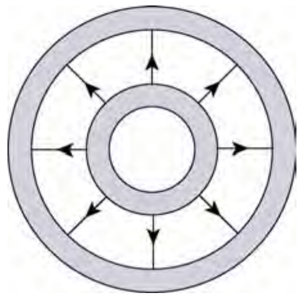
1. (a) We note that the electric field points leftward at both points. Using $\vec{F} = q_0\vec{E}$, and orienting our x axis rightward (so $\hat{\text{i}}$ points right in the figure), we find

$$\vec{F} = \left(+1.6\times10^{-19}\,\text{C}\right)\left(-40\frac{\text{N}}{\text{C}}\hat{\text{i}}\right) = (-6.4\times10^{-18}\,\text{N})\hat{\text{i}}$$

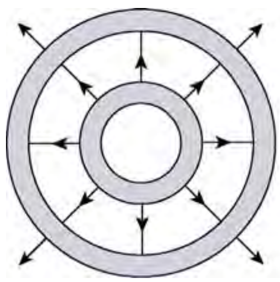
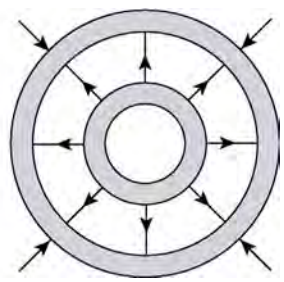
which means the magnitude of the force on the proton is 6.4×10^{-18} N and its direction $(-\hat{\text{i}})$ is leftward.

(b) As the discussion in §22-2 makes clear, the field strength is proportional to the "crowdedness" of the field lines. It is seen that the lines are twice as crowded at A than at B, so we conclude that $E_A = 2E_B$. Thus, $E_B = 20$ N/C.

2. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge which resides on the larger shell. The following sketch is for $q_1 = q_2$.



The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).

3. Since the magnitude of the electric field produced by a point charge q is given by $E = |q|/4\pi\varepsilon_0 r^2$, where r is the distance from the charge to the point where the field has magnitude E, the magnitude of the charge is

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.50\,\mathrm{m})^2\,(2.0\,\mathrm{N/C})}{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}} = 5.6\times10^{-11}\,\mathrm{C}.$$

4. We find the charge magnitude $|q|$ from $E = |q|/4\pi\varepsilon_0 r^2$:

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(1.00\,\mathrm{N/C})(1.00\,\mathrm{m})^2}{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}} = 1.11\times10^{-10}\,\mathrm{C}.$$

5. Since the charge is uniformly distributed throughout a sphere, the electric field at the surface is exactly the same as it would be if the charge were all at the center. That is, the magnitude of the field is

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

where q is the magnitude of the total charge and R is the sphere radius.

(a) The magnitude of the total charge is Ze, so

$$E = \frac{Ze}{4\pi\varepsilon_0 R^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)(94)\left(1.60\times10^{-19}\ \mathrm{C}\right)}{\left(6.64\times10^{-15}\ \mathrm{m}\right)^2} = 3.07\times10^{21}\ \mathrm{N/C}.$$

(b) The field is normal to the surface and since the charge is positive, it points outward from the surface.

6. With x_1 = 6.00 cm and x_2 = 21.00 cm, the point midway between the two charges is located at x = 13.5 cm. The values of the charge are $q_1 = -q_2 = -\,2.00 \times 10^{-7}$ C, and the magnitudes and directions of the individual fields are given by:

$$\vec{E}_1 = -\frac{|q_1|}{4\pi\varepsilon_0 (x-x_1)^2}\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)|-2.00\times10^{-7}\,\text{C}|}{\left(0.135\ \text{m}-0.060\ \text{m}\right)^2}\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

$$\vec{E}_2 = -\frac{q_2}{4\pi\varepsilon_0 (x-x_2)^2}\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-7}\,\text{C})}{\left(0.135\ \text{m}-0.210\ \text{m}\right)^2}\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

Thus, the net electric field is

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = -(6.39\times10^5\ \text{N/C})\hat{\text{i}}$$

7. At points between the charges, the individual electric fields are in the same direction and do not cancel. Since charge $q_2 = -4.00\ q_1$ located at x_2 = 70 cm has a greater magnitude than $q_1 = 2.1 \times 10^{-8}$ C located at x_1 = 20 cm, a point of zero field must be closer to q_1 than to q_2. It must be to the left of q_1.

Let x be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_2|}{(x-x_2)^2} - \frac{|q_1|}{\left(x-x_1\right)^2}\right).$$

If the field is to vanish, then

$$\frac{|q_2|}{(x-x_2)^2} = \frac{|q_1|}{\left(x-x_1\right)^2} \quad \Rightarrow \quad \frac{|q_2|}{|q_1|} = \frac{(x-x_2)^2}{\left(x-x_1\right)^2}.$$

Taking the square root of both sides, noting that $|q_2|/|q_1| = 4$, we obtain

$$\frac{x - 70\text{ cm}}{x - 20\text{ cm}} = \pm 2.0\,.$$

Choosing –2.0 for consistency, the value of x is found to be x = –30 cm.

8. (a) The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 22-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $\left|\vec{E}_1\right|$ is everywhere bigger than $\left|\vec{E}_2\right|$ in this region. In the region between the charges ($0 < x < L$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > L$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\text{net}} = \left(|\vec{E}_2| - |\vec{E}_1|\right)\hat{\text{i}}\ .$$

Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\text{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > L$ region:

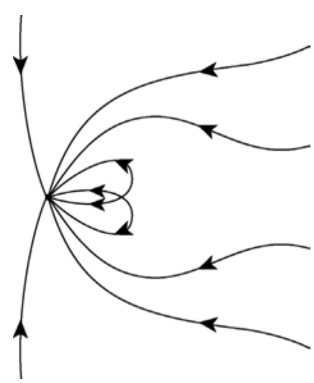
$$|\vec{E}_1| = |\vec{E}_2| \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-L)^2}$$

which leads to

$$\frac{x-L}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{2}{5}}.$$

Thus, we obtain $x = \dfrac{L}{1-\sqrt{2/5}} \approx 2.72L$.

(b) A sketch of the field lines is shown in the figure below:



9. The x component of the electric field at the center of the square is given by

$$
\begin{aligned}
E_x &= \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}-\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\
&= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(|q_1|+|q_2|-|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\
&= 0.
\end{aligned}
$$

Similarly, the y component of the electric field is

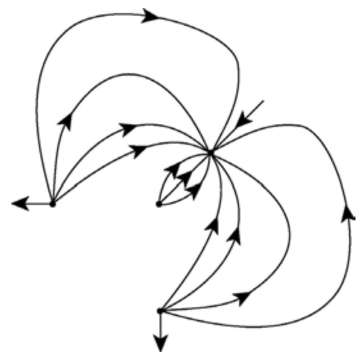
$$
\begin{aligned}
E_y &= \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}+\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\
&= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(-|q_1|+|q_2|+|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\
&= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.0\times10^{-8}\ \text{C})}{(0.050\ \text{m})^2/2}\frac{1}{\sqrt{2}} = 1.02\times10^5\ \text{N/C}.
\end{aligned}
$$

Thus, the electric field at the center of the square is $\vec{E} = E_y\hat{\text{j}} = (1.02\times10^5\ \text{N/C})\hat{\text{j}}$.

10. We place the origin of our coordinate system at point P and orient our y axis in the direction of the $q_4 = -12q$ charge (passing through the $q_3 = +3q$ charge). The x axis is perpendicular to the y axis, and thus passes through the identical $q_1 = q_2 = +5q$ charges. The individual magnitudes $|\vec{E}_1|$, $|\vec{E}_2|$, $|\vec{E}_3|$, and $|\vec{E}_4|$ are figured from Eq. 22-3, where the absolute value signs for q_1, q_2, and q_3 are unnecessary since those charges are positive (assuming $q > 0$). We note that the contribution from q_1 cancels that of q_2 (that is, $|\vec{E}_1| = |\vec{E}_2|$), and the net field (if there is any) should be along the y axis, with magnitude equal to

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_4|}{(2d)^2} - \frac{q_3}{d^2}\right)\hat{\mathrm{j}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{12q}{4d^2} - \frac{3q}{d^2}\right)\hat{\mathrm{j}}$$

which is seen to be zero. A rough sketch of the field lines is shown below:



11. (a) The vertical components of the individual fields (due to the two charges) cancel, by symmetry. Using d = 3.00 m and y = 4.00 m, the horizontal components (both pointing to the $-x$ direction) add to give a magnitude of

$$E_{x,\text{net}} = \frac{2|q|d}{4\pi\varepsilon_0(d^2+y^2)^{3/2}} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\ \text{C})(3.00\ \text{m})}{[(3.00\ \text{m})^2+(4.00\ \text{m})^2]^{3/2}} .$$

$$=1.38\times10^{-10}\ \text{N/C} .$$

(b) The net electric field points in the $-x$ direction, or 180° counterclockwise from the $+x$ axis.

12. For it to be possible for the net field to vanish at some $x > 0$, the two individual fields (caused by q_1 and q_2) must point in opposite directions for $x > 0$. Given their locations in the figure, we conclude they are therefore oppositely charged. Further, since the net field points more strongly leftward for the small positive x (where it is very close to q_2) then we conclude that q_2 is the negative-valued charge. Thus, q_1 is a positive-valued charge. We write each charge as a multiple of some positive number ξ (not determined at this point). Since the problem states the absolute value of their ratio, and we have already inferred their signs, we have $q_1 = 4\xi$ and $q_2 = -\xi$. Using Eq. 22-3 for the individual fields, we find

$$E_{\text{net}} = E_1 + E_2 = \frac{4\xi}{4\pi\varepsilon_o (L+x)^2} - \frac{\xi}{4\pi\varepsilon_o x^2}$$

for points along the positive x axis. Setting $E_{\text{net}} = 0$ at $x = 20$ cm (see graph) immediately leads to $L = 20$ cm.

(a) If we differentiate E_{net} with respect to x and set equal to zero (in order to find where it is maximum), we obtain (after some simplification) that location:

$$x = \left(\frac{2}{3}\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{4} + \frac{1}{3}\right)L = 1.70(20 \text{ cm}) = 34 \text{ cm}.$$

We note that the result for part (a) does not depend on the particular value of ξ.

