EXTREME CORRELATION IN CRYPTOCURRENCY MARKETS*

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ABSTRACT

In this paper, we study the contemporaneous tail dependence structure in a pairwise comparison of the ten largest cryptocurrencies, namely Bitcoin, Dash, Dogecoin, Ethereum, Litecoin, Monero, Namecoin, Novacoin, Peercoin, and Ripple. We apply multivariate extreme value theory and we estimate a bias-corrected extreme correlation coefficient. Our findings reveal clear patterns of significantly high bivariate dependency in the distribution tails of some of the most basic and widespread cryptocurrencies, primarily over various downside constraints. This means that extreme correlation is not related to cryptocurrency market volatility *per se*, but to the trend of the cryptocurrency market. Therefore, extreme correlation increases in bear markets, but not in bull markets for these pairs. Interestingly, there is also a significant number of pairs which exhibit a weak level of dependency in distribution tails.

Keywords: Bitcoin; cryptocurrencies; extremes; tail dependence; downside risk
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1. INTRODUCTION

It is generally acknowledged that cryptocurrencies behave fundamentally dissimilarly to traditional fiat currencies. Several studies reveal that cryptocurrency returns present non-Gaussian behaviour and heavier distribution tails (Chu et al., 2017; Osterrieder and Lorenz., 2017; Phillip et al., 2017; and Gkillas and Katsiampa, 2018), are prone to speculative bubbles (Cheah and Fry, 2015; Fry and Cheah, 2016; Balcombe and Fraser, 2017; Corbet et al., 2017) and violate the hypothesis of efficient markets (Urquhart, 2016; Bariviera, 2017; Nadarajah and Chu, 2017; Caporale et al., 2018). Such behavior could lead to high gains but could also result in the occurrence of extreme downside shocks and high losses increasing the likelihood of a potential market collapse in an integrated investment environment. Although market participants are always aware of such various downside constraints, when extreme adverse market movements occur, investors in cryptocurrency markets seem to be uninformed about the aversion to extreme risk. It also seems that they are ignorant of the extreme risk that they are exposed to since they are carried away by the current speculative frenzy in cryptocurrency markets.

On the other hand, notwithstanding the rapid growth of cryptocurrency markets, research on cryptocurrency dependencies is rather limited.[2] Some studies consider the relationship between cryptocurrencies and a variety of other financial assets, mostly for Bitcoin (Baur et al., 2018; Bouri et al., 2017a; Bouri et al., 2017b; Bouri et al., 2017c; Corbet et al., 2018; Dyhrberg, 2016; Dyhsberd 2016b). Balcilar et al. (2017) investigate the causal relation between trading volume and Bitcoin price returns and volatility in distribution tails. Likewise, among the few studies that examine the relationships among cryptocurrencies are those by Osterrieder et al. (2017) and Ciaian et al. (2018). The former observes that there is a weak level of dependency among the majority cryptocurrencies examined. They provide findings for a limited period of analysis (from June 2014 to September 2016) and a limited number of cryptocurrencies. The latter, involving a significantly higher number of cryptocurrencies, concludes that Bitcoin and Altcoin markets are interdependent both in the short run and the long run.[3] Ciaian et al. (2018) investigate the relationships among 6 major altcoins, 10 minor altcoins and two

[2] See Brandvold et al. (2015) for a historical overview on cryptocurrencies. See Corbet et al. (2018) for a systematic review of the empirical literature of cryptocurrencies.
[3] Altcoins refer to alternative cryptocurrencies due to the fact that it is an abbreviation of Bitcoin alternative.

altcoin price indices. Nevertheless, the investigation of the extreme dependencies of cryptocurrencies is greatly unexplored, particularly during the current extremely volatile period. Furthermore, it is worth mentioning that when dealing with cryptocurrencies, it is crucial to take into serious consideration the fact that they exhibit heavy distribution tails and extremely volatile behavior. This means that their dependency constitutes a complex function which can easily lead us to wrong conclusions unless their statistical properties are clearly specified.[4]

In this paper, we contribute to the literature in numerous ways. Firstly, we study the contemporaneous tail dependence structure in a pairwise comparison of the ten largest cryptocurrencies (in terms of their market capitalization), namely Bitcoin, Dash, Dogecoin, Ethereum, Litecoin, Monero, Namecoin, Novacoin, Peercoin, and Ripple, by applying multivariate extreme value theory. Extreme value theory provides the proper statistical tools to deal with extremes, and therefore, it is the appropriate approach to scrutinize effectively the tail dependence structure of return exceedances. In this way, we only consider the relevant information of extremes avoiding critical misleading results (Longin, 2005). Secondly, we estimate a bias-corrected extreme correlation coefficient, following the study of Gkillas and Longin (2018), in order to reduce the estimation bias. Thirdly, we extend the work of Gkillas and Katsiampa (2018) to a bivariate framework, providing findings on connectedness among extreme movements of cryptocurrency markets. Fourthly, we extend the study of Osterrieder et al. (2017) by including a much longer period of analysis and significantly greater number of cryptocurrencies.

In our study, we find clear patterns of significantly high bivariate dependency in the distribution tails of some of the most basic and widespread cryptocurrencies, primarily over various downside constraints. This means that extreme correlation is not related to cryptocurrency market volatility *per se*, but to the trend of the cryptocurrency market. Therefore, extreme correlation only increases in bear markets and not in bull markets for a significant number of pairs. However, there is also a non-negligible number of pairs which exhibit a weak level of dependency in extremes. Such findings are highly important for investors, regulators and supervisors. Especially, our

[4] Extreme value theory has been widely used to model data distributions departure from normality characterized by heavy tails. For several applications of extreme value theory in finance, see Embrecht et al., (1997), McNeil and Frey (2000), Kalyvas et al. (2004), Longin (2000), Tsai and Chen (2011), Ourir and Snoussi (2012), Ghorbel and Trabelsi (2014), Qin and Liu (2014).

implications are vital during periods of speculative frenzy, which was established due to high gains observed in Bitcoin where investors desire to get positions in other cryptocurrencies, as well.

This paper is organized as follows: Section 2 presents the modelling of extremes. Section 3 describes the data and data adjustments. Section 4 provides the empirical analysis. Section 5 concludes the paper discussing some economic implications.

2. METHODOLOGY

Initially, we fit a general Pareto distribution (GPD) for each marginal distribution of the return exceedances of the ten largest cryptocurrencies under consideration by using the peaks-over-threshold method. Secondly, we model the tail dependence structure via a bivariate threshold excess model by estimating a bias-corrected extreme correlation coefficient.

Let us consider as a general case a sequence of independent and identically distributed random variables $X = \{X_1, X_2, \dots, X_n\}$ with a continuous cumulative distribution function F_X, for $i = 1, 2, .., n$. Over a high threshold $u > 0$, the excess distribution $(X - u)$ is denoted by F_X^u and can be formed as follows:

$$F_X^u(x) = P(X - u \leq x | X > u) \;\; = \frac{F_X(u + x) - F_X(u)}{1 - F_X(u)}, \quad 0 \leq x \leq x_{F_X} - u \qquad (1)$$

where $x = X - u$ represents the exceedances and $x_{F_X} \leq \infty$ is the right endpoint of F_X. Balkema and De Haan (1974) and Pickands (1975) proved that for a large class of underlying distributions, the excess distribution F_X^u can be approximated by a GPD, $F_X^u \approx G_{\xi,\sigma}(x)$ as $u \to \infty$.[5]

Therefore, using the peaks-over-threshold method, we model the excess distribution $(X - u)$ via a GPD, defined as:

$$G_{\xi,\sigma}(x) = 1 - \zeta \left\{1 + \frac{\xi x}{\sigma}\right\}^{-1/\xi}, \qquad x > u \qquad (2)$$

where x represents the exceedances, ζ is the tail probability close to the empirical probability of exceedances, related to the number of exceedances over the threshold u, $\sigma > 0$ represents the scale parameter and $\xi \in \mathbb{R}$ is the tail index or the shape parameter.

[5] The GPD has also been further studied by Davison (1984), Castillo (1997, 2009), Chen et al. (2017), among others.

For $x \geq 0$ if $\xi \geq 0$, and $x \in [0, -\sigma/\xi]$, if $\xi < 0$. When $\xi > 0$, the $G_{\xi,\sigma}(x)$ corresponds to a heavy tail distribution (Fréchet type distribution). When $\xi = 0$, the $G_{\xi,\sigma}(x)$ corresponds to an exponentially declining tail distribution (thin-tailed distribution or Gumbel type distribution). When $\xi < 0$, the $G_{\xi,\sigma}(x)$ corresponds to a distribution with no tail (finite-distribution or Weibull type distribution).[6]

Then, following Longin and Solnik (2001), we model the dependence structure of (x_1, x_2) exceedances by a Gumbel-Hougaard copula,[7] via the logistic model.[8] This model contains both asymptotic independence and total dependence and is parsimonious since we only need the dependence parameter α to model the bivariate dependence structure of exceedances. The logistic model is defined by:

$$G_\alpha(x_1, x_2) = exp\left\{-\log\left[-1/\log G_1^{u_1}(x_1)\right]^{-1/a} - \log\left[-1/\log G_2^{u_2}(x_2)\right]^{-1/a}\right\}^a \quad (3)$$

where the extreme correlation coefficient ρ is equal to $(1 - \alpha^2)$. The special cases where α is equal to 1 and α is equal to 0 correspond to asymptotic independence, in which the ρ is equal to 0, and total dependence, in which ρ is equal to 1 (Tiago de Oliveira, 1973), respectively.

Furthermore, in each bivariate model, we apply the parametric bootstrap bias-corrected approach developed by Gkillas and Longin (2018) by simulating from a logistic type model following Stephenson (2003). Applying this procedure, we are able to reduce the estimation bias due to the limited number of return exceedances moving towards the distribution tails. The bias-corrected bootstrap coefficient for the extreme correlation ρ is defined as follows:

$$\rho = B^{-1} \sum_{j=1}^{B} \tilde{\rho}_j^b(\rho) \quad (4)$$

where B corresponds to the length of the bootstrap samples.

