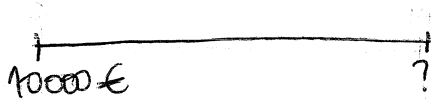
Ficha 5 - Matemática financeira

① $i = 14\%$
$n = 20$ anos
$C_0 = 10.000$?
$F = ?$

$$F = P(1+i)^n = 10000(1+0{,}14)^{20} = 137434{,}9€$$



③

$$f = P(1+i)^n \Leftrightarrow 74144 = 20000(1+i)^{10} \Leftrightarrow \sqrt[10]{\frac{74144}{20000}} - 1 = i \Leftrightarrow i = 14\%$$

④

$$P = f(1+i)^{-n} = 100000(1+0{,}1)^{-10} = 38554{,}3€$$

$i = 10\%$

⑤

$i = 10\%$

$$P = 20000(1+0{,}1)^{-1} + 30000(1+0{,}1)^{-2} + 40000(1+0{,}1)^{-3} = 73027{,}8€$$

⑥

$i = 10\%$

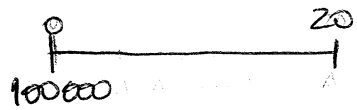
$$f = A\frac{(1+i)^n - 1}{i} = 2000\,\frac{(1+0{,}1)^{40} - 1}{0{,}1} = 885185{,}1€$$

⑦

$i = 12\%$

$$A = f\frac{i}{(1+i)^n - 1} = 100000\,\frac{0.12}{(1+0{,}12)^{10} - 1} = 5698{,}4€$$

(8)



$i = 10\%$

$$A = P\frac{i(1+i)^n}{(1+i)^n-1} = 100000\frac{0{,}1(1+0{,}1)^{20}}{(1+0{,}1)^{20}-1} = 11746{,}0\ € \text{ / año}$$

(9)

$$i_{ef,P} = \frac{i_n}{P} \Rightarrow i_{ef,\text{mes}} = \frac{12\%}{12} = 1\%$$

$$A = 100000\frac{0{,}01(1+0{,}01)^{240}}{(1+0{,}01)^{240}-1} = 1101{,}1\ € \text{ / mes}$$

(10)

$i_n = 12\%$

Cap. trimestral

$$i_{ef,trim} = \frac{12\%}{4} = 3\%$$

$$(1+i_{ef}) = (1+i_{ef,P})^P \Rightarrow i_{ef} = (1+0{,}03)^4 - 1 = 12{,}55\%$$

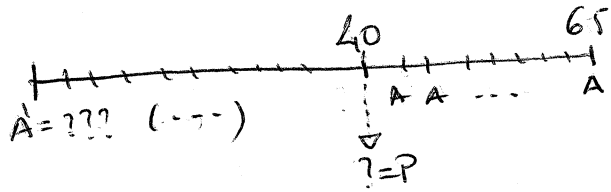
Cap. mensual:

$$i_{ef,m} = \frac{12\%}{12} = 1\%$$

$$(1+i_{ef}) = (1+0{,}01)^{12} \Rightarrow i_{ef} = 12{,}68\%$$

(12)

$A = 38000\ €$ / año
$n = 25$ años
$i = 8\%$



$$P = A\frac{(1+i)^n-1}{i(1+i)^n} = 38000\frac{(1+0{,}08)^{25}-1}{0{,}08(1+0{,}08)^{25}} = 405641{,}5\ €$$

$$A' = F\frac{i}{(1+i)^n-1} = 405641{,}5\cdot\frac{0{,}08}{(1+0{,}08)^{40}-1} = 1565{,}8\ €$$

Ficha 5

(14) $P = 750€$

$i = 18\%$

a)

Ano	Capital	Juro	Juro acumulado
1	750	135	135
2	885	159,3	294,3
3	1044,3	187,974	482,274
4	1232,3	221,81	704,083

ou Calcular f_2 e f_3 e fazer a diferença

b) Ao fim de 4 anos, juros recebidos $= 704,83$

ou

$f = P(1+i)^n = 750(1+0,18)^4 = 1454,083$

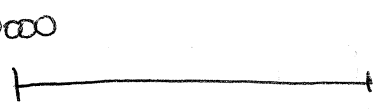
$1454,083 - 750 = 704,083$

(17) $C_i = 100000€$

$P = 15000€$

Prestação mensal

$i = 10\%$



$100000 = 15000 + 15000(1+0,10)^{-1} + 20000(1+0,10)^{-2} + A(1+0,1)^{-3} + A(1+0,1)^{-4} \Leftrightarrow$

