GUIÃO 3 (pg 20)

6.4.5

1) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{-1+\sqrt{x^2+y^2+1}} = \lim_{z\to 0} \frac{z}{-1+\sqrt{z+1}} \underset{R.C}{\overset{\left(\frac{0}{0}\right)}{=}} \lim_{z\to 0} \frac{(z)'}{(-1+\sqrt{z+1})'} = (*)$

$x^2+y^2 = z$

q$^{\text{do}}$ $(x,y)\to(0,0)$, tem-se $z\to 0$

$(*) = \lim_{z\to 0} \frac{1}{\frac{1}{2}(z+1)^{\frac{1}{2}-1}(z+1)'} = \lim_{z\to 0} \frac{1}{\frac{1}{2}(z+1)^{-\frac{1}{2}}} = \lim_{z\to 0} 2\sqrt{z+1} = 2\times 1 = 2$

6.4.6 Pg (20)

$g: \mathbb{R}^2 \to \mathbb{R}$, $g(x,y) = \begin{cases} \dfrac{x^2 \sin y + y^2 \sin x}{x^2+y^2}, & \text{se } (x,y) \neq (0,0) \\ 0, & \text{se } (x,y) = (0,0) \end{cases}$

(i) Para $(x,y) = (0,0)$:

g é contínua em (0,0) se $\lim_{(x,y)\to(0,0)} g(x,y) = g(0,0)$

Ora,

- $g(0,0) = 0$
- $\lim_{(x,y)\to(0,0)} g(x,y) = \lim_{(x,y)\to(0,0)} \dfrac{x^2 \sin y + y^2 \sin x}{x^2+y^2} =$

$= \lim_{(x,y)\to(0,0)} \left(\dfrac{x^2 \sin y}{x^2+y^2} + \dfrac{y^2 \sin x}{x^2+y^2} \right) = \lim_{(x,y)\to(0,0)} \dfrac{x^2 \sin y}{x^2+y^2} + \lim_{(x,y)\to(0,0)} \dfrac{y^2 \sin x}{x^2+y^2} =$

$= \lim_{(x,y)\to(0,0)} \underset{\substack{\downarrow \\ 0}}{\sin y} \underbrace{\dfrac{x^2}{x^2+y^2}}_{\text{função limitada}} + \lim_{(x,y)\to(0,0)} \underset{\substack{\downarrow \\ 0}}{\sin x} \underbrace{\dfrac{y^2}{x^2+y^2}}_{\text{função limitada}} = 0 + 0 = 0$

C.aux:

$x^2 + y^2 \geq x^2$

$\dfrac{1}{x^2+y^2} \leq \dfrac{1}{x^2}$

$\dfrac{x^2}{x^2+y^2} \leq \dfrac{x^2}{x^2}$

$0 \leq \dfrac{x^2}{x^2+y^2} \leq 1$ logo $\dfrac{x^2}{x^2+y^2}$ é função limitada $\forall (x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$

Como $\lim_{(x,y)\to(0,0)} g(x,y) = g(0,0) = 0$, g é contínua em (0,0)

(ii) Para $(x,y) \neq (0,0)$:

$g(x,y) = \dfrac{x^2 \sin y + y^2 \sin x}{x^2+y^2}$ é contínua pois é a soma, produto e quociente (em que o denominador não se anula) de funções contínuas (função trigonométrica seno e polinomiais).

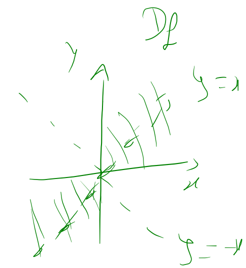
6.4.4 (pg 20)

$$f(x,y) = \frac{\sqrt{xy}}{x^2-y^2}$$

$$D_f = \{(x,y)\in\mathbb{R}^2 : xy \geq 0 \wedge x^2-y^2 \neq 0\} = \{(x,y)\in\mathbb{R}^2 : xy \geq 0 \wedge y^2 \neq x^2\}$$
$$= \{(x,y)\in\mathbb{R}^2 : xy \geq 0 \wedge y \neq |x|\}$$

NOTA: $xy \geq 0$
$\Leftrightarrow \underset{1^{\circ}Q}{(x\geq 0 \wedge y \geq 0)} \vee \underset{3^{\circ}Q}{(x \leq 0 \wedge y \leq 0)}$

