What stochastic processes reasonably captures commodity price dynamics

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Abstract

This article explores stochastic models for commodity prices, emphasizing spot and forward dynamics under market realities like seasonality, storage costs, and supply shocks. We compare Ornstein-Uhlenbeck, jump-diffusion, and multi-factor models, highlighting their ability to capture key features such as fat tails, mean reversion, and the Samuelson effect.

Preface

I´ve come across a question I would like to discuss/explore on a fairly “new” topic for me – *commodities.*

The question came from “what model I should use for a commodity producer which assumes the firm's cash flows to vary stochastically according to some sensible process.”

I am deeply happy for you to be here reading this new series on my adventure.

Introduction

How can a commodity be defined? The answer depends on the perspective taken. An economist might describe it as a *consumption good* whose scarcity and has significant consequences for both global and national economic development. A banker, on the other hand, would note that a commodity is *not a financial asset*; it does not generate a stream of predictable cash flows and thus cannot be valued using standard net present value techniques. From an ecologist's viewpoint, a commodity is a *natural resource* whose original state should be protected and preserved, emphasizing its environmental and ecological significance. An academic might argue that in an era of high currency volatility, even among the most stable currencies, a commodity serves as a reliable numéraire—a reference unit against which portfolio values can be measured.

Let´s note some assumptions or key properties for commodity markets:

- Commodity spot prices are defined by the intersections of supply and demand curves in each location, as opposed to the net present value of receivable cash flows.
- Demand for commodities is generally inelastic to prices, given the indispensable nature of the good. Inventories when they exist in sufficient volumes allow a smooth balance of supply and demand over time to be created.
- Physical transactions – which were the only ones prior to the introduction of financial trades – still have a crucial importance today. Among other virtues, they provide a reference spot price or index against which derivative transactions are financially settled.

- Supply is defined by production and inventory. But, in the case of energy commodities, underground reserves also play a role since they have an impact on long-term prices.
- Financial transactions (Forwards, Futures, Options) represent today a huge volume. They involve prices closely related to spot prices because physical delivery is a choice that is left to the buyer. Consequently, the understanding of spot markets and their characteristics is a necessary step in the analysis of commodities and commodity derivatives.
- For most energy commodities, the balancing of supply and demand now takes place both at the regional level and at the world level. This explains the explosion of shipping and freight markets and the emergence of new trading strategies such as the rerouting of an LNG (Liquid Natural Gas) tanker to countries where gas prices exhibit a momentary spike.

Let´s now discuss the modeling of the dynamics of commodity spot prices and forward curves.

Some *desirable* model features

When modelling commodity prices especially oil prices it becomes quite different to do so against equity returns, particularly because commodity prices are influenced by physical supply and demand dynamics, storage costs, convenience yield—factors that are either absent or far less influential in equity markets.
One could attempt to extract forward-looking information about the market's perception of future spot prices from the forward curve. However, under the risk-neutral measure $\mathbb{Q}$, this interpretation does not strictly hold. In a frictionless and arbitrage-free world, the forward price reflects the spot price adjusted for storage costs, risk-free interest rate, and the convenience yield.

$$F_0 = S_0 \times e^{(r+u-y)T} \tag{1.1}$$

Many accepted alternatives to GBM for modeling commodities prices use variations of exponential Ornstein-Uhlenbeck processes (Gibson and Schwartz (1990), Schwartz (1997), Smith and Schwartz (July 2000), etc.).

One could assume the randomness of future, spot and forward prices, although true, I think we could make this assumption a bit "better" by claiming they *random-like behavior*.

