

1. (a) The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70 \text{ pC}}{20 \text{ V}} = 3.5 \text{ pF}.$$

(b) The capacitance is independent of  $q$ ; it is still 3.5 pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200 \text{ pC}}{3.5 \text{ pF}} = 57 \text{ V}.$$

2. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then  $q = CV$ , and this is the same as the total charge that has passed through the battery. Thus,

$$q = (25 \times 10^{-6} \text{ F})(120 \text{ V}) = 3.0 \times 10^{-3} \text{ C}.$$

3. For a given potential difference  $V$ , the charge on the surface of the plate is

$$q = Ne = (nAd)e$$

where  $d$  is the depth from which the electrons come in the plate, and  $n$  is the density of conduction electrons. The charge collected on the plate is related to the capacitance and the potential difference by  $q = CV$  (Eq. 25-1). Combining the two expressions leads to

$$\frac{C}{A} = ne \frac{d}{V}.$$

With  $d/V = d_s/V_s = 5.0 \times 10^{-14} \text{ m/V}$  and  $n = 8.49 \times 10^{28} / \text{m}^3$  (see, for example, Sample Problem 25-1), we obtain

$$\frac{C}{A} = (8.49 \times 10^{28} / \text{m}^3)(1.6 \times 10^{-19} \text{ C})(5.0 \times 10^{-14} \text{ m/V}) = 6.79 \times 10^{-4} \text{ F/m}^2.$$

4. We use  $C = A\epsilon_0/d$ .

(a) The distance between the plates is

$$d = \frac{A\epsilon_0}{C} = \frac{(1.00\text{m}^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}{1.00\text{F}} = 8.85 \times 10^{-12} \text{ m}.$$

(b) Since  $d$  is much less than the size of an atom ( $\sim 10^{-10} \text{ m}$ ), this capacitor cannot be constructed.

5. (a) The capacitance of a parallel-plate capacitor is given by  $C = \epsilon_0 A/d$ , where  $A$  is the area of each plate and  $d$  is the plate separation. Since the plates are circular, the plate area is  $A = \pi R^2$ , where  $R$  is the radius of a plate. Thus,

$$C = \frac{\epsilon_0 \pi R^2}{d} = \frac{(8.85 \times 10^{-12} \text{ F/m}) \pi (8.2 \times 10^{-2} \text{ m})^2}{1.3 \times 10^{-3} \text{ m}} = 1.44 \times 10^{-10} \text{ F} = 144 \text{ pF}.$$

(b) The charge on the positive plate is given by  $q = CV$ , where  $V$  is the potential difference across the plates. Thus,

$$q = (1.44 \times 10^{-10} \text{ F})(120 \text{ V}) = 1.73 \times 10^{-8} \text{ C} = 17.3 \text{ nC}.$$

6. (a) We use Eq. 25-17:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} = \frac{(40.0 \text{ mm})(38.0 \text{ mm})}{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0 \text{ mm} - 38.0 \text{ mm})} = 84.5 \text{ pF}.$$

(b) Let the area required be  $A$ . Then  $C = \epsilon_0 A / (b - a)$ , or

$$A = \frac{C(b-a)}{\epsilon_0} = \frac{(84.5 \text{ pF})(40.0 \text{ mm} - 38.0 \text{ mm})}{\left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = 191 \text{ cm}^2.$$

7. Assuming conservation of volume, we find the radius of the combined spheres, then use  $C = 4\pi\epsilon_0 R$  to find the capacitance. When the drops combine, the volume is doubled. It is then  $V = 2(4\pi/3)R^3$ . The new radius  $R'$  is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \quad \Rightarrow \quad R' = 2^{1/3}R.$$

The new capacitance is

$$C' = 4\pi\epsilon_0 R' = 4\pi\epsilon_0 2^{1/3}R = 5.04\pi\epsilon_0 R.$$

With  $R = 2.00$  mm, we obtain  $C = 5.04\pi(8.85 \times 10^{-12} \text{ F/m})(2.00 \times 10^{-3} \text{ m}) = 2.80 \times 10^{-13} \text{ F}$ .

8. The equivalent capacitance is

$$C_{\text{eq}} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 4.00 \mu\text{F} + \frac{(10.0 \mu\text{F})(5.00 \mu\text{F})}{10.0 \mu\text{F} + 5.00 \mu\text{F}} = 7.33 \mu\text{F}.$$



9. The equivalent capacitance is

$$C_{\text{eq}} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = \frac{(10.0 \mu\text{F} + 5.00 \mu\text{F})(4.00 \mu\text{F})}{10.0 \mu\text{F} + 5.00 \mu\text{F} + 4.00 \mu\text{F}} = 3.16 \mu\text{F}.$$

10. The equivalent capacitance is given by  $C_{\text{eq}} = q/V$ , where  $q$  is the total charge on all the capacitors and  $V$  is the potential difference across any one of them. For  $N$  identical capacitors in parallel,  $C_{\text{eq}} = NC$ , where  $C$  is the capacitance of one of them. Thus,  $NC = q/V$  and

$$N = \frac{q}{VC} = \frac{1.00\text{C}}{(110\text{V})(1.00 \times 10^{-6}\text{F})} = 9.09 \times 10^3.$$

11. The charge that passes through meter  $A$  is

$$q = C_{\text{eq}} V = 3CV = 3(25.0 \mu\text{F})(4200 \text{ V}) = 0.315 \text{ C}.$$

12. (a) The potential difference across  $C_1$  is  $V_1 = 10.0$  V. Thus,

$$q_1 = C_1 V_1 = (10.0 \mu\text{F})(10.0 \text{ V}) = 1.00 \times 10^{-4} \text{ C}.$$

(b) Let  $C = 10.0 \mu\text{F}$ . We first consider the three-capacitor combination consisting of  $C_2$  and its two closest neighbors, each of capacitance  $C$ . The equivalent capacitance of this combination is

$$C_{\text{eq}} = C + \frac{C_2 C}{C + C_2} = 1.50 C.$$

Also, the voltage drop across this combination is

$$V = \frac{C V_1}{C + C_{\text{eq}}} = \frac{C V_1}{C + 1.50 C} = 0.40 V_1.$$

Since this voltage difference is divided equally between  $C_2$  and the one connected in series with it, the voltage difference across  $C_2$  satisfies  $V_2 = V/2 = V_1/5$ . Thus

$$q_2 = C_2 V_2 = (10.0 \mu\text{F}) \left( \frac{10.0 \text{ V}}{5} \right) = 2.00 \times 10^{-5} \text{ C}.$$

13. (a) and (b) The original potential difference  $V_1$  across  $C_1$  is

$$V_1 = \frac{C_{\text{eq}} V}{C_1 + C_2} = \frac{(3.16 \mu\text{F})(100.0 \text{ V})}{10.0 \mu\text{F} + 5.00 \mu\text{F}} = 21.1 \text{ V}.$$

Thus  $\Delta V_1 = 100.0 \text{ V} - 21.1 \text{ V} = 78.9 \text{ V}$  and

$$\Delta q_1 = C_1 \Delta V_1 = (10.0 \mu\text{F})(78.9 \text{ V}) = 7.89 \times 10^{-4} \text{ C}.$$

