

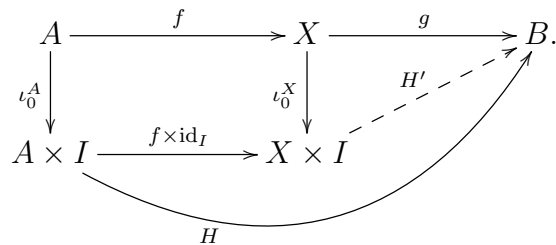
Sample HTML file: produced with lwarp, with mathematical formulae displayed with MathJax, and xymatrix commutative diagrams displayed with XyJax-v3

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The responsibility for this HTML file is João Faria Martins' only.
 Corrections, suggestions, etc. are very welcome, and can be sent to: j.fariamartins@leeds.ac.uk.
This is strictly a non-specialist experimental file.

See the links below for:

- “stand-alone” version of this webpage,
- “split” version of this webpage (with sans serif fonts),

- pdf version of this page,
- Latex source for the stand alone version of this page `Sample_Lwarp_Xy-jax.tex`,
- Latex source for the split version of this page, with sans serif fonts: `Sample_Lwarp_Xy-jax-split.tex`,
- CSS files for the stand-alone version of this page: `lwarp.css` (automatically provided by *lwarpmk*) and `my_file_stand-alone.css` (minor tweak to adjust margins),
- CSS files for the split version of this page: `lwarp_formal.css` (automatically provided by *lwarpmk*) and `my_file_split.css` (minor tweak enforcing sans serif fonts).
- Configuration file required for browsers to find Xy-Jaxv3, `lwarp-with-Xy-jax_v3.txt`. This script is essentially the original Lwarp MathJax emulation code, `lwarp_mathjax.txt` (automatically created by `lwarp v0.902`), with two minimal additions, as shown.

1 Context: using Lwarp to compile latex to HTML, so that xymatrix diagrams are displayed with Xy-Jax

This HTML file (that you are reading) was produced with the `lwarp.sty` package, available with **TeXLive**.¹ In particular the preamble contains:

```
\documentclass[a4paper,12pt]{article}
\usepackage[mathjax]{lwarp}
```

The version of the software was:

```
pdfTeX, Version 3.141592653-2.6-1.40.22 (TeX Live 2022/dev/Debian)
lwarpmk: v0.903 Automated make for the LaTeX Lwarp package.
```

The source latex file can be found here: `Sample_Lwarp_Xy-jax.tex`.

This HTML file also makes use of the following configuration script, so that **Xy-pic** diagrams, particularly *xymatrix* commutative diagrams, can be displayed by using **Xy-Jax-v3**. (Or at least a good sample of them: there seem to be some compatibility issues.)

`lwarp-with-Xy-jax_v3.txt`.

This script file has minimal additions (two in total) in the original Lwarp MathJax emulation code, that is automatically created by `lwarp v0.902`, namely:

`lwarp_mathjax.txt`,

(Two lines were added to the script above, as shown, in order that browsers can find Xy-Jaxv3.) This modification was produced under items 5 and 6 of the licence <https://www.ctan.org/license/lppl1.3> (The LATEX Project Public License 1.3).

Credit is due to instructions in: <https://github.com/sonoisa/XyJax-v3> (the official Xy-Jaxv3 documentation) and in "40.11 `lwarp_mathjax.txt`" of the official Lwarp documentation.

If using Xy-pic diagrams displayed with **XyJax-v3**, insert the following in the preamble of your latex file:

¹See also <https://people.bath.ac.uk/feb/lwarp/lwarp-intro.html>, for a quick introduction on how to use Lwarp, and without which this explanation that you are reading would not have been written.

```
\usepackage[all]{xy}
\MathJaxFilename{lwarp-with-Xy-jax_v3.txt}
```

Compile .tex to .html using:

1. *pdflatex Sample_Lwarp_Xy-jax.tex* (possibly twice) : creates Sample_Lwarp_Xy-jax.pdf (usual pdf file, that can also be shared) and Sample_Lwarp_Xy-jax_html.tex. (And some other aux files.)
2. *lwarpmk html* : converts Sample_Lwarp_Xy-jax_html.tex to Sample_Lwarp_Xy-jax_html.html. And then creates Sample_Lwarp_Xy-jax.html (possibly split into different HTML files if such option is chosen).

