Single Qubit Operations

Single Qubit $>$ Operation

Given two bits a, b we have that

$$a > b \quad \Leftrightarrow \quad a = 1 \wedge b = 0 \quad \Leftrightarrow \quad a \ \wedge \neg b \tag{1}$$

Using this result we can construct a quantum circuit which outputs $a \ \wedge \neg b$ in the following way:

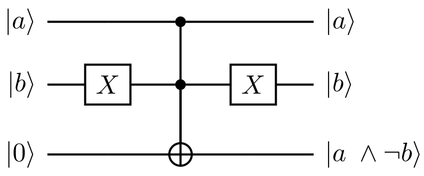

Figure 1: Quantum circuit for $>$-operation for single bit comparison.

Single Qubit $=$ Operation

Given two bits a, b we have that

$$a = b \quad \Leftrightarrow \quad \neg(a \neq b) \quad \Leftrightarrow \quad \neg(a \oplus b) \tag{2}$$

Where $\oplus$ is the XOR operator.
The quantum circuit for equality is as follows:

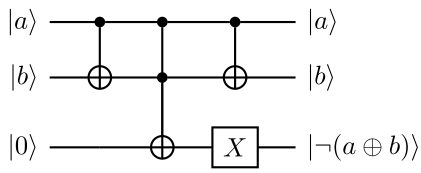

Figure 2: Quantum circuit for $=$-operation for single bit comparison.

Quantum Comparison

Given two qubits a, b and two ancillas qubits the Quantum Comparison operator is

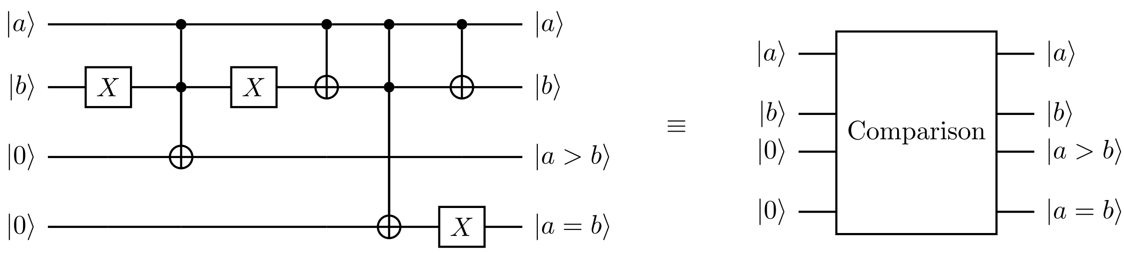

Figure 3: Quantum circuit with both $>$ and $=$ operations performed.

Multiple Qubit Comparison

Let a, b be two binary numbers such that $a = a_n a_{n-1} \dots a_1 a_0$ and $b = b_n b_{n-1} \dots b_1 b_0$.
Then $a > b$ if and only if

$$a_n > b_n \oplus (a_n = b_n \wedge a_{n-1} > b_{n-1}) \oplus \cdots \oplus (a_n = b_n \wedge \cdots \wedge a_1 = b_1 \wedge a_0 > b_0) \tag{3}$$

With the above result in mind and using the *Comparison* operator, the circuit for computing $a > b$ is detailed bellow:

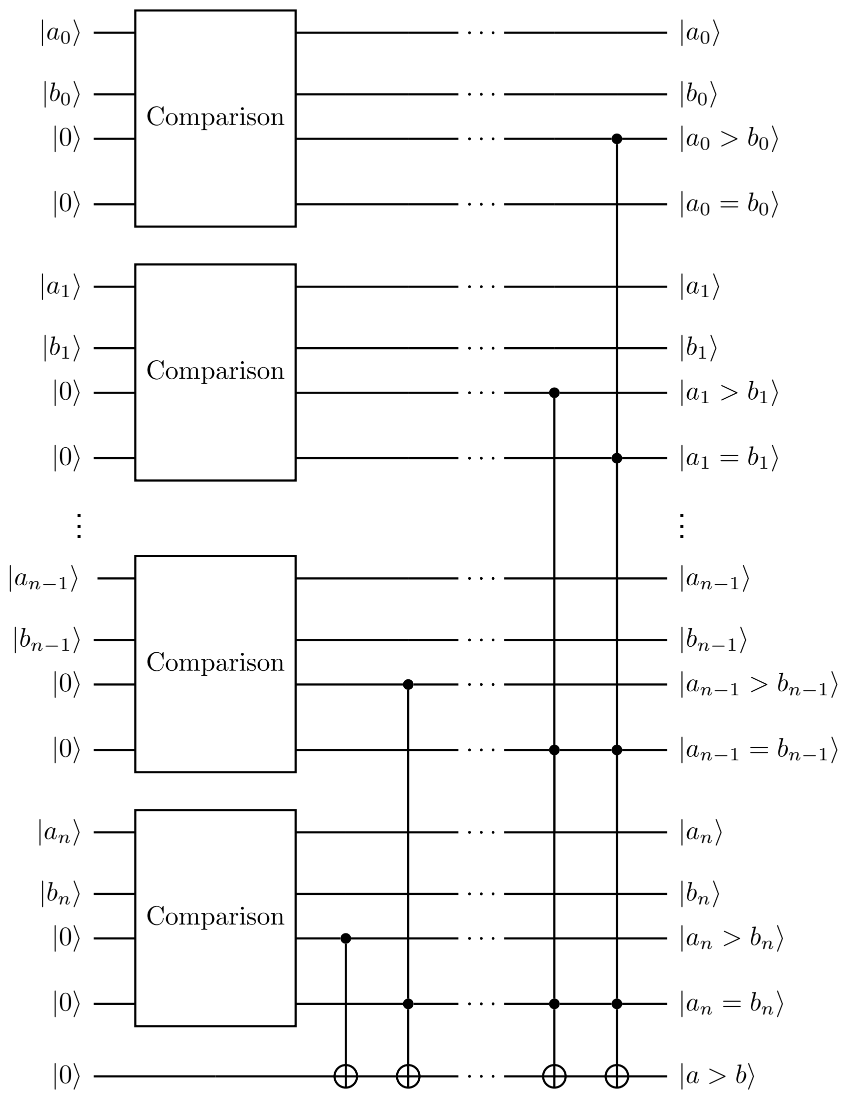

Figure 4: Quantum circuit for comparing numbers requiring $n + 1$ bits in the binary representation.

Necessary Qubits

Given two integer numbers i, j with binary representation, using a signed bit, with size n_i, n_j respectively, one can build a quantum circuit using the elements above using $2n$ input qubits for the binary representation and $2n + 1$ ancilla qubits, where $n = \max(n_i, n_j)$.