Commodity	Underlying	Source deviation	Mean	Standard	Skewness	Kurtosis
Crude oil	Nearby Futures	NYMEX	0.0794	0.3507	0.0832	6.2057
Brent	Nearby Futures	IPE	0.0803	0.3325	−0.1647	6.0807
Natural gas	Nearby Futures	NYMEX	0.1197	0.666	−1.0125	30.7429
Heating fuel	Nearby Futures	NYMEX	0.0789	0.3457	−0.9507	11.1104
Unleaded gasoline	Nearby Futures	NYMEX	0.0654	0.3412	−0.3282	4.7057
Corn	Nearby Futures	CBOT	−0.0136	0.2599	−2.9179	51.2049
Soybeans	Nearby Futures	CBOT	−0.0451	0.2261	−1.2871	19.0947
Soymeal	Nearby Futures	CBOT	−0.0232	0.2571	−1.116	15.782
Soy oil	Nearby Futures	CBOT	−0.0643	0.2053	−0.0798	5.0402
Wheat	Nearby Futures	CBOT	−0.0141	0.3072	−0.7754	59.6257
Oats	Nearby Futures	CBOT	−0.0396	0.3292	0.3196	23.5515
Coffee	Nearby Futures	CSCE	0.026	0.4806	0.4458	10.1014
Aluminum	Spot	LME	0.0239	0.1854	−0.1132	6.0372
Copper	Spot	LME	−0.0066	0.2442	−0.3599	7.5366
Zinc	Spot	LME	0.0236	0.2161	−0.8367	12.4078
Nickel	Spot	LME	0.0476	0.2831	−0.0768	5.3854
Tin	Spot	LME	0.0037	0.1849	−0.3114	6.7626
Lead	Spot	LME	0.0277	0.2607	0.125	6.0316

Figure 1 - First four moments of commodity price returns over the period July 1993–November 2000

Commodity prices exhibit significantly higher annualized volatility than equities or interest rates, with natural gas and coffee among the most volatile due to storage limitations and weather risks. In contrast, metals are less volatile, as they are easier to store. Additionally, all commodities return distributions display excess kurtosis, indicating fat tails and a higher likelihood of extreme price movements.

Stochastic processes, traditionally applied to equity and interest rate markets, can be adapted to commodity markets, which have unique features like seasonality, storage costs, and convenience yields. A single commodity spot price is treated as a stochastic process $S(t)$, with modeling goals structured around three core challenges:

- Choosing an appropriate mathematical process (e.g., GBM) that:
  - Matches the empirical distribution of historical prices (particularly higher moments like skewness and kurtosis).
  - Reflects the dynamic properties of price changes over time intervals.
- Estimating model parameters from market data once the stochastic "backbone" is selected. These parameters (e.g., volatility, mean-reversion rate) must be inferred

through calibration techniques such as maximum likelihood or least squares, requiring access to liquid markets and high-quality data.

- Ensuring robust estimation, particularly when models are complex (e.g., involving multiple parameters), by leveraging all available observations. The text underscores that parameter estimation improves significantly with data richness, and that disciplined use of market information is essential for model reliability.

Modeling commodity prices is inherently complex due to their random evolution over time and the need to account not only for spot prices $S(t)$ but also for the entire forward curve $\{F^T(t)\}_{T>t}$, which includes an infinite number of *tenors.*

A crucial requirement for any realistic model is that it must be arbitrage-free across the forward curve. The model must be constructed under the economic constraint that no arbitrage opportunities should exist between forward prices of different maturities. In well-functioning markets, arbitrageurs act quickly to eliminate any such mispricings, restoring prices to their fair values between F^{T_1} and F^{T_2}.[1]

This requirement increases the complexity of modeling but is necessary to ensure market realism. Regardless of the market's efficiency, if a model leaves arbitrage opportunities open, traders will exploit them, and the model will yield invalid or misleading results.

[1] Being arbitrage-free does not make things simpler, quite the opposite. When one builds a model, there will *at any time* exist at least one market participant to take advantage of the arbitrage opportunities that a firm is leaving open through an inappropriate model.

Models

Ornstein-Uhlenbeck Process

The Ornstein-Uhlenbeck process is a classical mean-reverting model originally developed for interest rate modeling (Vasicek 1977), but it can also be applied in commodity markets, especially for assets that can exhibit *negative prices*[2] and avoiding the unrealistic behavior of Brownian motion and Geometric Brownian motion, which allow for a drift to infinity. In contrast to stock prices which grow on average – since the investor is rewarded for the time value of his money incremented by a risk premium – commodity prices do not generally exhibit trends over long periods (let's remove oil for this last statement).

$$dS_t = k\,(\theta - S_t)dt + \sigma dW_t \tag{1.2}$$

Where k is the speed of mean reversion, θ the long-term mean level; k, θ and σ are positive constants. This formulation ensures that $S_t < \theta$, the expected change is positive and if $S_t > \theta$ then the expected change will be negative, thus enforcing reversion toward a long-term average.

Commodities also often display mean-reverting behavior, reflecting their economic nature: prices tend to fluctuate around marginal production costs rather than growing indefinitely. This behavior has been widely documented and already incorporated in the literature.

