

$$A^{(0)} = \begin{bmatrix} \textcircled{4} & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 4 \end{bmatrix} \quad \begin{aligned} \text{pivot} &= a_{11} = 4 \\ m_{L2} &= -\frac{1}{2} \\ m_{L3} &= \frac{1}{4} \end{aligned}$$

$$L_2 \leftarrow L_2 - m_{L2} \times L_1$$

$$-2 - (-1/2) \times 4 = 0$$

$$4 - (-1/2) \times -2 = 3$$

$$-2 - (-1/2) \times 1 = -\frac{3}{2}$$

$$L_3 \leftarrow L_3 - m_{L3} \times L_1$$

$$1 - 1/4 \times 4 = 0$$

$$-2 - 1/4 \times -2 = -3/2$$

$$4 - 1/4 \times 1 = 15/4$$

$$A^{(1)} = \begin{bmatrix} 4 & -2 & 1 \\ 0 & \textcircled{3} & -3/2 \\ 0 & -3/2 & 15/4 \end{bmatrix}$$

$$\text{pivot} = a_{22} = 3$$

$$m_{L3} = \frac{-3/2}{3} = -\frac{1}{2}$$

$$L_3 \leftarrow L_3 - m_{L3} \times L_2$$

$$0 - (-1/2) \times 0 = 0$$

$$-3/2 - (-1/2) \times 3 = 0$$

$$15/4 - (-1/2) \times -3/2 = 12/4 = 3$$

$$A^{(2)} = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 3 & -3/2 \\ 0 & 0 & 3 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/4 & -1/2 & 1 \end{bmatrix}$$

$$Ly = b$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/4 & -1/2 & 1 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 11 \\ -16 \\ 17 \end{bmatrix}$$

$$\begin{cases} y_1 = 11 \\ -1/2 y_1 + y_2 = -16 \\ 1/4 y_1 - 1/2 y_2 + y_3 = 17 \end{cases} \quad y = \begin{bmatrix} 11 \\ -21 \\ 9 \end{bmatrix}$$

$$-1/2 \cdot 11 + y_2 = -16 \Rightarrow y_2 = -16 + \frac{11}{2} = -\frac{21}{2}$$

$$y_2 = -16 + \frac{11}{2} = -\frac{21}{2}$$

$$y_2 = -\frac{21}{2}$$

$$\frac{11}{4} - \frac{1}{2} \cdot \left(-\frac{21}{2}\right) + y_3 = 17$$

$$\frac{11}{4} + \frac{21}{4} + y_3 = 17$$

$$y_3 = 17 - 8 = 9$$

1	1
0 3 1 0 0 0	0 0 0 0 0 0

$$Ux = y$$

$$\begin{bmatrix} 4 & -2 & 1 \\ 0 & 3 & -3/2 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11 \\ -21/2 \\ 9 \end{bmatrix}$$

$$4x_1 - 2x_2 + x_3 = 11$$

$$3x_2 - 3/2 x_3 = -21/2$$

$$3x_3 = 9$$

$$x_3 = 3$$

$$3x_2 - \frac{9}{2} = -21/2$$

$$3x_2 = -\frac{12}{2}$$

$$3x_2 = -6$$

$$x_2 = -2$$

$$4x_1 - 2(-2) + 3 = 11$$

$$4x_1 + 7 = 11$$

$$x_1 = 1$$

$$x = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

