Robotic Locomotion & Kinematics

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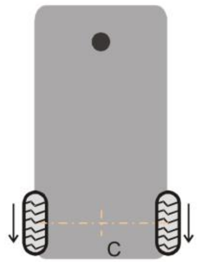
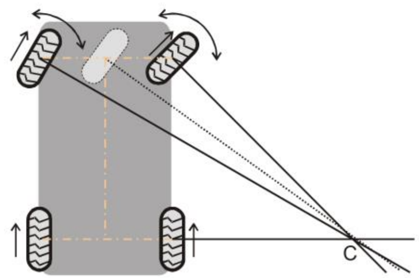
Locomotion

(Mobile Robot)

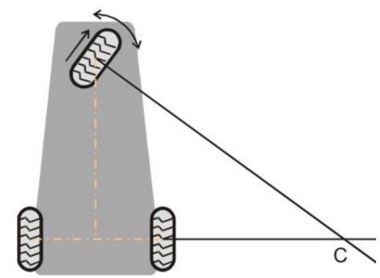
Locomotion (i) - Wheels



Differential drive
(wheelchair)

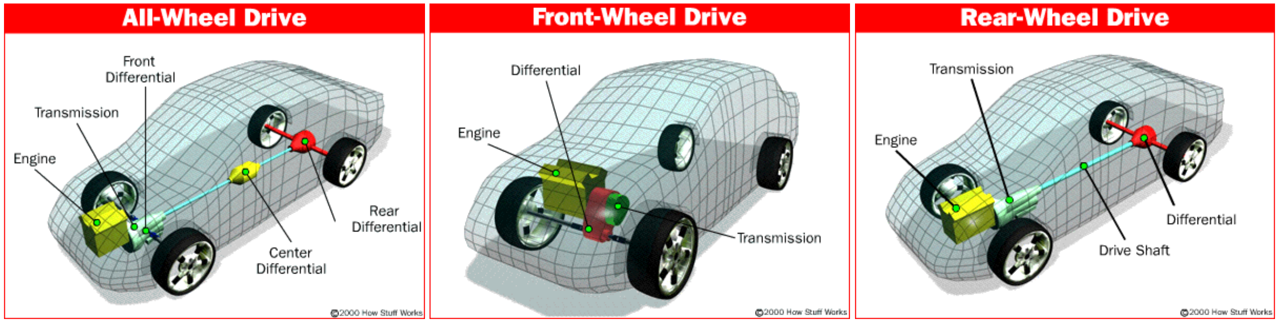


Ackerman
(normal car drive)

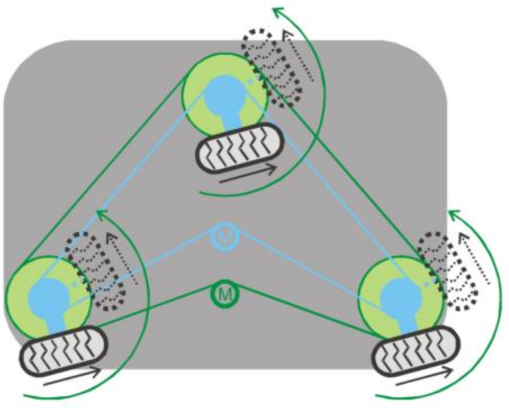


Tricycle

Ackerman
(normal car drive)

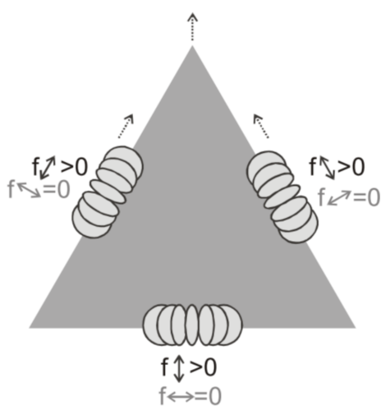


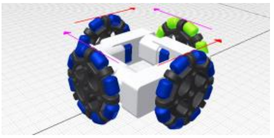
Locomotion (ii) – Omni / Omniwheels



Synchronous
(Quasi omnidrive)

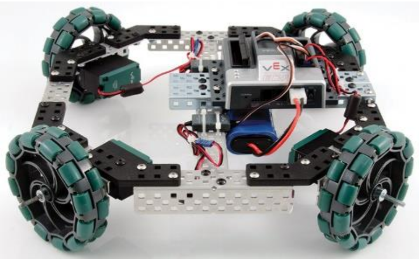
Locomotion (iii) – Omni / Omniwheels

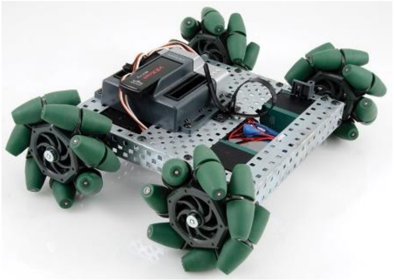




Omni Wheels Holonomic Drive

Mecanum Wheel





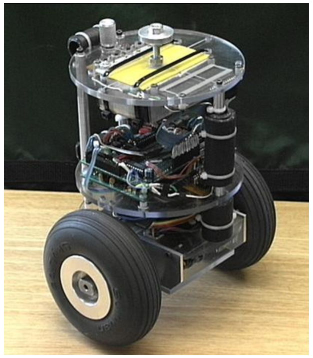
https://www.servomagazine.com/magazine/article/a-look-at-holonomic-locomotion
https://www.chiefdelphi.com/t/help-with-omni-wheels/83764/21

https://www.vicenzathunders.com/products

Mecannum (Omni) Wheels

- http://youtu.be/o-j9TReI1aQ
- http://youtu.be/Yng7JB6swuI
- http://youtu.be/TXTo16KKm8Q
- Omni Lifter Airtrax Cobra - http://youtu.be/IlmKcohyXG0
- Lego Omni Wheel - http://youtu.be/7fevklHUaIk

