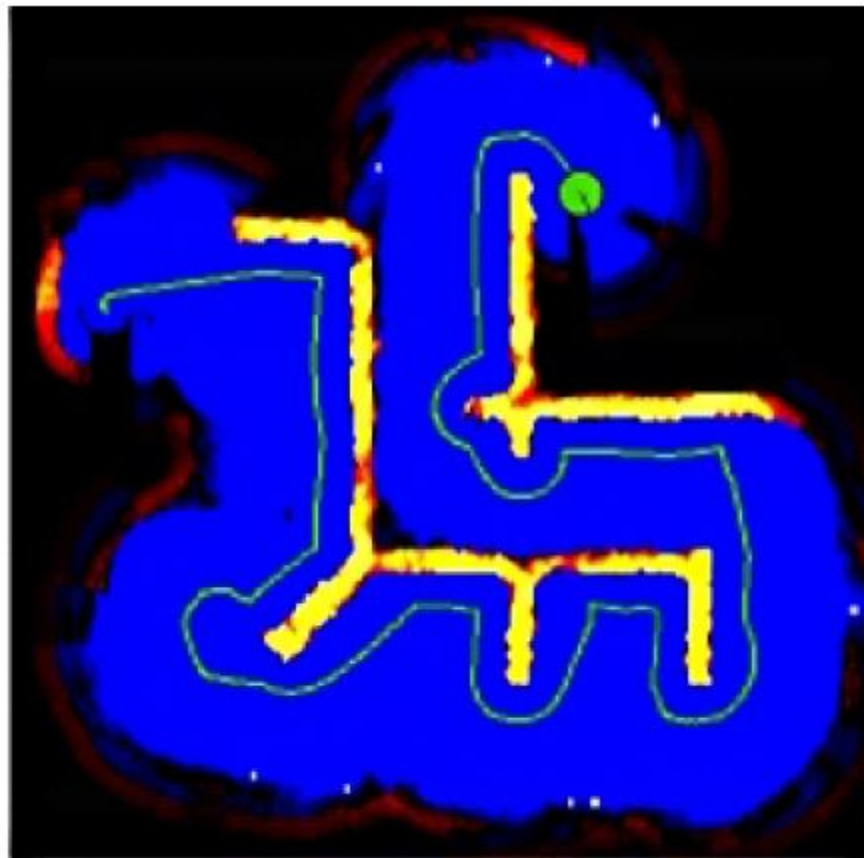
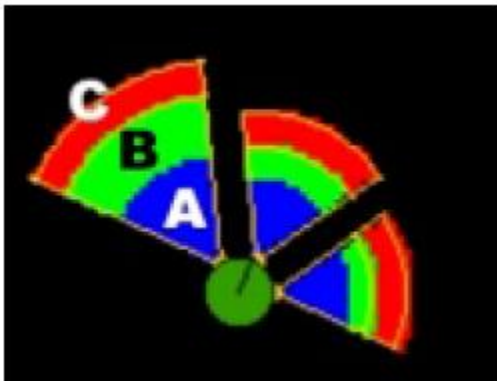
Mapping

- **Mapping is the process of building an internal estimate of the metric map of the environment**
- **What should be represented?**
 - Each cell is occupied or unoccupied
 - Each cell has a continuous value (high-occupied, low unoccupied)
 - Probability that each cell is occupied $P(H)$ and probability that each cell is unoccupied $P(\sim H)$
 $0 \leq P(H) \leq 1$
 $1 - P(H) = P(\sim H)$

Mapping

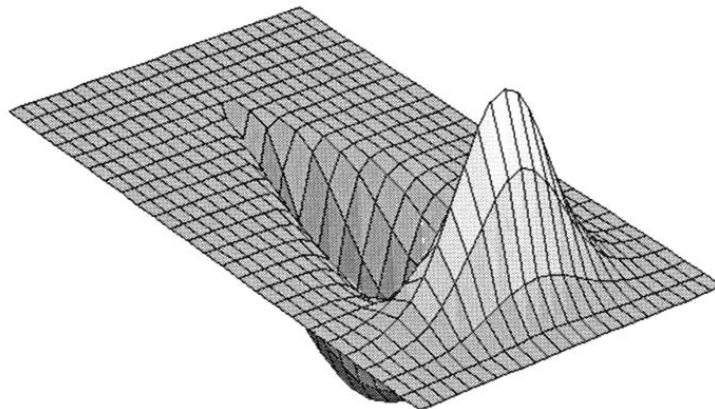
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Mapping



