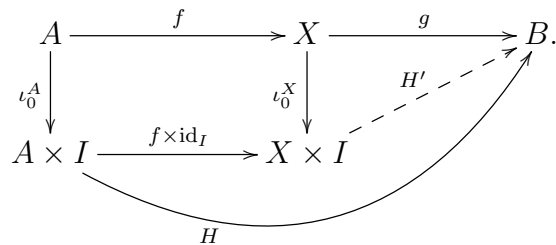# Sample HTML file: produced with lwarp, with mathematical formulae displayed with MathJax, and xymatrix commutative diagrams displayed with XyJax-v3

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Last modified: May 23, 2023

Contents

$$\begin{array}{ccc}
A \xrightarrow{\;f\;} & X \xrightarrow{\;g\;} & B. \\
\iota_0^A \downarrow & \iota_0^X \downarrow & \\
A \times I \xrightarrow{f \times \mathrm{id}_I} & X \times I & \\
\end{array}$$

The responsibility for this HTML file is João Faria Martins' only.
Corrections, suggestions, etc. are very welcome, and can be sent to: j.fariamartins@leeds.ac.uk.
This is strictly a non-specialist experimental file.

See the links below for:

- "stand-alone" version of this webpage,

- "split" version of this webpage (with sans serif fonts),

- pdf version of this page,

- Latex source for the stand alone version of this page Sample_Lwarp_Xy-jax.tex,

- Latex source for the split version of this page, with sans serif fonts: Sample_Lwarp_Xy-jax-split.tex,

- CSS files for the stand-alone version of this page: lwarp.css (automatically provided by *lwarpmk*) and my_file_stand-alone.css (minor tweak to adjust margins),

- CSS files for the split version of this page: lwarp_formal.css (automatically provided by *lwarpmk*) and my_file_split.css (minor tweak enforcing sans serif fonts).

- Configuration file required for browsers to find Xy-Jaxv3, lwarp-with-Xy-jax_v3.txt. This script is essentially the original Lwarp MathJax emulation code, lwarp_mathjax.txt (automatically created by lwarp v0.902), with two minimal additions, as shown.

1 Context: using Lwarp to compile latex to HTML, so that xymatrix diagrams are displayed with Xy-Jax

This HTML file (that you are reading) was produced with the lwarp.sty package, available with **TeXLive**. [1] In particular the preamble contains:

```
\documentclass[a4paper,12pt]{article}
\usepackage[mathjax]{lwarp}
```

The version of the software was:

```
pdfTeX, Version 3.141592653-2.6-1.40.22 (TeX Live 2022/dev/Debian)
lwarpmk: v0.903  Automated make for the LaTeX Lwarp package.
```

The source latex file can be found here: Sample_Lwarp_Xy-jax.tex.

This HTML file also makes use of the following configuration script, so that **Xy-pic** diagrams, particularly *xymatrix* commmutative diagrams, can be displayed by using **Xy-Jax-v3**. (Or at least a good sample of them: there seem to be some compatibility issues.)

lwarp-with-Xy-jax_v3.txt.

This script file has minimal additions (two in total) in the original Lwarp MathJax emulation code, that is automatically created by lwarp v0.902), namely:

lwarp_mathjax.txt,

(Two lines were added to the script above, as shown, in order that browsers can find Xy-Jaxv3.) This modification was produced under items 5 and 6 of the licence `https://www.ctan.org/license/lppl1.3` (The LATEX Project Public License 1.3).
Credit is due to instructions in: `https://github.com/sonoisa/XyJax-v3` (the official Xy-Jaxv3 documentation) and in "40.11 lwarp_mathjax.txt" of the official Lwarp documentation.

If using Xy-pic diagrams displayed with **XyJax-v3**, insert the following in the preamble of your latex file:

---

[1] See also `https://people.bath.ac.uk/feb/lwarp/lwarp-intro.html`, for a quick introduction on how to use Lwarp, and without which this explanation that you are reading would not have been written.

```
\usepackage[all]{xy}
\MathJaxFilename{lwarp-with-Xy-jax_v3.txt}
```

Compile .tex to .html using:

1. *pdflatex Sample_ Lwarp_ Xy-jax.tex* (possibly twice) : creates Sample_Lwarp_Xy-jax.pdf (usual pdf file, that can also be shared) and Sample_Lwarp_Xy-jax_html.tex. (And some other aux files.)