Note: Use *lwarpmk html1* to force a recompile.

Note: In case of error, with *lwarpmk html*, use *lwarpmk pdftohtml* to create HTML file.

Note: When using *bibtex* files, then the following is required:

- *bibtex* Sample_Lwarp_Xy-jax_html.aux
- *pdflatex* Sample_Lwarp_Xy-jax_html.tex

The following steps are only needed once (any time tikz figures change). Only needed with tikz.

3. *lwarpmk limages* : If there are images done with tikz, tikz-cd, etc.
4. Repeat 2 (now using *lwarpmk html1*) and 3 (to finish creating the images).

Note. *It is advisable to use a recent version of **TexLive**.*

Warnings:

- The usage of **XyJax-v3** software with lwarp is experimental.
- Not all accessibility features of commutative diagrams displayed with XyJax-v3 work. Among the accessibility features that seem to work well are:
 1. View original latex source.
 2. Zoom maths expression.

2 Examples: xymatrix diagrams displayed with Xy-Jaxv3 using Lwarp

This is an example of a HTML file with:

- Mathematical formulae displayed with **MathJax**:

$$x^2 + y^2 = 1, \tag{1}$$

$$(x \xrightarrow{\gamma} y) \in \pi_1(X, X_0),$$

$$\cdots \rightarrow \pi_i(E_b, x) \xrightarrow{\iota} \pi_i(E, x) \xrightarrow{\partial} \pi_i(B, b) \xrightarrow{\delta} \pi_{i-1}(E_b, x) \rightarrow \cdots \xrightarrow{\iota} \pi_1(E, x) \xrightarrow{\partial} \pi_1(B, b) \xrightarrow{\delta_x} \pi_0(E_b) \xrightarrow{\iota} \pi_0(B)$$

- To see how lecture notes look like with Lwarp, see subsections 6.1 and 6.2.
- This configuration is compatible with AMS-CD commutative diagrams, e.g. this diagram from the amscd package manual:

$$\begin{array}{ccc} A & \xrightarrow[a]{b} & B \\ l \downarrow r & & l \uparrow r \\ C & \xleftarrow[b]{a} & D \end{array}$$

- Commutative diagrams drawn with **Xy-pic** displayed by using **XyJax-v3**.
E.g.

$$\begin{array}{ccc} S_3 & \xrightarrow{f} & S_4 \\ & \searrow g \cong & \downarrow \pi \\ & & S_4/V, \end{array}$$

$$\begin{array}{ccccc} & & Q & & \\ & \swarrow & \downarrow & \searrow & \\ Q(\omega) & & Q(\gamma) & & Q(\beta) \\ & \searrow & \downarrow & \swarrow & \\ & & Q(\alpha, \beta, \gamma) & & Q(\alpha) \end{array}$$

(123).

$$\begin{array}{ccccc} A & \xrightarrow{f} & X & \xrightarrow{g} & B, \\ \iota_0^A \downarrow & & \downarrow \iota_0^X & \nearrow H' & \\ A \times I & \xrightarrow{f \times \text{id}_I} & X \times I & & \\ & \searrow H & & & \end{array}$$