Finally, we provide empirical findings for an extended range of fixed thresholds as well as for optimal thresholds. Fixed thresholds are defined with various

[6] See Gomes and Guillou (2014) for an analytical overview of the literature on modelling univariate extremes.
[7] Taking into account the suggestions by Balakrishna and Lai (2009), we refer to the Gumpel copula as the Gumbel–Hougaard copula, on account of the fact that it first appeared in the works of Gumbel (1960, 1961), while a derivation of this has been provided by Hougaard (1986).
[8] See Ledford and Tawn (1996) for the estimation of the logistic function via the maximum likelihood method.

predetermined tail probability levels across the entire range of each statistical distribution tail. Optimal thresholds are computed as in Gkillas et al. (2017) via the bootstrap goodness-of-fit test of Villasenor-Alva and Gonzalez-Estrada (2009) for 999 bootstrap samples.[9] To this end, we apply an iterative k-step algorithm with a rolling threshold and we select the threshold that corresponds to the maximal p-value of the goodness-of-fit test (p_j), as follows:

$$u \mathrel{\widehat{=}} \max_{j=1,\ldots,k} \{p_1, \ldots, p_k\}, \qquad j \in \{1, \ldots, k\} \tag{5}$$

This test is an intersection-union goodness-of-fit test for the null hypothesis $H_0\colon F_X^u(x) \sim G_{\xi,\sigma}(x)$ defined by two sub-classes of GPD. A^+ stands for $H_0^+\colon F_X^u(x) \sim G_{\xi,\sigma}(x)$ with $\xi \geq 0$ and A^- stands for $H_0^-\colon F_X^u(x) \sim G_{\xi,\sigma}(x)$ with $\xi < 0$. Thus, $H_0\colon F \in (A^+ \cup A^-)$ is rejected whenever both hypotheses H_0^+ and H_0^- are rejected. Optimal thresholds are different at each distribution tail.[10]

3. DATA AND DATA ADJUSTMENTS

We analyse the tail dependence structure of the ten largest cryptocurrencies starting from the common oldest date available in the parenthesis to 21rd February 2018, in a pairwise comparison. More specifically, our dataset comprises daily closing prices for Bitcoin (28 April, 2013), Dash (14 February, 2014), Dogecoin (15 December, 2013), Ethereum (28 April, 2013), Litecoin (28 April, 2013), Monero (21 May, 2014), Namecoin (28 April, 2013), Novacoin (28 April, 2013), Peercoin (28 April, 2013), and Ripple (4 August, 2013). The closing prices are collected using the "*crypto*" package in R retrieved from the Coinmarketcap database (https://coinmarketcap.com). We end up with this sample so as to obtain as long a period as possible. In this paper, we study ninety (90) combinations of cryptocurrencies in a pairwise comparison: forty-five for left distribution tail i.e., negative return exceedances and forty-five for right distribution tail and positive return exceedances for several fixed and optimal threshold levels.

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[9] See Scarrott and Macdonald (2012) on alternative procedures regarding the threshold selection in extreme value theory.

[10] The selection of the threshold u is a critical factor in extreme value analysis. A low value of threshold u, can lead to a significant estimation bias by characterizing observations as exceedances not belonging to the distribution tails. Conversely, a high value of threshold u leads to inefficiency by reducing the estimation sample and increasing standard errors. In this paper, we check the stability of our estimates at optimal threshold levels, by considering two additional procedures of the computation of optimal thresholds, based on Jansen and de Vries (1991) and Danielsson et al. (2001).We still find the same dependency patterns at optimal threshold levels.

Furthermore, for each log-return series, we apply the data adjustment approach developed by Gallant et al. (1992) to de-trend the mean and variance. We also build on the approach of MacNeil and Frey (2000), taking heteroskedasticity reflecting volatility clustering into account via an autoregressive moving average model with exponential generalized autoregressive conditional heteroskedasticity errors (ARMA-t-EGARCH). Therefore, we limit the sample bias observed at serially-correlated and clustered data, thus achieving stationarity in the series.[11]

4. EMPIRICAL RESULTS

We present the estimation results of the bivariate tail dependence structure of the ten largest cryptocurrencies in Tables 1-5. These tables are colour gradation tables representing the bias-corrected extreme correlation coefficient ρ for 999 bootstrap iterations, for each tail probability level ζ as well as for optimal threshold levels. More specifically, Table 1 corresponds to tail probability ζ_1 equal to 0.35, Table 2 to tail probability ζ_2 equal to 0.25, Table 3 to tail probability ζ_3 equal to 0.15, Table 4 to tail probability ζ_4 equal to 0.05 and finally Table 5 corresponds to the optimal threshold levels. In each table, panel A represents left distribution tail and negative return exceedances, while panel B the right distribution tail and positive return exceedances. High values of the ρ are represented with a lighter colour and vice-versa. Therefore, these tables demonstrably depict the bivariate dependence patterns in the cryptocurrency markets and efficiently represent the evolution of the extreme correlation for different threshold levels across the entire range of the conditional distribution.

We use Table 5 as an example in order to describe our results. This table is highly important for our study in that it represents the bivariate dependence patterns at optimal threshold levels. As for panel A, the highest value of the extreme correlation coefficient ρ exists in the pair of Namecoin and Peercoin, in which ρ is equal to 0.8279, while higher values of ρ also exist among the pairs, whereby one of the cryptocurrencies is either Bitcoin or Litecoin. On the other hand, the lower value of the extreme correlation coefficient ρ exists in the pair of Ethereum and Monero, in which ρ is equal to 0.0544, while lower values of ρ also exist among the pairs, for Ethereum, Monero,

[11] See also Longin and Pagliardi (2016) and Gkillas and Longin (2018).

Namecoin or Novacoin. Some pairs (consisting of Dash) show a weak level of dependency. As for panel B, the highest value of ρ exists in the pair of Bitcoin and Litecoin, while higher values of ρ also exist among currencies where either Bitcoin or Ethereum comprises the pair. On the other hand, the lowest value of ρ exists in the pair of Peercoin and Ripple, while for 86% of the total pairs, ρ does not exceed 0.3. Regarding negative return exceedances, the corresponding percentage is equal to 40%. This means that a major cryptocurrency, such as Bitcoin, collapses alongside other major cryptocurrencies.

Furthermore, in Table 6, we report some summarized descriptive statistics for the estimates of Tables 1-5. We keep the same representation form as in previous tables; therefore, panel A is for negative return exceedances and panel B is for positive ones. In this table, we report the maximum (Max), the minimum ($Min.$), the frequency ($f\%$) and the number of observations ($Obs.$) for the extreme correlation coefficient ρ in some specific subintervals. We report empirical results for each tail probability level ζ as well as for optimal threshold levels across the asymptotic independence, in which ρ is equal to 0, whilst total dependence is represented by ρ being equal to 1. We observe that a great number of pairs corresponding to higher values of the extreme correlation coefficient ρ declines moving towards the distribution tails. Conversely, the number of pairs corresponding to lower values of ρ increases as we move towards the distribution tails. However, there is a significant almost constant number of pairs which correspond to an extreme correlation $\rho \in [0.6, 0.8)$ across the left tails and negative return exceedances across all pairs. We do not observe the same patterns in right tails and positive return exceedances.

More specifically, for panel A, the number of pairs is equal to 56% of the total number of pairs for the subinterval, where $\rho \in [0.4, 0.6)$ and is equal to 42% for the subinterval of $\rho \in [0.6, 0.8)$. For these percentages, the tail probability ζ is equal to 0.35. The corresponding percentages, for the tail probability ζ equal to 0.05, are 27% and 20% for $\rho \in [0.4, 0.6)$ and $\rho \in [0.6, 0.8)$, respectively. As for panel B, the number of pairs is equal to 58% for the subinterval, where $\rho \in [0.4, 0.6)$ and is equal to 22% for the subinterval in which $\rho \in [0.6, 0.8)$ for ζ equal to 0.35. The corresponding percentages, for the tail probability ζ equal to 0.05, are 13% and 2% for $\rho \in [0.4, 0.6)$ and $\rho \in [0.6, 0.8)$, respectively. At optimal threshold levels, for panel A, the number of pairs is equal to 40% of the total number of pairs when $\rho \in [0, 0.2)$, whilst it is equal to 22% for $\rho \in [0.6, 0.8)$ and 2% for $\rho \in [0.8, 1)$. In case of panel B, the number of pairs is equal to 44% for $\rho \in [0, 0.2)$, equal to 2% for $\rho \in [0.6, 0.8)$ and equal to 0% for $\rho \in [0.8, 1)$.

Overall, our findings could be encapsulated as follows: (i) We observe that the level of dependency declines as we move towards the distribution tails. However, the level of dependency declines faster at the right tails than the left tails. (ii) There is a significant number of pairs which exhibit a strong level of dependency at optimal threshold levels, especially during various downside constraints. This means that extreme correlation is not related to cryptocurrency market volatility *per se*, and hence, can be related to the trend of the cryptocurrency market. Therefore, extreme correlation only increases in bear markets and not in bull markets for a significant number of pairs, especially for the pairs which consist of some major cryptocurrencies. (iii) There is a significant number of pairs which exhibit a weak level of dependency at optimal threshold levels. In fiat currencies, low values of the extreme correlation coefficient ρ suggest that there are significant diversification benefits on market breakdowns. However, the fact that cryptocurrencies present extremely volatile behaviour (see Blau, 2018, Chu et al., 2017 and Katsiampa, 2016) and quite heavier distributions implies unusually high values of tail risk. Ibragimov and Prokhorov (2016) mention that in such cases, diversification increases portfolio riskiness in terms of tail risk. Hence, investors in cryptocurrency markets should be utterly attentive and precautious, considering the immensely high risk they are exposed to (Gkillas and Katsiampa, 2018).

A detailed representation of our results is also provided in Figures 1-5. These capture the asymmetry between negative and positive return exceedances presenting the difference between the dependence structure of left and right distribution tail for each of the Tables 1-5. A bar below the horizontal axis in each figure displays that the extreme correlation of negative return exceedances (ρ^-) is greater than the correlation of positive exceedances (ρ^+). On the other hand, a bar above the horizontal axis means that the extreme correlation of negative return exceedances is lower than the one of positive return exceedances. As shown by the figures for the majority of cryptocurrency pairs, the extreme correlation between negative return exceedances is always greater than the extreme correlation between positive return exceedances. Furthermore, this asymmetry tends to increase moving towards the distribution tails. In particular, in Figure 1, which corresponds to fixed threshold levels for tail probability equal to 0.35, we show that the greatest negative difference is highlighted in the pair of Bitcoin and Litecoin, while the greatest positive difference exists between Dogecoin and Peercoin. On the other hand, moving towards the distribution tails at fixed threshold levels for tail probability equal to 0.05, in Figure 4, we observe that the greatest negative difference exists between the pair Bitcoin and Monero, while the greatest positive difference exists between Ethereum and Novacoin. Considering Figure 5, which corresponds to the optimal threshold levels, the greatest negative difference exists between the pair of Dogecoin and Bitcoin, while the greatest positive difference is between Bitcoin and Peercoin.

In most of our findings, the difference between the dependence structure of left and right distribution tail is statistically different from zero at 5% confidence levels (in all figures), as indicated by a Wald test we conducted.

5. CONCLUSIONS

In this paper, we employ multivariate extreme value theory to investigate the tail dependence structure of the returns of the ten largest cryptocurrencies, namely Bitcoin, Dash, Dogecoin, Ethereum, Litecoin, Monero, Namecoin, Novacoin, Peercoin, and Ripple, in a pairwise comparison. To this end, we estimate an extreme correlation coefficient studying ninety (90) combinations of cryptocurrencies -forty-five (45) for each distribution tail- at several fixed thresholds as well as for optimal threshold levels. Our findings reveal clear patterns of significantly high bivariate dependency in the distribution tails of some of the most basic and widespread cryptocurrencies during extremely volatile periods, primarily over various downside constraints. This means that extreme correlation is not related to cryptocurrency market volatility *per se*, but to the trend of the cryptocurrency market. Thus, extreme correlation sharply increases in bear markets, yet not in bull ones for these pairs. However, there is a non-negligible number of pairs which exhibit a weak level of dependency in distribution tails.