(b) Now we are asked to set $\xi = 3e$, where $e = 1.60 \times 10^{-19}$ C, and evaluate E_{net} at the value of x (converted to meters) found in part (a). The result is 2.2×10^{-8} N/C .

13. By symmetry we see the contributions from the two charges $q_1 = q_2 = +e$ cancel each other, and we simply use Eq. 22-3 to compute magnitude of the field due to $q_3 = +2e$.

(a) The magnitude of the net electric field is

$$|\vec{E}_{\text{net}}| = \frac{1}{4\pi\varepsilon_0}\frac{2e}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2e}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{a^2}$$
$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.60\times10^{-19}\,\text{C})}{(6.00\times10^{-6}\ \text{m})^2} = 160\ \text{N/C}.$$

(b) This field points at 45.0°, counterclockwise from the x axis.

14. The field of each charge has magnitude

$$E = \frac{kq}{r^2} = k\frac{e}{(0.020\,\text{m})^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{1.60\times10^{-19}\,\text{C}}{(0.020\ \text{m})^2} = 3.6\times10^{-6}\ \text{N/C}.$$

The directions are indicated in standard format below. We use the magnitude-angle notation (convenient if one is using a vector-capable calculator in polar mode) and write (starting with the proton on the left and moving around clockwise) the contributions to $\vec{E}_{\text{net}}$ as follows:

$$(E\angle -20°) + (E\angle 130°) + (E\angle -100°) + (E\angle -150°) + (E\angle 0°).$$

This yields $(3.93\times10^{-6}\angle -76.4°)$, with the N/C unit understood.

(a) The result above shows that the magnitude of the net electric field is $|\vec{E}_{\text{net}}| = 3.93\times10^{-6}$ N/C.

(b) Similarly, the direction of $\vec{E}_{\text{net}}$ is –76.4° from the x axis.

15. (a) The electron e_c is a distance $r = z = 0.020$ m away. Thus,

$$E_C = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\,\mathrm{C})}{(0.020\ \mathrm{m})^2} = 3.60\times10^{-6}\ \mathrm{N/C}\,.$$

(b) The horizontal components of the individual fields (due to the two e_s charges) cancel, and the vertical components add to give

$$E_{\mathrm{s,net}} = \frac{2ez}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} = \frac{2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\ \mathrm{C})(0.020\ \mathrm{m})}{[(0.020\ \mathrm{m})^2+(0.020\ \mathrm{m})^2]^{3/2}}$$
$$= 2.55\times10^{-6}\ \mathrm{N/C}\,.$$

(c) Calculation similar to that shown in part (a) now leads to a stronger field $E_c = 3.60\times10^{-4}\ \mathrm{N/C}$ from the central charge.

(d) The field due to the side charges may be obtained from calculation similar to that shown in part (b). The result is $E_{\mathrm{s,\,net}} = 7.09 \times 10^{-7}$ N/C.

(e) Since E_c is inversely proportional to z^2, this is a simple result of the fact that z is now much smaller than in part (a). For the net effect due to the side charges, it is the "trigonometric factor" for the y component (here expressed as $z/\sqrt{r}$) which shrinks almost linearly (as z decreases) for very small z, plus the fact that the x components cancel, which leads to the decreasing value of $E_{\mathrm{s,\,net}}$.

16. The net field components along the *x* and *y* axes are

$$E_{\text{net},\,x} = \frac{q_1}{4\pi\varepsilon_0 R^2} - \frac{q_2\cos\theta}{4\pi\varepsilon_0 R^2}, \quad E_{\text{net},\,y} = -\frac{q_2\sin\theta}{4\pi\varepsilon_0 R^2}.$$

The magnitude is the square root of the sum of the components-squared. Setting the magnitude equal to $E = 2.00 \times 10^5$ N/C, squaring and simplifying, we obtain

$$E^2 = \frac{q_1^2 + q_1^2 - 2q_1q_2\cos\theta}{(4\pi\varepsilon_0 R^2)^2}.$$

With $R = 0.500$ m, $q_1 = 2.00 \times 10^{-6}$ C and $q_2 = 6.00 \times 10^{-6}$ C, we can solve this expression for $\cos\theta$ and then take the inverse cosine to find the angle:

$$\theta = \cos^{-1}\left(\frac{q_1^2 + q_1^2 - (4\pi\varepsilon_0 R^2)^2 E^2}{2q_1q_2}\right).$$

There are two answers.

(a) The positive value of angle is $\theta = 67.8°$.

(b) The positive value of angle is $\theta = -67.8°$.

17. We make the assumption that bead 2 is in the lower half of the circle, partly because it would be awkward for bead 1 to "slide through" bead 2 if it were in the path of bead 1 (which is the upper half of the circle) and partly to eliminate a second solution to the problem (which would have opposite angle and charge for bead 2). We note that the net y component of the electric field evaluated at the origin is negative (points *down*) for all positions of bead 1, which implies (with our assumption in the previous sentence) that bead 2 is a negative charge.

(a) When bead 1 is on the $+y$ axis, there is no x component of the net electric field, which implies bead 2 is on the $-y$ axis, so its angle is –90°.

(b) Since the downward component of the net field, when bead 1 is on the $+y$ axis, is of largest magnitude, then bead 1 must be a positive charge (so that its field is in the same direction as that of bead 2, in that situation). Comparing the values of E_y at 0° and at 90° we see that the absolute values of the charges on beads 1 and 2 must be in the ratio of 5 to 4. This checks with the 180° value from the E_x graph, which further confirms our belief that bead 1 is positively charged. In fact, the 180° value from the E_x graph allows us to solve for its charge (using Eq. 22-3):

$$q_1 = 4\pi\varepsilon_o r^2 E = 4\pi\left(8.854\times10^{-12}\,\frac{\text{C}^2}{\text{N m}^2}\right)(0.60\text{ m})^2\left(5.0\times10^4\,\frac{\text{N}}{\text{C}}\right) = 2.0\times10^{-6}\text{ C}\,.$$

(c) Similarly, the 0° value from the E_y graph allows us to solve for the charge of bead 2:

$$q_2 = 4\pi\varepsilon_o r^2 E = 4\pi\left(8.854\times10^{-12}\,\frac{\text{C}^2}{\text{N m}^2}\right)(0.60\text{ m})^2\left(-4.0\times10^4\,\frac{\text{N}}{\text{C}}\right) = -1.6\times10^{-6}\text{ C}\,.$$

18. According to the problem statement, E_{act} is Eq. 22-5 (with $z = 5d$)

$$E_{\text{act}} = \frac{q}{4\pi\varepsilon_0 (4.5d)^2} - \frac{q}{4\pi\varepsilon_0 (5.5d)^2} = \frac{160}{9801} \cdot \frac{q}{4\pi\varepsilon_0 d^2}$$

and E_{approx} is

$$E_{\text{approx}} = \frac{2qd}{4\pi\varepsilon_0 (5d)^3} = \frac{2}{125} \cdot \frac{q}{4\pi\varepsilon_0 d^2}.$$

The ratio is

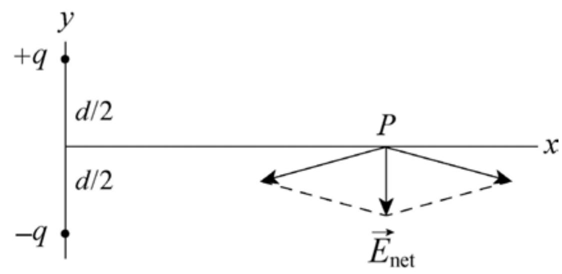
$$\frac{E_{\text{approx}}}{E_{\text{act}}} = 0.9801 \approx 0.98.$$

19. (a) Consider the figure below. The magnitude of the net electric field at point *P* is

$$\left|\vec{E}_{\text{net}}\right| = 2E_1 \sin\theta = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(d/2)^2+r^2}\right]\frac{d/2}{\sqrt{(d/2)^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[(d/2)^2+r^2\right]^{3/2}}$$

For $r \gg d$, we write $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$|\vec{E}_{\text{net}}| \approx \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}.$$



(b) From the figure, it is clear that the net electric field at point *P* points in the $-\hat{\mathrm{j}}$ direction, or –90° from the +*x* axis.

20. Referring to Eq. 22-6, we use the binomial expansion (see Appendix E) but keeping higher order terms than are shown in Eq. 22-7:

$$E = \frac{q}{4\pi\varepsilon_o z^2}\left(\left(1 + \frac{d}{z} + \frac{3}{4}\frac{d^2}{z^2} + \frac{1}{2}\frac{d^3}{z^3} + \ldots\right) - \left(1 - \frac{d}{z} + \frac{3}{4}\frac{d^2}{z^2} - \frac{1}{2}\frac{d^3}{z^3} + \ldots\right)\right)$$

$$= \frac{q\,d}{2\pi\varepsilon_o z^3} + \frac{q\,d^3}{4\pi\varepsilon_o z^5} + \ldots$$

Therefore, in the terminology of the problem, $E_{\text{next}} = q\,d^3/\,4\pi\varepsilon_0 z^5$.

21. Think of the quadrupole as composed of two dipoles, each with dipole moment of magnitude $p = qd$. The moments point in opposite directions and produce fields in opposite directions at points on the quadrupole axis. Consider the point P on the axis, a distance z to the right of the quadrupole center and take a rightward pointing field to be positive. Then, the field produced by the right dipole of the pair is $qd/2\pi\varepsilon_0(z - d/2)^3$ and the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$. Use the binomial expansions

$$(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$$

$$(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$$

to obtain

$$E = \frac{qd}{2\pi\varepsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4}.$$

Let $Q = 2qd^2$. We have $E = \dfrac{3Q}{4\pi\varepsilon_0 z^4}$.

22. (a) We use the usual notation for the linear charge density: $\lambda = q/L$. The arc length is $L = r\theta$ with θ is expressed in radians. Thus,

$$L = (0.0400 \text{ m})(0.698 \text{ rad}) = 0.0279 \text{ m}.$$

With $q = -300(1.602 \times 10^{-19} \text{ C})$, we obtain $\lambda = -1.72 \times 10^{-15}$ C/m.

