[Folha resumo 1]

Exercício 4.3.5

1. $y^{(4)} + y'' = 0$

Equação característica : $r^4 + r^2 = 0$

$$r^4 + r^2 = 0$$
$$r^2 (r^2 + 1) = 0$$
$$r^2 = 0 \lor r^2 + 1 = 0$$
$$\underbrace{r = 0}_{\text{tem multiplicidade 2}} \lor \underbrace{r = \pm i}_{\alpha = 0,\ \beta = 1}$$

$e^0, x e^0$

$= 1, x$

$e^{0x} \cos(1x), e^{0x} \operatorname{sen}(1x)$

$\cos x, \operatorname{sen} x$

$SFS = \{1, x, \cos x, \operatorname{sen} x\}$

Solução geral da EDO : $y = 1C_1 + xC_2 + C_3 \cos x + C_4 \operatorname{sen} x, \quad C_1, C_2, C_3, C_4 \in \mathbb{R}$

2. $y^{(4)} - 3y''' - y'' + 3y' = 0$

$r^4 - 3r^3 - r^2 + 3r = 0$

$\Leftrightarrow r(r^3 - 3r^2 - r + 3) = 0$

	1	-3	-1	3
1		1	-2	-3
	1	-2	-3	0

$\Leftrightarrow r(r-1)(r^2 - 2r - 3) = 0$

$\Leftrightarrow r = 0 \lor r - 1 = 0 \lor r^2 - 2r - 3 = 0$

$r = 0 \lor r = 1 \lor r = \dfrac{2 \pm \sqrt{4 - 4(-3)}}{2}$

$r = 0 \lor r = 1 \lor r = \dfrac{2-4}{2} \lor r = \dfrac{2+4}{2}$

$r = 0 \lor r = 1 \lor r = -1 \lor r = 3$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$e^{0x} \quad e^{1x} \quad e^{-x} \quad e^{3x}$

$SFS = \{1, e^{x}, e^{-x}, e^{3x}\}$

Solução geral: $y = c_1 + c_2 e^{x} + C_3 e^{-x} + c_4 e^{3x}$, $c_1, c_2, c_3, c_4 \in \mathbb{R}$

3. $y'' + 2y' + 5y = 0$

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$r^2 + 2r + 5 = 0$

$\Rightarrow r = \dfrac{-2 \pm \sqrt{4 - 4 \times 5}}{2}$

$\Rightarrow r = \dfrac{-2 \pm 4i}{2}$

$\Rightarrow r = -1 \pm 2i$

$\alpha = -1$ e $\beta = 2$

$\downarrow$

$e^{-x}\cos(2x)$ e $e^{-x}\operatorname{sen}(2x)$

$SFS = \{ e^{-x}\cos(2x) , e^{-x}\operatorname{sen}(2x) \}$

<u>Solução geral</u> :

$y = C_1 e^{-x}\cos(2x) + C_2 e^{-x}\operatorname{sen}(2x)$

$C_1, C_2 \in \mathbb{R}$

2

4. $2y^{(5)} - 8y^{(4)} + 8y''' = 0$

$2r^5 - 8r^4 + 8r^3 = 0$

$\Leftrightarrow r^3(2r^2 - 8r + 8) = 0$

$\Leftrightarrow r^3 = 0 \vee r = \dfrac{8 \pm \sqrt{64 - 4\times 2\times 8}}{2\times 2}$

$\Leftrightarrow \underbrace{r = 0}_{\text{raiz de multiplicidade 3}} \vee \underbrace{r = 2}_{\text{raiz de multiplicidade 2}}$

$\Downarrow$

$e^{0x}, xe^{0x}, x^2e^{0x}$

e^{2x}, xe^{2x}

$1, x, x^2$

$SFS = \{1, x, x^2, e^{2x}, xe^{2x}\}$

Solução geral: $y = c_1 + c_2x + c_3x^2 + c_4e^{2x} + c_5xe^{2x}$

$c_1, c_2, c_3, c_4, c_5 \in \mathbb{R}$