$$\begin{array}{ccc}
\bigoplus_{y \in H_0} \mathbf{H}_1(x, y) \otimes \mathbf{H}'_1(y, z) & \xrightarrow{\text{proj}} & \int^{y \in H_0} \mathbf{H}_1(x, y) \otimes \mathbf{H}'_1(y, z) \\
\downarrow \bigoplus_{y \in H_0} \eta_{(x, y)} \otimes \eta'_{(y, z)} & & \downarrow \int^{y \in Y} \eta_{(x, y)} \otimes \eta'_{(y, z)} \\
\bigoplus_{y \in H_0} \mathbf{H}_2(x, y) \otimes \mathbf{H}'_2(y, z) & \xrightarrow{\text{proj}} & \int^{y \in H_0} \mathbf{H}_2(x, y) \otimes \mathbf{H}'_2(y, z) , \\
\int^{y \in Y} \overline{\mathbf{H}}_{(X, Y)}^{M_1}(-, y) \otimes \overline{\mathbf{H}}_{(Y, Z)}^{M_2}(y, -) & \xrightarrow{\eta_{(X, Y, Z)}^{M_1, M_2}} & \overline{\mathbf{H}}_{(X, Z)}^{M_1 \times_Y M_2} \\
\parallel \begin{array}{c} \overline{2\mathbf{H}}_{(X, Y)}^{\mathbf{W}_1} \bullet \overline{2\mathbf{H}}_{(Y, Z)}^{\mathbf{W}_2} \\ \Downarrow \end{array} & & \parallel \begin{array}{c} \overline{2\mathbf{H}}_{(X, Z)}^{\mathbf{W}_1 \#_0 \mathbf{W}_2} \\ \Downarrow \end{array} \\
\int^{y \in Y} \overline{\mathbf{H}}_{(X, Y)}^{N_1}(-, y) \otimes \overline{\mathbf{H}}_{(Y, Z)}^{N_2}(y, -) & \xrightarrow{\eta_{(X, Y, Z)}^{N_1, N_2}} & \overline{\mathbf{H}}_{(X, Z)}^{N_1 \times_Y N_2} .
\end{array}$$

3 Issues when using XyJax-v3 and Lwarp for typesetting xymatrix diagrams

Please do contact me if you know the solution to any of these issues. Remember: this is stricly a non-specialist file!

- The commands of the form:

`\stackrel{(*)}{\implies}`

or

`\left (x \xrightarrow{f} y \right)`

do not seem to work well inside

`\begin{equation} \end{equation}.`

So use instead, for example,

`$$ \stackrel{(*)}{\implies} $$`

`$$ \left (x \xrightarrow{f} y \right) $$`

. Here is how it looks:

$$\begin{array}{c}
\stackrel{(*)}{\implies} \\
\left(x \xrightarrow{f} y \right) .
\end{array}$$

- Sometimes there are errors if there are mathematical commands in the title of sections, subsections, etc.
- Lwarpmk gives error messages if it finds *xymatrix* inside:

`\begin{equation} \end{equation}.`

(However a html file can still be produced with *lwarpmk pdftohtml*.)

So avoid:

`\begin{equation} \xymatrix{ } \end{equation},`

and use instead:

`\[\xymatrix{ } \],`

or

`$$ \xymatrix{ } $$.`

- The 2-cell option does not appear to work with Xy-Jaxv3. However xymatrix diagrams can still be compiled as svg figures, with alt text; see 5.1.

$$\begin{array}{ccc}
 T \times T & \xrightarrow{\tau} & T \times T \\
 & \searrow \sqcup & \downarrow \sqcup \\
 & & T.
 \end{array}$$

- **Not all accessibility functions of MathJax work with Xy-Jaxv3, only some can be used, e.g. zoom and source latex code.**
Accessibility features of Xy-jax are not officially supported.

- References to section, chapters etc, have glitches **if inside mathematics environments**. For instance, the links provided do not appear to work well. E.g.

$$\exp(x + yi) = e^x (\cos(y) + i \sin(y)) \quad (2)$$

$e^{i\pi} + 1 = (\cos(\pi) + i \sin(\pi)) + 1$, using Equation (2). (Just for testing) equations (1), (3)
= 0. A reference to second section 2

Outside maths environments all seems to work fine:

Using Equation (2). (Just for testing) equations (1), (3)
A reference to second section 2

- When compiling xymatrix, or tikz, diagrams as sgv figures, with Alt-Text (which sometimes is necessary) it sometimes happens that the figure is not cropped correctly. This seems to depend strongly on the operating system being used.

4 More general notes

- **pdf** and **HTML** automatically coexist: here is the pdf version of this HTML file `Sample_Lwarp_Xy-jax.pdf`
- It is possible to split an HTML page into sub-pages: credit <https://people.bath.ac.uk/feb/lwarp/lwarp-intro.html>. Instructions can be seen in the source latex file of this HTML file `Sample_Lwarp_Xy-jax.tex`.
- Additions are required in the .tex file in order that **MathJax** displays commands, macros and definitions correctly. E.g. write:

```
\DeclareMathOperator{\Sym}{Sym}
\def \Mon {{\mathbf{Mon}}}
\newcommand{\Q}{{\mathbb{Q}}}
```

and then:

```
\begin{warpHTML}
\CustomizeMathJax{\DeclareMathOperator{\Sym}{Sym}}
\CustomizeMathJax{\def \Mon {{\mathbf{Mon}}}}
\CustomizeMathJax{\newcommand{\Q}{{\mathbb{Q}}}}
\end{warpHTML}
```

This means that you can use slightly different versions of commands for pdf and for html.