(b) We consider the same charge distributed over an area $A = \pi r^2 = \pi(0.0200 \text{ m})^2$ and obtain $\sigma = q/A = -3.82 \times 10^{-14}$ C/m².

(c) Now the area is four times larger than in the previous part ($A_{\text{sphere}} = 4\pi r^2$) and thus obtain an answer that is one-fourth as big:

$$\sigma = q/A_{\text{sphere}} = -9.56 \times 10^{-15} \text{ C/m}^2.$$

(d) Finally, we consider that same charge spread throughout a volume of $V = 4\pi r^3/3$ and obtain the charge density $\rho = q/V = -1.43 \times 10^{-12}$ C/m^3.

23. We use Eq. 22-3, assuming both charges are positive. At *P*, we have

$$E_{\text{left ring}} = E_{\text{right ring}} \quad \Rightarrow \quad \frac{q_1 R}{4\pi\varepsilon_0 \left(R^2 + R^2\right)^{3/2}} = \frac{q_2 (2R)}{4\pi\varepsilon_0 [(2R)^2 + R^2]^{3/2}}$$

Simplifying, we obtain

$$\frac{q_1}{q_2} = 2\left(\frac{2}{5}\right)^{3/2} \approx 0.506.$$

24. Studying Sample Problem 22-3, we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\bigg|_{-\theta}^{\theta}$$

along the symmetry axis, with λ = $q/r\theta$ with θ in radians. In this problem, each charged quarter-circle produces a field of magnitude

$$|\vec{E}| = \frac{|q|}{r\pi/2}\frac{1}{4\pi\varepsilon_0 r}\sin\theta\bigg|_{-\pi/4}^{\pi/4} = \frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}\,|q|}{\pi r^2}.$$

That produced by the positive quarter-circle points at – 45°, and that of the negative quarter-circle points at +45°.

(a) The magnitude of the net field is

$$\begin{aligned}E_{\text{net},x} &= 2\left(\frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}\,|q|}{\pi r^2}\right)\cos 45° = \frac{1}{4\pi\varepsilon_0}\frac{4|q|}{\pi r^2}\\ &= \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)4(4.50\times10^{-12}\,\text{C})}{\pi(5.00\times10^{-2}\ \text{m})^2} = 20.6\ \text{N/C}.\end{aligned}$$

(b) By symmetry, the net field points vertically downward in the $-\hat{\text{j}}$ direction, or –90° counterclockwise from the $+x$ axis.

25. From symmetry, we see that the net field at P is twice the field caused by the upper semicircular charge $+q = \lambda \cdot \pi R$ (and that it points downward). Adapting the steps leading to Eq. 22-21, we find

$$\vec{E}_{\text{net}} = 2(-\hat{\text{j}})\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_{-90^\circ}^{90^\circ} = -\left(\frac{q}{\varepsilon_0 \pi^2 R^2}\right)\hat{\text{j}}.$$

(a) With $R = 8.50 \times 10^{-2}$ m and $q = 1.50 \times 10^{-8}$ C, $|\vec{E}_{\text{net}}| = 23.8$ N/C.

(b) The net electric field $\vec{E}_{\text{net}}$ points in the $-\hat{\text{j}}$ direction, or $-90°$ counterclockwise from the $+x$ axis.

26. We find the maximum by differentiating Eq. 22-16 and setting the result equal to zero.

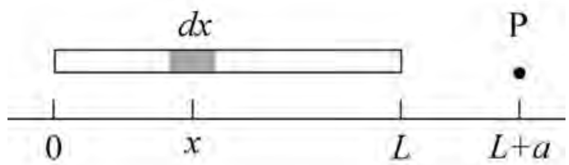
$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right)=\frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}}=0$$

which leads to $z = R/\sqrt{2}$. With R = 2.40 cm, we have z = 1.70 cm.

27. (a) The linear charge density is the charge per unit length of rod. Since the charge is uniformly distributed on the rod,

$$\lambda = \frac{-q}{L} = \frac{-4.23\times10^{-15}\ \text{C}}{0.0815\ \text{m}} = -5.19\times10^{-14}\ \text{C/m}.$$

(b) We position the *x* axis along the rod with the origin at the left end of the rod, as shown in the diagram.



Let *dx* be an infinitesimal length of rod at *x*. The charge in this segment is $dq = \lambda\, dx$. The charge *dq* may be considered to be a point charge. The electric field it produces at point *P* has only an *x* component and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{\left(L+a-x\right)^2}.$$

The total electric field produced at *P* by the whole rod is the integral

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{\left(L+a-x\right)^2} = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{L+a-x}\bigg|_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{L+a}\right)$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{a\left(L+a\right)} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a\left(L+a\right)},$$

upon substituting $-q = \lambda L$. With q = 4.23 × 10^{-15} C, L =0.0815 m and a = 0.120 m, we obtain $E_x = -1.57\times10^{-3}\ \text{N/C}$, or $|E_x| = 1.57\times10^{-3}\ \text{N/C}$.

(c) The negative sign in E_x indicates that the field points in the –*x* direction, or –180° counterclockwise form the +*x* axis.

(d) If *a* is much larger than *L*, the quantity *L* + *a* in the denominator can be approximated by *a* and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2}.$$

Since $a = 50\ \text{m} \gg L = 0.0815\ \text{m}$, the above approximation applies and we have $E_x = -1.52\times10^{-8}\ \text{N/C}$, or $|E_x| = 1.52\times10^{-8}\ \text{N/C}$.

(e) For a particle of charge $-q = -4.23\times10^{-15}\ \text{C}$, the electric field at a distance *a* = 50 m away has a magnitude $|E_x| = 1.52\times10^{-8}\ \text{N/C}$.

28. We use Eq. 22-16, with "*q*" denoting the charge on the larger ring:

$$\frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}+\frac{qz}{4\pi\varepsilon_0[z^2+(3R)^2]^{3/2}}=0 \quad\Rightarrow\quad q=-Q\left(\frac{13}{5}\right)^{3/2}=-4.19Q\,.$$

Note: we set $z = 2R$ in the above calculation.

29. The smallest arc is of length $L_1 = \pi r_1/2 = \pi R/2$; the middle-sized arc has length $L_2 = \pi r_2/2 = \pi(2R)/2 = \pi R$; and, the largest arc has $L_3 = \pi(3R)/2$. The charge per unit length for each arc is $\lambda = q/L$ where each charge q is specified in the figure. Following the steps that lead to Eq. 22-21 in Sample Problem 22-3, we find

$$E_{\text{net}} = \frac{\lambda_1(2\sin 45°)}{4\pi\varepsilon_0 r_1} + \frac{\lambda_2(2\sin 45°)}{4\pi\varepsilon_0 r_2} + \frac{\lambda_3(2\sin 45°)}{4\pi\varepsilon_0 r_3} = \frac{Q}{\sqrt{2}\pi^2\varepsilon_0 R^2}$$

which yields $E_{\text{net}} = 1.62 \times 10^6$ N/C .

(b) The direction is – 45º, measured counterclockwise from the $+x$ axis.

30. (a) It is clear from symmetry (also from Eq. 22-16) that the field vanishes at the center.

(b) The result ($E = 0$) for points infinitely far away can be reasoned directly from Eq. 22-16 (it goes as $1/z^2$ as $z \to \infty$) or by recalling the starting point of its derivation (Eq. 22-11, which makes it clearer that the field strength decreases as $1/r^2$ at distant points).

(c) Differentiating Eq. 22-16 and setting equal to zero (to obtain the location where it is maximum) leads to

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right)=\frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}}=0 \quad\Rightarrow\quad z=+\frac{R}{\sqrt{2}}=0.707R\,.$$

(d) Plugging this value back into Eq. 22-16 with the values stated in the problem, we find $E_{max} = 3.46 \times 10^7$ N/C.

31. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22-3 illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ if θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}.$$

The problem asks for the ratio $E_{\text{particle}} / E_{\text{arc}}$ where E_{particle} is given by Eq. 22-3:

$$\frac{E_{\text{particle}}}{E_{\text{arc}}} = \frac{Q/4\pi\varepsilon_0 R^2}{2Q\sin(\theta/2)/4\pi\varepsilon_0 R^2\theta} = \frac{\theta}{2\sin(\theta/2)}.$$

With $\theta = \pi$, we have

$$\frac{E_{\text{particle}}}{E_{\text{arc}}} = \frac{\pi}{2} \approx 1.57.$$

32. We assume $q > 0$. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element dx of the rod contains charge $dq = \lambda\, dx$. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin\theta = R/r$ where $r = \sqrt{x^2 + R^2}$.

(a) Using Eq. 22-3 (with the 2 and $\sin\theta$ factors just discussed) the magnitude is

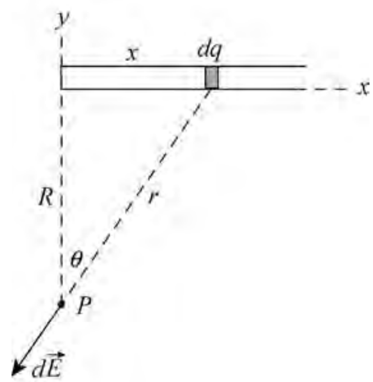
$$\begin{aligned}
|\vec{E}| &= 2\int_0^{L/2}\left(\frac{dq}{4\pi\varepsilon_0 r^2}\right)\sin\theta = \frac{2}{4\pi\varepsilon_0}\int_0^{L/2}\left(\frac{\lambda\, dx}{x^2+R^2}\right)\left(\frac{y}{\sqrt{x^2+R^2}}\right)\\
&= \frac{\lambda R}{2\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{(q/L)R}{2\pi\varepsilon_0}\cdot\left.\frac{x}{R^2\sqrt{x^2+R^2}}\right|_0^{L/2}\\
&= \frac{q}{2\pi\varepsilon_0 LR}\frac{L/2}{\sqrt{(L/2)^2+R^2}} = \frac{q}{2\pi\varepsilon_0 R}\frac{1}{\sqrt{L^2+4R^2}}
\end{aligned}$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals). With $q = 7.81\times10^{-12}$ C , $L = 0.145$ m and $R = 0.0600$ m, we have $|\vec{E}| = 12.4$ N/C .

(b) As noted above, the electric field $\vec{E}$ points in the $+y$ direction, or $+90°$ counterclockwise from the $+x$ axis.