Our analysis is of utmost importance for both investors and policy-makers. On the one hand, a cryptocurrency portfolio comprising the most widespread cryptocurrencies involves a high level of dependency, which in turn means that investors should be particularly cautious in avoiding positioning in more than one of these cryptocurrencies. In a possible collapse, these major cryptocurrencies altogether represent 85% of the total cryptocurrency capitalisation at present. Interestingly, a portfolio consisting of cryptocurrencies which exhibit a weak level of dependency does not necessarily mean diversification benefits. Cryptocurrencies present rather heavier distribution tails, implying unusually high values of tail risk. Following Ibragimov and Prokhorov (2016), we indicate that in those cases, diversification increases portfolio riskiness in terms of tail riskiness.

Finally, policy-makers and regulators should also be particularly cautious in a market which hit an all-time high of more than $800 billion in January 2018. In a dramatic panic selling (as the recent painful experience of a huge cryptocurrency sell-off), this high market capitalization, which is expected to reach a total value of $1 trillion per year, could lead to uncontrolled consequences and bankruptcies.

REFERENCES

Balakrishna, N., Lai, C.D., 2009. Continuous bivariate distributions, Continuous Bivariate Distributions. Springer New York, New York, NY. doi:10.1007/b101765

Balcilar, M., Bouri, E., Gupta, R., Roubaud, D., 2017. Can volume predict Bitcoin returns and volatility? A quantiles-based approach. Econ. Model. 64, 74–81. doi:10.1016/J.ECONMOD.2017.03.019

Balcombe, K., Fraser, I., 2017. Do bubbles have an explosive signature in markov switching models? Econ. Model. 66, 81–100. doi:10.1016/j.econmod.2017.06.001

Balkema, A.A., de Haan, L., 1974. Residual Life Time at Great Age. Ann. Probab. doi:10.1214/aop/1176996548

Bariviera, A.F., Basgall, M.J., Hasperué, W., Naiouf, M., 2017. Some stylized facts of the Bitcoin market. Phys. A Stat. Mech. its Appl. 484, 82–90. doi:10.1016/j.physa.2017.04.159

Baur, D.G., Hong, K.H., Lee, A.D., 2018. Bitcoin: Medium of exchange or speculative assets? J. Int. Financ. Mark. Institutions Money 54, 177–189. doi:10.1016/j.intfin.2017.12.004

Blau, B.M., 2018. Price dynamics and speculative trading in Bitcoin. Res. Int. Bus. Financ. 43, 15–21. doi:10.1016/j.ribaf.2017.07.183

Bouri, E., Gupta, R., Tiwari, A.K., Roubaud, D., 2017. Does Bitcoin hedge global uncertainty? Evidence from wavelet-based quantile-in-quantile regressions. Financ. Res. Lett. 23, 87–95. doi:10.1016/j.frl.2017.02.009

Bouri, E., Jalkh, N., Molnár, P., Roubaud, D., 2017. Bitcoin for energy commodities before and after the December 2013 crash: diversifier, hedge or safe haven? Appl. Econ. 49, 5063–5073. doi:10.1080/00036846.2017.1299102

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., Hagfors, L.I., 2017. On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? Financ. Res. Lett. 20, 192–198. doi:10.1016/j.frl.2016.09.025

Brandvold, M., Molnár, P., Vagstad, K., Andreas Valstad, O.C., 2015. Price discovery on Bitcoin exchanges. J. Int. Financ. Mark. Institutions Money 36, 18–35. doi:10.1016/j.intfin.2015.02.010

Caporale, G.M., Gil-Alana, L., Plastun, A., 2018. Persistence in the cryptocurrency market. Res. Int. Bus. Financ. 46, 141–148. doi:10.1016/j.ribaf.2018.01.002

Castillo, E., Hadi, A.S., 1997. Fitting the Generalized Pareto Distribution to Data. J. Am. Stat. Assoc. 92, 1609–1620. doi:10.1080/01621459.1997.10473683

Cheah, E.T., Fry, J., 2015. Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. Econ. Lett. 130, 32–36. doi:10.1016/j.econlet.2015.02.029

Chen, H., Cheng, W., Zhao, J., Zhao, X., 2017. Parameter estimation for generalized Pareto distribution by generalized probability weighted moment-equations. Commun. Stat. Simul. Comput. 46, 7761–7776. doi:10.1080/03610918.2016.1249884

Chu, J., Chan, S., Nadarajah, S., Osterrieder, J., 2017. GARCH Modelling of Cryptocurrencies. J. Risk Financ. Manag. 10, 17. doi:10.3390/jrfm10040017

Ciaian, P., Rajcaniova, M., Kancs, D., 2018. Virtual relationships: Short- and long-run evidence from BitCoin and altcoin markets. J. Int. Financ. Mark. Institutions Money 52, 173–195. doi:10.1016/j.intfin.2017.11.001

Corbet, S., Lucey, B.M., Urquhart, A., Yarovaya, L., 2018. Cryptocurrencies as a Financial Asset: A Systematic Analysis. SSRN Electron. J. doi:10.2139/ssrn.3143122

Corbet, S., Meegan, A., Larkin, C., Lucey, B., Yarovaya, L., 2018. Exploring the dynamic relationships between cryptocurrencies and other financial assets. Econ. Lett. 165, 28–34. doi:10.1016/j.econlet.2018.01.004

Danielsson, J., De Haan, L., Peng, L., De Vries, C.G., 2001. Using a Bootstrap Method to Choose the Sample Fraction in Tail Index Estimation. J. Multivar. Anal. 76, 226–248. doi:10.1006/jmva.2000.1903

Davison, A.C., 1984. Modelling Excesses over High Thresholds, with an Application, in: Statistical Extremes and Applications. Springer Netherlands, Dordrecht, pp. 461–482. doi:10.1007/978-94-017-3069-3_34

del Castillo, J., Daoudi, J., 2009. Estimation of the generalized Pareto distribution. Stat. Probab. Lett. 79, 684–688. doi:10.1016/j.spl.2008.10.021

Dyhrberg, A.H., 2016. Bitcoin, gold and the dollar - A GARCH volatility analysis. Financ. Res. Lett. 16, 85–92. doi:10.1016/j.frl.2015.10.008

Embrechts, P., Klüppelberg, C., Mikosch, T., 1997. Modelling Extremal Events : for Insurance and Finance. Springer Berlin Heidelberg.

Fry, J., Cheah, E.T., 2016. Negative bubbles and shocks in cryptocurrency markets. Int. Rev. Financ. Anal. 47, 343–352. doi:10.1016/j.irfa.2016.02.008

Gallant, A.R., Rossi, P.E., Tauchen, G., 1992. Stock Prices and Volume. Rev. Financ. Stud. 5, 199–242. doi:10.1093/rfs/5.2.199

Ghorbel, A., Trabelsi, A., 2014. Energy portfolio risk management using time-varying extreme value copula methods. Econ. Model. 38, 470–485. doi:10.1016/J.ECONMOD.2013.12.023

Gkillas, K., Katsiampa, P., 2018. An application of extreme value theory to cryptocurrencies. Econ. Lett. 164, 109–111. doi:10.1016/j.econlet.2018.01.020

Gkillas, K., Longin, F., 2018. Is Bitcoin the New Digital Gold? Evidence From Extreme Price Movements in Financial Markets. SSRN Electron. J. doi:10.2139/ssrn.3245571

Gkillas, K., Longin, F., 2018. Financial market activity under capital controls: lessons from extreme events. Econ. Lett. 171, 10–13. doi:10.1016/j.econlet.2018.07.004

Gkillas, K., Longin, F.M., Tsagkanos, A., 2017. Asymmetric Exceedance-Time Model: An Optimal Threshold Approach Based on Extreme Value Theory. SSRN Electron. J. doi:10.2139/ssrn.3016145

Gomes, M.I., Guillou, A., 2015. Extreme Value Theory and Statistics of Univariate Extremes: A Review. Int. Stat. Rev. 83, 263–292. doi:10.1111/insr.12058

Gumbel, E.J., 1961. Bivariate Logistic Distributions. J. Am. Stat. Assoc. 56, 335–349. doi:10.1080/01621459.1961.10482117

Gumbel, E.J., 1960. Bivariate Exponential Distributions. J. Am. Stat. Assoc. 55, 698–707. doi:10.1080/01621459.1960.10483368

Hougaard, P., 1986. A class of multivanate failure time distributions. Biometrika 73, 671–678. doi:10.1093/biomet/73.3.671

Ibragimov, R., Prokhorov, A., 2016. Heavy tails and copulas: Limits of diversification revisited. Econ. Lett. 149, 102–107. doi:10.1016/j.econlet.2016.10.024

Jansen, D.W., de Vries, C.G., 1991. On the Frequency of Large Stock Returns: Putting Booms and Busts into Perspective. Rev. Econ. Stat. 73, 18. doi:10.2307/2109682

Kalyvas, L., Siriopoulos, C., Dritsakis, N., 2004. Reaching extreme events with conditional and unconditional models. WSEAS Trans. Bus. Econ. 156–161.

Katsiampa, P., 2017. Volatility estimation for Bitcoin: A comparison of GARCH models. Econ. Lett. 158, 3–6. doi:10.1016/j.econlet.2017.06.023

Ledford, A., Tawn, J.A., 1996. Statistics for near independence in multivariate extreme values. Biometrika 83, 169–187. doi:10.1093/biomet/83.1.169

Longin, F., 2005. The choice of the distribution of asset returns: How extreme value theory can help?, in: Journal of Banking and Finance. North-Holland, pp. 1017–1035. doi:https://doi.org/10.1016/j.jbankfin.2004.08.011

Longin, F.M., 2000. From value at risk to stress testing: The extreme value approach. J. Bank. Financ. 24, 1097–1130. doi:10.1016/S0378-4266(99)00077-1

Longin, F., Solnik, B., 2001. Extreme Correlation of International Equity Markets. J. Finance 56, 649–676. doi:10.1111/0022-1082.00340

McNeil, A.J., Frey, R., 2000. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. J. Empir. Financ. 7, 271–300. doi:10.1016/S0927-5398(00)00012-8

Nadarajah, S., Chu, J., 2017. On the inefficiency of Bitcoin. Econ. Lett. 150, 6–9. doi:10.1016/J.ECONLET.2016.10.033

Osterr, J., Lorenz, J., 2017. Bitcoin and cryptocurrencies—not for the faint-hearted. Int. Financ. Bank. 4, 56–94. doi:10.5296/ifb.v4i1.10451

Osterrieder, J., Lorenz, J., 2017. A statistical risk assessment of Bitcoin and its extreme tail behavior. Ann. Financ. Econ. 12, 1750003. doi:10.1142/S2010495217500038

Ourir, A., Snoussi, W., 2012. Markets liquidity risk under extremal dependence: Analysis with VaRs methods. Econ. Model. 29, 1830–1836. doi:10.1016/J.ECONMOD.2012.05.036

Phillip, A., Chan, J., Peiris, S., 2018. A new look at Cryptocurrencies. Econ. Lett. 163, 6–9. doi:10.1016/j.econlet.2017.11.020

Pickands, J., 1975. Statistical Inference Using Extreme Order Statistics. Ann. Stat. 3, 119–131. doi:10.1214/aos/1176343003

Qin, X., Liu, L., 2014. Extremes, return level and identification of currency crises. Econ. Model. 37, 439–450. doi:10.1016/J.ECONMOD.2013.11.035

Scarrott, C., Macdonald, A., 2012. A review of extreme value threshold estimation and uncertainty quantification. Revstat - Stat. J. 10, 33–60.