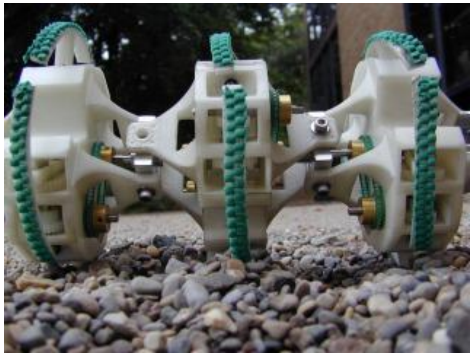
Locomotion (iv)



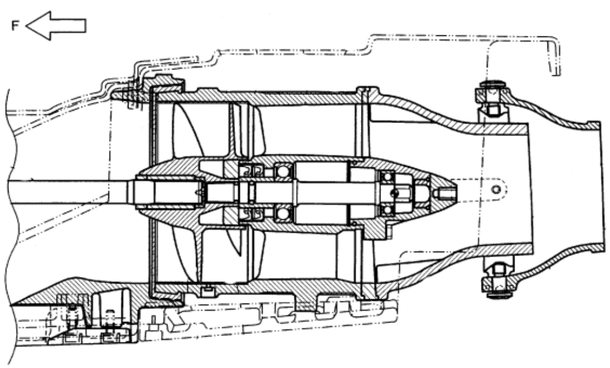
Nbot / segway

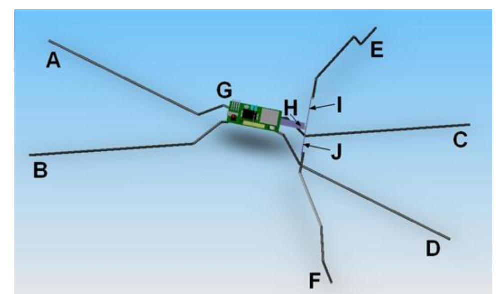
Track Drive
(military tank like)

Soryu

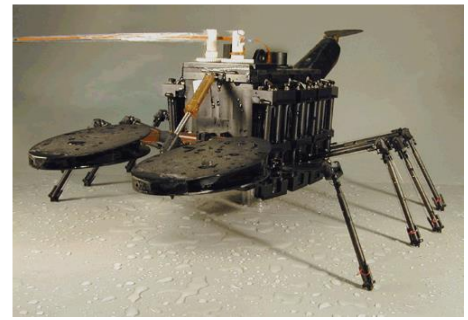


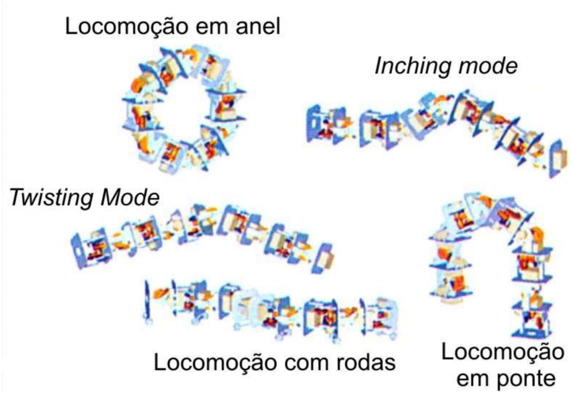
Locomotion (v) – water related

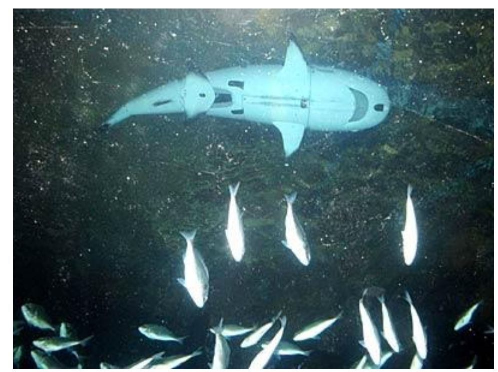




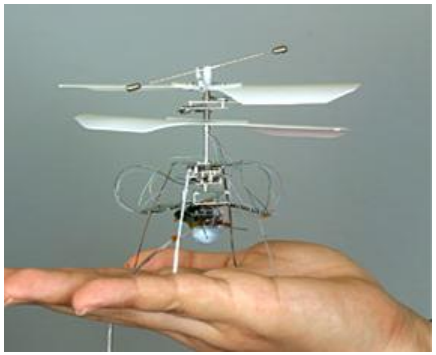
Locomotion (vi)

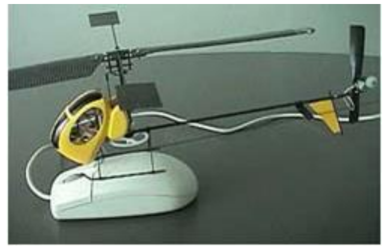






Locomotion (vii) – Heli, Jet, Mars, other...



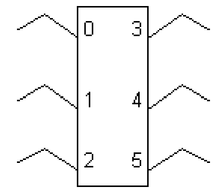


Other motion principles

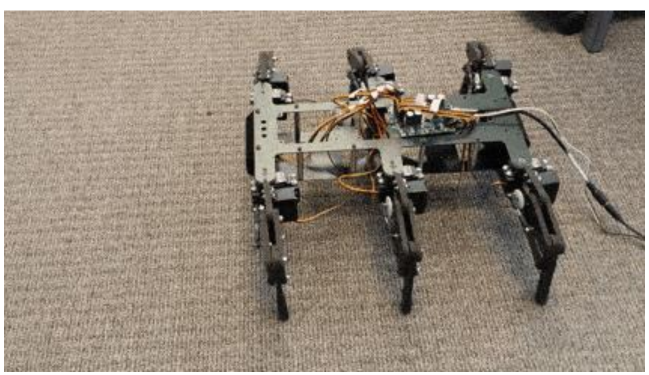
- Jet / Propellers (air/water)
  - Dynamic (ruder, plane wings - *pressure difference*)
  - Helicopter
  - Wings / directive fins / rudders (air/water)
- Rocket / Solar Wind / Ion Thrusters
- Wind...
- ...
- Keep a sharp eye –
  many, MANY other options !!!

Legged Locomotion Example: 6 legs

Forward Sequence:
•Lift legs 0, 2 and 4.
•Pull back legs 1, 3 and 5 to push robot forwards.
•Lower legs 0, 2 and 4.
•Lift legs 1, 3 and 5.
•Move legs 1, 3 and 5 forward to reset them.
•Pull back legs 0, 2 and 4 to push robot forwards.
•Lower legs 1, 3 and 5.
•Lift legs 0, 2 and 4.
•Move legs 0, 2 and 4 forward to reset them.
•Lower legs 0, 2 and 4.



