Circuit Theory and Electronics Fundamentals

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Department of Electrical and Computer Engineering, Aerospace Engineering,
Técnico, University of Lisbon

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1 Introduction

The objective of this laboratory assignment is to study a circuit in which we have the following components: two current sources (I_b and I_d), one of them voltage dependent (I_b), two voltage sources (V_a and V_c), one of them current dependent (V_c) and seven resistors.

The circuit is represented with resort to *LibreOffice Draw* and can be viewed in figure 1.

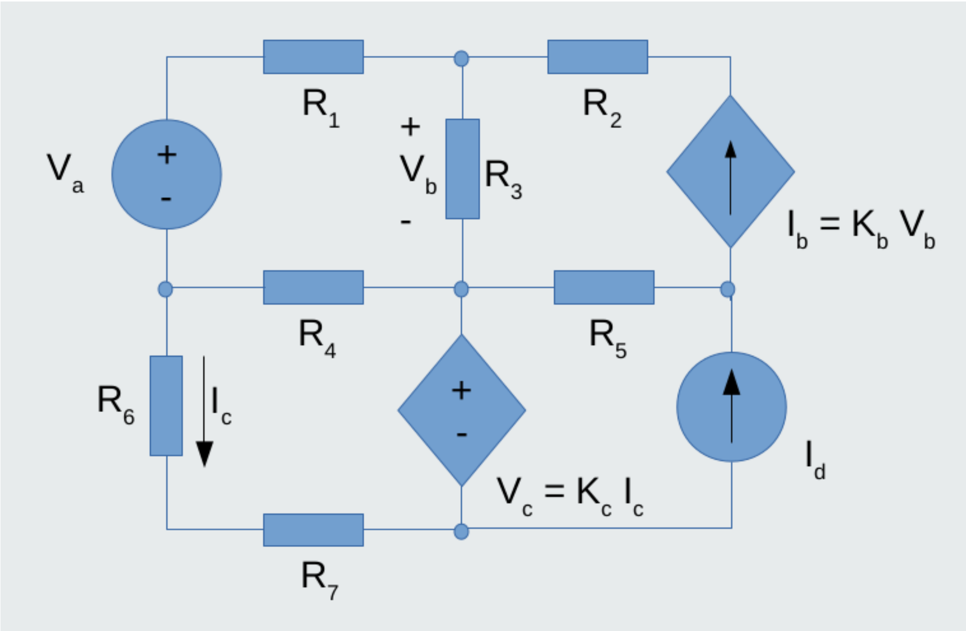

Figure 1: Studied Circuit

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, using the two methods required, mesh and nodes. The known values can be checked in the table below.

R_1	1.01080769792 kOhm
R_2	2.07664633274 kOhm
R_3	3.12595649013 kOhm
R_4	4.18722214507 kOhm
R_5	3.0841699201 kOhm
R_6	2.00179338129 kOhm
R_7	1.04556537884 kOhm
V_a	5.00120775651 V
I_d	1.03136220214 mA
K_b	7.12593545434 mS
K_c	8.24048597287 kOhm

Table 1: Known Data

2.1 Mesh Method

A mesh is a loop that contains no other loops within it. That said, the mesh method is able to determine the current in each of the said loops, resorting to the Kirchoff Voltage Law (KVL). Having those values it is a matter of applying Ohms Law to the resistances in the mesh in order to determine the voltage in every node in the circuit, given that the values for the resistances are provided. This method is not common in automation because it is necessary to identify the different meshes, to which there may be a necessity for an outside observer.

Analyzing our circuit in particular, we can identify four meshes, to which we wrote the equations, stipulating a current flow for each mesh, as shown in figure 2.

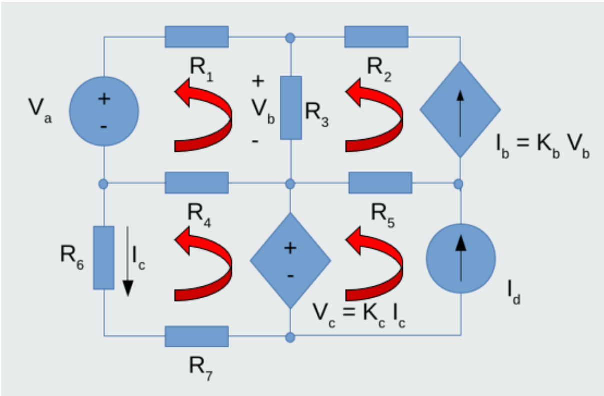

Figure 2: Mesh Method applied to the circuit

After simplyfying the mesh method, the system 2.1 was obtained as follows

$$\begin{cases} R_1 I_a + R_3(I_a - I_b) + R_4(I_a - I_c) = V_a \\ R_3(I_a - I_b) + \frac{I_b}{K_b} = 0 \\ R_4(I_a - I_c) - R_6 I_c - R_7 I_c + K_c I_c = 0 \end{cases}$$