2. *lwarpmk html* : converts Sample_Lwarp_Xy-jax_html.tex to Sample_Lwarp_Xy-jax_html.html. And then creates Sample_Lwarp_Xy-jax.html (possibly split into different HTML files if such option is chosen).

    Note: Use *lwarpmk html1* to force a recompile.

    Note: In case of error, with *lwarpmk html*, use *lwarpmk pdftohtml* to create HTML file.

    Note: When using *bibtex* files, then the following is required:

    - bibtex Sample_Lwarp_Xy-jax_html.aux
    - *pdflatex* Sample_Lwarp_Xy-jax_html.tex

    The following steps are only needed once (any time tikz figures change). Only needed with tikz.

3. *lwarpmk limages* : If there are images done with tikz, tikz-cd, etc.

4. Repeat 2 (now using *lwarpmk html1*) and 3 (to finish creating the images).

Note. *It is advisable to use a recent version of* **TexLive**.

Warnings:

- The usage of **XyJax-v3** software with lwarp is experimental.

- Not all accessibility features of commutative diagrams displayed with XyJax-v3 work. Among the accessibility features that seem to work well are:

    1. View original latex source.
    2. Zoom maths expression.

2 Examples: xymatrix diagrams displayed with Xy-Jaxv3 using Lwarp

This is an example of a HTML file with:

- Mathematical formulae displayed with **MathJax**:

$$x^2 + y^2 = 1, \tag{1}$$

$$\left(x \xrightarrow{\gamma} y\right) \in \pi_1(X, X_0),$$

$$\cdots \to \pi_i(E_b, x) \xrightarrow{\iota} \pi_i(E, x) \xrightarrow{\partial} \pi_i(B, b) \xrightarrow{\delta} \pi_{i-1}(E_b, x) \to \ldots \xrightarrow{\iota} \pi_1(E, x) \xrightarrow{\partial} \pi_1(B, b) \xrightarrow{\delta_x} \pi_0(E_b) \xrightarrow{\iota} \pi_0($$

3

- To see how lecture notes look like with Lwarp, see subsections 6.1 and 6.2.

- This configuration is compatible with AMS-CD commutative diagrams, e.g. this diagram from the amscd package manual:

$$
\begin{CD}
A @>a>b> B \\
@VlVrV @AlArA \\
C @<a<b< D
\end{CD}
$$

- Commutative diagrams drawn with **Xy-pic** displayed by using **XyJax-v3**. E.g.

$$
\begin{array}{ccc}
S_3 & \overset{f}{\hookrightarrow} & S_4 \\
& {}_{g}\searrow{}^{\cong} & \downarrow {}^{\pi} \\
& & S_4/V,
\end{array}
$$

$$
\begin{array}{ccccc}
& & \mathbb{Q} & & \\
\mathbb{Q}(\omega) & & \mathbb{Q}(\gamma) \quad \mathbb{Q}(\beta) & & \mathbb{Q}(\alpha), \\
& & \mathbb{Q}(\alpha,\beta,\gamma) & &
\end{array}
$$

(123).

$$
\begin{array}{ccc}
\bullet \longrightarrow \star & & \star \longrightarrow \square \\
\big\backslash \quad \big/ & = & \big\backslash \quad \big/ \\
\square & & \bullet
\end{array}
$$
,

$$
\begin{array}{ccccc}
A & \overset{f}{\longrightarrow} & X & \overset{g}{\longrightarrow} & B, \\
\downarrow {}_{\iota_0{}^{A}} & & \downarrow {}_{\iota_0{}^{X}} & {}_{H'} \nearrow & \\
A \times I & \underset{f \times \mathrm{id}_I}{\longrightarrow} & X \times I & &
\end{array}
$$
$$
{}_{H}
$$