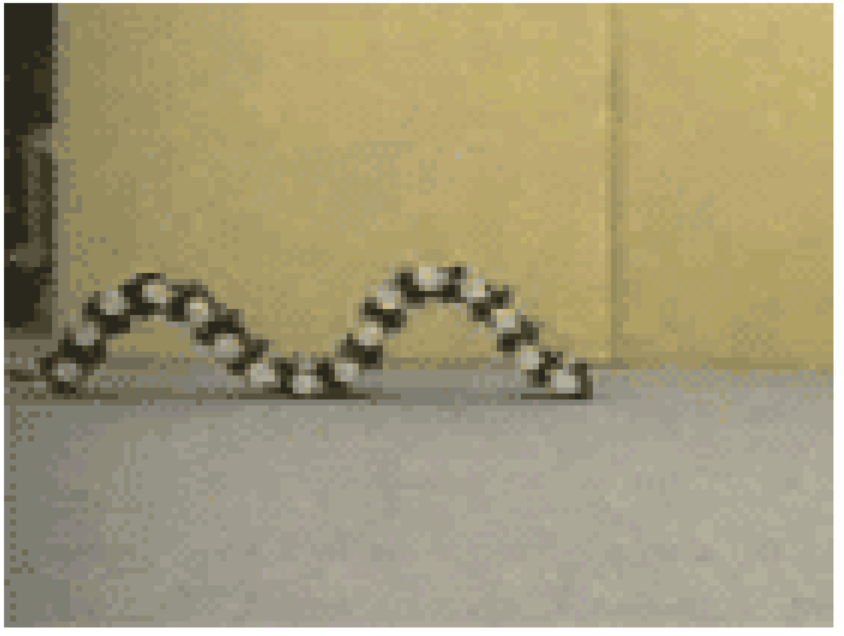
Turn Right Sequence:
•Lift legs 1, 3 and 5.
•Push back legs 0 and 2 to turn robot.
•Lower legs 1, 3 and 5.
•Lift legs 0 and 2.
•Move legs 0 and 2 forward to reset them.
•Lower legs 0 and 2.



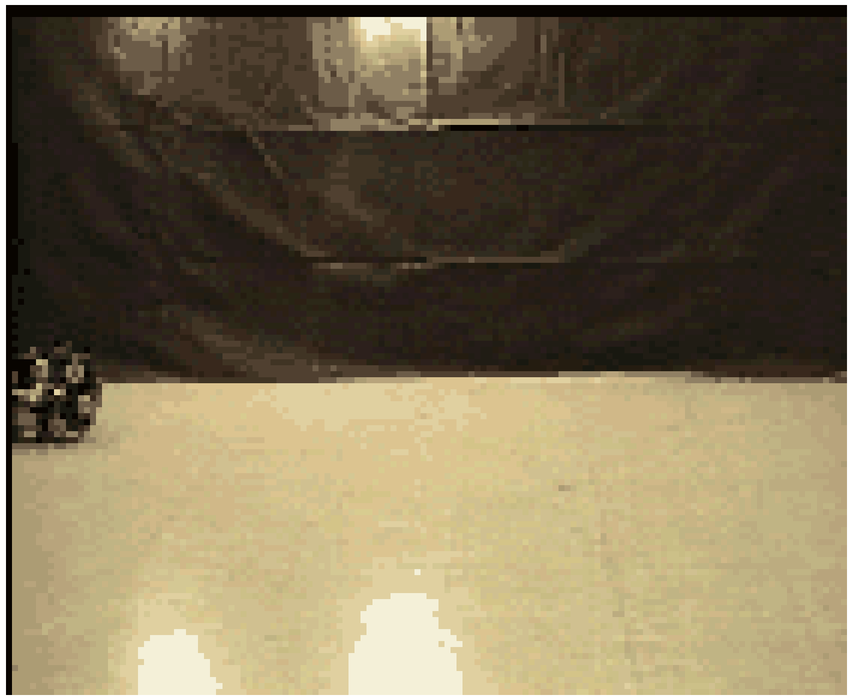
http://www.mcdonald.org.uk/andrew/archive/robot/comp/movement.htm
http://www.youtube.com/watch?v=ZhB4nFN_mRI
http://youtu.be/-Soq9qpK5Ac