14. The two  $6.0 \mu\text{F}$  capacitors are in parallel and are consequently equivalent to  $C_{\text{eq}} = 12 \mu\text{F}$ . Thus, the total charge stored (before the squeezing) is

$$q_{\text{total}} = C_{\text{eq}} V = (12 \mu\text{F}) (10.0 \text{V}) = 120 \mu\text{C}.$$

(a) and (b) As a result of the squeezing, one of the capacitors is now  $12 \mu\text{F}$  (due to the inverse proportionality between  $C$  and  $d$  in Eq. 25-9) which represents an increase of  $6.0 \mu\text{F}$  and thus a charge increase of

$$\Delta q_{\text{total}} = \Delta C_{\text{eq}} V = (6.0 \mu\text{F}) (10.0 \text{V}) = 60 \mu\text{C} .$$

15. The charge initially on the charged capacitor is given by  $q = C_1 V_0$ , where  $C_1 = 100 \text{ pF}$  is the capacitance and  $V_0 = 50 \text{ V}$  is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is  $q_1 = C_1 V$ , where  $V = 35 \text{ V}$  is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is  $q_2 = q - q_1$ , where  $C_2$  is the capacitance of the second capacitor. Substituting  $C_1 V_0$  for  $q$  and  $C_1 V$  for  $q_1$ , we obtain  $q_2 = C_1 (V_0 - V)$ . The potential difference across the second capacitor is also  $V$ , so the capacitance is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V} C_1 = \frac{50 \text{ V} - 35 \text{ V}}{35 \text{ V}} (100 \text{ pF}) = 43 \text{ pF}.$$

16. We note that the voltage across  $C_3$  is  $V_3 = (12 \text{ V} - 2 \text{ V} - 5 \text{ V}) = 5 \text{ V}$ . Thus, its charge is  $q_3 = C_3 V_3 = 4 \text{ } \mu\text{C}$ .

(a) Therefore, since  $C_1$ ,  $C_2$  and  $C_3$  are in series (so they have the same charge), then

$$C_1 = \frac{4 \text{ } \mu\text{C}}{2 \text{ V}} = 2.0 \text{ } \mu\text{F}.$$

(b) Similarly,  $C_2 = 4/5 = 0.80 \text{ } \mu\text{F}$ .



17. (a) First, the equivalent capacitance of the two  $4.00\ \mu\text{F}$  capacitors connected in series is given by  $4.00\ \mu\text{F}/2 = 2.00\ \mu\text{F}$ . This combination is then connected in parallel with two other  $2.00\text{-}\mu\text{F}$  capacitors (one on each side), resulting in an equivalent capacitance  $C = 3(2.00\ \mu\text{F}) = 6.00\ \mu\text{F}$ . This is now seen to be in series with another combination, which consists of the two  $3.0\text{-}\mu\text{F}$  capacitors connected in parallel (which are themselves equivalent to  $C' = 2(3.00\ \mu\text{F}) = 6.00\ \mu\text{F}$ ). Thus, the equivalent capacitance of the circuit is

$$C_{\text{eq}} = \frac{CC'}{C + C'} = \frac{(6.00\ \mu\text{F})(6.00\ \mu\text{F})}{6.00\ \mu\text{F} + 6.00\ \mu\text{F}} = 3.00\ \mu\text{F}.$$

(b) Let  $V = 20.0\ \text{V}$  be the potential difference supplied by the battery. Then

$$q = C_{\text{eq}}V = (3.00\ \mu\text{F})(20.0\ \text{V}) = 6.00 \times 10^{-5}\ \text{C}.$$

(c) The potential difference across  $C_1$  is given by

$$V_1 = \frac{CV}{C + C'} = \frac{(6.00\ \mu\text{F})(20.0\ \text{V})}{6.00\ \mu\text{F} + 6.00\ \mu\text{F}} = 10.0\ \text{V}.$$

(d) The charge carried by  $C_1$  is  $q_1 = C_1V_1 = (3.00\ \mu\text{F})(10.0\ \text{V}) = 3.00 \times 10^{-5}\ \text{C}$ .

(e) The potential difference across  $C_2$  is given by  $V_2 = V - V_1 = 20.0\ \text{V} - 10.0\ \text{V} = 10.0\ \text{V}$ .

(f) The charge carried by  $C_2$  is  $q_2 = C_2V_2 = (2.00\ \mu\text{F})(10.0\ \text{V}) = 2.00 \times 10^{-5}\ \text{C}$ .

(g) Since this voltage difference  $V_2$  is divided equally between  $C_3$  and the other  $4.00\text{-}\mu\text{F}$  capacitors connected in series with it, the voltage difference across  $C_3$  is given by  $V_3 = V_2/2 = 10.0\ \text{V}/2 = 5.00\ \text{V}$ .

(h) Thus,  $q_3 = C_3V_3 = (4.00\ \mu\text{F})(5.00\ \text{V}) = 2.00 \times 10^{-5}\ \text{C}$ .

18. We determine each capacitance from the slope of the appropriate line in the graph. Thus,  $C_1 = (12 \mu\text{C})/(2.0 \text{ V}) = 6.0 \mu\text{F}$ . Similarly,  $C_2 = 4.0 \mu\text{F}$  and  $C_3 = 2.0 \mu\text{F}$ . The total equivalent capacitance is given by

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)},$$

or

$$C_{123} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3} = \frac{(6.0 \mu\text{F})(4.0 \mu\text{F} + 2.0 \mu\text{F})}{6.0 \mu\text{F} + 4.0 \mu\text{F} + 2.0 \mu\text{F}} = \frac{36}{12} \mu\text{F} = 3.0 \mu\text{F}.$$

This implies that the charge on capacitor 1 is  $q_1 = (3.0 \mu\text{F})(6.0 \text{ V}) = 18 \mu\text{C}$ . The voltage across capacitor 1 is therefore  $V_1 = (18 \mu\text{C})/(6.0 \mu\text{F}) = 3.0 \text{ V}$ . From the discussion in section 25-4, we conclude that the voltage across capacitor 2 must be  $6.0 \text{ V} - 3.0 \text{ V} = 3.0 \text{ V}$ . Consequently, the charge on capacitor 2 is  $(4.0 \mu\text{F})(3.0 \text{ V}) = 12 \mu\text{C}$ .

19. (a) After the switches are closed, the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from  $a$  to  $b$  is given by  $V_{ab} = Q/C_{\text{eq}}$ , where  $Q$  is the net charge on the combination and  $C_{\text{eq}}$  is the equivalent capacitance. The equivalent capacitance is  $C_{\text{eq}} = C_1 + C_2 = 4.0 \times 10^{-6} \text{ F}$ . The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1 V = (1.0 \times 10^{-6} \text{ F})(100 \text{ V}) = 1.0 \times 10^{-4} \text{ C}$$

and the charge on capacitor 2 is

$$q_2 = C_2 V = (3.0 \times 10^{-6} \text{ F})(100 \text{ V}) = 3.0 \times 10^{-4} \text{ C},$$

so the net charge on the combination is  $3.0 \times 10^{-4} \text{ C} - 1.0 \times 10^{-4} \text{ C} = 2.0 \times 10^{-4} \text{ C}$ . The potential difference is

$$V_{ab} = \frac{2.0 \times 10^{-4} \text{ C}}{4.0 \times 10^{-6} \text{ F}} = 50 \text{ V}.$$

(b) The charge on capacitor 1 is now  $q_1 = C_1 V_{ab} = (1.0 \times 10^{-6} \text{ F})(50 \text{ V}) = 5.0 \times 10^{-5} \text{ C}$ .