$\Leftrightarrow 1,4343A = 54.834,7 \Leftrightarrow A = 38231€$

(16)

a) $(1+0{,}165) = (1+i_{ef,sem})^2 \Rightarrow i_{ef,sem} = 7{,}94\%$

b) $(1+i_{ef}) = (1+0{,}08)^3 \Rightarrow i_{ef} = 25{,}97\%$

c) $(1+0{,}075)^3 = (1+i_{ef,sem})^4 \Rightarrow i_{ef,trim} = 5{,}57\%$

(19) $P = 150000€$
A, trim
$i_n = 12\%$
$n = 2$ anos

a) $i_{eq,trim} = \frac{12\%}{4} = 3\%$

Trimestre	Capital dívida início do ano	Pagamento	Juro	Amortização	Capital dívida final do ano
1	150000€	21368,5	4500	16868,5	133131,5
2	133131,5	"	3994	17374,5	115757
3	115757	"	3472,71	17895,79	97861,21
4	97861,21	"	2935,8	18432,7	79428,5
5	79428,5	"	2382,9	18985,6	60442,9
6	60442,9	"	1813,3	19555,2	40887,7
7	40887,7	"	1226,6	20141,9	20746
8	20746	"	622,4	20746	0

$A = 150000 \cdot \frac{0,03(1+0,03)^8}{(1+0,03)^8-1} = 21368,5€$

Juro = Cap. dívida $\times 0,03$

b) $(1+i_{ef}) = (1+i_{eq,trim})^4 \Leftrightarrow i_{ef} = 1,03^4 - 1 = 12,55\%$

(18) $P = 40000€$
A, anual
$i = 12\%$

a) $A = P \frac{i(1+i)^n}{(1+i)^n - 1} = 40000 \cdot \frac{0,12(1+0,12)^4}{(1+0,12)^4-1} = 13169,4€$

b) c)

Ano	Cap. dívida início	Amort	Juro	AR	Cap. dív. final
1	40000	13169,4	4800	8369,4	31630,6
2	31630,6	"	3795,7	9373,7	22256,9
3	22256,9	"	2670,8	10498,6	11758,3
4	11758,3	"	1411	11758,4	$\simeq 0$

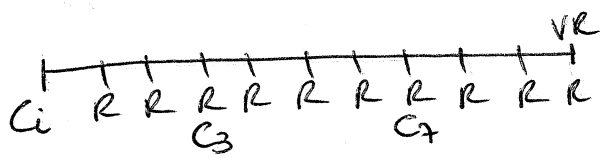
Ficha 6

④ $i_n = 12\%$.

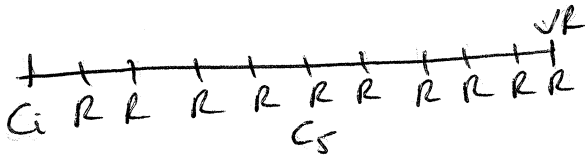
$i_{ef} = 12\%$.

Máquina A	Máquina B
$C_i = 20000€$	$C_i = 35000€$
$R = 14000€$	$R = 3000€$
$C_3 = 4000€$	$C_5 = 5000€$
$C_7 = 4000€$	$VR = 12000€$
$VR = 5000€$	$n = 10$ anos
$n = 10$ anos	

(A)



(B)

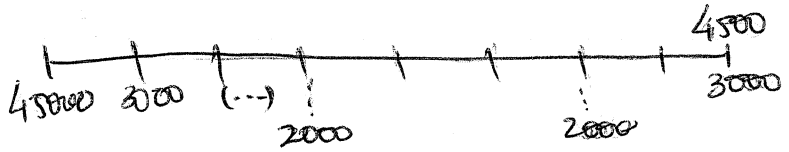


$$VAL_A = -20000 - 14000 \cdot \frac{(1+0,12)^{10}-1}{0,12(1+0,12)^{10}} - 4000(1+0,12)^{-3} - 4000(1+0,12)^{-7} + 5000(1+0,12)^{-10} = -102.149,77€$$