It was then morphed into the system below 1 and solved with the aid of *Octave*.

$$\begin{bmatrix} R_1 + R_3 + R_4 & -R_3 & -R_4 \\ R_3 & -R_3 + \frac{1}{K_b} & 0 \\ R_4 & 0 & K_c - R_4 - R_6 - R_7 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

It was possible, after the said, to obtain the values for the voltage and current sources, as can be verified in the following table 2, with the current measured in mA and the voltage in V.

Name	Value [mA]
I_a	2.224637e-01
I_b	2.329201e-01
I_c	-9.260366e-01
I_d	1.031362e+00

Table 2: Mesh method results.

2.2 Node Method

The most crucial part in this method is to properly identify all the nodes in a circuit, which are the regions that connect two or more elements of a circuit. Then, with the aid of a system of equations provided by the Kirchoff Current Law (KCL) in the nodes unrelated to voltage sources and some extra equalities, it is possible to determine the voltage in every node. Due to the fact that we can only use KCL like previously explained, the aforementioned extra equalities must be create some sort of connection between the nodes not analyzed and the current sources. Contrary to the Mesh method, it is simple to automate, given its straightforward approach, making it achievable to obtain values in a simulation.

Firstly, all the nodes were numbered and represented as shown in figure 3.

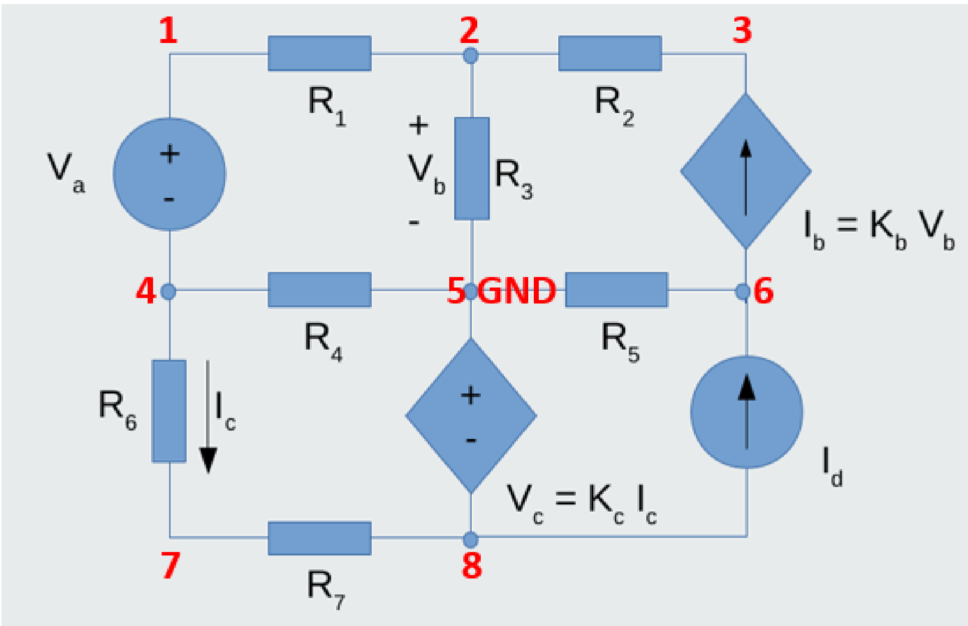

Figure 3: Nodes Method applied to the circuit

Then, the system of equations was written using the numeration stipulated above. There was a voltage source V_T created to aid the analysis of the simulation.

$$\begin{cases} G_1(V_1-V_2)+G_2(V_3-V_2)-G_3V_b=0 \\ G_2(V_2-V_3)+V_b=0 \\ G_5(V_5-V_6)-V_b=-I_d \\ G_6(V_4-V_7)-\frac{V_c}{K_c}=0 \\ G_7/(V_8-V_7)+\frac{V_c}{K_c}=0 \\ V_1-V_4=V_a \\ V_2-V_5-V_b=0 \\ V_5=0 \\ V_5-V_8-V_c=0 \\ G_1(V_2-V_1)+G_4(V_5-V_4)-\frac{V_c}{K_c}=0 \end{cases}$$