https://www.phidgets.com/?view=articles&article=HexWalkerFirstSteps

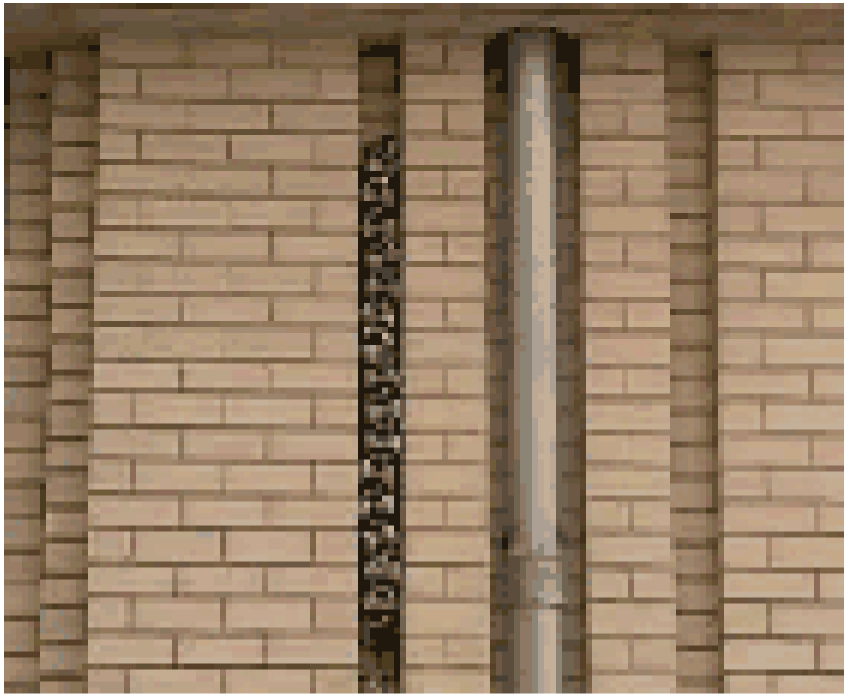
Locomotion animations



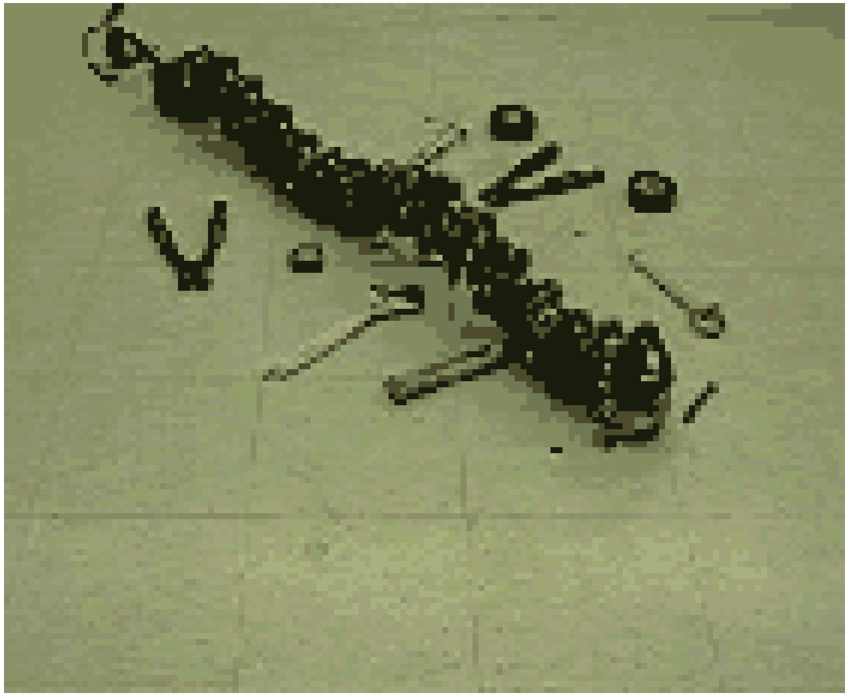
Locomotion animations (ii)



Locomotion animations (iii)



Locomotion animations (iv)



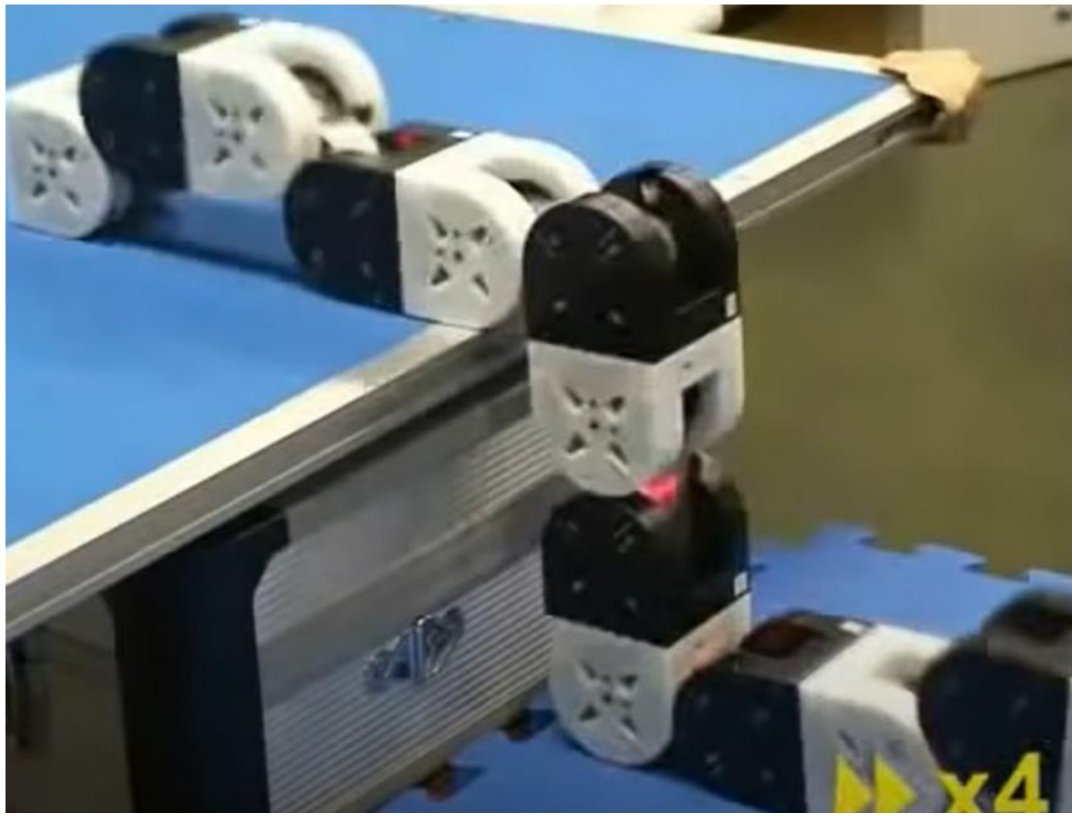
Locomotion animations (v) - caterpillar

http://www.youtube.com/watch?v=vfw4nduVU_E

Locomotion animations (vi) - rolling

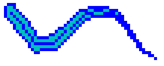
http://www.youtube.com/watch?v=f3rr1lcFe3Q&feature=related

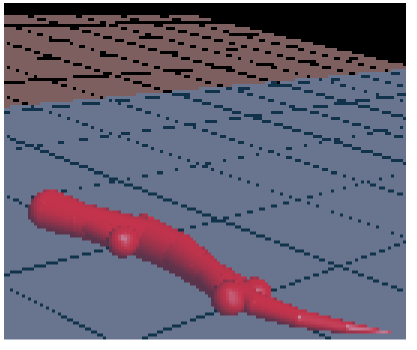
Snake Morphing Adapt Locomotion...