So, in order for prices to remain positive we apply the *log* to (1.2):

$$\frac{dS_t}{S_t} = k\,(\theta - \ln(S_t))dt + \sigma dW_t \tag{1.3}$$

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[2] In April 2020, WTI oil futures turned negative for the first time in history (−\$37.63/barrel) due to a collapse in demand from COVID-19 lockdowns and a simultaneous supply glut. This extreme movement occurred as traders exited positions to avoid physical delivery—a common practice around futures roll dates—yet it was significantly amplified by sharply falling demand and a global shortage of storage capacity.

Under $\mathbb{Q}$, the forward price $F(t,T)$ becomes:

$$F(t,T) = \exp\left\{e^{-k(T-t)}\ln(S_t) + \left(1 - e^{-k(T-t)}\right)\left(L - \frac{\sigma^2}{2k}\right) + \frac{\sigma^2}{4k}(1 - e^{2k(T-t)}\right\} \quad (1.4)$$

Differentiating (1.4) we get the volatility of $F(t,T)$:

$$\sigma_F\,(t,T) = \sigma\, e^{-k(T-t)} \quad (1.5)$$

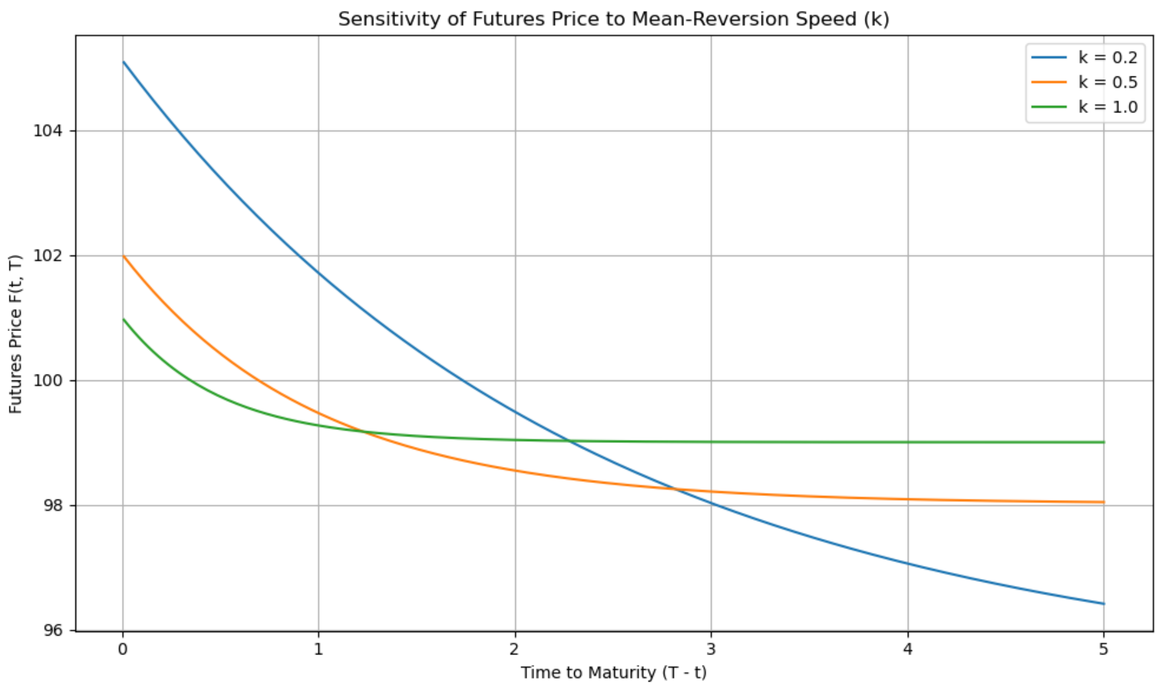


Figure 2 Sensitivity of future prices to k

Here´s a demonstration on the sensitivity of the futures price $F(t,T)$ to different values of the mean-reversion speed k.

- Higher k values (faster mean reversion) cause the futures price to approach the long-term mean more rapidly.
- Slower mean reversion (lower k) results in more persistence of the current spot price over time.