Mapping

- **Conditional probabilities**
 - We want to determine $P(H|s)$
 - Probability cell is occupied given a certain measure s
 - Let's start by determining $P(s|H)$
 - Probability of getting measure s if there is H is occupied
 - In general $P(H|s)$ is not equal to $P(s|H)$
 - This is the sensor model



Mapping

- **Conditional probabilities**

- We want to determine $P(H|s)$
- From Bayes' Rule:

$$P(H | s) = \frac{P(s | H).P(H)}{P(s | H)P(H) + P(s | \sim H)P(\sim H)}$$

- $P(s|H)$ and $P(s|\sim H)$ are known from the sensor model
- $P(H)$ and $P(\sim H)$ are unconditional probabilities or prior probabilities
 - If no information is available $P(H)=P(\sim H)=0.5$ can be assumed

Mapping

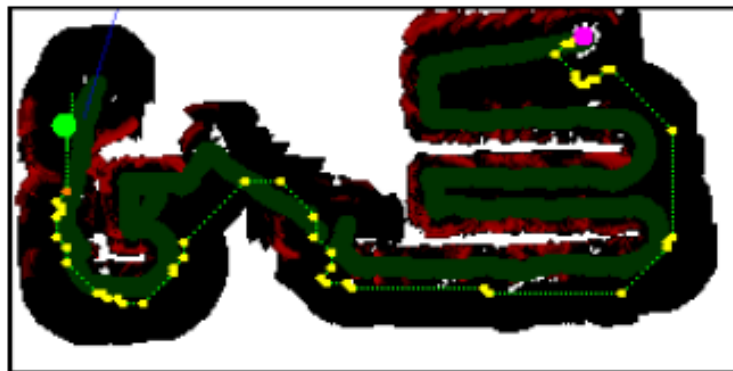
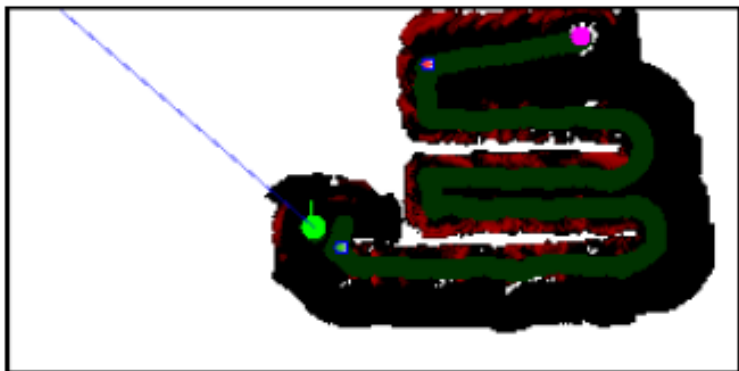
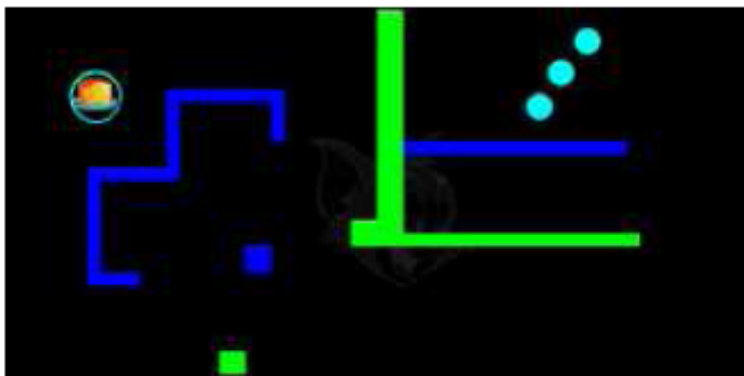
- **Updating with the Bayes' rule**

- How to fuse the computed probabilities with new sensor readings?

$$P(H | s_n) = \frac{P(s_n | H).P(H | s_{n-1})}{P(s_n | H)P(H | s_{n-1}) + P(s_n | \sim H)P(\sim H | s_{n-1})}$$

- This is the recursive version of the update formula
- Each time a new observation is made it can be employed to update the occupancy grid

Mapping



Mapping

- **Dempster-Schafer Theory**
 - Belief functions instead of probabilities
 - Measure belief mass
 - Each sensor contributes with a belief mass of 1.0
 - Can distribute the mass to a set of propositions
 - Set of propositions is the Frame of Discernement (FOD)
 - In the case of occupancy grid $FOD = \{\text{Occupied}, \text{Empty}\}$
 - It may include non exclusive propositions
 - A sensor reading may be considered as ambiguous