http://youtu.be/v6W-sEpJEqY

Locomotion animations – Biology inspired





Less common robots

Mtran Modular robot:
http://www.youtube.com/watch?v=4oSavAHf0dg
http://unit.aist.go.jp/is/frrg/dsysd/mtran3/mtran3.htm

Robotic Chair (reassembles itself):
http://youtu.be/t5pvZoZwzh0

LS3 - Legged Squad Support System ("dog"):
http://youtu.be/R7ezXBEBE6U
http://youtu.be/cNZPRsrwumQ

Ibot Robotic Wheel chair (climbs stairs):
http://youtu.be/O7otewMk9pc
https://youtu.be/9qYz2wsVKYE (more recent)

Origami Robot:
https://spectrum.ieee.org/origami-robot-folds-itself-up-does-cool-stuff-dissolves-into-nothing
https://youtu.be/f0CluQiwLRg
https://youtu.be/Awufipq9JnQ

Summary

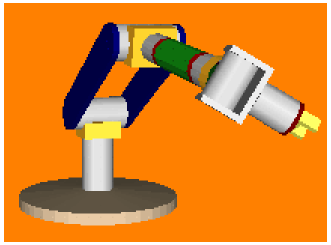
- Many locomotion types…
  - According to environment…
- Most common and relevant to this course: Differential drive
  - OmniWheels a also easy!

Kinematics & Dynamics

Degrees of Freedom

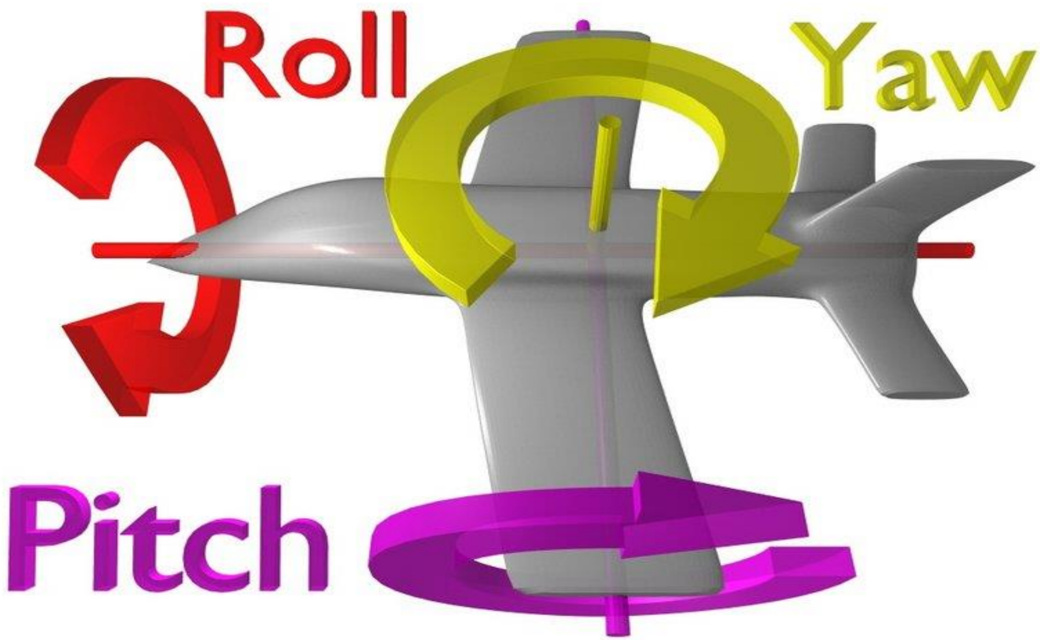
Mechanical engineering, aeronautical engineering, and robotics, the six DOFs of a rigid body have special names:
This is a typical robot arm which has 7 DOF (including surge at the end of the arm).

1. Moving up and down (heaving);
2. Moving left and right (swaying);
3. Moving forward and backward (surging);
4. Tilting up and down (pitching);
5. Turning left and right (yawing);
6. Tilting side to side (rolling).



Obs: Euler angles!!!

Degrees of Freedom (ii)



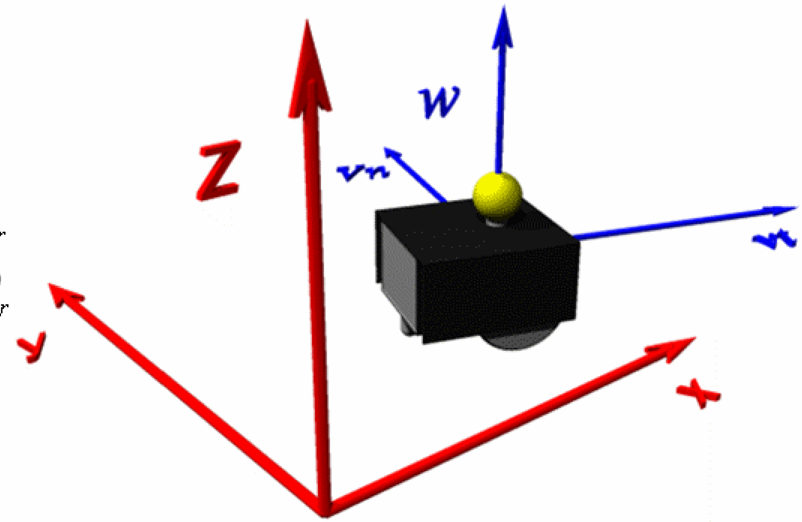
2D Robot dynamic model

- State Vector:

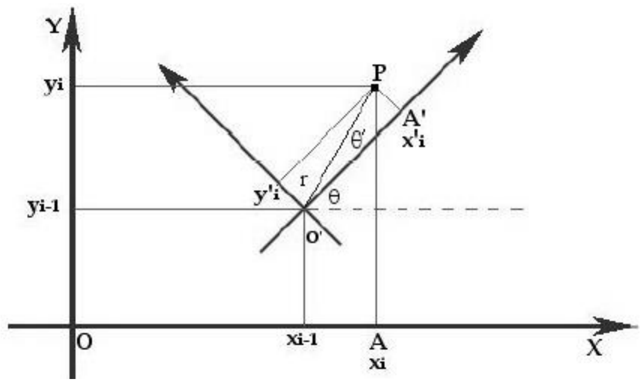
$$X_r^T = [x_r\, y_r\, \theta_r\, v_{rt}\, v_{rn}\, \omega_r]$$