Stephenson, A., 2003. Simulating Multivariate Extreme Value Distributions of Logistic Type. Extremes 6, 49–59. doi:10.1023/A:1026277229992

Tiago de Oliveira, 1973. Statistical extremes-A Survey. Center of Applied Mathematics, Lisbon.

Tsai, M.-S., Chen, L.-C., 2011. The calculation of capital requirement using Extreme Value Theory. Econ. Model. 28, 390–395. doi:10.1016/J.ECONMOD.2010.08.010

Urquhart, A., 2017. Price clustering in Bitcoin. Econ. Lett. 159, 145–148. doi:10.1016/j.econlet.2017.07.035

Urquhart, A., 2016. The Inefficiency of Bitcoin. Econ. Lett. 150, 1–7. doi:10.1016/j.econlet.2016.09.019

Villaseñor-Alva, J.A., González-Estrada, E., 2009. A bootstrap goodness of fit test for the generalized Pareto distribution. Comput. Stat. {&} Data Anal. 53, 3835–3841. doi:10.1016/j.csda.2009.04.001

TABLE 1. Bivariate dependency scores in cryptocurrency markets at fixed threshold levels for tail probability equal to 0.35

Panel A: Negative return exceedances

	Bitcoin	Dash	Dogecoin	Ethereum	Litecoin	Monero	Namecoin	Novacoin	Peercoin	Ripple
Ripple	0.6161	0.4153	0.4088	0.4276	0.6415	0.4182	0.6182	0.5543	0.6169	1
Peercoin	0.7597	0.4132	0.4139	0.4465	0.7589	0.4359	0.8158	0.7371	1	
Novacoin	0.7186	0.4111	0.4156	0.4255	0.7191	0.4371	0.7353	1		
Namecoin	0.7885	0.4347	0.4135	0.4281	0.7971	0.4105	1			
Monero	0.6404	0.6283	0.5817	0.4122	0.6089	1				
Litecoin	0.8227	0.4255	0.4325	0.4045	1					
Ethereum	0.5594	0.6087	0.5442	1						
Dogecoin	0.7187	0.4324	1							
Dash	0.6357	1								
Bitcoin	1									

ρ: 1.00, 0.75, 0.50, 0.25, 0.00

Panel B: Positive return exceedances

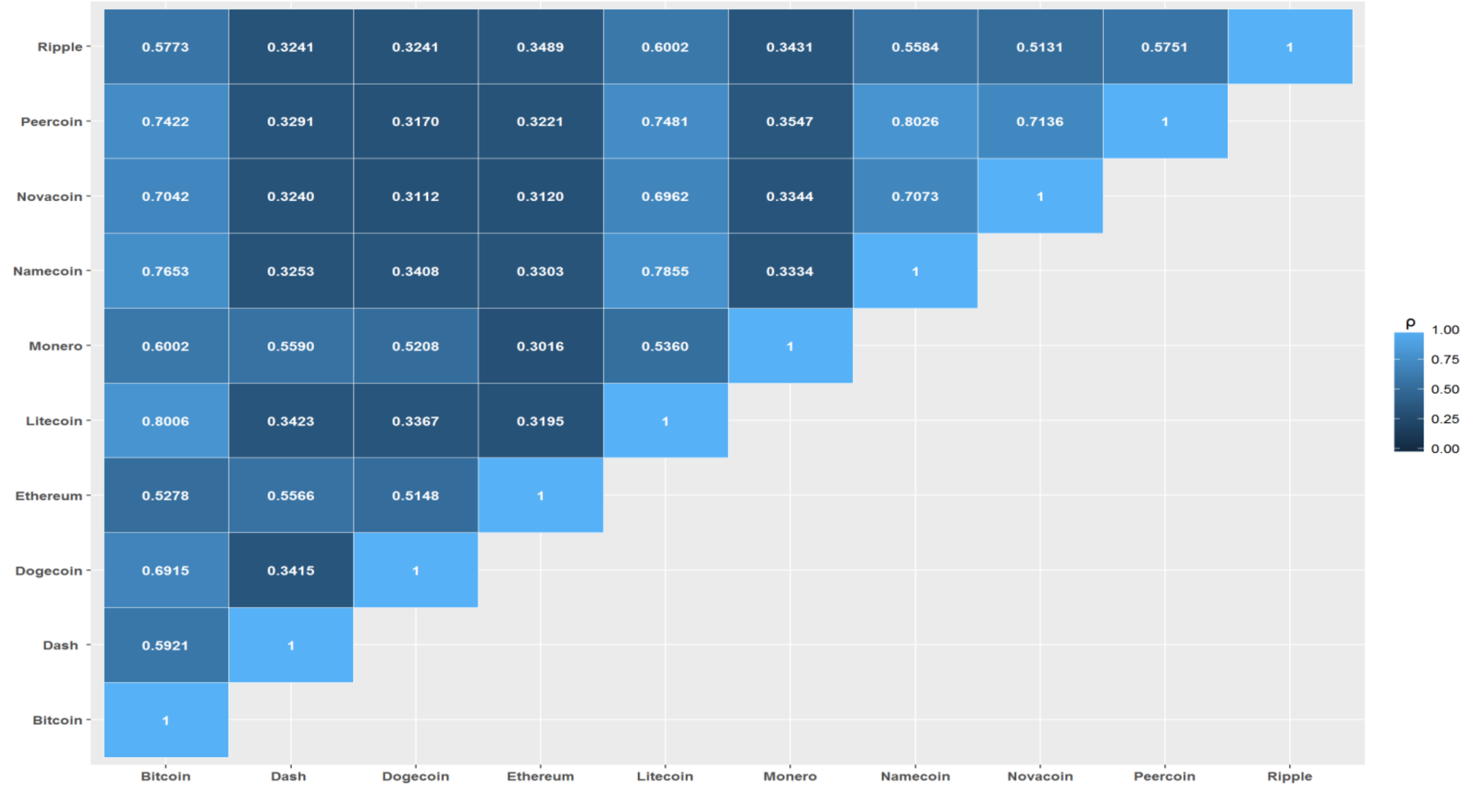
	Bitcoin	Dash	Dogecoin	Ethereum	Litecoin	Monero	Namecoin	Novacoin	Peercoin	Ripple
Ripple	0.5691	0.4007	0.4433	0.4121	0.5777	0.3979	0.5775	0.5371	0.5804	1
Peercoin	0.6728	0.4256	0.3954	0.3875	0.6901	0.4094	0.8016	0.7108	1	
Novacoin	0.6743	0.3980	0.3979	0.3885	0.6708	0.4164	0.7338	1		
Namecoin	0.7157	0.4296	0.4148	0.4030	0.7156	0.4153	1			
Monero	0.5651	0.5737	0.5345	0.3951	0.5482	1				
Litecoin	0.7300	0.4205	0.4241	0.3953	1					
Ethereum	0.5338	0.5818	0.5432	1						
Dogecoin	0.6524	0.4284	1							
Dash	0.5721	1								
Bitcoin	1									

ρ: 1.00, 0.75, 0.50, 0.25, 0.00

Note. This table is a colour gradation table representing the bias-corrected extreme correlation coefficient ρ for 999 bootstrap samples, for the tail probability level ζ_1 equal to 0.35. Panel A represents left distribution tail and negative return exceedances. Panel B represents the right tails and positive return exceedances. Ninety (90) combinations of cryptocurrencies are represented: forty-five combinations for each distribution tail; considering daily closing prices for the ten largest cryptocurrencies in a pairwise comparison. The cryptocurrencies under consideration are: Bitcoin, Dash, Dogecoin, Ethereum, Litecoin, Monero, Namecoin, Novacoin, Peercoin and Ripple. High values of the ρ are represented with a lighter colour. Low values of ρ are represented with a darker colour.

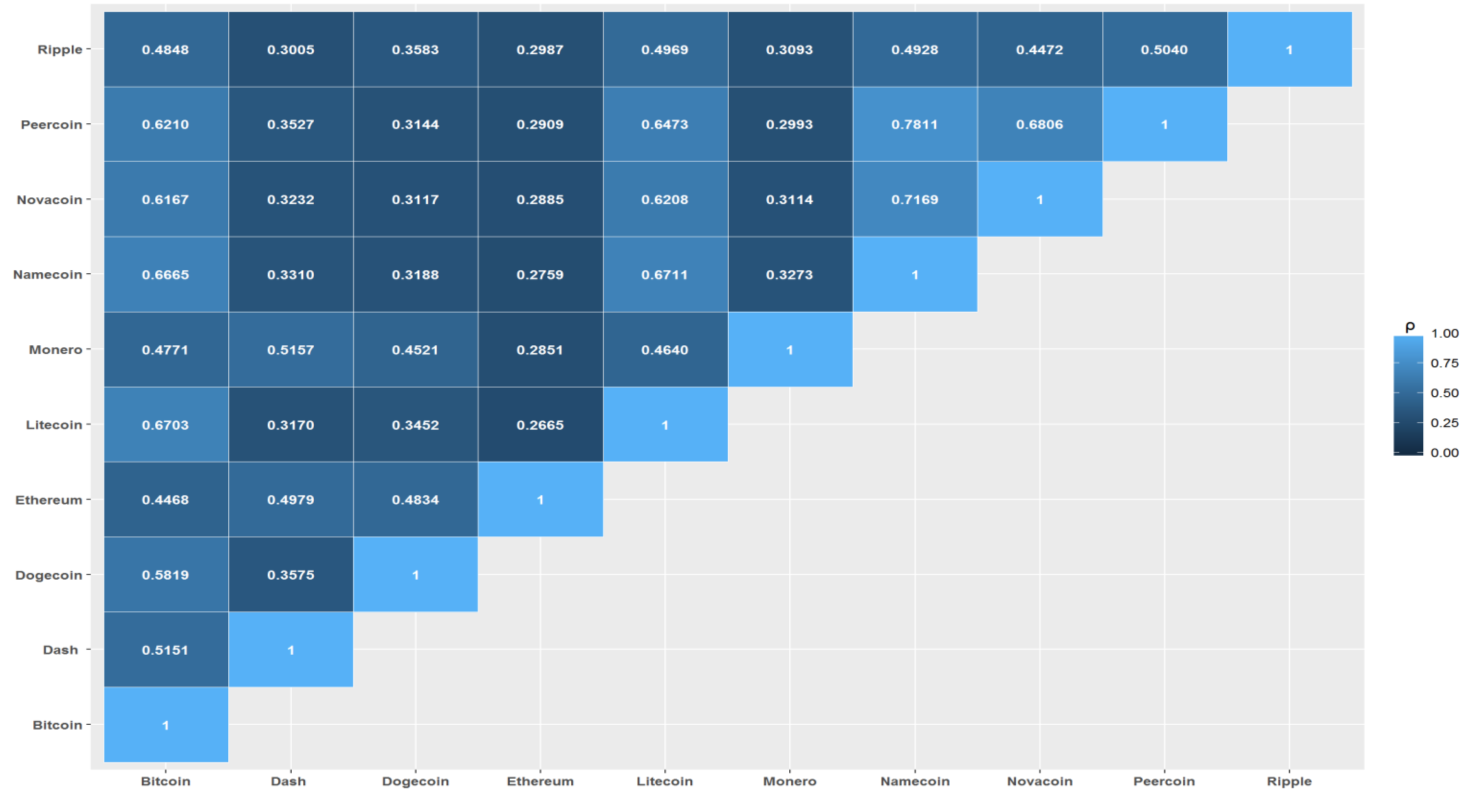
TABLE 2. Bivariate dependency scores in cryptocurrency markets at fixed threshold levels for tail probability equal to 0.25