33. Consider an infinitesimal section of the rod of length dx, a distance x from the left end, as shown in the following diagram. It contains charge $dq = \lambda\, dx$ and is a distance r from P. The magnitude of the field it produces at P is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}.$$



The x and the y components are

$$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\sin\theta$$

and

$$dE_y = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\cos\theta\,,$$

respectively. We use θ as the variable of integration and substitute $r = R/\cos\ \theta$, $x = R\tan\theta$ and $dx = (R/\cos^2\ \theta)\ d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus,

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\sin\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\cos\ \theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

and

$$E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\sin\ \theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}.$$

We notice that $E_x = E_y$ no matter what the value of R. Thus, $\vec{E}$ makes an angle of 45° with the rod for all values of R.

34. From Eq. 22-26, we obtain

$$E=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)=\frac{5.3\times10^{-6}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}\left[1-\frac{12\,\mathrm{cm}}{\sqrt{\left(12\,\mathrm{cm}\right)^2+\left(2.5\,\mathrm{cm}\right)^2}}\right]=6.3\times10^{3}\ \mathrm{N/C}.$$

35. At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 22-26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of z such that $E/E_c = 1/2$. This means

$$1 - \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2} \Rightarrow \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2}.$$

Squaring both sides, then multiplying them by $z^2 + R^2$, we obtain $z^2 = (z^2/4) + (R^2/4)$. Thus, $z^2 = R^2/3$, or $z = R\big/\sqrt{3}$. With $R = 0.600$ m, we have $z = 0.346$ m.

36. We write Eq. 22-26 as

$$\frac{E}{E_{\max}} = 1 - \frac{z}{(z^2 + R^2)^{1/2}}$$

and note that this ratio is $\frac{1}{2}$ (according to the graph shown in the figure) when z = 4.0 cm. Solving this for R we obtain $R = z\sqrt{3}$ = 6.9 cm.

37. We use Eq. 22-26, noting that the disk in figure (b) is effectively equivalent to the disk in figure (a) plus a concentric smaller disk (of radius R/2) with the opposite value of σ. That is,

$$E_{(b)} = E_{(a)} - \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{2R}{\sqrt{(2R)^2 + (R/2)^2}}\right)$$

where

$$E_{(a)} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{2R}{\sqrt{(2R)^2 + R^2}}\right) .$$

We find the relative difference and simplify:

$$\frac{E_{(a)} - E_{(b)}}{E_{(a)}} = \frac{1-2/\sqrt{4+1/4}}{1-2/\sqrt{4+1}} = \frac{1-2/\sqrt{17/4}}{1-2/\sqrt{5}} = \frac{0.0299}{0.1056} = 0.283$$

or approximately 28%.

38. From $dA = 2\pi r\, dr$ (which can be thought of as the differential of $A = \pi r^2$) and $dq = \sigma\, dA$ (from the definition of the surface charge density σ), we have

$$dq = \left(\frac{Q}{\pi R^2}\right) 2\pi r\, dr$$

where we have used the fact that the disk is uniformly charged to set the surface charge density equal to the total charge (Q) divided by the total area (πR^2). We next set $r = 0.0050$ m and make the approximation $dr \approx 30 \times 10^{-6}$ m. Thus we get $dq \approx 2.4 \times 10^{-16}$ C.

39. The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{(1.60\times 10^{-19}\ \text{C})(2.00\times 10^{4}\ \text{N/C})}{9.11\times 10^{-31}\ \text{kg}} = 3.51\times 10^{15}\ \text{m/s}^2\ .$$

40. Eq. 22-28 gives

$$\vec{E} = \frac{\vec{F}}{q} = \frac{m\vec{a}}{(-e)} = -\left(\frac{m}{e}\right)\vec{a}$$

using Newton’s second law.

(a) With *east* being the $\hat{\mathrm{i}}$ direction, we have

$$\vec{E} = -\left(\frac{9.11\times10^{-31}\ \mathrm{kg}}{1.60\times10^{-19}\ \mathrm{C}}\right)\left(1.80\times10^{9}\ \mathrm{m/s^2}\ \hat{\mathrm{i}}\right) = (-0.0102\ \mathrm{N/C})\,\hat{\mathrm{i}}$$

which means the field has a magnitude of 0.0102 N/C

(b) The result shows that the field $\vec{E}$ is directed in the $-x$ direction, or westward.

41. We combine Eq. 22-9 and Eq. 22-28 (in absolute values).

$$F = |q|E = |q|\left(\frac{p}{2\pi\varepsilon_0 z^3}\right) = \frac{2kep}{z^3}$$

where we have used Eq. 21-5 for the constant k in the last step. Thus, we obtain

$$F = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(3.6\times10^{-29}\text{C}\cdot\text{m}\right)}{\left(25\times10^{-9}\,\text{m}\right)^3} = 6.6\times10^{-15}\ \text{N}\,.$$

If the dipole is oriented such that $\vec{p}$ is in the $+z$ direction, then $\vec{F}$ points in the $-z$ direction.

42. (a) Vertical equilibrium of forces leads to the equality

$$q\left|\vec{E}\right| = mg \;\Rightarrow\; \left|\vec{E}\right| = \frac{mg}{2e}.$$

Substituting the values given in the problem, we obtain

$$\left|\vec{E}\right| = \frac{mg}{2e} = \frac{(6.64\times10^{-27}\ \text{kg})(9.8\ \text{m/s}^2)}{2(1.6\times10^{-19}\,\text{C})} = 2.03\times10^{-7}\ \text{N/C}\,.$$

(b) Since the force of gravity is downward, then $q\vec{E}$ must point upward. Since $q > 0$ in this situation, this implies $\vec{E}$ must itself point upward.

43. (a) The magnitude of the force on the particle is given by $F = qE$, where q is the magnitude of the charge carried by the particle and E is the magnitude of the electric field at the location of the particle. Thus,

$$E = \frac{F}{q} = \frac{3.0\times10^{-6}\text{ N}}{2.0\times10^{-9}\text{ C}} = 1.5\times10^{3}\text{ N/C}.$$

The force points downward and the charge is negative, so the field points upward.

(b) The magnitude of the electrostatic force on a proton is

$$F_{el} = eE = \left(1.60\times10^{-19}\text{ C}\right)\left(1.5\times10^{3}\text{ N/C}\right) = 2.4\times10^{-16}\text{N}.$$

(c) A proton is positively charged, so the force is in the same direction as the field, upward.

(d) The magnitude of the gravitational force on the proton is

$$F_g = mg = \left(1.67\times10^{-27}\text{ kg}\right)\left(9.8\text{ m/s}^2\right) = 1.6\times10^{-26}\text{N}.$$

The force is downward.

(e) The ratio of the forces is

$$\frac{F_{el}}{F_g} = \frac{2.4\times10^{-16}\text{ N}}{1.64\times10^{-26}\text{ N}} = 1.5\times10^{10}.$$

44. (a) $F_e = Ee = (3.0 \times 10^6 \text{ N/C})(1.6 \times 10^{-19} \text{ C}) = 4.8 \times 10^{-13}$ N.

(b) $F_i = Eq_{\text{ion}} = Ee = (3.0 \times 10^6 \text{ N/C})(1.6 \times 10^{-19} \text{ C}) = 4.8 \times 10^{-13}$ N.

45. (a) The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus,

$$a = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{4}\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 1.92\times10^{12}\text{ m/s}^2 .$$

(b) We assume the proton starts from rest and use the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$) to show that

$$v = \sqrt{2ax} = \sqrt{2\left(1.92\times10^{12}\text{ m/s}^2\right)\left(0.0100\text{ m}\right)} = 1.96\times10^{5}\text{ m/s}.$$

46. (a) The initial direction of motion is taken to be the $+x$ direction (this is also the direction of $\vec{E}$). We use $v_f^2 - v_i^2 = 2a\Delta x$ with $v_f = 0$ and $\vec{a} = \vec{F}/m = -e\vec{E}/m_e$ to solve for distance Δx:

$$\Delta x = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-\left(9.11\times10^{-31}\text{ kg}\right)\left(5.00\times10^{6}\text{ m/s}\right)^2}{-2\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{3}\text{ N/C}\right)} = 7.12\times10^{-2}\text{ m}.$$

(b) Eq. 2-17 leads to

$$t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_i} = \frac{2\left(7.12\times10^{-2}\text{ m}\right)}{5.00\times10^{6}\text{ m/s}} = 2.85\times10^{-8}\text{ s}.$$

(c) Using $\Delta v^2 = 2a\Delta x$ with the new value of Δx, we find

$$\frac{\Delta K}{K_i} = \frac{\Delta\left(\frac{1}{2}m_e v^2\right)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta x}{v_i^2} = \frac{-2eE\Delta x}{m_e v_i^2}$$
$$= \frac{-2\left(1.60\times10^{-19}\text{C}\right)\left(1.00\times10^{3}\text{ N/C}\right)\left(8.00\times10^{-3}\text{m}\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(5.00\times10^{6}\text{ m/s}\right)^2} = -0.112.$$

Thus, the fraction of the initial kinetic energy lost in the region is 0.112 or 11.2%.

47. When the drop is in equilibrium, the force of gravity is balanced by the force of the electric field: $mg = -qE$, where m is the mass of the drop, q is the charge on the drop, and E is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where r is its radius and ρ is its mass density. Thus,

$$q = -\frac{mg}{E} = -\frac{4\pi r^3 \rho g}{3E} = -\frac{4\pi\left(1.64\times10^{-6}\,\text{m}\right)^3\left(851\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)}{3\left(1.92\times10^5\,\text{N/C}\right)} = -8.0\times10^{-19}\,\text{C}$$

and $q/e = (-8.0 \times 10^{-19}\text{ C})/(1.60 \times 10^{-19}\text{ C}) = -5$, or $q = -5e$.

48. We assume there are no forces or force-components along the x direction. We combine Eq. 22-28 with Newton's second law, then use Eq. 4-21 to determine time t followed by Eq. 4-23 to determine the final velocity (with $-g$ replaced by the a_y of this problem); for these purposes, the velocity components *given* in the problem statement are re-labeled as v_{0x} and v_{0y} respectively.

