Exercício 4.3.10

1. $y' \operatorname{sen} x + y \cos x = \operatorname{sen}^2 x$

1º) obter y_h

$y' \operatorname{sen} x + y \cos x = 0$

$\Leftrightarrow \operatorname{sen} x \, y' = -y \cos x$

$\Rightarrow \frac{1}{y} y' = -\frac{\cos x}{\operatorname{sen} x}$

$\ln|y| = -\ln|\operatorname{sen} x| + C$

$$\boxed{y_h = C_1 \cdot \frac{1}{\operatorname{sen} x}, \quad C_1 \in \mathbb{R}}$$

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2º Obter y_p pelo MVC:

$$y_p(x) = c_1(x) \cdot \frac{1}{\operatorname{sen} x}$$

$$y_p'(x) = c_1'(x) \cdot \frac{1}{\operatorname{sen} x} + c_1(x) \cdot \left(- \frac{1 \cdot \cos x}{\operatorname{sen}^2 x} \right)$$

Substituindo,

$$\left(\frac{c_1'(x)}{\operatorname{sen} x} - \frac{c_1(x) \cos x}{\operatorname{sen}^2 x} \right) \operatorname{sen} x + \frac{c_1(x)}{\operatorname{sen} x} \cdot \cos x = \operatorname{sen}^2 x$$

$$\Rightarrow \quad c_1'(x) = \operatorname{sen}^2 x \qquad\qquad \operatorname{sen}^2 x = \frac{1}{2}(1 - \cos(2x))$$

$$c_1(x) = \int \operatorname{sen}^2 x \, dx$$

$$= \int \frac{1}{2}(1 - \cos(2x))\, dx = \frac{1}{2}x - \frac{1}{4}\operatorname{sen}(2x) + C, \; C \in \mathbb{R}$$

$$c_1(x) = \frac{1}{2}x - \frac{1}{4}\operatorname{sen}(2x)$$

$$y_p(x) = \left(\frac{1}{2}x - \frac{1}{4}\operatorname{sen}(2x) \right) \cdot \frac{1}{\operatorname{sen} x}$$

$$= \frac{x}{2 \operatorname{sen} x} - \frac{1}{2}\cos x$$

3º Solução geral

$$y = y_h + y_p$$

$$\boxed{y = c_1 \cdot \frac{1}{\operatorname{sen} x} + \frac{x}{2 \operatorname{sen} x} - \frac{1}{2}\cos x \quad , \; c_1 \in \mathbb{R}}$$

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2. $xy' - y = x - 1 \quad , \quad x > 0$

1°) obter y_h:

$xy' - y = 0$

$\Rightarrow xy' = y$

$\Rightarrow \frac{1}{y}y' = \frac{1}{x}$

$\ln|y| = \ln|x| + c, \; c \in \mathbb{R}$

$$\boxed{y_h = c_1 x \quad , x > 0}$$

2° obter y_p pelo M.V.C:

$y_p(x) = c_1(x)\,x$

$y_p'(x) = c_1'(x) . x + c_1(x)$

Substituindo

$x\left(c_1'(x) . x + c_1(x)\right) - c_1(x) . x = x - 1$

$x^2 c_1'(x) = x - 1$

$\Rightarrow c_1'(x) = \frac{1}{x} - \frac{1}{x^2} \quad \Rightarrow c_1(x) = \ln|x| + \frac{1}{x} + c$

$\therefore y_p(x) = \left(\ln|x| + \frac{1}{x}\right)x$

$y_p(x) = x \ln x + 1$

3° Obter solução geral

$$\boxed{y = c_1 x + x\ln x + 1 \quad , \; c_1 \in \mathbb{R}}$$

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3. $xy' + y - e^{x} = 0 \quad , x > 0$

$\Leftrightarrow xy' + y = e^{x}$

1° y_h

$xy' + y = 0$

$\Leftrightarrow xy' = -y \Leftrightarrow \frac{1}{y}\,dy = -\frac{1}{x}\,dx$

$\ln|y| = -\ln|x| + c \Rightarrow y = C_1 \cdot \frac{1}{x}$

2° y_p

$y_p(x) = C_1(x) \cdot \frac{1}{x}$

$y_p'(x) = C_1'(x) \cdot \frac{1}{x} + C_1(x)\left(-\frac{1}{x^2}\right)$

Subst.

$x\left(C_1'(x) \cdot \frac{1}{x} - C_1(x) \cdot \frac{1}{x^2}\right) + C_1(x) \cdot \frac{1}{x} = e^{x}$

$C_1'(x) = e^{x} \Rightarrow C_1(x) = e^{x} + C \, , \, C \in \mathbb{R}$

$y_p(x) = e^{x} \cdot \frac{1}{x}$

3° Soluç.

$y = y_h + y_p$

$y = C_1 \cdot \frac{1}{x} + \frac{e^{x}}{x}$

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