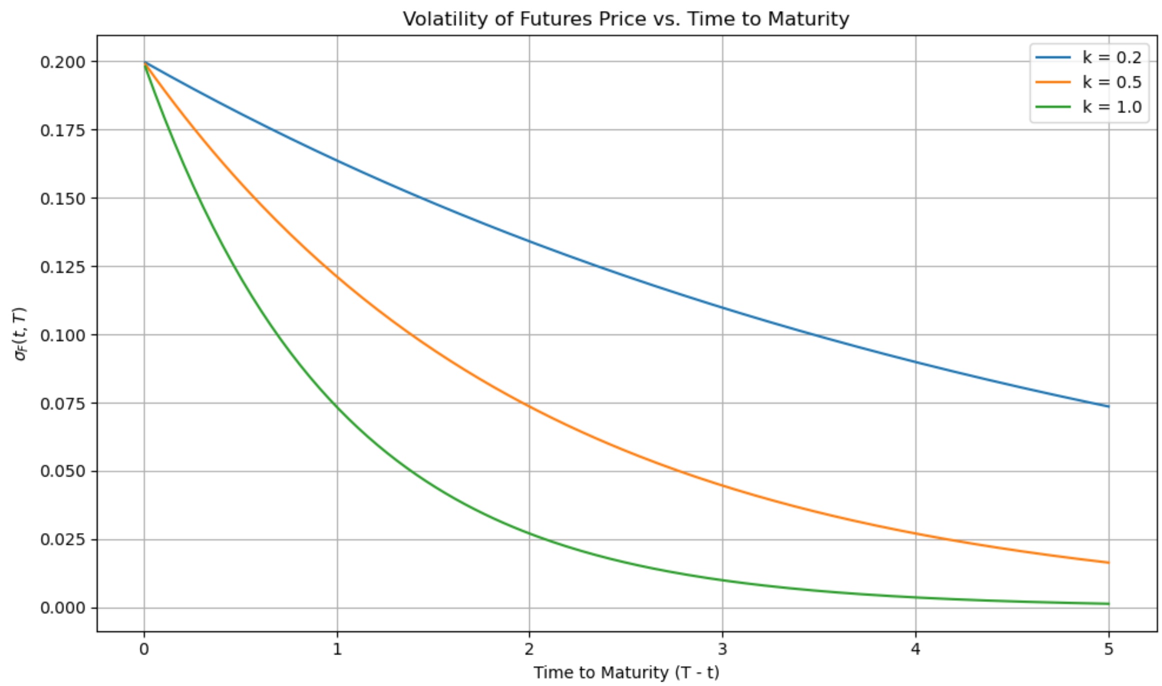


Figure 3 Volatility of future prices to T

This illustration is essentially known as the *Samuelson effect* on how the volatility of the futures price $\sigma F(t, T)$ varies with time to maturity for different speeds of mean reversion k:

- As $(T - t)$ increases, volatility decreases, illustrating the Samuelson effect.
- Faster mean reversion $(k = 0.1)$ causes volatility to decay more quickly than slower mean reversion $(k = 0.2)$.

Equation (1.5) implies that forward contract volatility decreases for long maturities (fig.3), which agrees with the Samuelson effect. However, this volatility goes to zero for long times to maturity, a property which is clearly a limit of the model since this is not observed in practice.

Seasonality

What about commodities that exhibit clear seasonal patterns in their prices. To model this, let´s combine mean-reversion and seasonality under $\mathbb{P}$.