(c) The charge on capacitor 2 is now  $q_2 = C_2 V_{ab} = (3.0 \times 10^{-6} \text{ F})(50 \text{ V}) = 1.5 \times 10^{-4} \text{ C}$ .

20. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if  $Q = C_1 V_{\text{bat}} = 100 \mu\text{C}$ , and  $q_1$ ,  $q_2$  and  $q_3$  are the charges on  $C_1$ ,  $C_2$  and  $C_3$  after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2 + q_3 .$$

Since the parallel pair  $C_2$  and  $C_3$  are identical, it is clear that  $q_2 = q_3$ . They are in parallel with  $C_1$  so that  $V_1 = V_3$ , or

$$\frac{q_1}{C_1} = \frac{q_3}{C_3}$$

which leads to  $q_1 = q_3/2$ . Therefore,

$$Q = (q_3 / 2) + q_3 + q_3 = 5q_3 / 2$$

which yields  $q_3 = 2Q/5 = 2(100 \mu\text{C})/5 = 40 \mu\text{C}$  and consequently  $q_1 = q_3/2 = 20 \mu\text{C}$ .

21. Eq. 23-14 applies to each of these capacitors. Bearing in mind that  $\sigma = q/A$ , we find the total charge to be

$$q_{\text{total}} = q_1 + q_2 = \sigma_1 A_1 + \sigma_2 A_2 = \epsilon_0 E_1 A_1 + \epsilon_0 E_2 A_2 = 3.6 \text{ pC}$$

where we have been careful to convert  $\text{cm}^2$  to  $\text{m}^2$  by dividing by  $10^4$ .

22. Using Equation 25-14, the capacitances are

$$C_1 = \frac{2\pi\epsilon_0 L_1}{\ln(b_1/a_1)} = \frac{2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.050 \text{ m})}{\ln(15 \text{ mm}/5.0 \text{ mm})} = 2.53 \text{ pF}$$

$$C_2 = \frac{2\pi\epsilon_0 L_2}{\ln(b_2/a_2)} = \frac{2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.090 \text{ m})}{\ln(10 \text{ mm}/2.5 \text{ mm})} = 3.61 \text{ pF} .$$

Initially, the total equivalent capacitance is

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2} \Rightarrow C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2.53 \text{ pF})(3.61 \text{ pF})}{2.53 \text{ pF} + 3.61 \text{ pF}} = 1.49 \text{ pF} ,$$

and the charge on the positive plate of each one is  $(1.49 \text{ pF})(10 \text{ V}) = 14.9 \text{ pC}$ . Next, capacitor 2 is modified as described in the problem, with the effect that

$$C'_2 = \frac{2\pi\epsilon_0 L_2}{\ln(b'_2/a_2)} = \frac{2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.090 \text{ m})}{\ln(25 \text{ mm}/2.5 \text{ mm})} = 2.17 \text{ pF} .$$

The new total equivalent capacitance is

$$C'_{12} = \frac{C_1 C'_2}{C_1 + C'_2} = \frac{(2.53 \text{ pF})(2.17 \text{ pF})}{2.53 \text{ pF} + 2.17 \text{ pF}} = 1.17 \text{ pF}$$

and the new charge on the positive plate of each one is  $(1.17 \text{ pF})(10 \text{ V}) = 11.7 \text{ pC}$ . Thus we see that the charge transferred from the battery (considered in absolute value) as a result of the modification is  $14.9 \text{ pC} - 11.7 \text{ pC} = 3.2 \text{ pC}$ .

(a) This charge, divided by  $e$  gives the number of electrons that pass point  $P$ . Thus,

$$N = \frac{3.2 \times 10^{-12} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 2.0 \times 10^7 .$$

(b) These electrons move rightwards in the figure (that is, away from the battery) since the positive plates (the ones closest to point  $P$ ) of the capacitors have suffered a *decrease* in their positive charges. The usual reason for a metal plate to be positive is that it has more protons than electrons. Thus, in this problem some electrons have “returned” to the positive plates (making them less positive).

23. (a) and (b) We note that the charge on  $C_3$  is  $q_3 = 12 \mu\text{C} - 8.0 \mu\text{C} = 4.0 \mu\text{C}$ . Since the charge on  $C_4$  is  $q_4 = 8.0 \mu\text{C}$ , then the voltage across it is  $q_4/C_4 = 2.0 \text{ V}$ . Consequently, the voltage  $V_3$  across  $C_3$  is  $2.0 \text{ V} \Rightarrow C_3 = q_3/V_3 = 2.0 \mu\text{F}$ .

Now  $C_3$  and  $C_4$  are in parallel and are thus equivalent to  $6 \mu\text{F}$  capacitor which would then be in series with  $C_2$ ; thus, Eq 25-20 leads to an equivalence of  $2.0 \mu\text{F}$  which is to be thought of as being in series with the unknown  $C_1$ . We know that the total effective capacitance of the circuit (in the sense of what the battery “sees” when it is hooked up) is  $(12 \mu\text{C})/V_{\text{battery}} = 4\mu\text{F}/3$ . Using Eq 25-20 again, we find

$$\frac{1}{2 \mu\text{F}} + \frac{1}{C_1} = \frac{3}{4 \mu\text{F}} \Rightarrow C_1 = 4.0 \mu\text{F} .$$

24. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of  $(n - 1)$  identical single capacitors connected in parallel. Each capacitor has surface area  $A$  and plate separation  $d$  so its capacitance is given by  $C_0 = \epsilon_0 A/d$ . Thus, the total capacitance of the combination is

$$C = (n-1)C_0 = \frac{(n-1)\epsilon_0 A}{d} = \frac{(8-1)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.25 \times 10^{-4} \text{ m}^2)}{3.40 \times 10^{-3} \text{ m}} = 2.28 \times 10^{-12} \text{ F}.$$



25. We note that the total equivalent capacitance is  $C_{123} = [(C_3)^{-1} + (C_1 + C_2)^{-1}]^{-1} = 6 \mu\text{F}$ .

(a) Thus, the charge that passed point  $a$  is  $C_{123} V_{\text{batt}} = (6 \mu\text{F})(12 \text{ V}) = 72 \mu\text{C}$ . Dividing this by the value  $e = 1.60 \times 10^{-19} \text{ C}$  gives the number of electrons:  $4.5 \times 10^{14}$ , which travel to the left – towards the positive terminal of the battery.

(b) The equivalent capacitance of the parallel pair is  $C_{12} = C_1 + C_2 = 12 \mu\text{F}$ . Thus, the voltage across the pair (which is the same as the voltage across  $C_1$  and  $C_2$  individually) is

$$\frac{72 \mu\text{C}}{12 \mu\text{F}} = 6 \text{ V}.$$

Thus, the charge on  $C_1$  is  $q_1 = (4 \mu\text{F})(6 \text{ V}) = 24 \mu\text{C}$ , and dividing this by  $e$  gives  $N_1 = q_1 / e = 1.5 \times 10^{14}$ , the number of electrons that have passed (upward) through point  $b$ .