- Dinamics:

$$\dot{x}_r = v_{rt}\cos\theta_r - v_{rn}\sin\theta_r$$
$$\dot{y}_r = v_{rt}\sin\theta_r + v_{rn}\cos\theta_r$$
$$\dot{\theta}_r = \omega_r$$
$$\dot{v}_{rt} = \alpha\left(v_{ref} - v_{rt}\right)$$
$$\dot{v}_{rn} = 0$$
$$\dot{\omega}_r = \gamma\left(\omega_{ref} - \omega_r\right)$$



2D movement



$$x_i = \overline{OA} = x_{i-1} + r*\cos(\theta+\theta') = x_{i-1} + r*\cos(\theta)*\cos(\theta) - r*\sin(\theta)*\sin(\theta) . \quad (1)$$

$$y_i = \overline{AP} = y_{i-1} + r*\sin(\theta+\theta') = y_{i-1} + r*\sin(\theta)*\cos(\theta') + r*\cos(\theta)*\sin(\theta') \quad (2)$$

$$x_i' = \overline{O'A'} = r*\cos(\theta') . \quad (3)$$

$$y_i' = \overline{A'P} = r*\sin(\theta') . \quad (4)$$

$$x_i = x_i'\cos(\theta) - y_i'\sin(\theta) + x_{i-1} . \quad (5)$$

$$y_i = x_i'\sin(\theta) + y_i'\cos(\theta) + y_{i-1} . \quad (6)$$

$$\theta_i = \theta_I' + \theta_{i-1} . \quad (7)$$

$$x_i = d_i*\cos(\theta_i) + x_{i-1} . \quad (8)$$

$$y_i = d_i*\sin(\theta_i) + y_{i-1} . \quad (9)$$

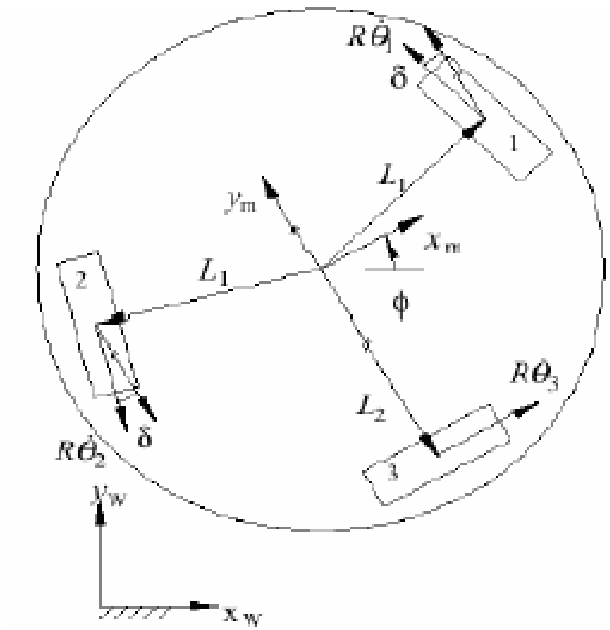
Simulation of Mobile Robots in Virtual Environments
Jesús Savage, Emmanuel Hernández, Gabriel Vázquez, Humberto Espinosa, Edna Márquez

Position: $x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$.

Speed: $\dfrac{dx(t)}{dt} = a_1 + 2a_2 t + 3a_3 t^2$.

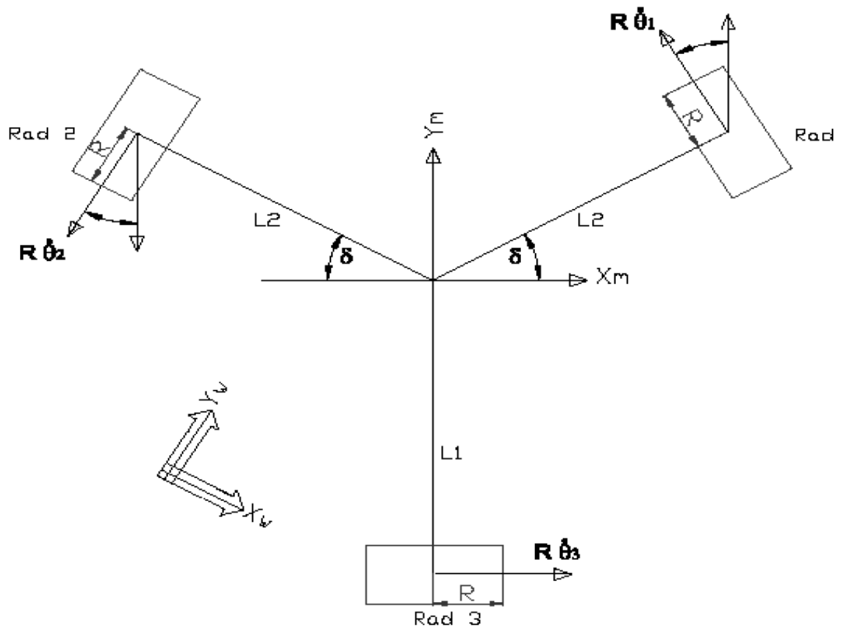
Acceleration: $\dfrac{dx(t)^2}{dt^2} = 2a_2 + 6a_3 t$.