Panel A: Negative return exceedances



	Bitcoin	Dash	Dogecoin	Ethereum	Litecoin	Monero	Namecoin	Novacoin	Peercoin	Ripple
Ripple	0.5773	0.3241	0.3241	0.3489	0.6002	0.3431	0.5584	0.5131	0.5751	1
Peercoin	0.7422	0.3291	0.3170	0.3221	0.7481	0.3547	0.8026	0.7136	1	
Novacoin	0.7042	0.3240	0.3112	0.3120	0.6962	0.3344	0.7073	1		
Namecoin	0.7653	0.3253	0.3408	0.3303	0.7855	0.3334	1			
Monero	0.6002	0.5590	0.5208	0.3016	0.5360	1				
Litecoin	0.8006	0.3423	0.3367	0.3195	1					
Ethereum	0.5278	0.5566	0.5148	1						
Dogecoin	0.6915	0.3415	1							
Dash	0.5921	1								
Bitcoin	1									

Panel B: Positive return exceedances



	Bitcoin	Dash	Dogecoin	Ethereum	Litecoin	Monero	Namecoin	Novacoin	Peercoin	Ripple
Ripple	0.4848	0.3005	0.3583	0.2987	0.4969	0.3093	0.4928	0.4472	0.5040	1
Peercoin	0.6210	0.3527	0.3144	0.2909	0.6473	0.2993	0.7811	0.6806	1	
Novacoin	0.6167	0.3232	0.3117	0.2885	0.6208	0.3114	0.7169	1		
Namecoin	0.6665	0.3310	0.3188	0.2759	0.6711	0.3273	1			
Monero	0.4771	0.5157	0.4521	0.2851	0.4640	1				
Litecoin	0.6703	0.3170	0.3452	0.2665	1					
Ethereum	0.4468	0.4979	0.4834	1						
Dogecoin	0.5819	0.3575	1							
Dash	0.5151	1								
Bitcoin	1									

Note. This table is a colour gradation table representing the bias-corrected extreme correlation coefficient ρ for 999 bootstrap samples, for the tail probability level ζ_2 equal to 0.25. Panel A represents left distribution tail and negative return exceedances. Panel B represents the right tails and positive return exceedances. Ninety (90) combinations of cryptocurrencies are represented: forty-five combinations for each distribution tail; considering daily closing prices for the ten largest cryptocurrencies in a pairwise comparison. The cryptocurrencies under consideration are: Bitcoin, Dash, Dogecoin, Ethereum, Litecoin, Monero, Namecoin, Novacoin, Peercoin and Ripple. High values of the ρ are represented with a lighter colour. Low values of ρ are represented with a darker colour.

TABLE 3. Bivariate dependency scores in cryptocurrency markets at fixed threshold levels for tail probability equal to 0.15

Panel A: Negative return exceedances

	Bitcoin	Dash	Dogecoin	Ethereum	Litecoin	Monero	Namecoin	Novacoin	Peercoin	Ripple
Ripple	0.5291	0.2176	0.2211	0.2228	0.5286	0.2173	0.5066	0.4496	0.4917	1
Peercoin	0.7122	0.2352	0.2460	0.2317	0.7382	0.2359	0.7986	0.6795	1	
Novacoin	0.6680	0.2629	0.2201	0.2057	0.6589	0.2331	0.6825	1		
Namecoin	0.7398	0.2343	0.2434	0.2434	0.7635	0.2580	1			
Monero	0.5702	0.5180	0.4880	0.2104	0.5055	1				
Litecoin	0.7882	0.2698	0.2470	0.2481	1					
Ethereum	0.4608	0.5111	0.4863	1						
Dogecoin	0.6256	0.2688	1							
Dash	0.5567	1								
Bitcoin	1									

Panel B: Positive return exceedances

	Bitcoin	Dash	Dogecoin	Ethereum	Litecoin	Monero	Namecoin	Novacoin	Peercoin	Ripple
Ripple	0.4037	0.2110	0.2960	0.1941	0.4209	0.1915	0.4187	0.3688	0.4078	1
Peercoin	0.5196	0.2367	0.2340	0.1961	0.5748	0.2154	0.7518	0.6169	1	
Novacoin	0.5095	0.2075	0.2276	0.2237	0.5333	0.2162	0.6695	1		
Namecoin	0.5583	0.2332	0.2112	0.1911	0.5922	0.2351	1			
Monero	0.3509	0.4265	0.3630	0.1877	0.3399	1				
Litecoin	0.5765	0.2091	0.2405	0.1610	1					
Ethereum	0.3397	0.3932	0.3857	1						
Dogecoin	0.4946	0.2483	1							
Dash	0.4240	1								
Bitcoin	1									

Note. This table is a colour gradation table representing the bias-corrected extreme correlation coefficient ρ for 999 bootstrap samples, for the tail probability level ζ_2 equal to 0.15. Panel A represents left distribution tail and negative return exceedances. Panel B represents the right tails and positive return exceedances. Ninety (90) combinations of cryptocurrencies are represented: forty-five combinations for each distribution tail; considering daily closing prices for the ten largest cryptocurrencies in a pairwise comparison. The cryptocurrencies under consideration are: Bitcoin, Dash, Dogecoin, Ethereum, Litecoin, Monero, Namecoin, Novacoin, Peercoin and Ripple. High values of the ρ are represented with a lighter colour. Low values of ρ are represented with a darker colour.

TABLE 4. Bivariate dependency scores in cryptocurrency markets at fixed threshold levels for tail probability equal to 0.05

Panel A: Negative return exceedances

	Bitcoin	Dash	Dogecoin	Ethereum	Litecoin	Monero	Namecoin	Novacoin	Peercoin	Ripple
Ripple	0.4370	0.0906	0.1350	0.1049	0.4326	0.1686	0.4664	0.3189	0.3474	1
Peercoin	0.6995	0.1558	0.1875	0.1302	0.7419	0.1343	0.8324	0.6428	1	
Novacoin	0.6180	0.1259	0.1599	0.0549	0.6560	0.1513	0.6751	1		
Namecoin	0.7622	0.1263	0.1412	0.0798	0.7581	0.0979	1			
Monero	0.5395	0.4536	0.4404	0.0545	0.5125	1				
Litecoin	0.7808	0.1558	0.1583	0.1298	1					
Ethereum	0.4308	0.4015	0.4365	1						
Dogecoin	0.5775	0.1916	1							
Dash	0.4993	1								
Bitcoin	1									

ρ: 1.00, 0.75, 0.50, 0.25, 0.00

Panel B: Positive return exceedances

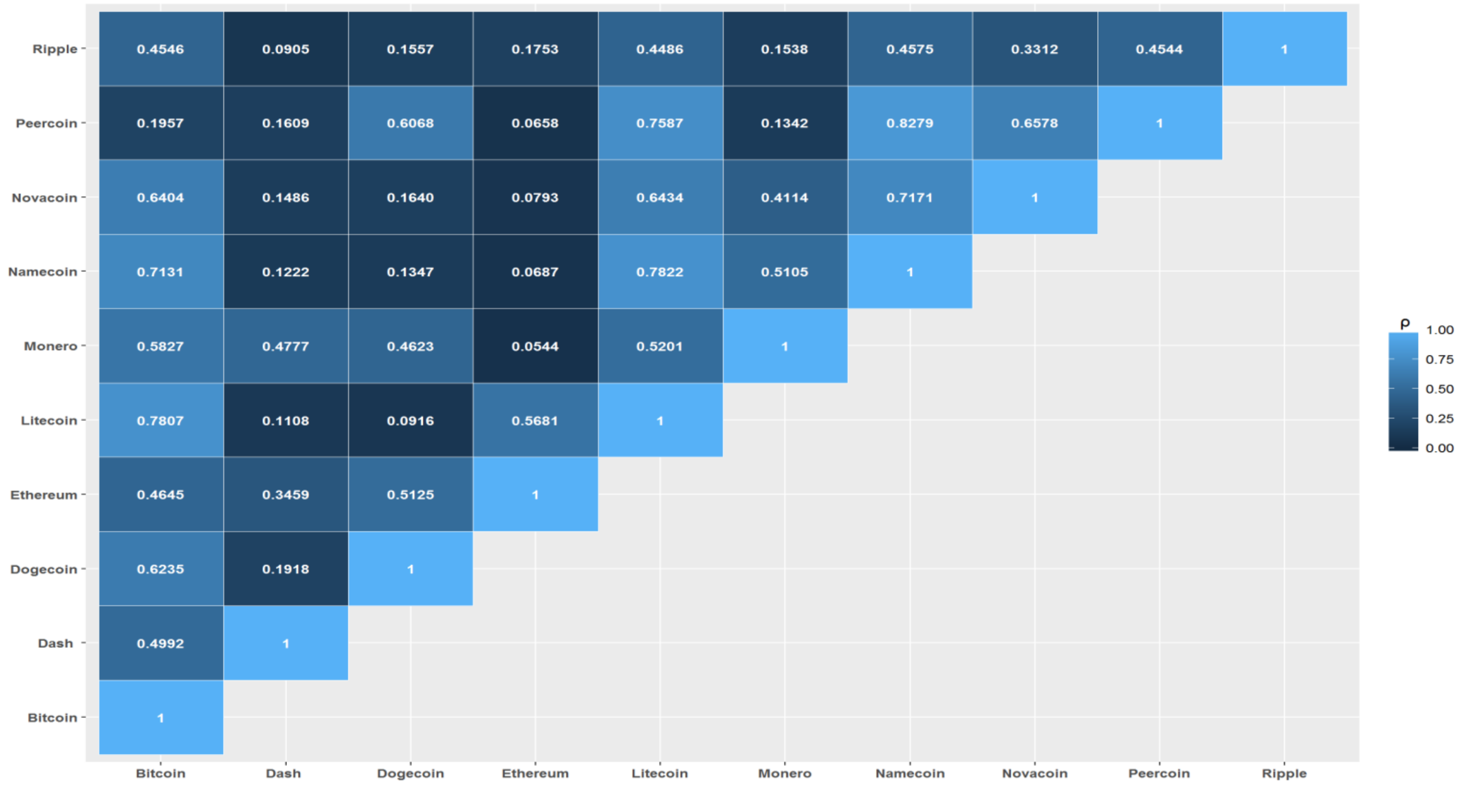
	Bitcoin	Dash	Dogecoin	Ethereum	Litecoin	Monero	Namecoin	Novacoin	Peercoin	Ripple
Ripple	0.2515	0.0735	0.1657	0.0586	0.2984	0.0384	0.2718	0.2024	0.2966	1
Peercoin	0.4212	0.0732	0.1228	0.0874	0.4687	0.0579	0.6631	0.4959	1	
Novacoin	0.3175	0.0736	0.1034	0.1914	0.3846	0.0925	0.5943	1		
Namecoin	0.3834	0.0375	0.0902	0.0602	0.5033	0.0570	1			
Monero	0.2796	0.3594	0.2065	0.1118	0.1725	1				
Litecoin	0.4655	0.0738	0.1027	0.0300	1					
Ethereum	0.1671	0.1679	0.1478	1						
Dogecoin	0.3543	0.1420	1							
Dash	0.3083	1								
Bitcoin	1									