(c) Similarly, the charge on  $C_2$  is  $q_2 = (8 \mu\text{F})(6 \text{ V}) = 48 \mu\text{C}$ , and dividing this by  $e$  gives  $N_2 = q_2 / e = 3.0 \times 10^{14}$ , the number of electrons which have passed (upward) through point  $c$ .

(d) Finally, since  $C_3$  is in series with the battery, its charge is the same that passed through the battery (the same as passed through the switch). Thus,  $4.5 \times 10^{14}$  electrons passed rightward through point  $d$ . By leaving the rightmost plate of  $C_3$ , that plate is then the positive plate of the fully charged capacitor – making its leftmost plate (the one closest to the negative terminal of the battery) the negative plate, as it should be.

(e) As stated in (b), the electrons travel up through point  $b$ .

(f) As stated in (c), the electrons travel up through point  $c$ .

26. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}.$$

Thus,  $C_{\text{eq}} = C_2 C_3 / (C_2 + C_3)$ . The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by  $q_2 / C_{\text{eq}}$ . The potential difference across capacitor 1 is  $q_1 / C_1$ , where  $q_1$  is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so  $q_1 / C_1 = q_2 / C_{\text{eq}}$ . Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If  $q_0$  is the original charge, conservation of charge yields  $q_1 + q_2 = q_0 = C_1 V_0$ , where  $V_0$  is the original potential difference across capacitor 1.

(a) Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{\text{eq}}}, \quad q_1 + q_2 = C_1 V_0$$

for  $q_1$  and  $q_2$ , we obtain

$$q_1 = \frac{C_1^2 V_0}{C_{\text{eq}} + C_1} = \frac{C_1^2 V_0}{\frac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3}.$$

With  $V_0 = 12.0 \text{ V}$ ,  $C_1 = 4.00 \text{ } \mu\text{F}$ ,  $C_2 = 6.00 \text{ } \mu\text{F}$  and  $C_3 = 3.00 \text{ } \mu\text{F}$ , we find  $C_{\text{eq}} = 2.00 \text{ } \mu\text{F}$  and  $q_1 = 32.0 \text{ } \mu\text{C}$ .

(b) The charge on capacitors 2 is

$$q_2 = C_1 V_0 - q_1 = (4.00 \text{ } \mu\text{F})(12.0 \text{ V}) - 32.0 \text{ } \mu\text{C} = 16.0 \text{ } \mu\text{C}.$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3 = C_1 V_0 - q_1 = (4.00 \text{ } \mu\text{F})(12.0 \text{ V}) - 32.0 \text{ } \mu\text{C} = 16.0 \text{ } \mu\text{C}.$$

27. (a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1 C_3 V}{C_1 + C_3} = \frac{(1.00 \mu\text{F})(3.00 \mu\text{F})(12.0 \text{V})}{1.00 \mu\text{F} + 3.00 \mu\text{F}} = 9.00 \mu\text{C}.$$

(b) Capacitors 2 and 4 are also in series:

$$q_2 = q_4 = \frac{C_2 C_4 V}{C_2 + C_4} = \frac{(2.00 \mu\text{F})(4.00 \mu\text{F})(12.0 \text{V})}{2.00 \mu\text{F} + 4.00 \mu\text{F}} = 16.0 \mu\text{C}.$$

(c)  $q_3 = q_1 = 9.00 \mu\text{C}$ .

(d)  $q_4 = q_2 = 16.0 \mu\text{C}$ .

(e) With switch 2 also closed, the potential difference  $V_1$  across  $C_1$  must equal the potential difference across  $C_2$  and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4} V = \frac{(3.00 \mu\text{F} + 4.00 \mu\text{F})(12.0 \text{V})}{1.00 \mu\text{F} + 2.00 \mu\text{F} + 3.00 \mu\text{F} + 4.00 \mu\text{F}} = 8.40 \text{V}.$$

Thus,  $q_1 = C_1 V_1 = (1.00 \mu\text{F})(8.40 \text{V}) = 8.40 \mu\text{C}$ .

(f) Similarly,  $q_2 = C_2 V_1 = (2.00 \mu\text{F})(8.40 \text{V}) = 16.8 \mu\text{C}$ .

(g)  $q_3 = C_3(V - V_1) = (3.00 \mu\text{F})(12.0 \text{V} - 8.40 \text{V}) = 10.8 \mu\text{C}$ .

(h)  $q_4 = C_4(V - V_1) = (4.00 \mu\text{F})(12.0 \text{V} - 8.40 \text{V}) = 14.4 \mu\text{C}$ .

28. Initially the capacitors  $C_1$ ,  $C_2$ , and  $C_3$  form a combination equivalent to a single capacitor which we denote  $C_{123}$ . This obeys the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)} .$$

Hence, using  $q = C_{123}V$  and the fact that  $q = q_1 = C_1 V_1$ , we arrive at

$$V_1 = \frac{q_1}{C_1} = \frac{q}{C_1} = \frac{C_{123}}{C_1} V = \frac{C_2 + C_3}{C_1 + C_2 + C_3} V .$$

(a) As  $C_3 \rightarrow \infty$  this expression becomes  $V_1 = V$ . Since the problem states that  $V_1$  approaches 10 volts in this limit, so we conclude  $V = 10$  V.

(b) and (c) At  $C_3 = 0$ , the graph indicates  $V_1 = 2.0$  V. The above expression consequently implies  $C_1 = 4C_2$ . Next we note that the graph shows that, at  $C_3 = 6.0 \mu\text{F}$ , the voltage across  $C_1$  is exactly half of the battery voltage. Thus,

$$\frac{1}{2} = \frac{C_2 + 6.0 \mu\text{F}}{C_1 + C_2 + 6.0 \mu\text{F}} = \frac{C_2 + 6.0 \mu\text{F}}{4C_2 + C_2 + 6.0 \mu\text{F}}$$

which leads to  $C_2 = 2.0 \mu\text{F}$ . We conclude, too, that  $C_1 = 8.0 \mu\text{F}$ .

29. The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference  $V$  across the capacitors is the same and the total energy is

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(2.0 \times 10^{-6} \text{ F} + 4.0 \times 10^{-6} \text{ F})(300 \text{ V})^2 = 0.27 \text{ J}.$$

30. (a) The capacitance is

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(40 \times 10^{-4} \text{ m}^2)}{1.0 \times 10^{-3} \text{ m}} = 3.5 \times 10^{-11} \text{ F} = 35 \text{ pF}.$$

(b)  $q = CV = (35 \text{ pF})(600 \text{ V}) = 2.1 \times 10^{-8} \text{ C} = 21 \text{ nC}.$

(c)  $U = \frac{1}{2} CV^2 = \frac{1}{2} (35 \text{ pF})(21 \text{ nC})^2 = 6.3 \times 10^{-6} \text{ J} = 6.3 \mu\text{J}.$

(d)  $E = V/d = 600 \text{ V}/1.0 \times 10^{-3} \text{ m} = 6.0 \times 10^5 \text{ V/m}.$

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3 \times 10^{-6} \text{ J}}{(40 \times 10^{-4} \text{ m}^2)(1.0 \times 10^{-3} \text{ m})} = 1.6 \text{ J/m}^3.$$