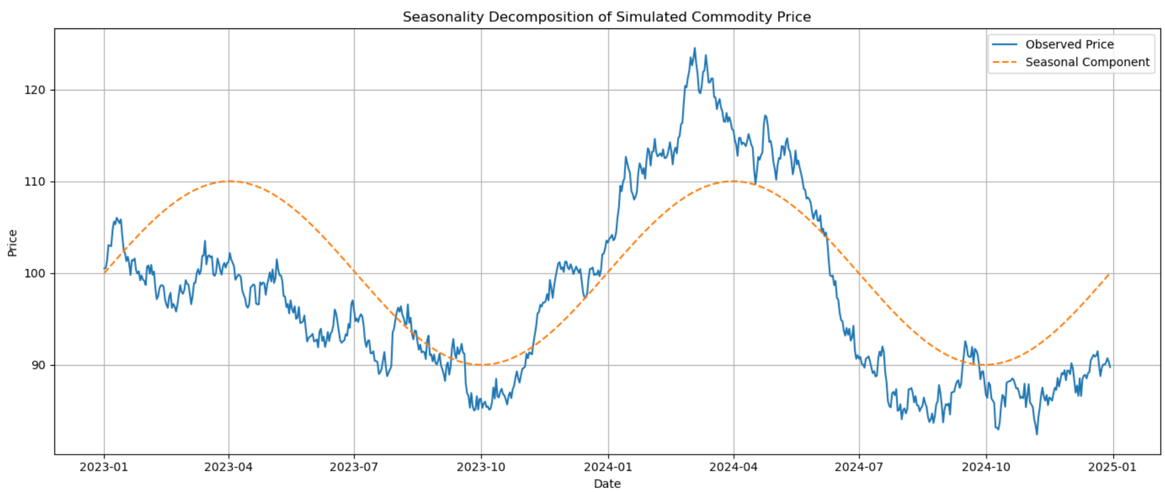


Figure 4 Seasonality decomposition of simulated commodity price

The model assumes the logarithm of the spot price $\ln(S_T)$:

$$\ln(S_T) = f(t) + X(t)$$

Where:

- $f(t)$: is a deterministic seasonal function, often sinusoidal (e.g., sin or cos), capturing periodic behavior.
- $X(t)$: is a mean-reverting Ornstein-Uhlenbeck process:

$$dX(t) = \big(\alpha - \beta X(t)\big)dt + \sigma dW_t \tag{1.6}$$

  ensuring reversion to the marginal cost of production.

This decomposition ensures positive prices, Analytical tractability, and the ability to separately capture seasonality and stochastic noise.

All parameters—α, β, σ—and the function $f(t)$ are empirically estimated from historical spot price data.

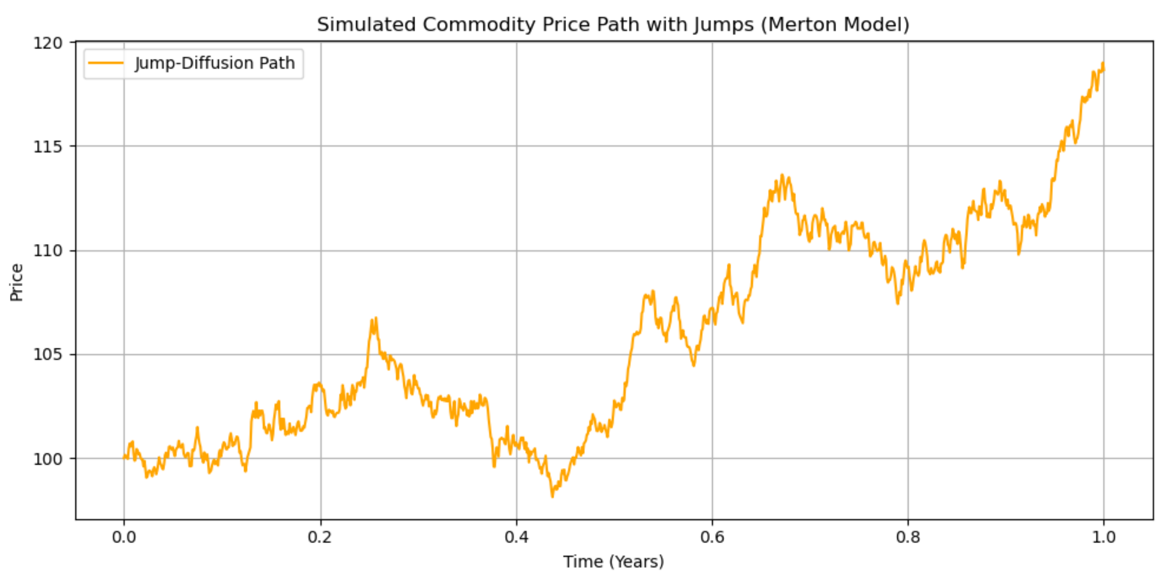
Jumps

To address key limitations of the Geometric Brownian Motion (GBM) model, particularly its inability to capture sudden**,** large price changes often observed in real markets. While GBM assumes continuous paths, empirical evidence, such as the 1987 stock market crash or sudden oil price spikes, shows that asset prices can experience discontinuous jumps.

To model this, Merton's jump-diffusion model (1976) is introduced. It extends the GBM framework by incorporating a jump component into the stochastic differential equation:

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW_t + U_t dN_t \tag{1.7}$$

N_t denotes a Poisson process with intensity accounting for the arrival of jumps. $U_t$is a real valued random variable (e.g., normally distributed), since the jump size may be positive or negative. $\mu\ and\ \sigma$ are parameters of the diffusion term that accounts for changes in commodity price return during ''ordinary'' days.