(a) We have $\vec{a} = q\vec{E}/m = -(e/m)\vec{E}$ which leads to

$$\vec{a} = -\left(\frac{1.60\times10^{-19}\,\text{C}}{9.11\times10^{-31}\text{kg}}\right)\left(120\,\frac{\text{N}}{\text{C}}\right)\hat{\text{j}} = -(2.1\times10^{13}\ \text{m/s}^2)\,\hat{\text{j}}.$$

(b) Since $v_x = v_{0x}$ in this problem (that is, $a_x = 0$), we obtain

$$t = \frac{\Delta x}{v_{0x}} = \frac{0.020\,\text{m}}{1.5\times10^5\ \text{m/s}} = 1.3\times10^{-7}\,\text{s}$$

$$v_y = v_{0y} + a_y t = 3.0\times10^3\ \text{m/s} + \left(-2.1\times10^{13}\ \text{m/s}^2\right)\left(1.3\times10^{-7}\,\text{s}\right)$$

which leads to $v_y = -2.8 \times 10^6$ m/s. Therefore, the final velocity is

$$\vec{v} = (1.5\times10^5\ \text{m/s})\,\hat{\text{i}} - (2.8\times10^6\ \text{m/s})\,\hat{\text{j}}.$$

49. (a) We use $\Delta x = v_{avg}t = vt/2$:

$$v = \frac{2\Delta x}{t} = \frac{2\left(2.0\times 10^{-2}\,\text{m}\right)}{1.5\times 10^{-8}\,\text{s}} = 2.7\times 10^{6}\ \text{m/s}.$$

(b) We use $\Delta x = \frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E = \frac{ma}{e} = \frac{2\Delta xm}{et^2} = \frac{2\left(2.0\times 10^{-2}\,\text{m}\right)\left(9.11\times 10^{-31}\,\text{kg}\right)}{\left(1.60\times 10^{-19}\,\text{C}\right)\left(1.5\times 10^{-8}\,\text{s}\right)^2} = 1.0\times 10^{3}\ \text{N/C}.$$

50. Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the $+y$ direction (which we will call "upward") leads to a downward acceleration. This is exactly like a projectile motion problem as treated in Chapter 4 (but with g replaced with $a = eE/m = 8.78 \times 10^{11}$ m/s^2). Thus, Eq. 4-21 gives

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{3.00\text{ m}}{(2.00\times10^6\text{ m/s})\cos 40.0°} = 1.96\times10^{-6}\text{ s}.$$

This leads (using Eq. 4-23) to

$$\begin{aligned} v_y &= v_0 \sin\theta_0 - at = (2.00\times10^6\text{ m/s})\sin 40.0° - (8.78\times10^{11}\text{ m/s}^2)(1.96\times10^{-6}\text{ s}) \\ &= -4.34\times10^5\text{ m/s}. \end{aligned}$$

Since the x component of velocity does not change, then the final velocity is

$$\vec{v} = (1.53\times10^6\text{ m/s})\,\hat{\text{i}} - (4.34\times10^5\text{ m/s})\,\hat{\text{j}}\ .$$

51. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where E is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time t is $x = \frac{1}{2} a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2} a_e t^2$. They pass each other when their coordinates are the same, or $\frac{1}{2} a_p t^2 = L + \frac{1}{2} a_e t^2$. This means $t^2 = 2L/(a_p - a_e)$ and

$$
\begin{aligned}
x &= \frac{a_p}{a_p - a_e} L = \frac{eE/m_p}{(eE/m_p) + (eE/m_e)} L = \left(\frac{m_e}{m_e + m_p} \right) L \\
&= \left(\frac{9.11\times10^{-31}\,\text{kg}}{9.11\times10^{-31}\,\text{kg} + 1.67\times10^{-27}\,\text{kg}} \right) (0.050\,\text{m}) \\
&= 2.7\times10^{-5}\,\text{m}.
\end{aligned}
$$

52. We are given $\sigma = 4.00 \times 10^{-6}$ C/m^2 and various values of z (in the notation of Eq. 22-26 which specifies the field E of the charged disk). Using this with $F = eE$ (the magnitude of Eq. 22-28 applied to the electron) and $F = ma$, we obtain $a = F/m = eE/m$.

(a) The magnitude of the acceleration at a distance R is

$$a = \frac{e\,\sigma\,(2-\sqrt{2}\,)}{4\,m\,\varepsilon_o} = 1.16 \times 10^{16}\ \text{m/s}^2 .$$

(b) At a distance R/100, $a = \dfrac{e\,\sigma\,(10001-\sqrt{10001}\,)}{20002\,m\,\varepsilon_o} = 3.94 \times 10^{16}\ \text{m/s}^2$.

(c) At a distance R/1000, $a = \dfrac{e\,\sigma\,(1000001-\sqrt{1000001}\,)}{2000002\,m\,\varepsilon_o} = 3.97 \times 10^{16}\ \text{m/s}^2$.

(d) The field due to the disk becomes more uniform as the electron nears the center point. One way to view this is to consider the forces exerted on the electron by the charges near the edge of the disk; the net force on the electron caused by those charges will decrease due to the fact that their contributions come closer to canceling out as the electron approaches the middle of the disk.

53. (a) Using Eq. 22-28, we find

$$\begin{aligned}\vec{F} &= \left(8.00\times10^{-5}\text{C}\right)\left(3.00\times10^{3}\ \text{N/C}\right)\hat{\text{i}} + \left(8.00\times10^{-5}\text{C}\right)\left(-600\,\text{N/C}\right)\hat{\text{j}} \\ &= \left(0.240\text{N}\right)\hat{\text{i}} - \left(0.0480\text{N}\right)\hat{\text{j}}.\end{aligned}$$

Therefore, the force has magnitude equal to

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(0.240\,\text{N}\right)^2 + \left(-0.0480\,\text{N}\right)^2} = 0.245\,\text{N}.$$

(b) The angle the force $\vec{F}$ makes with the $+x$ axis is

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.0480\,\text{N}}{0.240\,\text{N}}\right) = -11.3°$$

measured counterclockwise from the $+x$ axis.

(c) With m = 0.0100 kg, the (x, y) coordinates at t = 3.00 s can be found by combining Newton's second law with the kinematics equations of Chapters 2–4. The x coordinate is

$$x = \frac{1}{2}a_x t^2 = \frac{F_x t^2}{2m} = \frac{\left(0.240\ \text{N}\right)\left(3.00\ \text{s}\right)^2}{2\left(0.0100\ \text{kg}\right)} = 108\,\text{m}.$$

(d) Similarly, the y coordinate is

$$y = \frac{1}{2}a_y t^2 = \frac{F_y t^2}{2m} = \frac{\left(-0.0480\ \text{N}\right)\left(3.00\ \text{s}\right)^2}{2\left(0.0100\ \text{kg}\right)} = -21.6\,\text{m}.$$

54. (a) Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the same direction as the velocity leads to deceleration. Thus, with $t = 1.5 \times 10^{-9}$ s, we find

$$v = v_0 - |a|t = v_0 - \frac{eE}{m}t = 4.0\times10^4 \text{ m/s} - \frac{(1.6\times10^{-19}\text{ C})(50\text{ N/C})}{9.11\times10^{-31}\text{ kg}}(1.5\times10^{-9}\text{ s})$$
$$= 2.7\times10^4 \text{ m/s}.$$

(b) The displacement is equal to the distance since the electron does not change its direction of motion. The field is uniform, which implies the acceleration is constant. Thus,

$$d = \frac{v+v_0}{2}t = 5.0\times10^{-5}\text{ m}.$$

55. We take the charge $Q = 45.0\ \text{pC}$ of the bee to be concentrated as a particle at the center of the sphere. The magnitude of the induced charges on the sides of the grain is $|q| = 1.000\ \text{pC}$.

(a) The electrostatic force on the grain by the bee is

$$F = \frac{kQq}{(d+D/2)^2} + \frac{kQ(-q)}{(D/2)^2} = -kQ|q|\left[\frac{1}{(D/2)^2} - \frac{1}{(d+D/2)^2}\right]$$

where $D = 1.000\ \text{cm}$ is the diameter of the sphere representing the honeybee, and $d = 40.0\,\mu\text{m}$ is the diameter of the grain. Substituting the values, we obtain

$$\begin{aligned} F &= -\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(45.0\times10^{-12}\,\text{C})(1.000\times10^{-12}\,\text{C})\left[\frac{1}{(5.00\times10^{-3}\ \text{m})^2} - \frac{1}{(5.04\times10^{-3}\ \text{m})^2}\right] \\ &= -2.56\times10^{-10}\ \text{N}. \end{aligned}$$

The negative sign implies that the force between the bee and the grain is attractive. The magnitude of the force is $|F| = 2.56\times10^{-10}\ \text{N}$.

(b) Let $|Q'| = 45.0\ \text{pC}$ be the magnitude of the charge on the tip of the stigma. The force on the grain due to the stigma is

$$F' = \frac{k|Q'|q}{(d+D')^2} + \frac{k|Q'|(-q)}{(D')^2} = -k|Q'||q|\left[\frac{1}{(D')^2} - \frac{1}{(d+D')^2}\right]$$

where $D' = 1.000\ \text{mm}$ is the distance between the grain and the tip of the stigma. Substituting the values given, we have

$$\begin{aligned} F' &= -\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(45.0\times10^{-12}\,\text{C})(1.000\times10^{-12}\,\text{C})\left[\frac{1}{(1.000\times10^{-3}\ \text{m})^2} - \frac{1}{(1.040\times10^{-3}\ \text{m})^2}\right] \\ &= -3.06\times10^{-8}\ \text{N}. \end{aligned}$$

The negative sign implies that the force between the grain and the stigma is attractive. The magnitude of the force is $|F'| = 3.06\times10^{-8}\ \text{N}$.

(c) Since $|F'| > |F|$, the grain will move to the stigma.