ρ: 1.00, 0.75, 0.50, 0.25, 0.00

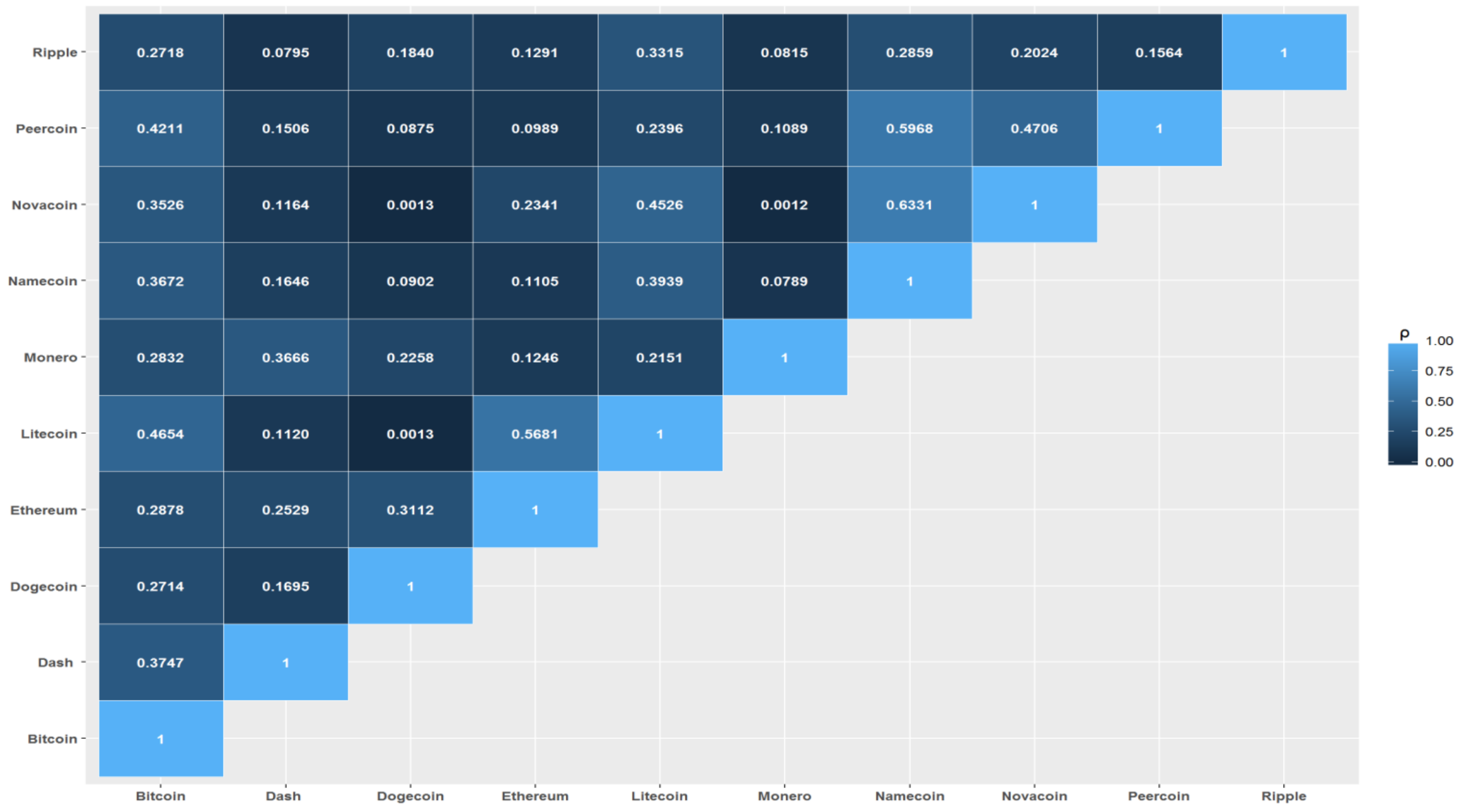
Note. This table is a colour gradation table representing the bias-corrected extreme correlation coefficient ρ for 999 bootstrap samples, for the tail probability level ζ_2 equal to 0.05. Panel A represents left distribution tail and negative return exceedances. Panel B represents the right tails and positive return exceedances. Ninety (90) combinations of cryptocurrencies are represented: forty-five combinations for each distribution tail; considering daily closing prices for the ten largest cryptocurrencies in a pairwise comparison. The cryptocurrencies under consideration are: Bitcoin, Dash, Dogecoin, Ethereum, Litecoin, Monero, Namecoin, Novacoin, Peercoin and Ripple. High values of the ρ are represented with a lighter colour. Low values of ρ are represented with a darker colour.

TABLE 5. Bivariate dependency patterns in cryptocurrency markets at optimal threshold levels

Panel A: Negative return exceedances



Panel B: Positive return exceedances



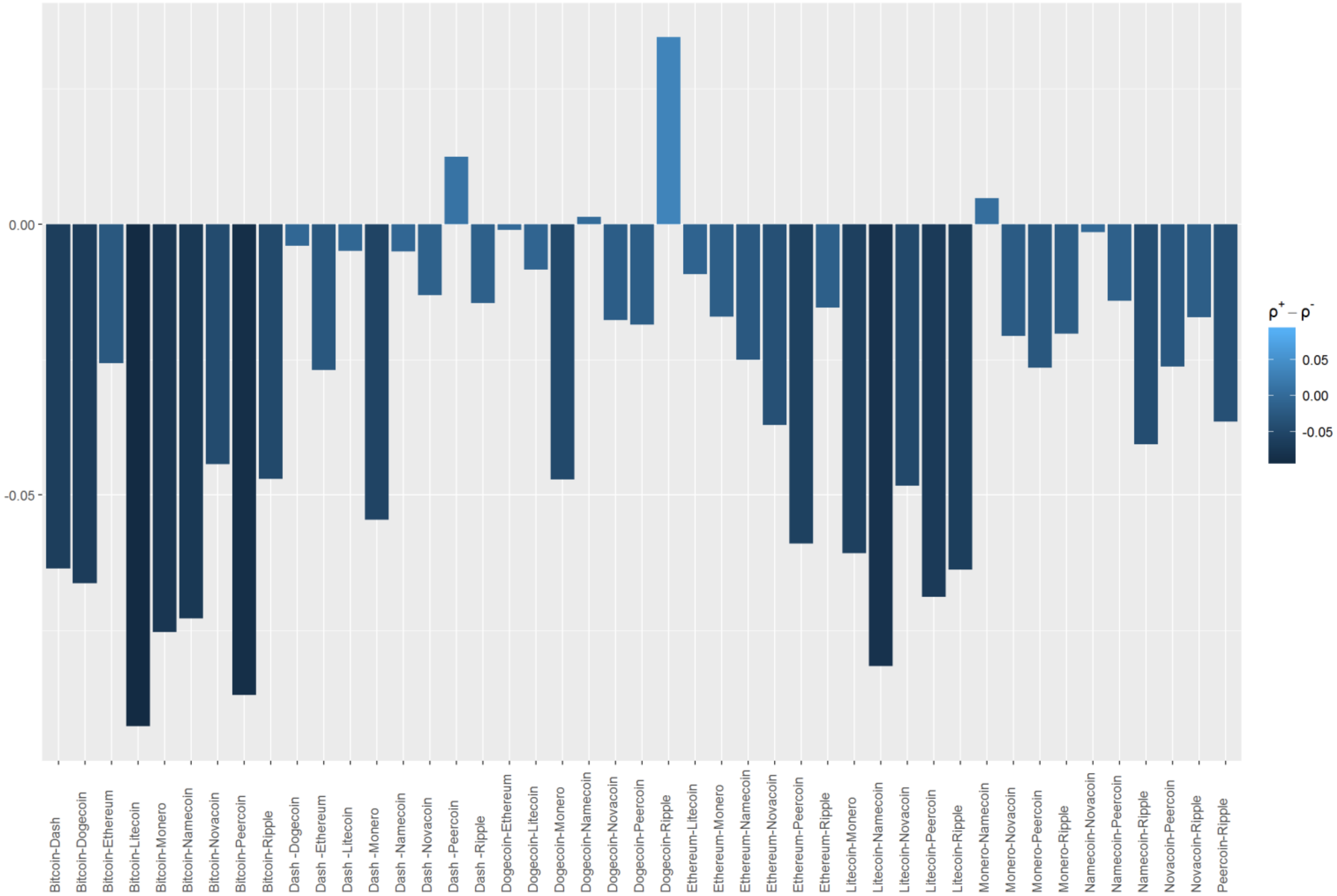
Note. This table is a colour gradation table representing the bias-corrected extreme correlation coefficient ρ for 999 bootstrap samples, at optimal threshold levels. Panel A represents left distribution tail and negative return exceedances. Panel B represents the right tails and positive return exceedances. Ninety (90) combinations of cryptocurrencies are represented: forty-five combinations for each distribution tail; considering daily closing prices for the ten largest cryptocurrencies in a pairwise comparison. The cryptocurrencies under consideration are: Bitcoin, Dash, Dogecoin, Ethereum, Litecoin, Monero, Namecoin, Novacoin, Peercoin and Ripple. High values of the ρ are represented with a lighter colour. Low values of ρ are represented with a darker colour.