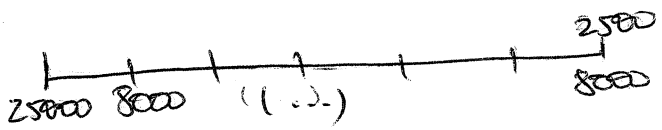
$$VAL_B = -35000 - 3000\frac{(1+0,12)^{10}-1}{0,12(1+0,12)^{10}} - 5000(1+0,12)^{-5} + 12000(1+0,12)^{-10} = -50924,12€$$

⑤ $i_{ef} = 6\%$

(A)



(B)



$$VAL_A = -45000 - 3000\frac{(1+0,06)^{8}-1}{0,06(1+0,06)^{8}} - 2000(1+0,06)^{-3} - 2000(1+0,06)^{-6} + 4500(1+0,06)^{-8} = -63895,2€$$

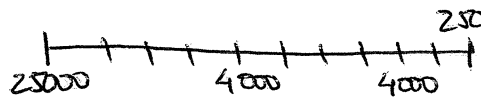
$$A_{(A)} = -63895,2 \cdot \frac{0,06(1+0,06)^{8}}{(1+0,06)^{8}-1} = -10289,48€/\text{ano}$$

$$VAL_B = -25000 - 8000\frac{(1+0,06)^{6}-1}{0,06(1+0,06)^{6}} + 25000(1+0,06)^{-6} = -62576,2€$$

$$A_{(B)} = -62576,2 \cdot \frac{0,06(1+0,06)^{6}}{(1+0,06)^{6}-1} = -12725,66€/\text{ano}$$

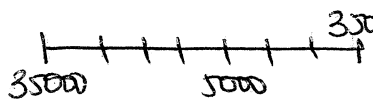
$i = 12\%$.

Modelo R.



$R = 5000€$

Modelo C.



$R = 2000€$

$$VAL_R = -25000 - 5000\frac{(1+0,12)^{10}-1}{0,12(1+0,12)^{10}} - 4000(1+0,12)^{-4} - 4000(1+0,12)^{-8} + 2500(1+0,12)^{-10} = -56603,79€$$

$$A_R = -56603,79\frac{0,12(1+0,12)^{10}}{(1+0,12)^{10}-1} = -10017,98€ \text{ / ano}$$

$$VAL_C = -35000 - 2000\frac{(1+0,12)^{7}-1}{0,12(1+0,12)^{7}} - 5000(1+0,12)^{-4} + 3500(1+0,12)^{-7} = -45721,88€$$

$$A_C = -45721,88\frac{0,12(1+0,12)^{7}}{(1+0,12)^{7}-1} = -10018,48€ \text{ / ano}$$

Máquina A:

$$VAL = -85000 + 26000\frac{(1+0,12)^{3}-1}{0,12(1+0,12)^{3}} + 18000\frac{(1+0,12)^{3}-1}{0,12(1+0,12)^{3}}\cdot(1+0,12)^{-3} = 8219,98€$$

$$FAE_A = 8219,98\cdot\frac{0,12(1+0,12)^{6}}{(1+0,12)^{6}-1} = 1999,31€ \text{ / ano}$$

Máquina B:

$$VAL = -65000 + 10000(1+0,12)^{-1} + 20000(1+0,12)^{-2} + 30000(1+0,12)^{-3} + 40000(1+0,12)^{-4} = 6646,58€$$

$$FAE_B = 6646,58\frac{0,12(1+0,12)^{4}}{(1+0,12)^{4}-1} = 2188,28€ \text{ / ano}$$

Máquina C:

$$VAL = -100500 + 30000\frac{(1+0,12)^{5}-1}{0,12(1+0,12)^{5}} = 7643,28€$$

$$FAE_C = 7643,28\cdot\frac{0,12(1+0,12)^{5}}{(1+0,12)^{5}-1} = 2120,32€ \text{ / ano}$$