Omniwheels



$$\begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} = \frac{1}{R} \begin{bmatrix} -\sin(\delta) & -\cos(\delta) & L_1 \\ -\sin(\delta) & \cos(\delta) & L_1 \\ 1 & 0 & L_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\phi} \end{Bmatrix}$$

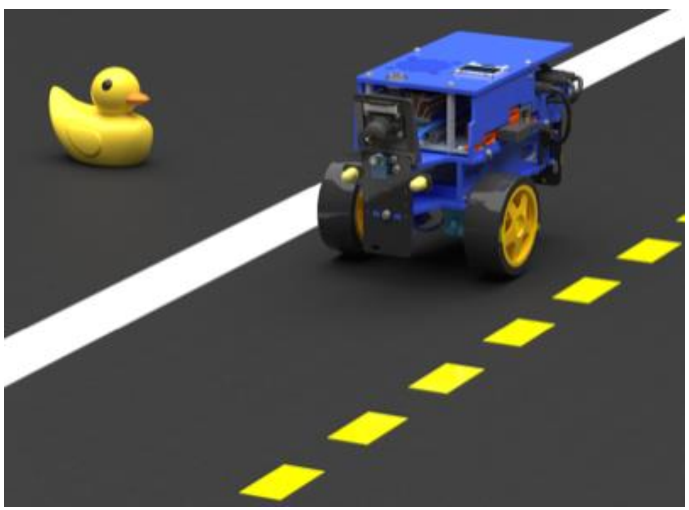
Omniwheels (ii)



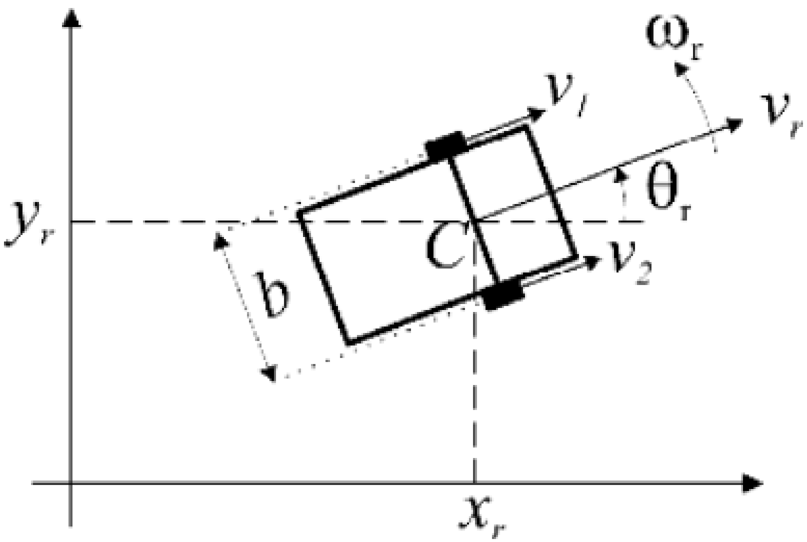
$$\left\{ \begin{array}{c} \dot{\Theta}_1 \\ \dot{\Theta}_2 \\ \dot{\Theta}_3 \end{array} \right\} = \frac{1}{R} \left[\begin{array}{ccc} -\sin(\delta+\Phi) & \cos(\delta+\Phi) & L_1 \\ -\sin(\delta-\Phi) & -\cos(\delta-\Phi) & L_1 \\ \cos(\Phi) & \sin(\Phi) & L_2 \end{array} \right] \left\{ \begin{array}{c} \dot{x_w} \\ \dot{y_w} \\ \dot{\Phi} \end{array} \right\}$$

Differential Drive Robot
("Wheelchair" drive)

- Math (kinematics) -



Differential ground robot



$$v_t = v_r$$

$$\begin{cases} v_t = \dfrac{(v_1 + v_2)}{2} \\ \omega_r = \dfrac{(v_1 - v_2)}{b} \end{cases}$$