TABLE 6. Tail probabilities and extreme correlation in cryptocurrency markets

Panel A: Negative return exceedances					Panel B: Positive return exceedances				
Range	Max	Min.	f%	Obs.	Range	Max	Min.	f%	Obs.
Tail probability equal to 0.35					Tail probability equal to 0.35				
$[0, 0.2)$	-	-	0%	0	$[0, 0.2)$	-	-	0%	0
$[0.2, 0.4)$	-	-	0%	0	$[0.2, 0.4)$	0.3980	0.3875	18%	8
$[0.4, 0.6)$	0.5817	0.4045	56%	25	$[0.4, 0.6)$	0.5818	0.4007	58%	26
$[0.6, 0.8)$	0.7971	0.6087	42%	19	$[0.6, 0.8)$	0.7338	0.6524	22%	10
$[0.8, 1)$	0.8227	0.8158	4%	2	$[0.8, 1)$	0.8016	0.8016	2%	1
All	0.8227	0.4045	100%	45	All	0.8016	0.3875	100%	45
Tail probability equal to 0.25					Tail probability equal to 0.25				
$[0, 0.2)$	-	-	0%	0	$[0, 0.2)$	-	-	0%	0
$[0.2, 0.4)$	0.3547	0.3016	47%	21	$[0.2, 0.4)$	0.3583	0.2665	47%	21
$[0.4, 0.6)$	0.5921	0.5131	24%	11	$[0.4, 0.6)$	0.5819	0.4468	31%	14
$[0.6, 0.8)$	0.7855	0.6002	27%	12	$[0.6, 0.8)$	0.7811	0.6167	22%	10
$[0.8, 1)$	0.8026	0.8006	4%	2	$[0.8, 1)$	0.0000	0.0000	0%	0
All	0.8026	0.3016	100%	45	All	0.7811	0.2665	100%	45
Tail probability equal to 0.15					Tail probability equal to 0.15				
$[0, 0.2)$	-	-	0%	0	$[0, 0.2)$	0.1961	0.1610	13%	6
$[0.2, 0.4)$	0.2698	0.2057	47%	21	$[0.2, 0.4)$	0.3932	0.2075	49%	22
$[0.4, 0.6)$	0.5702	0.4496	29%	13	$[0.4, 0.6)$	0.5922	0.4037	31%	14
$[0.6, 0.8)$	0.7986	0.6256	24%	11	$[0.6, 0.8)$	0.7518	0.6169	7%	3
$[0.8, 1)$	0.8500	0.8500	2%	1	$[0.8, 1)$	0.0000	0.0000	0%	0
All	0.8500	0.2057	100%	45	All	0.7518	0.1610	100%	45
Tail probability equal to 0.05					Tail probability equal to 0.05				
$[0, 0.2)$	0.1916	0.0545	47%	21	$[0, 0.2)$	0.1914	0.0300	56%	25
$[0.2, 0.4)$	0.3474	0.3189	4%	2	$[0.2, 0.4)$	0.3846	0.2024	29%	13
$[0.4, 0.6)$	0.5775	0.4015	27%	12	$[0.4, 0.6)$	0.5943	0.4212	13%	6
$[0.6, 0.8)$	0.7808	0.6180	20%	9	$[0.6, 0.8)$	0.6631	0.6631	2%	1
$[0.8, 1)$	0.9500	0.8324	4%	2	$[0.8, 1)$	0.0000	0.0000	0%	0
All	0.9500	0.0545	100%	45	All	0.6631	0.0300	100%	45
Optimal thesholds					Optimal thesholds				
$[0, 0.2)$	0.1957	0.0544	40%	18	$[0, 0.2)$	0.1840	0.0012	44%	20
$[0.2, 0.4)$	0.3459	0.3312	4%	2	$[0.2, 0.4)$	0.3939	0.2024	40%	18
$[0.4, 0.6)$	0.5827	0.4114	31%	14	$[0.4, 0.6)$	0.5968	0.4211	13%	6
$[0.6, 0.8)$	0.7822	0.6068	22%	10	$[0.6, 0.8)$	0.6331	0.6331	2%	1
$[0.8, 1)$	0.8279	0.8279	2%	1	$[0.8, 1)$	0.6331	0.0012	0%	0
All	0.8279	0.0544	100%	45	All	0.8279	0.0012	100%	45

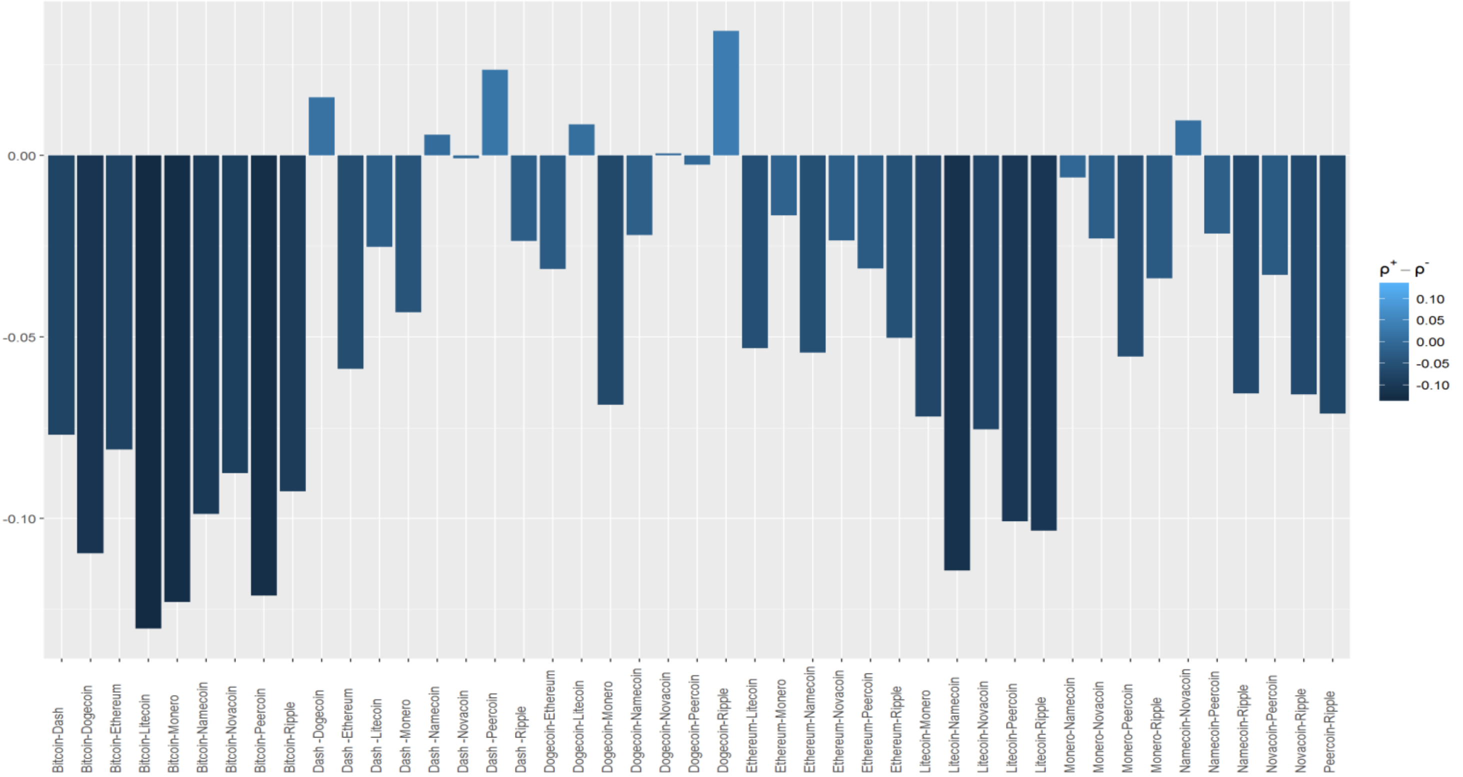
Note. This table reports summarized descriptive statistics for the estimates of Tables 1-5, across the asymptotic independence, in which ρ is equal to 0, whilst total dependence is represented by ρ being equal to 1. Panel A refers to negative return exceedances. Panel B refers to positive return exceedances. The maximum (Max), the minimum ($Min.$), the frequency ($f\%$) and the number of observations ($Obs.$) for the extreme correlation coefficient ρ in some specific subintervals are reported for each tail probability level ζ and optimal threshold levels.

FIGURE 1. Difference of extreme correlation between positive and negative return exceedances at fixed threshold levels for tail probability of 0.35



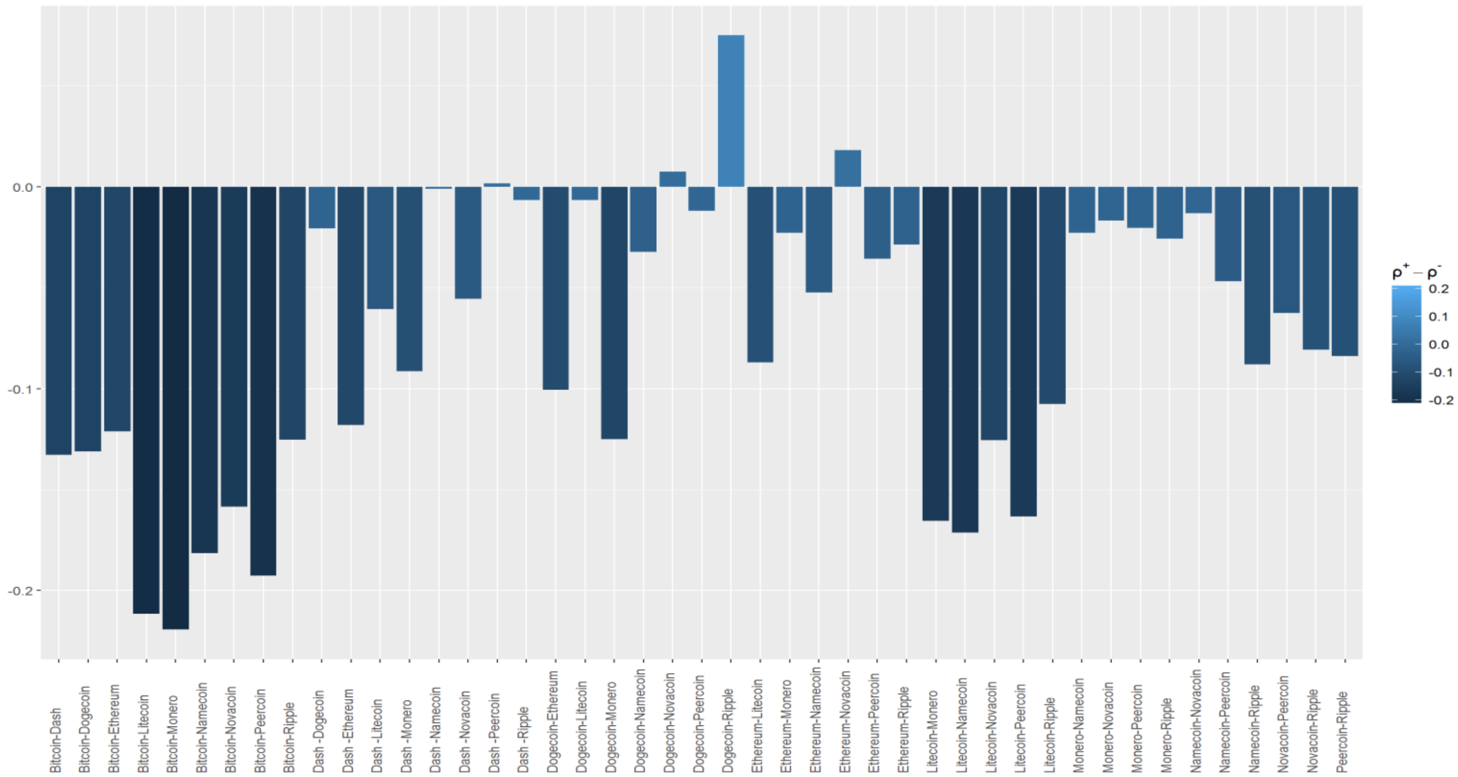
Note. This figure illustrates the asymmetry between negative and positive return exceedances via the difference between the dependence structure of left and right distribution tail for Table 1. A bar below the horizontal axis displays that the extreme correlation of negative return exceedances (ρ^-) is greater than the correlation of positive exceedances (ρ^+). A bar above the horizontal axis means that the extreme correlation of negative return exceedances is lower than the one of positive return exceedances.