State Variable Models for Commodity Prices

Gibson and Schwartz (1990) introduced a two-factor model for oil pricing, addressing the limitations of single-factor models like GBM. While previous sections dealt with stochastic modeling of the spot price alone, this framework introduces the convenience yield $y(t)$ as a second state variable, allowing for more realistic modeling of the term structure of futures prices.

$$\begin{cases} \dfrac{dS(t)}{S(t)} = \mu dt + \sigma_1 dW^1(t) \\ dy(t) = k\,\big(\alpha - y(t)\big)dt + \sigma_2 dW^2(t) \\ dW^1(t)\cdot dW^2(t) = \rho dt \end{cases} \tag{2.1}$$

μ notes the drift of spot price, α the long-term level of $y(t), \sigma_1$ and σ_2 volatilities of the spot and convenience yield, respectively and ρ the correlation between the two GBM.

This model allows both the GBM dynamics of the oil spot price and the Ornstein-Uhlenbeck mean-reverting behavior of the convenience yield, allowing it to evolve over time.

- Empirical observations of backwardation and contango in oil futures markets cannot be explained by a model with constant convenience yield.
- This model explains the dynamic relationship between spot and futures prices, which depends on the time-varying convenience yield.
- The stochastic yield accounts for the storage value and optionality embedded in holding the physical commodity.

A Stochastic Volatility Model for Commodity Prices

Eydeland and Geman (1998) extend the Heston (1993) stochastic volatility framework to commodity markets such as natural gas and electricity, where volatility is both high and time-varying. This model introduces mean-reversion in the spot price and allows the volatility to be stochastic, evolving as a second state variable. Following:

$$\begin{cases} dS_t = k\,(\alpha - \ln(S_t)\,S_t dt + \sigma(t) S_t dW_t^1 \\ d\sum(t) = b\left(c - \sum(t)\right)\,dt + e\sqrt{\sum(t)}\;dW_t^2 \\ dW_t^1 \cdot dW_t^2 = \rho dt \end{cases} \tag{3.1}$$

$\sum(t) = \sigma(t)^2$: the stochastic variance of the process, b, c, e are parameters controlling the mean-reversion and volatility of $\sum(t)$ and ρ is correlation between the Brownian motions driving the price and its volatility (typically negative for commodities) is in general negative since, in contrast to stock prices, the volatility of commodity prices tends to increase with prices – the inverse leverage effect which leads to a volatility smile ‘‘skewed’’ to the right.

A Three-State Variable Model for Oil Prices

By introducing a three-factor stochastic model, particularly suitable in bull cycles where traditional mean-reversion may be weak or absent. The framework builds upon earlier models with mean-reverting price processes and stochastic volatility, now incorporating a time-varying mean-reversion level L_t, which itself evolves stochastically.

$$\begin{cases} dS_t = k\,(\,L_t - \ln(S_t) S_t dt + \sigma(t) S_t\, dW_t^1 \\ \dfrac{dL_t}{L_t} = \mu\, dt + \sigma_2 dW_t^2 \\ d\sum(t) = b\left(c - \sum(t)\right) dt + e\sqrt{\sum(t) dW_t^3} \end{cases} \tag{4.1}$$

Were L_t is the stochastic long-term level toward which $\ln S_t$ mean reverts.

A positive drift in the second equation implies that the mean-reversion level L_t, toward which the commodity spot price S_t gravitates, tends to increase over time on average. However, the spot price itself may fluctuate significantly around this evolving target due to the arrival of positive or negative news, particularly concerning global or firm-specific reserve conditions. Importantly, the stochastic volatility $\sigma(t)$ can reach elevated levels on certain days, offering a plausible explanation for sharp movements in the spot price $S(t)$.

Dassios and Nagaradjasarma (1998) extended this process to price Asian options, leveraging its mathematical tractability. The model uses:

$$dS_t = rS_t dt + \sigma\sqrt{S_t}dW_t$$

And:

$$Y_t = \int_0^t S_u du$$

Where Y_t is the running average (crucial for Asian payoffs). Ribeiro and Hodges (2004) further validated this approach empirically, applying a two-factor model where the convenience yield is modeled as a CIR process and the volatility of spot price is proportional to its square root. They tested this on weekly crude oil futures data (1999–2003) and found that the CIR-based model performed comparably to the Gibson-Schwartz (1990) and Schwartz (1997) frameworks.

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