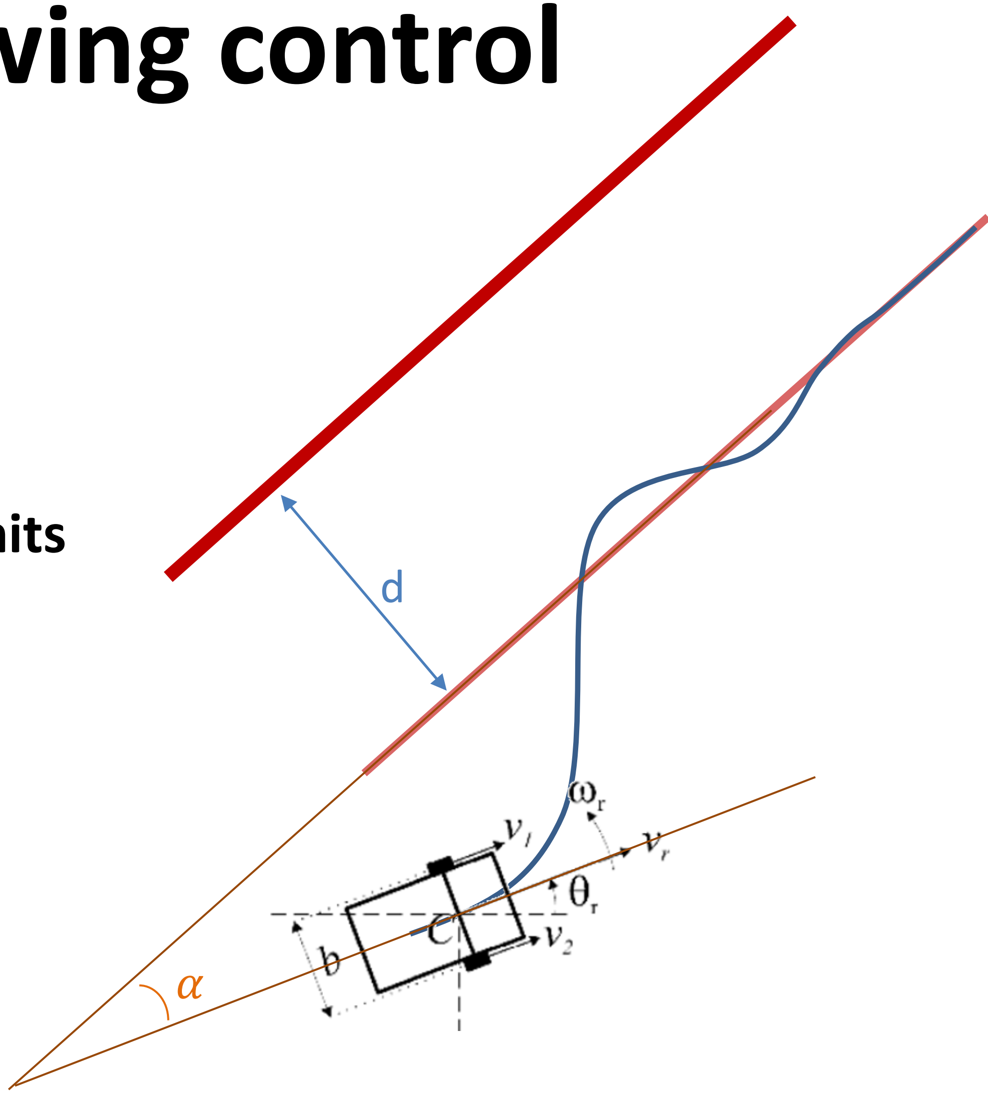
"Pose" $\Rightarrow (x, y, \theta)$

$$\begin{cases} x_r(t+1) = x_r(t) + v_r(t) \cdot \cos\theta_r(t) \cdot \Delta t \\ y_r(t+1) = y_r(t) + v_r(t) \cdot \sin\theta_r(t) \cdot \Delta t \\ \theta_r(t+1) = \theta_r(t) + \omega_r(t) \cdot \Delta t \end{cases}$$

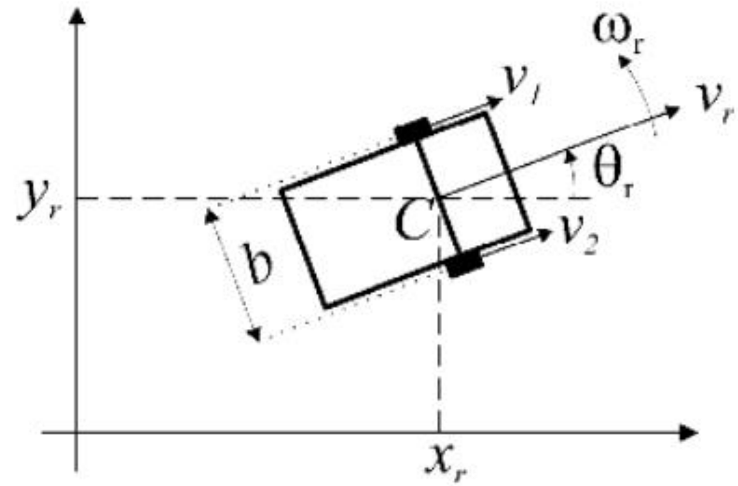
Wall following control

- **Always use floating point and SI units**
- **Many solutions…**
- **Example of a simple control:**
  - Have robot rotate proportionally to the difference of angle (robot and Wall to follow)
  - $\omega_r(t) = k \cdot \alpha(t-1)$

    (try k and –k)



$$\begin{cases} v = \frac{v_1+v_2}{2} \\ \omega = \frac{v_1-v_2}{b} \end{cases}$$



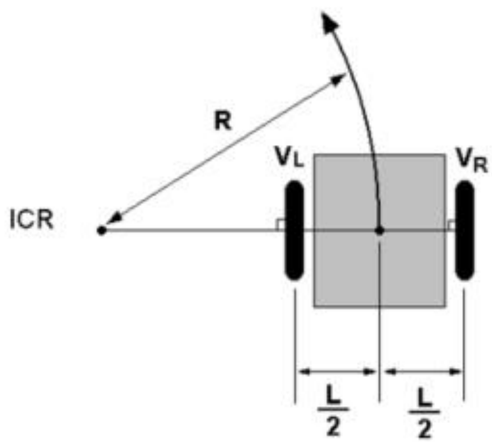
$$\begin{cases} \\ v_2 = v_1 - \omega b \end{cases}$$

$$\begin{cases} 2v = v_1 + v_1 - \omega b \\ \end{cases}$$

$$\begin{cases} v_1 = v + \omega b/2 \\ \end{cases}$$

$$\begin{cases} v_1 = v + \omega b/2 \\ v_2 = v - \omega b/2 \end{cases}$$

V, ω, v_L, v_R,



$$(V_R - V_L) / L = V_R / (R + \frac{L}{2})$$

$$R = \frac{L}{2} \frac{V_R + V_L}{V_R - V_L}$$

R : Radius of rotation

- Straight motion
  $R = \text{Infinity} \rightarrow V_R = V_L$

- Rotational motion
  $R = 0 \rightarrow V_R = -V_L$

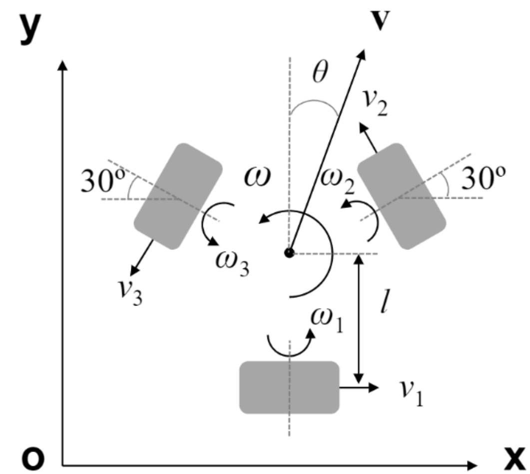
Velocity in the body coordinates :

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} v\sin\theta \\ v\cos\theta \end{bmatrix}$$

Velocity of the wheels :

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} r\omega_1 \\ r\omega_2 \\ r\omega_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & l \\ -\sin 30^\circ & \cos 30^\circ & l \\ -\sin 30^\circ & -\cos 30^\circ & l \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}$$

r : **Radius of the wheel**



Relationship between the world and body coordinates :

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} v_x^w \\ v_y^w \end{bmatrix}$$

ϕ : **Heading direction of the robot**

Velocity in the body coordinates

Velocity in the world coordinates

Velocity of the wheels :

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & l \\ -\sin 30^\circ & \cos 30^\circ & l \\ -\sin 30^\circ & -\cos 30^\circ & l \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x^w \\ v_y^w \\ \omega \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \cos\phi & l \\ -\sin(\phi+30^\circ) & \cos(\phi+30^\circ) & l \\ \sin(\phi-30^\circ) & -\cos(\phi-30^\circ) & l \end{bmatrix} \begin{bmatrix} v_x^w \\ v_y^w \\ \omega \end{bmatrix}$$

where

$$\phi = \int \omega\, dt$$

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