31. The energy stored by a capacitor is given by  $U = \frac{1}{2}CV^2$ , where  $V$  is the potential difference across its plates. We convert the given value of the energy to Joules. Since  $1 \text{ J} = 1 \text{ W} \cdot \text{s}$ , we multiply by  $(10^3 \text{ W/kW})(3600 \text{ s/h})$  to obtain  $10 \text{ kW} \cdot \text{h} = 3.6 \times 10^7 \text{ J}$ . Thus,

$$C = \frac{2U}{V^2} = \frac{2(3.6 \times 10^7 \text{ J})}{(1000 \text{ V})^2} = 72 \text{ F}.$$

32. Let  $\mathcal{V} = 1.00 \text{ m}^3$ . Using Eq. 25-25, the energy stored is

$$U = u\mathcal{V} = \frac{1}{2}\epsilon_0 E^2 \mathcal{V} = \frac{1}{2} \left( 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) (150 \text{ V/m})^2 (1.00 \text{ m}^3) = 9.96 \times 10^{-8} \text{ J}.$$



33. The energy per unit volume is

$$u = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 \left( \frac{e}{4\pi \varepsilon_0 r^2} \right)^2 = \frac{e^2}{32\pi^2 \varepsilon_0 r^4} .$$

(a) At  $r = 1.00 \times 10^{-3} \text{ m}$ , with  $e = 1.60 \times 10^{-19} \text{ C}$  and  $\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$ , we have  $u = 9.16 \times 10^{-18} \text{ J/m}^3$ .

(b) Similarly, at  $r = 1.00 \times 10^{-6} \text{ m}$ ,  $u = 9.16 \times 10^{-6} \text{ J/m}^3$ .

(c) At  $r = 1.00 \times 10^{-9} \text{ m}$ ,  $u = 9.16 \times 10^6 \text{ J/m}^3$ .

(d) At  $r = 1.00 \times 10^{-12} \text{ m}$ ,  $u = 9.16 \times 10^{18} \text{ J/m}^3$ .

(e) From the expression above  $u \propto r^{-4}$ . Thus, for  $r \rightarrow 0$ , the energy density  $u \rightarrow \infty$ .

34. (a) The potential difference across  $C_1$  (the same as across  $C_2$ ) is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(15.0 \mu\text{F})(100 \text{V})}{10.0 \mu\text{F} + 5.00 \mu\text{F} + 15.0 \mu\text{F}} = 50.0 \text{V}.$$

Also,  $V_3 = V - V_1 = V - V_2 = 100 \text{ V} - 50.0 \text{ V} = 50.0 \text{ V}$ . Thus,

$$q_1 = C_1 V_1 = (10.0 \mu\text{F})(50.0 \text{V}) = 5.00 \times 10^{-4} \text{ C}$$

$$q_2 = C_2 V_2 = (5.00 \mu\text{F})(50.0 \text{V}) = 2.50 \times 10^{-4} \text{ C}$$

$$q_3 = q_1 + q_2 = 5.00 \times 10^{-4} \text{ C} + 2.50 \times 10^{-4} \text{ C} = 7.50 \times 10^{-4} \text{ C}.$$

(b) The potential difference  $V_3$  was found in the course of solving for the charges in part (a). Its value is  $V_3 = 50.0 \text{ V}$ .

(c) The energy stored in  $C_3$  is  $U_3 = C_3 V_3^2 / 2 = (15.0 \mu\text{F})(50.0 \text{V})^2 / 2 = 1.88 \times 10^{-2} \text{ J}$ .

(d) From part (a), we have  $q_1 = 5.00 \times 10^{-4} \text{ C}$ , and

(e)  $V_1 = 50.0 \text{ V}$ , as shown in (a).

(f) The energy stored in  $C_1$  is

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} (10.0 \mu\text{F})(50.0 \text{V})^2 = 1.25 \times 10^{-2} \text{ J}.$$

(g) Again, from part (a),  $q_2 = 2.50 \times 10^{-4} \text{ C}$ .

(h)  $V_2 = 50.0 \text{ V}$ , as shown in (a).

(i) The energy stored in  $C_2$  is  $U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} (5.00 \mu\text{F})(50.0 \text{V})^2 = 6.25 \times 10^{-3} \text{ J}$ .

35. (a) Let  $q$  be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by  $\epsilon_0 A/d_i$ , the charge is  $q = CV = \epsilon_0 A V_i / d_i$ . After the plates are pulled apart, their separation is  $d_f$  and the potential difference is  $V_f$ . Then  $q = \epsilon_0 A V_f / d_f$  and

$$V_f = \frac{d_f}{\epsilon_0 A} q = \frac{d_f}{\epsilon_0 A} \frac{\epsilon_0 A}{d_i} V_i = \frac{d_f}{d_i} V_i.$$

With  $d_i = 3.00 \times 10^{-3} \text{ m}$ ,  $V_i = 6.00 \text{ V}$  and  $d_f = 8.00 \times 10^{-3} \text{ m}$ , we have  $V_f = 16.0 \text{ V}$ .

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2} C V_i^2 = \frac{\epsilon_0 A V_i^2}{2 d_i} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(8.50 \times 10^{-4} \text{ m}^2)(6.00 \text{ V})^2}{2(3.00 \times 10^{-3} \text{ m})} = 4.51 \times 10^{-11} \text{ J}.$$

(c) The final energy stored is

$$U_f = \frac{1}{2} \frac{\epsilon_0 A}{d_f} V_f^2 = \frac{1}{2} \frac{\epsilon_0 A}{d_f} \left( \frac{d_f}{d_i} V_i \right)^2 = \frac{d_f}{d_i} \left( \frac{\epsilon_0 A V_i^2}{2 d_i} \right) = \frac{d_f}{d_i} U_i.$$

With  $d_f / d_i = 8.00 / 3.00$ , we have  $U_f = 1.20 \times 10^{-10} \text{ J}$ .

(d) The work done to pull the plates apart is the difference in the energy:

$$W = U_f - U_i = 7.52 \times 10^{-11} \text{ J}.$$

36. (a) The charge  $q_3$  in the figure is  $q_3 = C_3 V = (4.00 \mu\text{F})(100 \text{ V}) = 4.00 \times 10^{-4} \text{ C}$ .

(b)  $V_3 = V = 100 \text{ V}$ .

(c) Using  $U_i = \frac{1}{2} C_i V_i^2$ , we have  $U_3 = \frac{1}{2} C_3 V_3^2 = 2.00 \times 10^{-2} \text{ J}$ .

(d) From the figure,

$$q_1 = q_2 = \frac{C_1 C_2 V}{C_1 + C_2} = \frac{(10.0 \mu\text{F})(5.00 \mu\text{F})(100 \text{ V})}{10.0 \mu\text{F} + 5.00 \mu\text{F}} = 3.33 \times 10^{-4} \text{ C}.$$

(e)  $V_1 = q_1 / C_1 = 3.33 \times 10^{-4} \text{ C} / 10.0 \mu\text{F} = 33.3 \text{ V}$ .

(f)  $U_1 = \frac{1}{2} C_1 V_1^2 = 5.55 \times 10^{-3} \text{ J}$ .

(g) From part (d), we have  $q_2 = q_1 = 3.33 \times 10^{-4} \text{ C}$ .

(h)  $V_2 = V - V_1 = 100 \text{ V} - 33.3 \text{ V} = 66.7 \text{ V}$ .

(i)  $U_2 = \frac{1}{2} C_2 V_2^2 = 1.11 \times 10^{-2} \text{ J}$ .

37. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across  $10\ \mu\text{F}$ , then the voltage across the  $20\ \mu\text{F}$  capacitor is 50 V and the voltage across the  $25\ \mu\text{F}$  capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 25-21 or Eq. 25-22, we sum the energies on the capacitors and obtain  $U_{\text{total}} = 0.095\ \text{J}$ .

38. (a) We calculate the charged surface area of the cylindrical volume as follows:

$$A = 2\pi rh + \pi r^2 = 2\pi(0.20 \text{ m})(0.10 \text{ m}) + \pi(0.20 \text{ m})^2 = 0.25 \text{ m}^2$$

where we note from the figure that although the bottom is charged, the top is not. Therefore, the charge is  $q = \sigma A = -0.50 \text{ } \mu\text{C}$  on the exterior surface, and consequently (according to the assumptions in the problem) that same charge  $q$  is induced in the interior of the fluid.

(b) By Eq. 25-21, the energy stored is

$$U = \frac{q^2}{2C} = \frac{(5.0 \times 10^{-7} \text{ C})^2}{2(35 \times 10^{-12} \text{ F})} = 3.6 \times 10^{-3} \text{ J}.$$

(c) Our result is within a factor of three of that needed to cause a spark. Our conclusion is that it will probably not cause a spark; however, there is not enough of a safety factor to be sure.

39. We use  $E = q / 4\pi\epsilon_0 R^2 = V / R$ . Thus

$$u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \left( \frac{V}{R} \right)^2 = \frac{1}{2} \left( 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) \left( \frac{8000 \text{ V}}{0.050 \text{ m}} \right)^2 = 0.11 \text{ J/m}^3.$$

40. (a) We use  $C = \epsilon_0 A/d$  to solve for  $d$ :

$$d = \frac{\epsilon_0 A}{C} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.35 \text{ m}^2)}{50 \times 10^{-12} \text{ F}} = 6.2 \times 10^{-2} \text{ m}.$$

(b) We use  $C \propto \kappa$ . The new capacitance is

$$C' = C(\kappa/\kappa_{\text{air}}) = (50 \text{ pf})(5.6/1.0) = 2.8 \times 10^2 \text{ pF}.$$



41. The capacitance with the dielectric in place is given by  $C = \kappa C_0$ , where  $C_0$  is the capacitance before the dielectric is inserted. The energy stored is given by  $U = \frac{1}{2} CV^2 = \frac{1}{2} \kappa C_0 V^2$ , so

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4 \times 10^{-6} \text{ J})}{(7.4 \times 10^{-12} \text{ F})(652 \text{ V})^2} = 4.7.$$

According to Table 25-1, you should use Pyrex.

42. If the original capacitance is given by  $C = \epsilon_0 A/d$ , then the new capacitance is  $C' = \epsilon_0 \kappa A/2d$ . Thus  $C'/C = \kappa/2$  or

$$\kappa = 2C'/C = 2(2.6 \text{ pF}/1.3 \text{ pF}) = 4.0.$$

43. The capacitance of a cylindrical capacitor is given by

$$C = \kappa C_0 = \frac{2\pi\kappa\epsilon_0 L}{\ln(b/a)},$$

where  $C_0$  is the capacitance without the dielectric,  $\kappa$  is the dielectric constant,  $L$  is the length,  $a$  is the inner radius, and  $b$  is the outer radius. The capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\epsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85 \times 10^{-12} \text{ F/m})}{\ln[(0.60 \text{ mm})/(0.10 \text{ mm})]} = 8.1 \times 10^{-11} \text{ F/m} = 81 \text{ pF/m}.$$

44. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know  $C_1$  and  $C_2$ . From Eq. 25-9,

$$C_2 = \frac{\epsilon_0 A}{d} = 2.21 \times 10^{-11} \text{ F} ,$$

and from Eq. 25-27,

$$C_1 = \frac{\kappa \epsilon_0 A}{d} = 6.64 \times 10^{-11} \text{ F} .$$

This leads to  $q_1 = C_1 V_1 = 8.00 \times 10^{-10} \text{ C}$  and  $q_2 = C_2 V_2 = 2.66 \times 10^{-10} \text{ C}$ . The addition of these gives the desired result:  $q_{\text{tot}} = 1.06 \times 10^{-9} \text{ C}$ . Alternatively, the circuit could be reduced to find the  $q_{\text{tot}}$ .

45. The capacitance is given by  $C = \kappa C_0 = \kappa \epsilon_0 A/d$ , where  $C_0$  is the capacitance without the dielectric,  $\kappa$  is the dielectric constant,  $A$  is the plate area, and  $d$  is the plate separation. The electric field between the plates is given by  $E = V/d$ , where  $V$  is the potential difference between the plates. Thus,  $d = V/E$  and  $C = \kappa \epsilon_0 AE/V$ . Thus,

$$A = \frac{CV}{\kappa \epsilon_0 E}.$$

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0 \times 10^{-8} \text{ F})(4.0 \times 10^3 \text{ V})}{2.8(8.85 \times 10^{-12} \text{ F/m})(18 \times 10^6 \text{ V/m})} = 0.63 \text{ m}^2.$$

46. (a) We use Eq. 25-14:

$$C = 2\pi\epsilon_0\kappa \frac{L}{\ln(b/a)} = \frac{(4.7)(0.15 \text{ m})}{2\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\ln(3.8 \text{ cm}/3.6 \text{ cm})} = 0.73 \text{ nF}.$$

(b) The breakdown potential is  $(14 \text{ kV/mm})(3.8 \text{ cm} - 3.6 \text{ cm}) = 28 \text{ kV}$ .

47. Using Eq. 25-29, with  $\sigma = q/A$ , we have

$$|\vec{E}| = \frac{q}{\kappa\epsilon_0 A} = 200 \times 10^3 \text{ N/C}$$

which yields  $q = 3.3 \times 10^{-7} \text{ C}$ . Eq. 25-21 and Eq. 25-27 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\epsilon_0 A} = 6.6 \times 10^{-5} \text{ J} .$$

48. The capacitor can be viewed as two capacitors  $C_1$  and  $C_2$  in parallel, each with surface area  $A/2$  and plate separation  $d$ , filled with dielectric materials with dielectric constants  $\kappa_1$  and  $\kappa_2$ , respectively. Thus, (in SI units),

$$\begin{aligned} C &= C_1 + C_2 = \frac{\epsilon_0 (A/2) \kappa_1}{d} + \frac{\epsilon_0 (A/2) \kappa_2}{d} = \frac{\epsilon_0 A}{d} \left( \frac{\kappa_1 + \kappa_2}{2} \right) \\ &= \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(5.56 \times 10^{-4} \text{ m}^2)}{5.56 \times 10^{-3} \text{ m}} \left( \frac{7.00 + 12.00}{2} \right) = 8.41 \times 10^{-12} \text{ F}. \end{aligned}$$



49. We assume there is charge  $q$  on one plate and charge  $-q$  on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1 \varepsilon_0 A},$$

where  $A$  is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2 \varepsilon_0 A}.$$

Let  $d/2$  be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\varepsilon_0 A} \left[ \frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right] = \frac{qd}{2\varepsilon_0 A} \frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

so

$$C = \frac{q}{V} = \frac{2\varepsilon_0 A}{d} \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$

This expression is exactly the same as that for  $C_{\text{eq}}$  of two capacitors in series, one with dielectric constant  $\kappa_1$  and the other with dielectric constant  $\kappa_2$ . Each has plate area  $A$  and plate separation  $d/2$ . Also we note that if  $\kappa_1 = \kappa_2$ , the expression reduces to  $C = \kappa_1 \varepsilon_0 A/d$ , the correct result for a parallel-plate capacitor with plate area  $A$ , plate separation  $d$ , and dielectric constant  $\kappa_1$ .