D_f



$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{\sqrt{xy}}{x^2-y^2} = ?$$

Considere-se os conj$^{\text{tos}}$ $A = \{(x,y) : y = 2x\}$

e $B = \{(x,y) : y = 0\}$

Ora,

• $$\lim_{\substack{(x,y)\to(0,0)\\ A}} \frac{\sqrt{xy}}{x^2-y^2} = \lim_{x\to 0} \frac{\sqrt{2x^2}}{x^2-4x^2} = \lim_{x\to 0} \frac{\sqrt{2}\,|x|}{-3x^2}$$

$$= -\frac{\sqrt{2}}{3} \lim_{x\to 0} \frac{|x|}{x^2}$$

$x>0$: $$= -\frac{\sqrt{2}}{3} \lim_{x\to 0^+} \frac{x}{x^2} = -\frac{\sqrt{2}}{3} \lim_{x\to 0^+} \underbrace{\frac{1}{x}}_{\to+\infty} = -\infty$$

$x<0$: $$= -\frac{\sqrt{2}}{3} \lim_{x\to 0^-} \frac{-x}{x^2} = +\frac{\sqrt{2}}{3} \lim_{x\to 0^-} \underbrace{\frac{1}{x}}_{\to-\infty} = -\infty$$

logo $\nexists \lim_{(x,y)\to(0,0)} f(x,y)$

(• $$\lim_{\substack{(x,y)\to(0,0)\\ B}} \frac{\sqrt{xy}}{x^2-y^2} = \lim_{x\to 0} \frac{\sqrt{0}}{x^2-0} = \lim_{x\to 0} \frac{0}{x^2} = \lim_{x\to 0} 0 = 0$$)

6.6.10 (pg 36)

$f: \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$

1) f é contínua em $(0,0)$ se $\lim\limits_{(x,y)\to(0,0)} f(x,y) = f(0,0)$

função limitada

função limitada

- $f(0,0) = 0$
- $$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \overset{\to 0}{x} \, \frac{x^2}{x^2+y^2} - \lim_{(x,y)\to(0,0)} \overset{\to 0}{y} \, \frac{y^2}{x^2+y^2}$$

$$= 0 - 0 = 0$$

$\therefore f$ é contínua em $(0,0)$

2) • $$f_x(0,0) = \frac{\partial f}{\partial x}(\overset{a}{0},\overset{b}{0}) = \lim_{h\to 0} \frac{f(\overset{a}{0+h}, \overset{b}{0}) - f(\overset{a}{0},\overset{b}{0})}{h} = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{h^3 - 0^3}{h^2+0^2} - 0}{h} = \text{T.P.C} = 1$$

• $$f_y(0,0) = \frac{\partial f}{\partial y}(\overset{a}{0},\overset{b}{0}) = \lim_{h\to 0} \frac{f(\overset{a}{0}, \overset{b}{0+h}) - f(\overset{a}{0},\overset{b}{0})}{h} = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{0^3 - h^3}{0^2+h^2} - 0}{h} = \text{T.P.C} = -1$$

6.6.1

③ $h(x,y)=\sqrt{xy+\frac{x}{y}}=\left(xy+\frac{x}{y}\right)^{\frac{1}{2}}$

$(u^n)'=n\,u^{n-1}\,u'$

- $\frac{\partial h}{\partial x}=\frac{1}{2}\left(xy+\frac{x}{y}\right)^{\frac{1}{2}-1}\left(xy+\frac{x}{y}\right)'_x$

$=\frac{1}{2}\left(xy+\frac{x}{y}\right)^{-\frac{1}{2}}\left(y+\frac{1}{y}\right)=\ldots$

c. aux

$\left(\frac{x}{y}\right)'_x=\left(\frac{1}{y}\cdot x\right)'_x$

$=\frac{1}{y}(x)'_x=\frac{1}{y}\times 1$

$=\frac{1}{y}$

- $\frac{\partial h}{\partial y}=\frac{1}{2}\left(xy+\frac{x}{y}\right)^{\frac{1}{2}-1}\left(xy+\frac{x}{y}\right)'_y$

$=\frac{1}{2}\left(xy+\frac{x}{y}\right)^{-\frac{1}{2}}\left(x-\frac{x}{y^2}\right)$

$=\ldots$

c. aux :

$\left(\frac{x}{y}\right)'_y=$

$=\frac{\overset{0}{x'}y-x\overset{=1}{y'}}{y^2}=-\frac{x}{y^2}$

(2) $g(x,y) = e^{2xy^3}$

- $\dfrac{\partial g(x,y)}{\partial x} = (2xy^3)'_x \, e^{2xy^3} = 2y^3 \, e^{2xy^3}$

- $\dfrac{\partial g(x,y)}{\partial y} = (2xy^3)'_y \, e^{2xy^2} = 2x \, 3y^2 \, e^{2xy^2} = 6xy^2 e^{2xy^2}$

NOTA:

$(u.v)' = u'v + uv'$

$(cu)' = \underbrace{c'}_{0}u + cu'$

$= cu'$