4

$$
\begin{array}{ccc}
\bigoplus_{y\in H_0} \mathbf{H}_1(x,y)\otimes \mathbf{H}'_1(y,z) & \xrightarrow{\ \ \mathrm{proj}\ \ } & \int^{y\in H_0} \mathbf{H}_1(x,y)\otimes \mathbf{H}'_1(y,z) \\[2mm]
\bigoplus_{y\in H_0}\eta_{(x,y)}\otimes\eta'_{(y,z)}\Big\downarrow & & \Big\downarrow \int^{y\in Y}\eta_{(x,y)}\otimes\eta'_{(y,z)} \\[2mm]
\bigoplus_{y\in H_0} \mathbf{H}_2(x,y)\otimes \mathbf{H}'_2(y,z) & \xrightarrow[\ \ \mathrm{proj}\ \ ]{} & \int^{y\in H_0} \mathbf{H}_2(x,y)\otimes \mathbf{H}'_2(y,z)\ ,
\end{array}
$$

$$
\begin{array}{ccc}
\int^{y\in Y}\overline{\mathbf{H}}^{M_1}_{(X,Y)}(-,y)\otimes\overline{\mathbf{H}}^{M_2}_{(Y,Z)}(y,-) & \overset{\eta^{M_1,M_2}_{(X,Y,Z)}}{\Longrightarrow} & \overline{\mathbf{H}}^{M_1\times_Y M_2}_{(X,Z)} \\[2mm]
\overline{\mathbf{2H}}^{\mathbf{W}_1}_{(X,Y)}\bullet\overline{\mathbf{2H}}^{\mathbf{W}_2}_{(Y,Z)}\Big\Downarrow & & \Big\Downarrow\overline{\mathbf{2H}}^{\mathbf{W}_1\#_0\mathbf{W}_2}_{(X,Z)} \\[2mm]
\int^{y\in Y}\overline{\mathbf{H}}^{N_1}_{(X,Y)}(-,y)\otimes\overline{\mathbf{H}}^{N_2}_{(Y,Z)}(y,-) & \overset{\eta^{N_1,N_2}_{(X,Y,Z)}}{\Longrightarrow} & \overline{\mathbf{H}}^{N_1\times_Y N_2}_{(X,Z)}\ .
\end{array}
$$

3 Issues when using XyJax-v3 and Lwarp for typesetting xymatrix diagrams

Please do contact me if you know the solution to any of these issues. Remember: this is stricly a non-specialist file!

- The commands of the form:

  `\stackrel{(*)}{\implies}`

  or

  `\left ( x \xrightarrow{f} y\right)`

  do not seem to work well inside

  `\begin{equation} \end{equation}`.

  So use instead, for example,

  `$$ \stackrel{(*)}{\implies} $$`

  `$$ \left ( x \xrightarrow{f} y \right)$$`

  . Here is how it looks:

$$
\overset{(*)}{\Longrightarrow}
$$
$$
\left(x \xrightarrow{f} y \right).
$$

- Sometimes there are errors if there are mathematical commands in the title of sections, subsections, etc.

- Lwarpmk gives error messages if it finds *xymatrix* inside:

  `\begin{equation} \end{equation}`.

  (However a html file can still be produced with *lwarpmk pdftohtml.*)

  So avoid:

  `\begin{equation} \xymatrix{    } \end{equation}`,

  and use instead:

  `\[ \xymatrix{   } \]`,

  or

  `$$ \xymatrix{   } $$`.

- The 2-cell option does not appear to work with Xy-Jaxv3. However xymatrix diagrams can still be compiled as svg figures, with alt text; see 5.1.

$$
\begin{array}{ccc}
T \times T & \xrightarrow{\ \tau\ } & T \times T \\
& \searrow \ \ \underset{\mathbf{R}}{\Leftarrow} \ \downarrow & \\
\sqcup & \sqcup & \\
& & T.
\end{array}
$$

- **Not all accessibility functions of MathJax work with Xy-Jaxv3, only some can be used, e.g. zoom and source latex code**.
  Accessibility features of Xy-jax are not officially supported.

- References to section, chapters etc, have glitches **if inside mathematics environments**. For instance, the links provided do not appear to work well. E.g.

$$
\exp(x + yi) = e^x \big(\cos(y) + i\sin(y) \big) \tag{2}
$$

$$
\begin{aligned}
e^{i\pi} + 1 &= \big(\cos(\pi) + i\sin(\pi) \big) + 1, \quad \text{using Equation (2). (Just for testing) equations (1), (3)} \\
&= 0. \quad \text{A reference to second section 2}
\end{aligned}
$$

  Outside maths environments all seems to work fine:

  Using Equation (2). (Just for testing) equations (1), (3)
  A reference to second section 2

- When compiling xymatrix, or tikz, diagrams as sgv figures, with Alt-Text (which sometimes is necessary) it sometimes happens that the figure is not cropped correctly. This seems to depend stronly on the operating system being used.