With  $A = 7.89 \times 10^{-4} \text{ m}^2$ ,  $d = 4.62 \times 10^{-3} \text{ m}$ ,  $\kappa_1 = 11.0$  and  $\kappa_2 = 12.0$ , the capacitance is

$$C = \frac{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(7.89 \times 10^{-4} \text{ m}^2)}{4.62 \times 10^{-3} \text{ m}} \frac{(11.0)(12.0)}{11.0 + 12.0} = 1.73 \times 10^{-11} \text{ F}.$$

50. Let  $C_1 = \varepsilon_0(A/2)\kappa_1/2d = \varepsilon_0 A \kappa_1/4d$ ,  $C_2 = \varepsilon_0(A/2)\kappa_2/d = \varepsilon_0 A \kappa_2/2d$ , and  $C_3 = \varepsilon_0 A \kappa_3/2d$ . Note that  $C_2$  and  $C_3$  are effectively connected in series, while  $C_1$  is effectively connected in parallel with the  $C_2$ - $C_3$  combination. Thus,

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A \kappa_1}{4d} + \frac{(\varepsilon_0 A/d) (\kappa_2/2) (\kappa_3/2)}{\kappa_2/2 + \kappa_3/2} = \frac{\varepsilon_0 A}{4d} \left( \kappa_1 + \frac{2\kappa_2 \kappa_3}{\kappa_2 + \kappa_3} \right).$$

With  $A = 1.05 \times 10^{-3} \text{ m}^2$ ,  $d = 3.56 \times 10^{-3} \text{ m}$ ,  $\kappa_1 = 21.0$ ,  $\kappa_2 = 42.0$  and  $\kappa_3 = 58.0$ , the capacitance is

$$C = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.05 \times 10^{-3} \text{ m}^2)}{4(3.56 \times 10^{-3} \text{ m})} \left( 21.0 + \frac{2(42.0)(58.0)}{42.0 + 58.0} \right) = 4.55 \times 10^{-11} \text{ F}.$$

51. (a) The electric field in the region between the plates is given by  $E = V/d$ , where  $V$  is the potential difference between the plates and  $d$  is the plate separation. The capacitance is given by  $C = \kappa\epsilon_0 A/d$ , where  $A$  is the plate area and  $\kappa$  is the dielectric constant, so  $d = \kappa\epsilon_0 A/C$  and

$$E = \frac{VC}{\kappa\epsilon_0 A} = \frac{(50 \text{ V})(100 \times 10^{-12} \text{ F})}{5.4(8.85 \times 10^{-12} \text{ F/m})(100 \times 10^{-4} \text{ m}^2)} = 1.0 \times 10^4 \text{ V/m}.$$

(b) The free charge on the plates is  $q_f = CV = (100 \times 10^{-12} \text{ F})(50 \text{ V}) = 5.0 \times 10^{-9} \text{ C}$ .

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is  $q/2\epsilon_0 A$ , the field between the plates is

$$E = \frac{q_f}{2\epsilon_0 A} + \frac{q_f}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \epsilon_0 AE = 5.0 \times 10^{-9} \text{ C} - (8.85 \times 10^{-12} \text{ F/m})(100 \times 10^{-4} \text{ m}^2)(1.0 \times 10^4 \text{ V/m}) \\ &= 4.1 \times 10^{-9} \text{ C} = 4.1 \text{ nC}. \end{aligned}$$

52. (a) The electric field  $E_1$  in the free space between the two plates is  $E_1 = q/\epsilon_0 A$  while that inside the slab is  $E_2 = E_1/\kappa = q/\kappa\epsilon_0 A$ . Thus,

$$V_0 = E_1 (d - b) + E_2 b = \left( \frac{q}{\epsilon_0 A} \right) \left( d - b + \frac{b}{\kappa} \right),$$

and the capacitance is

$$C = \frac{q}{V_0} = \frac{\epsilon_0 A \kappa}{\kappa (d - b) + b} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(115 \times 10^{-4} \text{ m}^2)(2.61)}{(2.61)(0.0124 \text{ m} - 0.00780 \text{ m}) + (0.00780 \text{ m})} = 13.4 \text{ pF}.$$

(b)  $q = CV = (13.4 \times 10^{-12} \text{ F})(85.5 \text{ V}) = 1.15 \text{ nC}$ .

(c) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\epsilon_0 A} = \frac{1.15 \times 10^{-9} \text{ C}}{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2})(115 \times 10^{-4} \text{ m}^2)} = 1.13 \times 10^4 \text{ N/C}.$$

(d) Using Eq. 25-34, we obtain

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13 \times 10^4 \text{ N/C}}{2.61} = 4.33 \times 10^3 \text{ N/C}.$$

53. (a) According to Eq. 25-17 the capacitance of an air-filled spherical capacitor is given by

$$C_0 = 4\pi\epsilon_0 \left( \frac{ab}{b-a} \right).$$

When the dielectric is inserted between the plates the capacitance is greater by a factor of the dielectric constant  $\kappa$ . Consequently, the new capacitance is

$$C = 4\pi\kappa\epsilon_0 \left( \frac{ab}{b-a} \right) = \frac{23.5}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2} \cdot \frac{(0.0120 \text{ m})(0.0170 \text{ m})}{0.0170 \text{ m} - 0.0120 \text{ m}} = 0.107 \text{ nF}.$$

(b) The charge on the positive plate is  $q = CV = (0.107 \text{ nF})(73.0 \text{ V}) = 7.79 \text{ nC}$ .

(c) Let the charge on the inner conductor be  $-q$ . Immediately adjacent to it is the induced charge  $q'$ . Since the electric field is less by a factor  $1/\kappa$  than the field when no dielectric is present, then  $-q + q' = -q/\kappa$ . Thus,

$$q' = \frac{\kappa - 1}{\kappa} q = 4\pi(\kappa - 1)\epsilon_0 \frac{ab}{b-a} V = \left( \frac{23.5 - 1.00}{23.5} \right) (7.79 \text{ nC}) = 7.45 \text{ nC}.$$

54. (a) We apply Gauss's law with dielectric:  $q/\epsilon_0 = \kappa EA$ , and solve for  $\kappa$ :

$$\kappa = \frac{q}{\epsilon_0 EA} = \frac{8.9 \times 10^{-7} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.4 \times 10^{-6} \text{ V/m})(100 \times 10^{-4} \text{ m}^2)} = 7.2.$$

(b) The charge induced is

$$q' = q \left( 1 - \frac{1}{\kappa} \right) = (8.9 \times 10^{-7} \text{ C}) \left( 1 - \frac{1}{7.2} \right) = 7.7 \times 10^{-7} \text{ C}.$$

55. (a) Initially, the capacitance is

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.12 \text{ m}^2)}{1.2 \times 10^{-2} \text{ m}} = 89 \text{ pF}.$$

(b) Working through Sample Problem 25-7 algebraically, we find:

$$C = \frac{\epsilon_0 A \kappa}{\kappa(d-b) + b} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.12 \text{ m}^2)(4.8)}{(4.8)(1.2 - 0.40)(10^{-2} \text{ m}) + (4.0 \times 10^{-3} \text{ m})} = 1.2 \times 10^2 \text{ pF}.$$

(c) Before the insertion,  $q = C_0 V = (89 \text{ pF})(120 \text{ V}) = 11 \text{ nC}$ .

