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PMT 07 - Lista de exercícios 9

Equações governantes:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\partial p}{\partial x} \quad \text{--- (1)}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\cdot\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\partial p}{\partial y} \quad \text{--- (2)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{--- (3)}$$

Derivando (1) em relação a y:

$$\rho\cdot\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\cdot\frac{\partial}{\partial y}\nabla^2 u-\frac{\partial^2 p}{\partial x\partial y}$$

$$\underbrace{\rho\cdot\frac{\partial^2 u}{\partial y\partial t}}_{\text{Termo transiente}}+\underbrace{\rho\left(\frac{\partial u}{\partial y}\cdot\frac{\partial u}{\partial x}+u\cdot\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial v}{\partial y}\cdot\frac{\partial u}{\partial y}+v\cdot\frac{\partial^2 u}{\partial y^2}\right)}_{\text{Termos convectivos}}=\underbrace{\mu\cdot\frac{\partial}{\partial y}\nabla^2 u}_{\text{Termos difusivos}}-\frac{\partial^2 p}{\partial x\partial y} \quad \text{--- (4)}$$

Derivando (2) em relação a x:

$$\rho\cdot\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t}+u\cdot\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\mu\cdot\frac{\partial}{\partial x}\nabla^2 v-\frac{\partial^2 p}{\partial x\partial y}$$

$$\underbrace{\rho\cdot\frac{\partial^2 v}{\partial x\partial t}}_{\text{Termo transiente}}+\underbrace{\rho\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+u\cdot\frac{\partial^2 v}{\partial x^2}+\frac{\partial v}{\partial x}\cdot\frac{\partial v}{\partial y}+v\cdot\frac{\partial^2 v}{\partial x\partial y}\right)}_{\text{Termos convectivos}}=\underbrace{\mu\cdot\frac{\partial}{\partial x}\nabla^2 v}_{\text{Termos difusivos}}-\frac{\partial^2 p}{\partial x\partial y} \quad \text{--- (5)}$$

Subtraindo (5) de (4)

→ Termos transientes:

$$\rho\frac{\partial^2 u}{\partial y\partial t}-\rho\frac{\partial^2 v}{\partial x\partial t}=\rho\frac{\partial}{\partial t}\overbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}^{\omega}=\underline{\rho\frac{\partial \omega}{\partial t}}$$

→ Termos Convectivos

$$\rho\left(\frac{\partial u}{\partial y}\frac{\partial u}{\partial x}+u\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial v}{\partial y}\frac{\partial u}{\partial y}+v\frac{\partial^2 u}{\partial y^2}\right)-\rho\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+u\frac{\partial^2 v}{\partial x^2}+\frac{\partial v}{\partial x}\cdot\frac{\partial v}{\partial y}+v\frac{\partial v^2}{\partial x\partial y}\right)$$

$$=\rho\left[u\left(\frac{\partial^2 u}{\partial y\partial x}-\frac{\partial^2 v}{\partial x^2}\right)+v\left(\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 v}{\partial x\partial y}\right)+\frac{\partial u}{\partial y}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial u}{\partial y}-\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}-\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right]$$

$$=\rho\left[u\cdot\frac{\partial}{\partial x}\underbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}_{\omega}+v\frac{\partial}{\partial y}\underbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}_{\omega}+\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\frac{\partial v}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]$$

$$=\rho\left[u\cdot\frac{\partial \omega}{\partial x}+v\cdot\frac{\partial \omega}{\partial y}+\underbrace{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}_{\substack{=0\\(\text{Eq. Continuidade})}}\underbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}_{\omega}\right]$$

$$=\underline{\rho\left(u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}\right)}$$

→ Termos Difusivos

$$\mu\cdot\frac{\partial}{\partial y}\nabla^2 u-\mu\cdot\frac{\partial}{\partial x}\nabla^2 v=\mu\cdot\nabla^2\overbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}^{\omega}=\underline{\mu\nabla^2\omega^2}$$

→ Termos de pressão:

$$\frac{-\partial^2 p}{\partial x\partial y}-\left(\frac{-\partial^2 p}{\partial x\partial y}\right)=\underline{0}$$

A equação resultante da subtração se torna:

$$\rho \frac{\partial w}{\partial t} + \rho \cdot u \frac{\partial w}{\partial x} + \rho \cdot v \cdot \frac{\partial w}{\partial y} = \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$