56. (a) Eq. 22-33 leads to $\tau = pE\sin 0° = 0.$

(b) With $\theta = 90°$, the equation gives

$$\tau = pE = \left(2\left(1.6\times10^{-19}\ \text{C}\right)\left(0.78\times10^{-9}\ \text{m}\right)\right)\left(3.4\times10^{6}\ \text{N/C}\right) = 8.5\times10^{-22}\ \text{N}\cdot\text{m}.$$

(c) Now the equation gives $\tau = pE\sin 180° = 0.$

57. (a) The magnitude of the dipole moment is

$$p = qd = \left(1.50\times10^{-9}\ \text{C}\right)\left(6.20\times10^{-6}\ \text{m}\right) = 9.30\times10^{-15}\ \text{C}\cdot\text{m}.$$

(b) Following the solution to part (c) of Sample Problem 22-5, we find

$$U\left(180°\right) - U\left(0\right) = 2pE = 2\left(9.30\times10^{-15}\,\text{C}\cdot\text{m}\right)\left(1100\ \text{N/C}\right) = 2.05\times10^{-11}\,\text{J}.$$

58. Using Eq. 22-35, considering θ as a variable, we note that it reaches its maximum value when θ = –90°: $\tau_{max} = pE$. Thus, with E = 40 N/C and $\tau_{max} = 100 \times 10^{-28}$ N·m (determined from the graph), we obtain the dipole moment: $p = 2.5 \times 10^{-28}$ C·m.

59. Eq. 22-35 $(\tau = -pE\sin\theta)$ captures the sense as well as the magnitude of the effect. That is, this is a restoring torque, trying to bring the tilted dipole back to its aligned equilibrium position. If the amplitude of the motion is small, we may replace $\sin\theta$ with θ in radians. Thus, $\tau \approx -pE\theta$. Since this exhibits a simple negative proportionality to the angle of rotation, the dipole oscillates in simple harmonic motion, like a torsional pendulum with torsion constant $\kappa = pE$. The angular frequency ω is given by

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I}$$

where I is the rotational inertia of the dipole. The frequency of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}.$$

60. Examining the lowest value on the graph, we have (using Eq. 22-38)

$$U = -\vec{p} \cdot \vec{E} = -1.00 \times 10^{-28} \text{ J}.$$

If $E = 20$ N/C, we find $p = 5.0 \times 10^{-28}$ C·m.

61. Following the solution to part (c) of Sample Problem 22-5, we find

$$
\begin{aligned}
W &= U(\theta_0 + \pi) - U(\theta_0) = -pE\left(\cos(\theta_0 + \pi) - \cos(\theta_0)\right) = 2pE\cos\theta_0 \\
&= 2(3.02\times10^{-25}\,\mathrm{C\cdot m})(46.0\ \mathrm{N/C})\cos 64.0^\circ \\
&= 1.22\times10^{-23}\ \mathrm{J}.
\end{aligned}
$$

62. Our approach (based on Eq. 22-29) consists of several steps. The first is to find an *approximate* value of e by taking differences between all the given data. The smallest difference is between the fifth and sixth values:

$$18.08 \times 10^{-19}\text{ C} - 16.48 \times 10^{-19}\text{ C} = 1.60 \times 10^{-19}\text{ C}$$

which we denote e_{approx}. The goal at this point is to assign integers n using this approximate value of e:

datum1	$\dfrac{6.563\times10^{-19}\text{C}}{e_{\text{approx}}}=4.10 \Rightarrow n_1=4$	datum6	$\dfrac{18.08\times10^{-19}\text{C}}{e_{\text{appeox}}}=11.30 \Rightarrow n_6=11$
datum2	$\dfrac{8.204\times10^{-19}\text{C}}{e_{\text{approx}}}=5.13 \Rightarrow n_2=5$	datum7	$\dfrac{19.71\times10^{-19}\text{C}}{e_{\text{approx}}}=12.32 \Rightarrow n_7=12$
datum3	$\dfrac{11.50\times10^{-19}\text{C}}{e_{\text{approx}}}=7.19 \Rightarrow n_3=7$	datum8	$\dfrac{22.89\times10^{-19}\text{C}}{e_{\text{approx}}}=14.31 \Rightarrow n_8=14$
datum4	$\dfrac{13.13\times10^{-19}\text{C}}{e_{\text{approx}}}=8.21 \Rightarrow n_4=8$	datum9	$\dfrac{26.13\times10^{-19}\text{C}}{e_{\text{approx}}}=16.33 \Rightarrow n_9=16$
datum5	$\dfrac{16.48\times10^{-19}\text{C}}{e_{\text{approx}}}=10.30 \Rightarrow n_5=10$		

Next, we construct a new data set (e_1, e_2, e_3 …) by dividing the given data by the respective exact integers n_i (for i = 1, 2, 3 …):

$$(e_1, e_2, e_3 \ldots) = \left(\frac{6.563\times10^{-19}\text{ C}}{n_1}, \frac{8.204\times10^{-19}\text{ C}}{n_2}, \frac{11.50\times10^{-19}\text{ C}}{n_3}\ldots\right)$$

which gives (carrying a few more figures than are significant)

$$\left(1.64075\times10^{-19}\text{ C},\ 1.6408\times10^{-19}\text{ C},\ 1.64286\times10^{-19}\text{ C}\ldots\right)$$

as the new data set (our experimental values for e). We compute the average and standard deviation of this set, obtaining

$$e_{\text{exptal}} = e_{\text{avg}} \pm \Delta e = (1.641 \pm 0.004)\times10^{-19}\text{ C}$$

which does not agree (to within one standard deviation) with the modern accepted value for e. The lower bound on this spread is $e_{\text{avg}} - \Delta e = 1.637 \times 10^{-19}$ C which is still about 2% too high.

63. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22-3 illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}\,.$$

Now, the arc length is $L = r\theta$ with θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}\ .$$

Thus, the problem requires $E_{\text{arc}} = \frac{1}{2}\,E_{\text{particle}}$ where E_{particle} is given by Eq. 22-3. Hence,

$$\frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2} \;\Rightarrow\; \sin\frac{\theta}{2} = \frac{\theta}{4}$$

where we note, again, that the angle is in radians. The approximate solution to this equation is $\theta = 3.791$ rad ≈ 217°.

64. Most of the individual fields, caused by diametrically opposite charges, will cancel, except for the pair that lie on the *x* axis passing through the center. This pair of charges produces a field pointing to the right

$$\vec{E} = \frac{3q}{4\pi\varepsilon_0 d^2}\hat{\mathrm{i}} = \frac{3e}{4\pi\varepsilon_0 d^2}\hat{\mathrm{i}} = \frac{3\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)}{\left(0.020\mathrm{m}\right)^2}\hat{\mathrm{i}} = 1.08\times10^{-5}\ \mathrm{N/C}\,.$$

65. (a) From symmetry, we see the net field component along the x axis is zero; the net field component along the y axis points upward. With θ = 60°,

$$E_{\text{net},y} = 2\frac{Q\sin\theta}{4\pi\varepsilon_0 a^2} .$$

Since sin(60°) = $\sqrt{3}/2$, we can write this as $E_{\text{net}} = kQ\sqrt{3}/a^2$ (using the notation of the constant k defined in Eq. 21-5). Numerically, this gives roughly 47 N/C.

(b) From symmetry, we see in this case that the net field component along the y axis is zero; the net field component along the x axis points rightward. With θ = 60°,

$$E_{\text{net},x} = 2\frac{Q\cos\theta}{4\pi\varepsilon_0 a^2} .$$

Since cos(60°) = 1/2, we can write this as $E_{\text{net}} = kQ/a^2$ (using the notation of Eq. 21-5). Thus, $E_{\text{net}} \approx 27$ N/C.

66. The two closest charges produce fields at the midpoint which cancel each other out. Thus, the only significant contribution is from the furthest charge, which is a distance $r = \sqrt{3}d/2$ away from that midpoint. Plugging this into Eq. 22-3 immediately gives the result:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0(\sqrt{3}d/2)^2} = \frac{4}{3}\frac{Q}{4\pi\varepsilon_0 d^2}.$$

67. (a) Since the two charges in question are of the same sign, the point $x = 2.0$ mm should be located in between them (so that the field vectors point in the opposite direction). Let the coordinate of the second particle be x' ($x' > 0$). Then, the magnitude of the field due to the charge $-q_1$ evaluated at x is given by $E = q_1/4\pi\varepsilon_0 x^2$, while that due to the second charge $-4q_1$ is $E' = 4q_1/4\pi\varepsilon_0(x' - x)^2$. We set the net field equal to zero:

$$\vec{E}_{\text{net}} = 0 \;\Rightarrow\; E = E'$$

so that

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{4q_1}{4\pi\varepsilon_0 (x' - x)^2}.$$

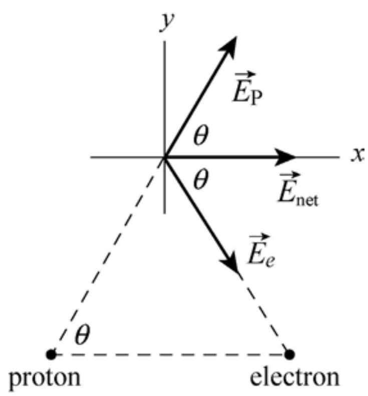
Thus, we obtain $x' = 3x = 3(2.0 \text{ mm}) = 6.0$ mm.

(b) In this case, with the second charge now positive, the electric field vectors produced by both charges are in the negative x direction, when evaluated at $x = 2.0$ mm. Therefore, the net field points in the negative x direction, or 180°, measured counterclockwise from the $+x$ axis.

68. We denote the electron with subscript e and the proton with p. From the figure below we see that

$$\left|\vec{E}_e\right| = \left|\vec{E}_p\right| = \frac{e}{4\pi\varepsilon_0 d^2}$$

where $d = 2.0 \times 10^{-6}$ m. We note that the components along the y axis cancel during the vector summation. With $k = 1/4\pi\varepsilon_0$ and $\theta = 60°$, the magnitude of the net electric field is obtained as follows:



$$\left|\vec{E}_{\text{net}}\right| = E_x = 2E_e\cos\theta = 2\left(\frac{e}{4\pi\varepsilon_0 d^2}\right)\cos\theta = 2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.6\times10^{-19}\ \text{C}\right)}{\left(2.0\times10^{-6}\ \text{m}\right)^2}\cos 60°$$

$$= 3.6\times10^2\ \text{N/C}.$$

69. On the one hand, the conclusion (that Q = +1.00 μC) is clear from symmetry. If a more in-depth justification is desired, one should use Eq. 22-3 for the electric field magnitudes of the three charges (each at the same distance $r = a/\sqrt{3}$ from C) and then find field components along suitably chosen axes, requiring each component-sum to be zero. If the y axis is vertical, then (assuming Q > 0) the component-sum along that axis leads to $2kq \sin 30° / r^2 = kQ / r^2$ where q refers to either of the charges at the bottom corners. This yields $Q = 2q \sin 30° = q$ and thus to the conclusion mentioned above.