FIGURE 2. Difference of extreme correlation between positive and negative return exceedances at fixed threshold levels for tail probability of 0.25



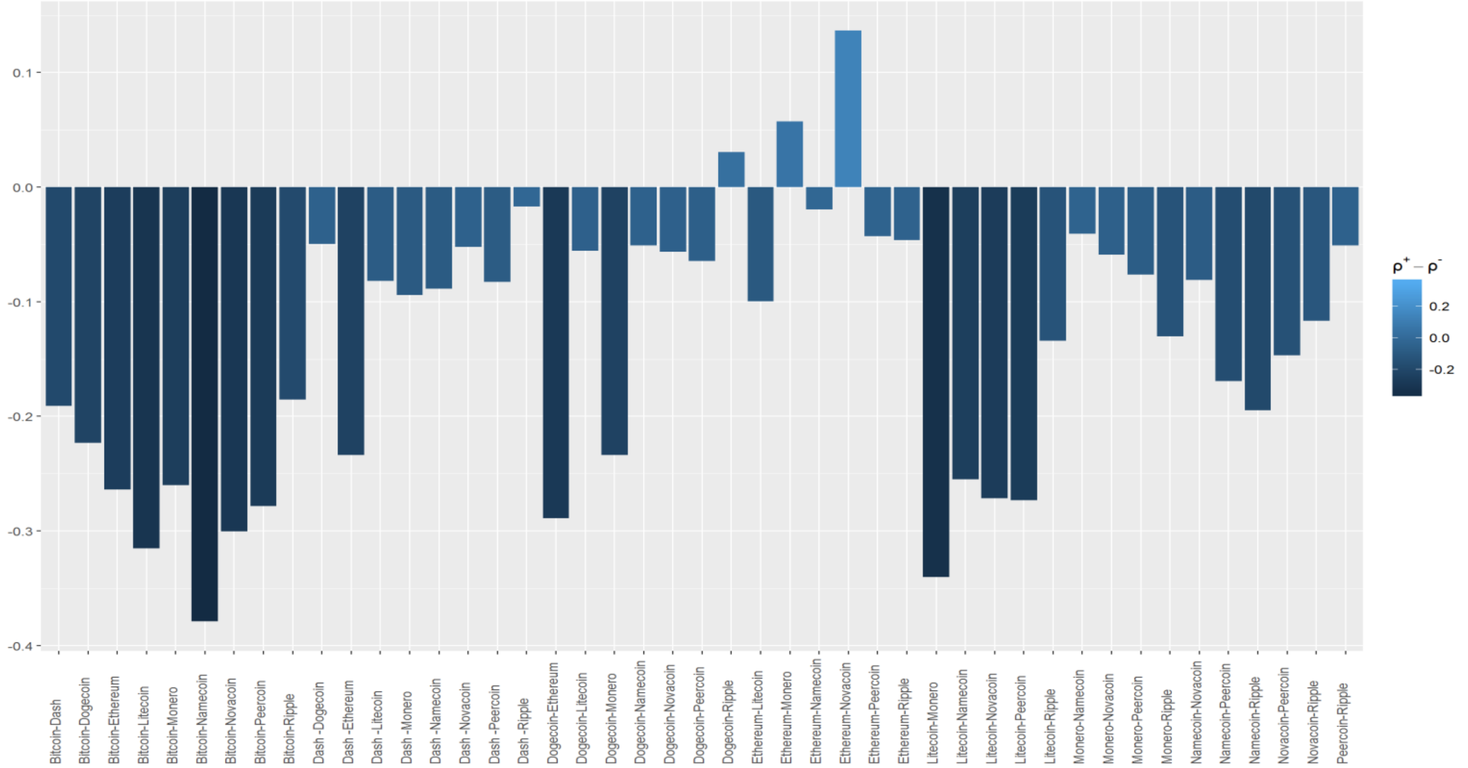
Note. This figure illustrates the asymmetry between negative and positive return exceedances via the difference between the dependence structure of left and right distribution tail for Table 2. A bar below the horizontal axis displays that the extreme correlation of negative return exceedances (ρ^-) is greater than the correlation of positive exceedances (ρ^+). A bar above the horizontal axis means that the extreme correlation of negative return exceedances is lower than the one of positive return exceedances.

FIGURE 3. Difference of extreme correlation between positive and negative return exceedances at fixed threshold levels for tail probability of 0.15



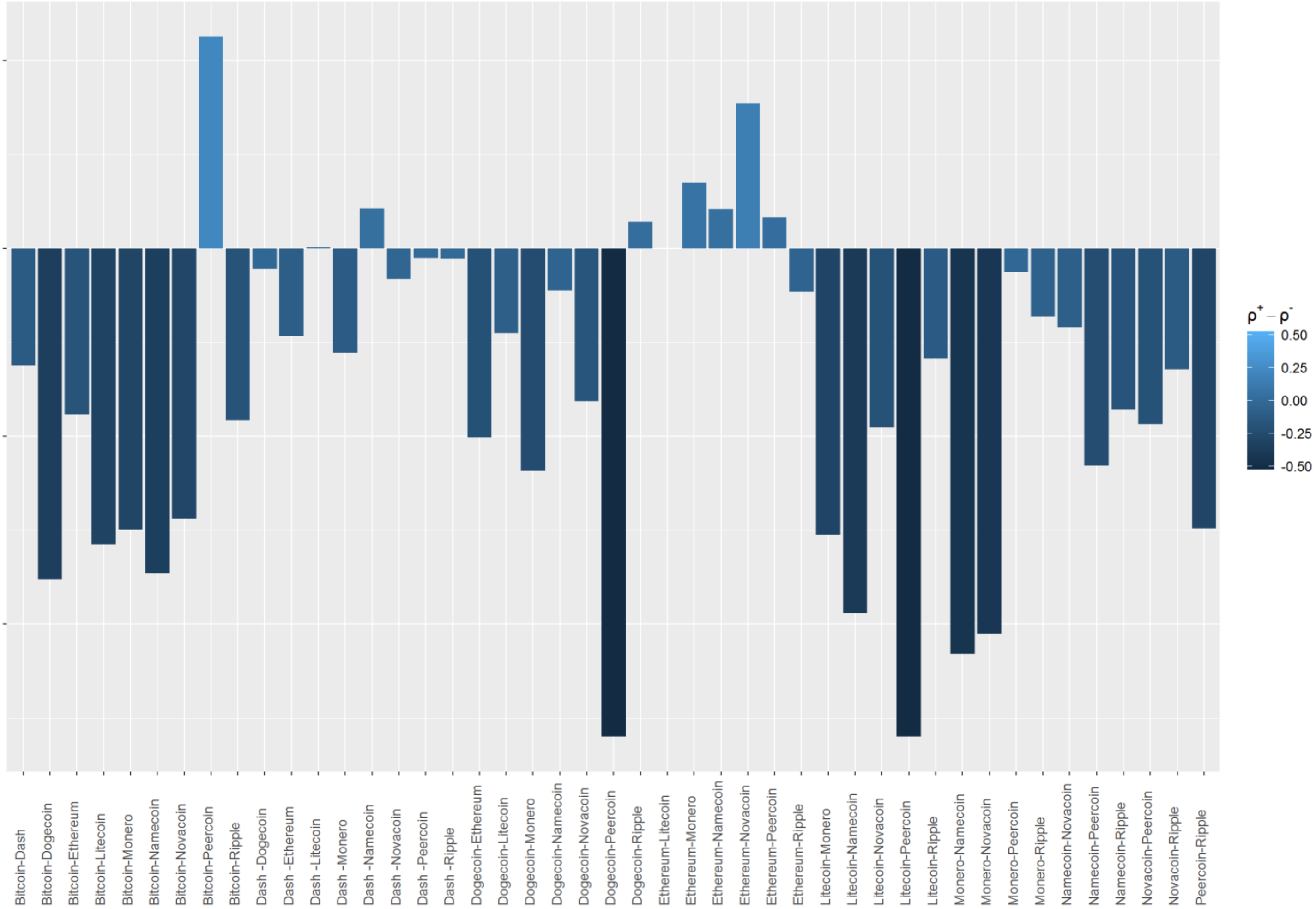
Note. This figure illustrates the asymmetry between negative and positive return exceedances via the difference between the dependence structure of left and right distribution tail for Table 3. A bar below the horizontal axis displays that the extreme correlation of negative return exceedances (ρ^-) is greater than the correlation of positive exceedances (ρ^+). A bar above the horizontal axis means that the extreme correlation of negative return exceedances is lower than the one of positive return exceedances.

FIGURE 4. Difference of extreme correlation between positive and negative return exceedances at fixed threshold levels for tail probability of 0.05



Note. This figure illustrates the asymmetry between negative and positive return exceedances via the difference between the dependence structure of left and right distribution tail for Table 4. A bar below the horizontal axis displays that the extreme correlation of negative return exceedances (ρ^{-}) is greater than the correlation of positive exceedances (ρ^{+}). A bar above the horizontal axis means that the extreme correlation of negative return exceedances is lower than the one of positive return exceedances.

FIGURE 5. Difference of extreme correlation between positive and negative return exceedances at optimal threshold levels



Note. This figure illustrates the asymmetry between negative and positive return exceedances via the difference between the dependence structure of left and right distribution tail for Table 5. A bar below the horizontal axis displays that the extreme correlation of negative return exceedances (ρ^-) is greater than the correlation of positive exceedances (ρ^+). A bar above the horizontal axis means that the extreme correlation of negative return exceedances is lower than the one of positive return exceedances.