The system above was then converted into a matrix equation 2 in order to have it solved by *Octave*.

$$\begin{bmatrix} G_1 & -G_1-G_2 & 0 & 0 & 0 & 0 & 0 & 0 & -G_3 & 0 \\ 0 & G_2 & -G_2 & 0 & 0 & 0 & 0 & 0 & K_b & 0 \\ 0 & 0 & 0 & 0 & G_5 & -G5 & 0 & 0 & -K_b & 0 \\ 0 & 0 & 0 & G_6 & 0 & 0 & -G_6 & 0 & 0 & -\frac{1}{K_c} \\ 0 & 0 & 0 & 0 & 0 & 0 & -G_7 & G_7 & 0 & \frac{1}{K_c} \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1 \\ -G_1 & G_1 & 0 & -G_4 & G_4 & 0 & 0 & 0 & 0 & -\frac{1}{K_c} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -I_d \\ 0 \\ 0 \\ V_a \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

With that, the values for V_1 up to V_8, V_b and V_c were achieved with the values afterwards displayed. The units for current and voltage are the same as in the mesh method.

Name	**Value [V]**
V_5	0.000000e+00
V_1	1.921818e-01
V_2	-3.268625e-02
V_3	-5.163789e-01
V_4	-4.809026e+00
V_6	3.899261e+00
V_7	-6.662760e+00
V_8	-7.630992e+00
V_a	5.001208e+00
V_b	-3.268625e-02
V_c	7.630992e+00

Table 3: Nodes method results.

It is now possible, with

$$V_b = \frac{I_b}{K_b} \tag{3}$$

and

$$V_c = K_c I_c \tag{4}$$

to obtain the values for all the voltages named in the circuit, V_a, V_b and V_c:

It is visible that the values obtain in both methods present virtually identical values.

In the next section, these methods will be compared with a simulation of the circuit, with the aim of testing this theoretical analysis and discussing its results.

3 Simulation Analysis

This simulation was run through a software called NGSpice. From it, the results obtained were the following.

Name	Value [A or V]
@g1[i]	-2.32920e-04
@i1[current]	1.031362e-03
@r1[i]	-2.22464e-04
@r2[i]	-2.32920e-04
@r3[i]	-1.04564e-05
@r4[i]	1.148500e-03
@r5[i]	1.264282e-03
@r6[i]	-9.26037e-04
@r7[i]	-9.26037e-04
v0	-4.80903e+00
v1	1.921818e-01
v2	-3.26862e-02
v3	-5.16379e-01
v4	-4.80903e+00
v6	3.899261e+00
v7	-6.66276e+00
v8	-7.63099e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Such results are sufficient for a thourough analysis of the entire circuit.

Firstly, by noting that V_b is the imposed voltage between nodes 2 and 5, one concludes that V_b is then given by V_2 given that node 5 is attached to ground. Likewise, and given that V_c is the imposed voltage between nodes 5 and 8 and since node 5 is connected to ground, V_c is then given by negative V_8.

$$V_b = V_2 \tag{5}$$

$$V_c = -V_8 \tag{6}$$

Secondly, by noting that currents passing through R_1, R_2 and R_6 are equal to I_a, I_b and I_c respectively, one can directly apply Ohms Law to determine these currents. While I_b and I_c could be determined through direct application of the law, it could also be determined by using the relation described in equations 3 and 4.

From such calculations, the following results were obtained.

I_a	2.40136e-04
I_b	2.51245e-04
I_c	9.76838e-04
V_b	3.4437e-02
V_c	7.979210e+00

Table 5: Table of results for the simulation in A and V

4 Conclusion

In this cirtcuit in specific there were only linear components, which enabled us to use the Kirchoff laws and subsequently the mesh and nodal analysis. Another consequence of this linearity is the superposition theroem, which is the basis for the mesh method, given it validates the concept of having more than one current going through a component, which can be viewed independently and allow for a method to succesfully discover all the unkowns in the circuit.

The nodal analysis is, as previously mentioned, the one that is easier to automize and is therefore included in *NGSpice*, the simulator used in this activity. It was, therefore, possible to draw conclusions that validate this method that allows for the proper examination of the board.

It is now possible to affirm that both theoretical methods are viable due to the fact that similar results were obtained when calculated by a simulator. The previously mentioned linearity of the system accompanied by the success of the *NGSpice* simulation prove this thesis.

Concluding, it is possible to analyze this circuit not only through simulation, but also theoretically using the Kirchoff Laws and the nodal and mesh methods because of circuits linearity.