70. (a) Let $E = \sigma/2\varepsilon_0 = 3 \times 10^6$ N/C. With $\sigma = |q|/A$, this leads to

$$|q| = \pi R^2\sigma = 2\pi\varepsilon_0 R^2 E = \frac{R^2E}{2k} = \frac{\left(2.5\times10^{-2}\,\mathrm{m}\right)^2\left(3.0\times10^6\ \mathrm{N/C}\right)}{2\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)} = 1.0\times10^{-7}\ \mathrm{C}\ ,$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\ \mathrm{N\cdot m^2/C^2}$.

(b) Setting up a simple proportionality (with the areas), the number of atoms is estimated to be

$$n = \frac{\pi\left(2.5\times10^{-2}\,\mathrm{m}\right)^2}{0.015\times10^{-18}\,\mathrm{m}^2} = 1.3\times10^{17}.$$

(c) The fraction is

$$\frac{q}{Ne} = \frac{1.0\times10^{-7}\,\mathrm{C}}{\left(1.3\times10^{17}\right)\left(1.6\times10^{-19}\,\mathrm{C}\right)} \approx 5.0\times10^{-6}.$$

71. (a) Using the density of water (ρ = 1000 kg/m^3), the weight mg of the spherical drop (of radius $r = 6.0 \times 10^{-7}$ m) is

$$W = \rho V g = \left(1000\,\text{kg/m}^3\right)\left(\frac{4\pi}{3}\left(6.0\times10^{-7}\text{ m}\right)^3\right)\left(9.8\,\text{m/s}^2\right) = 8.87\times10^{-15}\text{ N}.$$

(b) Vertical equilibrium of forces leads to $mg = qE = neE$, which we solve for n, the number of excess electrons:

$$n = \frac{mg}{eE} = \frac{8.87\times10^{-15}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(462\,\text{N/C}\right)} = 120.$$

72. Eq. 22-38 gives $U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$. We note that θ_i = 110° and θ_f = 70.0°. Therefore,

$$\Delta U = -pE\left(\cos 70.0° - \cos 110°\right) = -3.28 \times 10^{-21}\ \text{J}.$$

73. Studying Sample Problem 22-3, we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r} \sin\theta \Big|_{-\theta}^{\theta}$$

along the symmetry axis, where $\lambda = q/\ell = q/r\theta$ with θ in radians. Here ℓ is the length of the arc, given as $\ell = 4.0\,\mathrm{m}$. Therefore, $\theta = \ell/r = 4.0/2.0 = 2.0\,\mathrm{rad}$. Thus, with $q = 20 \times 10^{-9}$ C, we obtain

$$\left|\vec{E}\right| = \frac{(q/\ell)}{4\pi\varepsilon_0 r} \sin\theta \Big|_{-1.0\ \mathrm{rad}}^{1.0\ \mathrm{rad}} = 38\ \mathrm{N/C}\ .$$

74. (a) We combine Eq. 22-28 (in absolute value) with Newton's second law:

$$a = \frac{|q|E}{m} = \left(\frac{1.60\times10^{-19}\,\mathrm{C}}{9.11\times10^{-31}\,\mathrm{kg}}\right)\left(1.40\times10^{6}\ \frac{\mathrm{N}}{\mathrm{C}}\right) = 2.46\times10^{17}\ \mathrm{m/s^2}\,.$$

(b) With $v = \dfrac{c}{10} = 3.00\times10^{7}\ \mathrm{m/s}$, we use Eq. 2-11 to find

$$t = \frac{v - v_o}{a} = \frac{3.00\times10^{7}\ \mathrm{m/s}}{2.46\times10^{17}\ \mathrm{m/s^2}} = 1.22\times10^{-10}\ \mathrm{s}.$$

(c) Eq. 2-16 gives

$$\Delta x = \frac{v^2 - v_o^2}{2a} = \frac{\left(3.00\times10^{7}\ \mathrm{m/s}\right)^2}{2\left(2.46\times10^{17}\ \mathrm{m/s^2}\right)} = 1.83\times10^{-3}\ \mathrm{m}.$$

75. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o'clock ($-q$) and seven o'clock ($-7q$) positions is clearly equivalent to that of a single $-6q$ charge sitting at the seven o'clock position. Similarly, the net field due to just the charges in the six o'clock ($-6q$) and twelve o'clock ($-12q$) positions is the same as that due to a single $-6q$ charge sitting at the twelve o'clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o'clock, eight o'clock … twelve o'clock positions. Thus, the resultant field of all of these points, by symmetry, is directed toward the position midway between seven and twelve o'clock. Therefore, $\vec{E}_{\text{resultant}}$ points towards the nine-thirty position.

76. The electric field at a point on the axis of a uniformly charged ring, a distance z from the ring center, is given by

$$E = \frac{qz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}$$

where q is the charge on the ring and R is the radius of the ring (see Eq. 22-16). For q positive, the field points upward at points above the ring and downward at points below the ring. We take the positive direction to be upward. Then, the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}.$$

For small amplitude oscillations $z \ll R$ and z can be neglected in the denominator. Thus,

$$F = -\frac{eqz}{4\pi\varepsilon_0 R^3}.$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to z, just as if the electron were attached to a spring with spring constant $k = eq/4\pi\varepsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\varepsilon_0 mR^3}}$$

where m is the mass of the electron.

77. (a) Since $\vec{E}$ points down and we need an upward electric force (to cancel the downward pull of gravity), then we require the charge of the sphere to be negative. The magnitude of the charge is found by working with the absolute value of Eq. 22-28:

$$|q| = \frac{F}{E} = \frac{mg}{E} = \frac{4.4\,\text{N}}{150\ \text{N/C}} = 0.029\,\text{C},$$

or $q = -0.029$ C.

(b) The feasibility of this experiment may be studied by using Eq. 22-3 (using k for $1/4\pi\varepsilon_0$). We have $E = k|q|/r^2$ with

$$\rho_{\text{sulfur}}\left(\frac{4}{3}\pi r^3\right) = m_{\text{sphere}}$$

Since the mass of the sphere is 4.4/9.8 ≈ 0.45 kg and the density of sulfur is about 2.1×10^3 kg/m^3 (see Appendix F), then we obtain

$$r = \left(\frac{3m_{\text{sphere}}}{4\pi\rho_{\text{sulfur}}}\right)^{1/3} = 0.037\,\text{m} \;\Rightarrow\; E = k\frac{|q|}{r^2} \approx 2\times 10^{11}\ \text{N/C}$$

which is much too large a field to maintain in air.

78. The magnitude of the dipole moment is given by $p = qd$, where q is the positive charge in the dipole and d is the separation of the charges. For the dipole described in the problem,

$$p = (1.60 \times 10^{-19}\ \text{C})(4.30 \times 10^{-9}\ \text{m}) = 6.88 \times 10^{-28}\ \text{C} \cdot \text{m}.$$

The dipole moment is a vector that points from the negative toward the positive charge.

79. From the second measurement (at (2.0, 0)) we see that the charge must be somewhere on the x axis. A line passing through (3.0, 3.0) with slope $\tan^{-1}(3/4)$ will intersect the x axis at x = –1.0. Thus, the location of the particle is specified by the coordinates (in cm): (–1.0, 0).

(a) The x coordinate is x = –1.0 cm.

(b) Similarly, the y coordinate is y = 0.

(c) Using $k = 1/4\pi\varepsilon_0$, the field magnitude measured at (2.0, 0) (which is r = 0.030 m from the charge) is

$$\left|\vec{E}\right| = k\frac{q}{r^2} = 100 \text{ N/C}.$$

Therefore,

$$q = \frac{\left|\vec{E}\right| r^2}{k} = \frac{(100 \text{ N/C})(0.030 \text{ m})^2}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 1.0\times10^{-11}\text{C}.$$

80. We interpret the linear charge density, $\lambda = |Q|/L$, to indicate a positive quantity (so we can relate it to the magnitude of the field). Sample Problem 22-3 illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ with θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(|Q|/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(|Q|/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2|Q|\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} \ .$$

With $|Q| = 6.25\times10^{-12}$ C, $\theta = 2.40$ rad $= 137.5°$ and $R = 9.00\times10^{-2}$ m, the magnitude of the electric field is $E = 5.39$ N/C.

81. (a) From Eq. 22-38 (and the facts that $\hat{\mathrm{i}} \cdot \hat{\mathrm{i}} = 1$ and $\hat{\mathrm{j}} \cdot \hat{\mathrm{i}} = 0$), the potential energy is

$$U = -\vec{p} \cdot \vec{E} = -\left[\left(3.00\hat{\mathrm{i}} + 4.00\hat{\mathrm{j}}\right)\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\right]\cdot\left[\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}\right]$$
$$= -1.49\times10^{-26}\ \mathrm{J}.$$

(b) From Eq. 22-34 (and the facts that $\hat{\mathrm{i}} \times \hat{\mathrm{i}} = 0$ and $\hat{\mathrm{j}} \times \hat{\mathrm{i}} = -\hat{\mathrm{k}}$), the torque is

$$\vec{\tau} = \vec{p} \times \vec{E} = \left[\left(3.00\hat{\mathrm{i}} + 4.00\hat{\mathrm{j}}\right)\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\right]\times\left[\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}\right]$$
$$= \left(-1.98\times10^{-26}\,\mathrm{N\cdot m}\right)\hat{\mathrm{k}}.$$

(c) The work done is

$$W = \Delta U = \Delta\left(-\vec{p}\cdot\vec{E}\right) = \left(\vec{p}_i - \vec{p}_f\right)\cdot\vec{E}$$
$$= \left[\left(3.00\hat{\mathrm{i}} + 4.00\hat{\mathrm{j}}\right) - \left(-4.00\hat{\mathrm{i}} + 3.00\hat{\mathrm{j}}\right)\right]\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\right]\cdot\left[\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}\right]$$
$$= 3.47\times10^{-26}\ \mathrm{J}.$$

82. (a) From symmetry, we see the net force component along the *y* axis is zero.