4 More general notes

- **pdf** and **HTML** automatically coexist: here is the pdf version of this HTML file Sample_Lwarp_Xy-jax.pdf

- It is possible to split an HTML page into sub-pages: credit `https://people.bath.ac.uk/feb/lwarp/lwarp-intro.html`. Instructions can be seen in the source latex file of this HTML file Sample_Lwarp_Xy-jax.tex.

- Additions are required in the .tex file in order that **MathJax** displays commands, macros and definitions correctly. E.g. write:

```
\DeclareMathOperator{\Sym}{Sym}
\def \Mon {{\mathbf{Mon}}}
\newcommand{\Q}{\mathbb{Q}}
```

and then:

```
\begin{warpHTML}
  \CustomizeMathJax{\DeclareMathOperator{\Sym}{Sym}}
  \CustomizeMathJax{\def \Mon {{\mathbf{Mon}}}}
  \CustomizeMathJax{\newcommand{\Q}{\mathbb{Q}}}
\end{warpHTML}
```

This means that you can use slightly different versions of commands for pdf and for html.

- If using Xy-pic diagrams (if displayed as Xy-jax) put in the preamble of the tex file:

```
\MathJaxFilename{lwarp-with-Xy-jax_v3.txt}
```

You will also need this configuration file: lwarp-with-Xy-jax_v3.txt. (Explanation is in Section 1.)

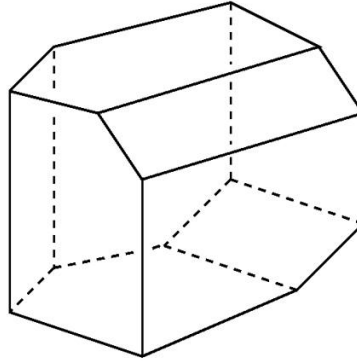- Sample use of Xy-pic compatible with lwarp (so that the resulting xymatrix code is readable by Xy-Jax): E.g.:

```
\[
\xymatrix
{
&& \Q\\ & \Q(\w) \ar@{<-}[ur] \\
& & \Q(\g)\ar@{<-}[uu] & \Q(\b)\ar@{<-}[uul] & \Q(\a)\ar@{<-}[uull]\\
&&\Q(\a,\b,\g)\ar@{<-}[uul]\ar@{<-}[u] \ar@{<-}[ur] \ar@{<-}[urr]
}
\],
```

or

```
\displaymathnormal{$$
\xymatrix{
&S_3 \ar@{^{(}->}[r]^f \ar[dr]_{g}^{\cong} &S_4 \ar@{->>}[d]^\pi
\\ &&S_4/V,
}
$$}.
```

- Figures, with alternative text, can be included like this:

```
\includegraphics[width = 0.4\textwidth,
alt={Type your alternative text here}]{Stasheff.png}
```


- Figures, including **tikz** and **tikz-cd figures**, can be compiled as figures with **Alt Text**. (The same option is also available for xymatrix diagrams, and may be preferable to Xy-Jaxv3, in the cases when it is sensible to provide a comprehensive alternative text to the figure/diagram, instead of relying mainly on the availability of the latex code, for accessibility.)
  Warning: In some operating systems, issues seem to exist with the conversion of tikz figures to svg: e.g. figures may be incorrectly cropped for the web-page.
  Size of xymatrix figures and tikz-cd figures seemingly then must be adjusted manually.

- When using **tikz** pictures use (note the Alt Text option):

```
\begin{figure}\ThisAltText{Alt text to your diagram}
\begin{tikzpicture}

\end{tikzpicture}
\end{figure}
```

or

```
\begin{center}\ThisAltText{Alt text to your diagram}
\begin{tikzpicture}

\end{tikzpicture}
\end{center}
```

Examples can be found towards the end of this file.

- When using **tikz-cd** use:

```
{\displaymathother
 \ThisAltText{ Alt text to your commutative diagram }
\[
    \begin{tikzcd} F(A) \arrow{r}{F(f)}& F(B) \\
    G(A)\arrow{r}[swap]{G(f)} \arrow{u}{\eta_A} & G(B)
    \arrow{u}[right]{\eta_B}
```

```
      \end{tikzcd}
  \]
  }
```

- It is possible to compile an **Xy-pic diagram** as a **figure with alt text**. Use:

```
{
\displaymathother
\ThisAltText{Write some alt text here.}
$$ \xymatrix{ } $$
}
```

  Examples can be found towards the end of this file.