Mapping

- **Dempster-Schafer Belief function properties**
 - $\text{Bel}(X)$ measures the likelihood that previous evidence supports X
 - $\text{Bel}(\{\}) = 0$
 - $\text{Bel}(\text{ set of all subsets of FOD }) = 1$
 - In the case of the occupancy grid:
 $\text{Bel} = m(\{\text{Occupied}\}), m(\{\text{Empty}\}), m(\text{dontknow})$
 $\text{dontknow} = \{\text{Occupied}, \text{Empty}\}$

Mapping

- **Belief function for sonar**
 - Region that supports evidence of having an obstacle
 - $m(\text{occupied}) = \text{evidence}$
 - $m(\text{empty}) = 0$
 - $m(\text{dontknow}) = 1 - \text{evidence}$
 - Region that supports evidence of being empty
 - $m(\text{occupied}) = 0$
 - $m(\text{empty}) = \text{evidence}$
 - $m(\text{dontknow}) = 1 - \text{evidence}$
 - The main difference from probabilities is that uncertainties in the reading count as belief mass for dontknow

Mapping

- Rule of combination**

- $Bel_1 = m(\text{occupied})=0.4, m(\text{empty})=0, m(\text{dontknow})=0.6$
- $Bel_2 = m(\text{occupied})=0.6, m(\text{empty})=0, m(\text{dontknow})=0.4$

Bel1	Dontknow=0,6	$\{\text{Occupied}\} \cap \{\text{dontknow}\} = \{\text{occupied}\}$ $0,6 * 0,6 = 0,36$	$\{\text{dontknow}\} \cap \{\text{dontknow}\} = \{\text{dontknow}\}$ $0,6 * 0,4 = 0,24$
	Occupied=0,4	$\{\text{Occupied}\} \cap \{\text{Occupied}\} = \{\text{occupied}\}$ $0,4 * 0,6 = 0,24$	$\{\text{dontknow}\} \cap \{\text{Occupied}\} = \{\text{occupied}\}$ $0,4 * 0,4 = 0,16$
		Occupied=0,6	dontknow=0,4
		Bel2	

- $Bel_3 = m(\text{occupied})=0,76, m(\text{empty})=0,0, m(\text{dontknow})=0,16$

Mapping

- Rule of combination**

- $Bel_1 = m(\text{occupied})=0.4, m(\text{empty})=0, m(\text{dontknow})=0.6$
- $Bel_2 = m(\text{occupied})=0.0, m(\text{empty})=0.6, m(\text{dontknow})=0.4$

Bel1	Dontknow=0,6	$\{\text{empty}\} \cap \{\text{dontknow}\} = \{\text{empty}\}$ $0,6 * 0,6 = 0,36$	$\{\text{dontknow}\} \cap \{\text{dontknow}\} = \{\text{dontknow}\}$ $0,6 * 0,4 = 0,24$
	Occupied=0,4	$\{\text{Occupied}\} \cap \{\text{empty}\} = \{\}$ $0,4 * 0,6 = 0,24$	$\{\text{dontknow}\} \cap \{\text{Occupied}\} = \{\text{occupied}\}$ $0,4 * 0,4 = 0,16$
		empty=0,6	dontknow=0,4
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Mapping

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Bel1	Dontknow=0,6	$\{\text{empty}\} \cap \{\text{dontknow}\} = \{\text{empty}\}$ $0,6 * 0,6 = 0,36$	$\{\text{dontknow}\} \cap \{\text{dontknow}\} = \{\text{dontknow}\}$ $0,6 * 0,4 = 0,24$
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		empty=0,6	dontknow=0,4
		Bel2	

- As $m(\{\})$ must be 0, a normalization is needed
- $Bel_3 =$
 - $m(\text{occupied}) = 0,16 / (1 - 0,24) = 0,21$
 - $m(\text{empty}) = 0,36 / (1 - 0,24) = 0,47$
 - $m(\text{dontknow}) = 0,24 / (1 - 0,24) = 0,32$

Mapping

- **Rule of combination**

$$m(C_k) = \frac{\sum_{A_i \cap B_j = C_k; C_k \neq \{\}} m(A_i).m(B_j)}{1 - \sum_{A_i \cap B_j = \{\}} m(A_i).m(B_j)}$$

- It must be repeated for every subset of the orthogonal sum

Additional Sources

Mapping

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