- If using Xy-pic diagrams (if displayed as Xy-jax) put in the preamble of the tex file:

```
\MathJaxFilename{lwarp-with-Xy-jax_v3.txt}
```

You will also need this configuration file: `lwarp-with-Xy-jax_v3.txt`. (Explanation is in Section 1.)

- Sample use of Xy-pic compatible with lwarp (so that the resulting xymatrix code is readable by Xy-Jax): E.g.:

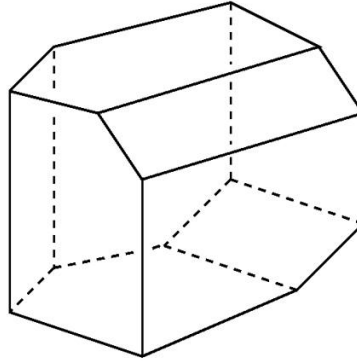
```
\[
\begin{matrix}
& \& \Q \& \& \Q(\backslash w) \ar@{<-}[ur] \& \\
& \& \Q(\backslash g) \ar@{<-}[uu] \& \Q(\backslash b) \ar@{<-}[uul] \& \Q(\backslash a) \ar@{<-}[uull] \& \\
& \& \Q(\backslash a, \backslash b, \backslash g) \ar@{<-}[uul] \ar@{<-}[u] \ar@{<-}[ur] \ar@{<-}[urr] \\
& \& \\
\end{matrix},
```

or

```
\displaymathnormal{$$$
\begin{matrix}
& S_3 \ar@{^{(->}}[r] \& f \ar[dr]_{\{g\}^{\backslash \text{cong}}} \& S_4 \ar@{->>}[d] \& \pi \\
& \& S_4/V, \\
& \\
\end{matrix}
$$$}.
```

- Figures, with alternative text, can be included like this:

```
\includegraphics[width = 0.4\textwidth,
alt={Type your alternative text here}]{Stasheff.png}
```



- Figures, including **tikz** and **tikz-cd** figures, can be compiled as figures with **Alt Text**. (The same option is also available for xymatrix diagrams, and may be preferable to Xy-Jaxv3, in the cases when it is sensible to provide a comprehensive alternative text to the figure/diagram, instead of relying mainly on the availability of the latex code, for accessibility.)

Warning: In some operating systems, issues seem to exist with the conversion of tikz figures to svg: e.g. figures may be incorrectly cropped for the web-page. Size of xymatrix figures and tikz-cd figures seemingly then must be adjusted manually.

- When using **tikz** pictures use (note the Alt Text option):

```
\begin{figure}\ThisAltText{Alt text to your diagram}
\begin{tikzpicture}

\end{tikzpicture}
\end{figure}
```

or

```
\begin{center}\ThisAltText{Alt text to your diagram}
\begin{tikzpicture}

\end{tikzpicture}
\end{center}
```

Examples can be found towards the end of this file.

- When using **tikz-cd** use:

```
{\displaymathother
\ThisAltText{ Alt text to your commutative diagram }
\[
\begin{tikzcd} F(A) \arrow{r}{F(f)} & F(B) \\
G(A) \arrow{r}[swap]{G(f)} & \arrow{u}{\eta_A} & G(B) \\
\arrow{u}[right]{\eta_B} \end{tikzcd}
```



```

\end{tikzcd}
\]
}

```

- It is possible to compile an **Xy-pic diagram** as a **figure with alt text**. Use:

```

{
\displaymathother
\ThisAltText{Write some alt text here.}
$$ \xymatrix{ } $$
}

```

Examples can be found towards the end of this file.

- To select your own .CSS files use:

```
\CSSFilename{your_file.css}
```

- Avoid good old Tex commands like

```
{\bf }, {\it }.
```

It seems that lwarp does not deal with them properly. Instead use:

```
\textbf{ }, \textit{ }.
```

(In fact better to avoid *italics* altogether for accessibility reasons: use **bold**.)