- **To select your own .CSS files use:**

```
\CSSFilename{your_file.css}
```

- Avoid good old Tex commands like

```
{\bf }, {\it }.
```

  It seems that lwarp does not deal with them properly. Instead use:

```
\textbf{ }, \textit{ }.
```

  (In fact better to avoid *italics* altogether for accessibility reasons: use **bold**.)

- Lwarp gives errors messages when compiling Xy-pic diagrams inside

```
\begin{equation} \end{equation}
```

  Use instead:

```
\[\xymatrix{} \]
or
\[\begin{xy} \xymatrix{} \end{xy}\]
```

  Or use *lwarpmk pdftohtml* if there are compilation errors.

- Figures created with tikz and xymatrix frequently have issues: e.g. they may be too small (so size needs to be adjusted), or incorrectly cropped. Seems to depend on operating system.

5   Some examples of tikz and xymatrix diagrams compiled as svg images, with alternative text

As written in Section 2, it is possible to display xymatrix diagrams by using XyJax-v3. In some cases, it may however be preferable to compile xymatrix diagrams as figures with **Alt Text**. (E.g. it may be sensible to instead provide a comprehensive alternative text to the figure/diagram, instead of relying mainly on the availability of the latex code, for accessibility.)

This option (svg image with alternative text) is also available for tikz and tikz-cd figures. Examples are below.

5.1 An xymatrix diagram compiled as an image with alt text

An *xymatrix* diagram compiled into a picture with alt text. Can utilize equation numbers, with no errors. The 2-cell option can be used.

The size of the svg image must be adjusted. Sometimes the image is not correctly cropped: depends on operating system.

$$
\begin{array}{c}
T \times T \xrightarrow{\ \tau\ } T \times T \\
\end{array}
\tag{3}
$$

5.2 A tikz image compiled as an image with alt text:

Size can be adjusted. Sometimes (depending on operating system, it seems) the image may be incorrectly cropped. See figure 1.
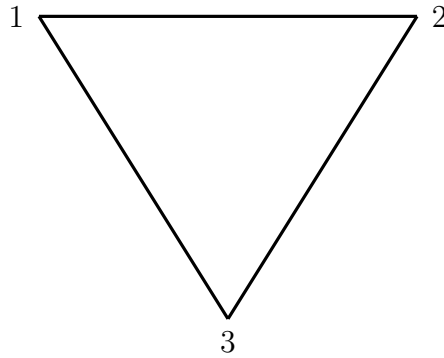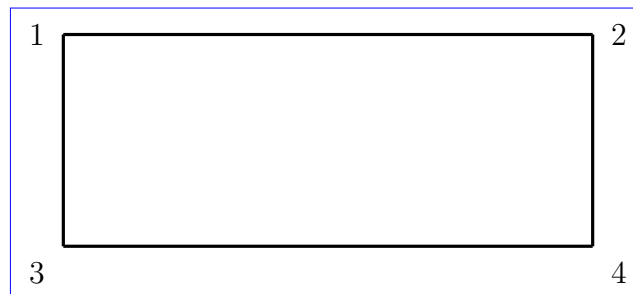
Figure 1: An equilateral triangle with its vertices labeled 1, 2 and 3

5.3 A tikz image compiled to an image with alt text, outside a float environment

5.4 A tikz-cd diagram compiled into a figure with alt text:

Issues with size of diagrams created with tikz-cd. Sometimes (depending on operating system) the image may be incorrectly cropped.

The following diagram commutes.

$$F(A) \xrightarrow{F(f)} F(B)$$

$$\eta_A \uparrow \qquad\qquad \uparrow \eta_B$$

$$G(A) \xrightarrow[G(f)]{} G(B)$$

6 Sample mathematics

6.1 Adjoint functors and coadjoint functors via universal arrows

Let $\mathcal{C}$ and $\mathcal{D}$ be categories. Let $G\colon \mathcal{D} \to \mathcal{C}$ be a functor.