(b) The net force component along the *x* axis points rightward. With $\theta = 60°$,

$$F_3 = 2\frac{q_3 q_1 \cos\theta}{4\pi\varepsilon_0 a^2}.$$

Since $\cos(60°) = 1/2$, we can write this as

$$F_3 = \frac{kq_3 q_1}{a^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-12}\ \text{C})(2.00\times10^{-12}\ \text{C})}{(0.0950\ \text{m})^2} = 9.96\times10^{-12}\ \text{N}.$$

83. A small section of the distribution that has charge dq is $\lambda\ dx$, where $\lambda = 9.0 \times 10^{-9}$ C/m. Its contribution to the field at $x_P = 4.0$ m is

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0\left(x - x_P\right)^2}$$

pointing in the $+x$ direction. Thus, we have

$$\vec{E} = \int_0^{3.0\,\mathrm{m}} \frac{\lambda\ dx}{4\pi\varepsilon_0\left(x - x_P\right)^2}\,\hat{\mathrm{i}}$$

which becomes, using the substitution $u = x - x_P$,

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int_{-4.0\,\mathrm{m}}^{-1.0\,\mathrm{m}} \frac{du}{u^2}\,\hat{\mathrm{i}} = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{-1.0\,\mathrm{m}} - \frac{-1}{-4.0\,\mathrm{m}}\right)\hat{\mathrm{i}}$$

which yields 61 N/C in the $+x$ direction.

84. Let q_1 denote the charge at $y = d$ and q_2 denote the charge at $y = -d$. The individual magnitudes $|\vec{E}_1|$ and $|\vec{E}_2|$ are figured from Eq. 22-3, where the absolute value signs for q are unnecessary since these charges are both positive. The distance from q_1 to a point on the x axis is the same as the distance from q_2 to a point on the x axis: $r = \sqrt{x^2 + d^2}$. By symmetry, the y component of the net field along the x axis is zero. The x component of the net field, evaluated at points on the positive x axis, is

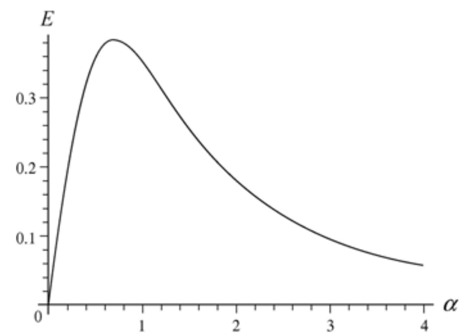
$$E_x = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{x^2 + d^2}\right)\left(\frac{x}{\sqrt{x^2 + d^2}}\right)$$

where the last factor is $\cos\theta = x/r$ with θ being the angle for each individual field as measured from the x axis.

(a) If we simplify the above expression, and plug in $x = \alpha d$, we obtain

$$E_x = \frac{q}{2\pi\varepsilon_0 d^2}\left(\frac{\alpha}{(\alpha^2 + 1)^{3/2}}\right).$$

(b) The graph of $E = E_x$ versus α is shown below. For the purposes of graphing, we set $d = 1$ m and $q = 5.56 \times 10^{-11}$ C.



(c) From the graph, we estimate E_{max} occurs at about $\alpha = 0.71$. More accurate computation shows that the maximum occurs at $\alpha = 1/\sqrt{2}$.

(d) The graph suggests that "half-height" points occur at $\alpha \approx 0.2$ and $\alpha \approx 2.0$. Further numerical exploration leads to the values: $\alpha = 0.2047$ and $\alpha = 1.9864$.

85. (a) For point *A*, we have (in SI units)

$$
\begin{aligned}
\vec{E}_A &= \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{q_2}{4\pi\varepsilon_0 r_2^2}\right]\left(-\hat{\text{i}}\right) \\
&= \frac{\left(8.99\times10^{9}\right)\left(1.00\times10^{-12}\,\text{C}\right)}{\left(5.00\times10^{-2}\right)^2}\left(-\hat{\text{i}}\right) + \frac{\left(8.99\times10^{9}\right)|-2.00\times10^{-12}\,\text{C}|}{\left(2\times5.00\times10^{-2}\right)^2}\left(\hat{\text{i}}\right) \ . \\
&= (-1.80\ \text{N/C})\hat{\text{i}}.
\end{aligned}
$$

(b) Similar considerations leads to

$$
\begin{aligned}
\vec{E}_B &= \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\text{i}} = \frac{\left(8.99\times10^{9}\right)\left(1.00\times10^{-12}\,\text{C}\right)}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\text{i}} + \frac{\left(8.99\times10^{9}\right)|-2.00\times10^{-12}\,\text{C}|}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\text{i}} \\
&= (43.2\ \text{N/C})\hat{\text{i}}.
\end{aligned}
$$

(c) For point C, we have

$$
\begin{aligned}
\vec{E}_C &= \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} - \frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\text{i}} = \frac{\left(8.99\times10^{9}\right)\left(1.00\times10^{-12}\,\text{C}\right)}{\left(2.00\times5.00\times10^{-2}\right)^2}\hat{\text{i}} - \frac{\left(8.99\times10^{9}\right)|-2.00\times10^{-12}\,\text{C}|}{\left(5.00\times10^{-2}\right)^2}\hat{\text{i}} \\
&= -(6.29\ \text{N/C})\hat{\text{i}}.
\end{aligned}
$$

(d) Although a sketch is not shown here, it would be somewhat similar to Fig. 22-5 in the textbook except that there would be twice as many field lines "coming into" the negative charge (which would destroy the simple up/down symmetry seen in Fig. 22-5).

86. (a) The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is $a = eE/m$, where E is the magnitude of the field and m is the mass of the electron. Its numerical value is

$$a = \frac{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(2.00\times10^{3}\,\mathrm{N/C}\right)}{9.11\times10^{-31}\,\mathrm{kg}} = 3.51\times10^{14}\ \mathrm{m/s^2}\,.$$

We put the origin of a coordinate system at the initial position of the electron. We take the x axis to be horizontal and positive to the right; take the y axis to be vertical and positive toward the top of the page. The kinematic equations are

$$x = v_0 t\cos\theta,\quad y = v_0 t\sin\theta - \frac{1}{2}at^2,\qquad \text{and}\qquad v_y = v_0\sin\theta - at.$$

First, we find the greatest y coordinate attained by the electron. If it is less than d, the electron does not hit the upper plate. If it is greater than d, it will hit the upper plate if the corresponding x coordinate is less than L. The greatest y coordinate occurs when $v_y = 0$. This means $v_0 \sin\theta - at = 0$ or $t = (v_0/a)\sin\theta$ and

$$y_{\max} = \frac{v_0^2\sin^2\theta}{a} - \frac{1}{2}a\frac{v_0^2\sin^2\theta}{a^2} = \frac{1}{2}\frac{v_0^2\sin^2\theta}{a} = \frac{\left(6.00\times10^{6}\ \mathrm{m/s}\right)^2\sin^2 45°}{2\left(3.51\times10^{14}\ \mathrm{m/s^2}\right)} = 2.56\times10^{-2}\,\mathrm{m}.$$

Since this is greater than $d = 2.00$ cm, the electron might hit the upper plate.

(b) Now, we find the x coordinate of the position of the electron when $y = d$. Since

$$v_0\sin\theta = \left(6.00\times10^{6}\ \mathrm{m/s}\right)\sin 45° = 4.24\times10^{6}\ \mathrm{m/s}$$

and

$$2ad = 2\left(3.51\times10^{14}\ \mathrm{m/s^2}\right)\left(0.0200\,\mathrm{m}\right) = 1.40\times10^{13}\ \mathrm{m^2/s^2}$$

the solution to $d = v_0 t\sin\theta - \frac{1}{2}at^2$ is

$$t = \frac{v_0\sin\theta - \sqrt{v_0^2\sin^2\theta - 2ad}}{a} = \frac{(4.24\times10^{6}\ \mathrm{m/s}) - \sqrt{\left(4.24\times10^{6}\ \mathrm{m/s}\right)^2 - 1.40\times10^{13}\ \mathrm{m^2/s^2}}}{3.51\times10^{14}\ \mathrm{m/s^2}}$$
$$= 6.43\times10^{-9}\,\mathrm{s}.$$

The negative root was used because we want the *earliest* time for which $y = d$. The x coordinate is

$$x = v_0 t\cos\theta = \left(6.00\times10^{6}\ \mathrm{m/s}\right)\left(6.43\times10^{-9}\,\mathrm{s}\right)\cos 45° = 2.72\times10^{-2}\,\mathrm{m}.$$

This is less than L so the electron hits the upper plate at $x = 2.72$ cm.

87. (a) If we subtract each value from the next larger value in the table, we find a set of numbers which are suggestive of a basic unit of charge: 1.64×10^{-19}, 3.3×10^{-19}, 1.63×10^{-19}, 3.35×10^{-19}, 1.6×10^{-19}, 1.63×10^{-19}, 3.18×10^{-19}, 3.24×10^{-19}, where the SI unit Coulomb is understood. These values are either close to a common $e \approx 1.6 \times 10^{-19}\,\mathrm{C}$ value or are double that. Taking this, then, as a crude approximation to our experimental e we divide it into all the values in the original data set and round to the nearest integer, obtaining n = 4,5,7,8,10,11,12,14, and 16.

(b) When we perform a least squares fit of the original data set versus these values for n we obtain the linear equation:

$$q = 7.18 \times 10^{-21} + 1.633 \times 10^{-19} n \,.$$

If we dismiss the constant term as unphysical (representing, say, systematic errors in our measurements) then we obtain $e = 1.63 \times 10^{-19}$ when we set $n = 1$ in this equation.

88. Since both charges are positive (and aligned along the z axis) we have

$$\left|\vec{E}_{\text{net}}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(z-d/2)^2} + \frac{q}{(z+d/2)^2}\right].$$

For $z \gg d$ we have $(z \pm d/2)^{-2} \approx z^{-2}$, so

$$\left|\vec{E}_{\text{net}}\right| \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z^2} + \frac{q}{z^2}\right) = \frac{2q}{4\pi\varepsilon_0 z^2}.$$