- Lwarp gives errors messages when compiling Xy-pic diagrams inside

```
\begin{equation} \end{equation}
```

Use instead:

```

\[ \xymatrix{ } \]
or
\[ \begin{xy} \xymatrix{ } \end{xy} \]

```

Or use *lwarpmk pdftohtml* if there are compilation errors.

- Figures created with tikz and xymatrix frequently have issues: e.g. they may be too small (so size needs to be adjusted), or incorrectly cropped. Seems to depend on operating system.

5 Some examples of tikz and xymatrix diagrams compiled as svg images, with alternative text

As written in Section 2, it is possible to display xymatrix diagrams by using XyJax-v3. In some cases, it may however be preferable to compile xymatrix diagrams as figures with **Alt Text**. (E.g. it may be sensible to instead provide a comprehensive alternative text to the figure/diagram, instead of relying mainly on the availability of the latex code, for accessibility.)

This option (svg image with alternative text) is also available for tikz and tikz-cd figures. Examples are below.

5.1 An xymatrix diagram compiled as an image with alt text

An *xymatrix* diagram compiled into a picture with alt text. Can utilize equation numbers, with no errors. The 2-cell option can be used.

The size of the svg image must be adjusted. Sometimes the image is not correctly cropped: depends on operating system.

$$\begin{array}{ccc}
 T \times T & \xrightarrow{\tau} & T \times T \\
 & \searrow \sqcup & \downarrow \sqcup \\
 & & T
 \end{array}
 \quad \text{with a 2-cell } \mathbf{R} \text{ from } T \times T \text{ to } T
 \tag{3}$$

5.2 A tikz image compiled as an image with alt text:

Size can be adjusted. Sometimes (depending on operating system, it seems) the image may be incorrectly cropped. See figure 1.

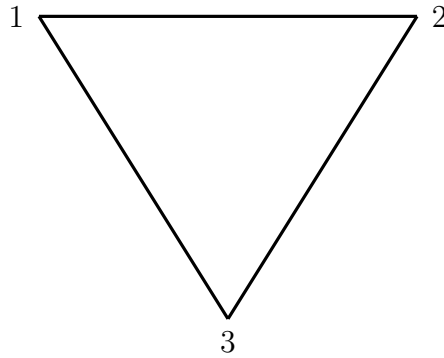
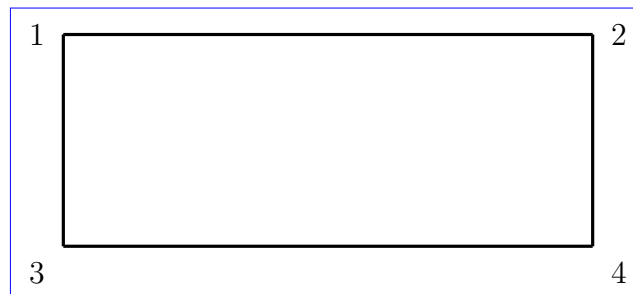


Figure 1: An equilateral triangle with its vertices labeled 1, 2 and 3

5.3 A tikz image compiled to an image with alt text, outside a float environment



5.4 A tikz-cd diagram compiled into a figure with alt text:

Issues with size of diagrams created with tikz-cd. Sometimes (depending on operating system) the image may be incorrectly cropped.

The following diagram commutes.

$$\begin{array}{ccc}
F(A) & \xrightarrow{F(f)} & F(B) \\
\eta_A \uparrow & & \uparrow \eta_B \\
G(A) & \xrightarrow{G(f)} & G(B)
\end{array}$$

6 Sample mathematics

6.1 Adjoint functors and coadjoint functors via universal arrows

Let \mathcal{C} and \mathcal{D} be categories. Let $G: \mathcal{D} \rightarrow \mathcal{C}$ be a functor.