Definition 1. *Let A be an object of $\mathcal{C}$. A **universal arrow** from A to $G\colon \mathcal{D} \to \mathcal{C}$ is a pair:*

$$\left(F_A, A \xrightarrow{\eta_A} G(F_A) \right),$$

where F_A is an object in $\mathcal{D}$ and $\eta_A\colon A \to G(F_A)$ is an arrow in $\mathcal{C}$, such that the following universal property is satisfied:

*For any object of B of $\mathcal{D}$ and any arrow $f\colon A \to G(B)$, in $\mathcal{C}$, there exists a **unique** arrow $\hat{f}\colon F_A \to B$, in $\mathcal{D}$, that makes the following diagram, in $\mathcal{C}$, commute:*

$$
\begin{array}{ccc}
A & \xrightarrow{\eta_A} & G(F_A) \\
 & \searrow^{f} & \downarrow^{G(\hat{f})} \\
 & & G(B)
\end{array}
$$

Exercise 2. *In the conditions of the previous definition, prove that if $\left(F_A, A \xrightarrow{\eta_A} G(F_A) \right)$ is a universal arrow from A to G, then we have a bijection:*

$$\phi_{A,B}\colon \hom_{\mathcal{C}} \left(A, G(B) \right) \longrightarrow \hom_{\mathcal{D}}(F_A, B),$$

such that

$$\left(f\colon A \to G(B) \right) \xmapsto{\phi_{(A,B)}} \left(\hat{f}\colon F_A \to B \right).$$

Moreover, prove that the bijection $\phi_{A,B}$ is natural in B. This means that given any arrow $g\colon B \to C$, in $\mathcal{D}$, the following diagram (in the category of sets) commutes:

$$
\begin{array}{ccc}
\hom_{\mathcal{C}}(A, G(B)) & \xrightarrow{\phi_{A,B}} & \hom_{\mathcal{D}}(F_A, B) \\
\downarrow^{m \mapsto G(g) \circ m} & & \downarrow^{n \mapsto g \circ n} \\
\hom_{\mathcal{C}}(A, G(C)) & \xrightarrow[\phi_{A,C}]{} & \hom_{\mathcal{D}}(F_A, C)
\end{array}
$$

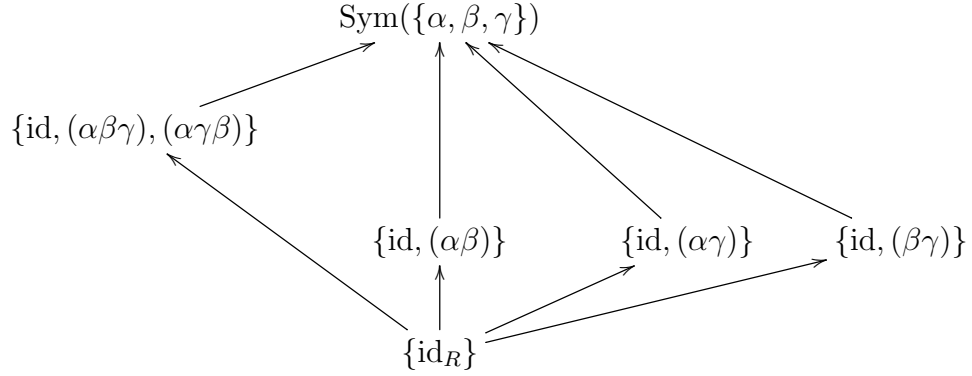## 6.2 The Galois correspondence for $f(t) = t^3 - 2$, over the rational field

Let $f(t) = t^3 - 2 \in \mathbb{Q}[t]$. Let $\omega = e^{\frac{2\pi i}{3}}$. Let $\alpha = \sqrt[3]{2}$, $\beta = \alpha\omega$ and $\gamma = \alpha\omega^2$. Hence the set of roots of f is $\{\alpha, \beta, \gamma\}$. The splitting field of f, over $\mathbb{Q}$ is:

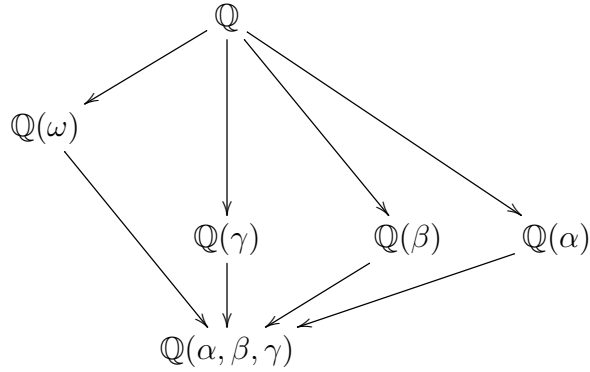$$\mathbb{Q}(\alpha, \beta, \gamma) = \mathbb{Q}(\alpha, \omega).$$

The monomorphism of groups,

$$\theta\colon \Gamma(f,\mathbb{Q}) \longrightarrow \mathrm{Sym}(\{\alpha,\beta,\gamma\})$$
$$\tau \longmapsto \begin{pmatrix} \{\alpha,\beta,\gamma\} \longrightarrow \{\alpha,\beta,\gamma\} \\ a \quad \longmapsto \quad \tau(a) \end{pmatrix},$$

is, in this case, an isomorphism. The diagram of subgroups of $\Gamma(f,\mathbb{Q}) \cong \mathrm{Sym}(\{\alpha,\beta,\gamma\})$ is below (note that inclusions go in the direction of arrows):

The corresponding diagram of intermediate fields $\mathbb{Q} \subseteq L \subseteq \mathbb{Q}(\alpha,\beta,\gamma)$ is:

Also note that $\{\mathrm{id}_R\}$, $\mathrm{Sym}(R)$ and $\{\mathrm{id},(\alpha\beta\gamma),(\alpha\gamma\beta)\} \cong A_3$ are the only normal subgroups of $\mathrm{Sym}(R)$. So, respectively,

$$\mathbb{Q}(\alpha,\beta,\gamma):\mathbb{Q}, \qquad \mathbb{Q}:\mathbb{Q} \quad \text{and} \quad \mathbb{Q}(\omega):\mathbb{Q} \tag{4}$$

are the only normal extensions of $\mathbb{Q}$ contained in $\mathbb{Q}(\alpha,\beta,\gamma)$. Note that the fact that the extensions in (4) are are normal also follows from the fact that they are the splitting fields of $p(t) = t^3 - 2$, $q(t) = t$ and $r(t) = t^2 + t + 1$ (which has ω and ω^2 as roots), over $\mathbb{Q}$.

Remark 3. *Also note that we have a series of subfields of $\mathbb{C}$ (we use $\leq$ to denote subfield):*

$$\mathbb{Q} \leq \mathbb{Q}(\omega) \leq \mathbb{Q}(\alpha,\beta\,\gamma) = \mathbb{Q}(\alpha,\omega).$$

Note that:

- *$\mathbb{Q}(\omega):\mathbb{Q}$ is a normal extension (since it is the splitting field of $t^2 + t + 1$, over $\mathbb{Q}$).*

- *$\mathbb{Q}(\omega,\alpha):\mathbb{Q}(\omega)$ is also a normal extension. This is because $\mathbb{Q}(\omega,\alpha)$ is the splitting field of $t^3 - 2$ over $\mathbb{Q}(\omega)$.*

And then it follows that:

- $\Gamma\big(\mathbb{Q}(\omega, \alpha), \mathbb{Q}(\omega)\big)$ *is normal in* $\Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q})$,

- *we have a series of subgroups of* $\Gamma\big(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}\big)$:

$$\{e\} = \Gamma\big(\mathbb{Q}(\omega, a) : \mathbb{Q}(\omega, \alpha)\big) \trianglelefteq \Gamma\big(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)\big) \trianglelefteq \Gamma\big(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}\big),$$

- *The quotient groups can be explicitly determined:*

  - $\Gamma\big(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}\big)/\Gamma\big(\mathbb{Q}(\omega, a) : \mathbb{Q}(\omega)\big) \cong \Gamma\big(\mathbb{Q}(\omega) : \mathbb{Q}\big) \cong \mathbb{Z}_2.$

    Where the last equation follows since $\mathbb{Q}(\omega) : \mathbb{Q}$ *is a normal extension of degree 2, since it is the splitting field of the irreducible polynomial* $t^2 + t + 1$, *over* $\mathbb{Q}$. *This* $\mathbb{Z}_2$ *is an abelian group.*

  - $\Gamma\big(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)\big)/\Gamma\big(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega, \alpha)\big) \cong \Gamma\big(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)\big) \cong \mathbb{Z}_3.$

    Where the last equation follows since $\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)$ *is a normal extension of degree 3. This is because it is the splitting field of the polynomial* $t^3 - 2$, *which is irreducible over* $\mathbb{Q}(\omega)$. *This is an abelian group.*

What was just shown is a general patern that exists any time we compute the splitting field of a polynomial that is soluble by radicals.