Definition 1. Let A be an object of \mathcal{C} . A **universal arrow** from A to $G: \mathcal{D} \rightarrow \mathcal{C}$ is a pair:

$$(F_A, A \xrightarrow{\eta_A} G(F_A)),$$

where F_A is an object in \mathcal{D} and $\eta_A: A \rightarrow G(F_A)$ is an arrow in \mathcal{C} , such that the following universal property is satisfied:

For any object of B of \mathcal{D} and any arrow $f: A \rightarrow G(B)$, in \mathcal{C} , there exists a **unique** arrow $\hat{f}: F_A \rightarrow B$, in \mathcal{D} , that makes the following diagram, in \mathcal{C} , commute:

$$\begin{array}{ccc}
A & \xrightarrow{\eta_A} & G(F_A) \\
& \searrow f & \downarrow G(\hat{f}) \\
& & G(B)
\end{array}$$

Exercise 2. In the conditions of the previous definition, prove that if $(F_A, A \xrightarrow{\eta_A} G(F_A))$ is a universal arrow from A to G , then we have a bijection:

$$\phi_{A,B}: \text{hom}_{\mathcal{C}}(A, G(B)) \longrightarrow \text{hom}_{\mathcal{D}}(F_A, B),$$

such that

$$(f: A \rightarrow G(B)) \xrightarrow{\phi_{(A,B)}} (\hat{f}: F_A \rightarrow B).$$

Moreover, prove that the bijection $\phi_{A,B}$ is natural in B . This means that given any arrow $g: B \rightarrow C$, in \mathcal{D} , the following diagram (in the category of sets) commutes:

$$\begin{array}{ccc}
\text{hom}_{\mathcal{C}}(A, G(B)) & \xrightarrow{\phi_{A,B}} & \text{hom}_{\mathcal{D}}(F_A, B) \\
\downarrow m \mapsto G(g) \circ m & & \downarrow n \mapsto g \circ n \\
\text{hom}_{\mathcal{C}}(A, G(C)) & \xrightarrow{\phi_{A,C}} & \text{hom}_{\mathcal{D}}(F_A, C)
\end{array}$$

6.2 The Galois correspondence for $f(t) = t^3 - 2$, over the rational field

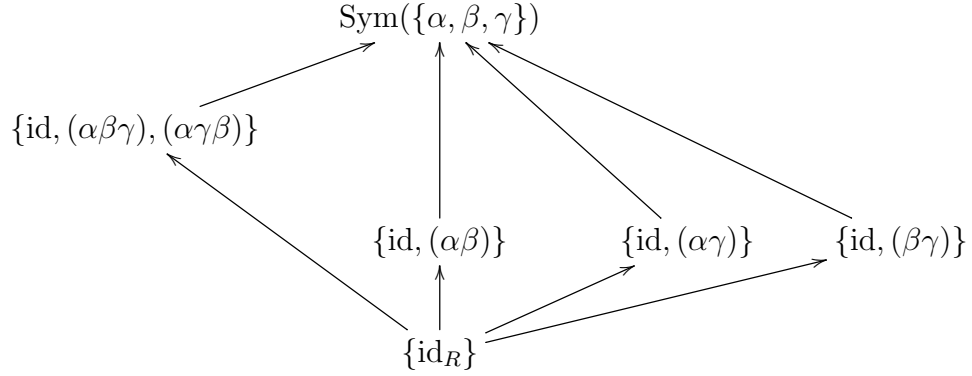
Let $f(t) = t^3 - 2 \in \mathbb{Q}[t]$. Let $\omega = e^{\frac{2\pi i}{3}}$. Let $\alpha = \sqrt[3]{2}$, $\beta = \alpha\omega$ and $\gamma = \alpha\omega^2$. Hence the set of roots of f is $\{\alpha, \beta, \gamma\}$. The splitting field of f , over \mathbb{Q} is:

$$\mathbb{Q}(\alpha, \beta, \gamma) = \mathbb{Q}(\alpha, \omega).$$

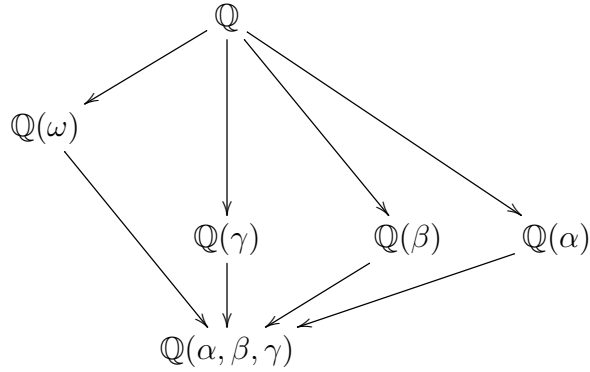
The monomorphism of groups,

$$\begin{aligned} \theta: \Gamma(f, \mathbb{Q}) &\longrightarrow \text{Sym}(\{\alpha, \beta, \gamma\}) \\ \tau &\longmapsto \left(\begin{array}{ccc} \{\alpha, \beta, \gamma\} & \longrightarrow & \{\alpha, \beta, \gamma\} \\ a & \longmapsto & \tau(a) \end{array} \right), \end{aligned}$$

is, in this case, an isomorphism. The diagram of subgroups of $\Gamma(f, \mathbb{Q}) \cong \text{Sym}(\{\alpha, \beta, \gamma\})$ is below (note that inclusions go in the direction of arrows):



The corresponding diagram of intermediate fields $\mathbb{Q} \subseteq L \subseteq \mathbb{Q}(\alpha, \beta, \gamma)$ is:



Also note that $\{\text{id}_R\}$, $\text{Sym}(R)$ and $\{\text{id}, (\alpha\beta\gamma), (\alpha\gamma\beta)\} \cong A_3$ are the only normal subgroups of $\text{Sym}(R)$. So, respectively,

$$\mathbb{Q}(\alpha, \beta, \gamma) : \mathbb{Q}, \quad \mathbb{Q} : \mathbb{Q} \quad \text{and} \quad \mathbb{Q}(\omega) : \mathbb{Q} \quad (4)$$

are the only normal extensions of \mathbb{Q} contained in $\mathbb{Q}(\alpha, \beta, \gamma)$. Note that the fact that the extensions in (4) are normal also follows from the fact that they are the splitting fields of $p(t) = t^3 - 2$, $q(t) = t$ and $r(t) = t^2 + t + 1$ (which has ω and ω^2 as roots), over \mathbb{Q} .

Remark 3. Also note that we have a series of subfields of \mathbb{C} (we use \leq to denote subfield):

$$\mathbb{Q} \leq \mathbb{Q}(\omega) \leq \mathbb{Q}(\alpha, \beta\gamma) = \mathbb{Q}(\alpha, \omega).$$

Note that:

- $\mathbb{Q}(\omega) : \mathbb{Q}$ is a normal extension (since it is the splitting field of $t^2 + t + 1$, over \mathbb{Q}).
- $\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)$ is also a normal extension. This is because $\mathbb{Q}(\omega, \alpha)$ is the splitting field of $t^3 - 2$ over $\mathbb{Q}(\omega)$.

And then it follows that:

- $\Gamma(\mathbb{Q}(\omega, \alpha), \mathbb{Q}(\omega))$ is normal in $\Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q})$,
- we have a series of subgroups of $\Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q})$:

$$\{e\} = \Gamma(\mathbb{Q}(\omega, a) : \mathbb{Q}(\omega, \alpha)) \trianglelefteq \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)) \trianglelefteq \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}),$$

- The quotient groups can be explicitly determined:

$$- \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}) / \Gamma(\mathbb{Q}(\omega, a) : \mathbb{Q}(\omega)) \cong \Gamma(\mathbb{Q}(\omega) : \mathbb{Q}) \cong \mathbb{Z}_2.$$

Where the last equation follows since $\mathbb{Q}(\omega) : \mathbb{Q}$ is a normal extension of degree 2, since it is the splitting field of the irreducible polynomial $t^2 + t + 1$, over \mathbb{Q} . This \mathbb{Z}_2 is an abelian group.

$$- \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)) / \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega, \alpha)) \cong \Gamma(\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)) \cong \mathbb{Z}_3.$$

Where the last equation follows since $\mathbb{Q}(\omega, \alpha) : \mathbb{Q}(\omega)$ is a normal extension of degree 3. This is because it is the splitting field of the polynomial $t^3 - 2$, which is irreducible over $\mathbb{Q}(\omega)$. This is an abelian group.

What was just shown is a general pattern that exists any time we compute the splitting field of a polynomial that is soluble by radicals.