(d) Since the battery is disconnected,  $q$  will remain the same after the insertion of the slab, with  $q = 11 \text{ nC}$ .

(e)  $E = q / \epsilon_0 A = 11 \times 10^{-9} \text{ C} / (8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2})(0.12 \text{ m}^2) = 10 \text{ kV} / \text{m}$ .

(f)  $E' = E/\kappa = (10 \text{ kV/m})/4.8 = 2.1 \text{ kV/m}$ .

(g) The potential difference across the plates is

$$V = E(d-b) + E'b = (10 \text{ kV/m})(0.012 \text{ m} - 0.0040 \text{ m}) + (2.1 \text{ kV/m})(0.40 \times 10^{-3} \text{ m}) = 88 \text{ V}.$$

(h) The work done is

$$W_{\text{ext}} = \Delta U = \frac{q^2}{2} \left( \frac{1}{C} - \frac{1}{C_0} \right) = \frac{(11 \times 10^{-9} \text{ C})^2}{2} \left( \frac{1}{89 \times 10^{-12} \text{ F}} - \frac{1}{120 \times 10^{-12} \text{ F}} \right) = -1.7 \times 10^{-7} \text{ J}.$$

56. (a) Eq. 25-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(200 \times 10^{-12} \text{ F})(7.0 \times 10^3 \text{ V})^2 = 4.9 \times 10^{-3} \text{ J}.$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.



57. Initially the capacitors  $C_1$ ,  $C_2$ , and  $C_3$  form a series combination equivalent to a single capacitor which we denote  $C_{123}$ . Solving the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_1 C_2 + C_2 C_3 + C_1 C_3}{C_1 C_2 C_3},$$

we obtain  $C_{123} = 2.40 \mu\text{F}$ . With  $V = 12.0 \text{ V}$ , we then obtain  $q = C_{123}V = 28.8 \mu\text{C}$ . In the final situation,  $C_2$  and  $C_4$  are in parallel and are thus effectively equivalent to  $C_{24} = 12.0 \mu\text{F}$ . Similar to the previous computation, we use

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_{24}} + \frac{1}{C_3} = \frac{C_1 C_{24} + C_{24} C_3 + C_1 C_3}{C_1 C_{24} C_3}$$

and find  $C_{1234} = 3.00 \mu\text{F}$ . Therefore, the final charge is  $q = C_{1234}V = 36.0 \mu\text{C}$ .

(a) This represents a change (relative to the initial charge) of  $\Delta q = 7.20 \mu\text{C}$ .

(b) The capacitor  $C_{24}$  which we imagined to replace the parallel pair  $C_2$  and  $C_4$  is in series with  $C_1$  and  $C_3$  and thus also has the final charge  $q = 36.0 \mu\text{C}$  found above. The voltage across  $C_{24}$  would be

$$V_{24} = \frac{q}{C_{24}} = \frac{36.0 \mu\text{C}}{12.0 \mu\text{F}} = 3.00 \text{ V}.$$

This is the same voltage across each of the parallel pair. In particular,  $V_4 = 3.00 \text{ V}$  implies that  $q_4 = C_4 V_4 = 18.0 \mu\text{C}$ .

(c) The battery supplies charges only to the plates where it is connected. The charges on the rest of the plates are due to electron transfers between them, in accord with the new distribution of voltages across the capacitors. So, the battery does not directly supply the charge on capacitor 4.

58. In series, their equivalent capacitance (and thus their total energy stored) is smaller than either one individually (by Eq. 25-20). In parallel, their equivalent capacitance (and thus their total energy stored) is larger than either one individually (by Eq. 25-19). Thus, the middle two values quoted in the problem must correspond to the individual capacitors. We use Eq. 25-22 and find

$$(a) \ 100 \ \mu\text{J} = \frac{1}{2} C_1 (10 \ \text{V})^2 \Rightarrow C_1 = 2.0 \ \mu\text{F}$$

$$(b) \ 300 \ \mu\text{J} = \frac{1}{2} C_2 (10 \ \text{V})^2 \Rightarrow C_2 = 6.0 \ \mu\text{F} .$$

59. Initially, the total equivalent capacitance is  $C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 3.0 \mu\text{F}$ , and the charge on the positive plate of each one is  $(3.0 \mu\text{F})(10 \text{ V}) = 30 \mu\text{C}$ . Next, the capacitor (call is  $C_1$ ) is squeezed as described in the problem, with the effect that the new value of  $C_1$  is  $12 \mu\text{F}$  (see Eq. 25-9). The new total equivalent capacitance then becomes

$$C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 4.0 \mu\text{F},$$

and the new charge on the positive plate of each one is  $(4.0 \mu\text{F})(10 \text{ V}) = 40 \mu\text{C}$ .

(a) Thus we see that the charge transferred from the battery as a result of the squeezing is  $40 \mu\text{C} - 30 \mu\text{C} = 10 \mu\text{C}$ .

(b) The total increase in positive charge (on the respective positive plates) stored on the capacitors is twice the value found in part (a) (since we are dealing with two capacitors in series):  $20 \mu\text{C}$ .

60. (a) We reduce the parallel group  $C_2$ ,  $C_3$  and  $C_4$ , and the parallel pair  $C_5$  and  $C_6$ , obtaining equivalent values  $C' = 12 \mu\text{F}$  and  $C'' = 12 \mu\text{F}$ , respectively. We then reduce the series group  $C_1$ ,  $C'$  and  $C''$  to obtain an equivalent capacitance of  $C_{\text{eq}} = 3 \mu\text{F}$  hooked to the battery. Thus, the charge stored in the system is  $q_{\text{sys}} = C_{\text{eq}} V_{\text{bat}} = 36 \mu\text{C}$  .

(b) Since  $q_{\text{sys}} = q_1$  then the voltage across  $C_1$  is

$$V_1 = \frac{q_1}{C_1} = \frac{36 \mu\text{C}}{6.0 \mu\text{F}} = 6.0 \text{ V} .$$

The voltage across the series-pair  $C'$  and  $C''$  is consequently  $V_{\text{bat}} - V_1 = 6.0 \text{ V}$ . Since  $C' = C''$ , we infer  $V' = V'' = 6.0/2 = 3.0 \text{ V}$ , which, in turn, is equal to  $V_4$ , the potential across  $C_4$ . Therefore,

$$q_4 = C_4 V_4 = (4.0 \mu\text{F})(3.0 \text{ V}) = 12 \mu\text{C} .$$

61. The pair  $C_3$  and  $C_4$  are in parallel and consequently equivalent to  $30\ \mu\text{F}$ . Since this numerical value is identical to that of the others (with which it is in series, with the battery), we observe that each has one-third the battery voltage across it. Hence,  $3.0\ \text{V}$  is across  $C_4$ , producing a charge

$$q_4 = C_4 V_4 = (15\ \mu\text{F})(3.0\ \text{V}) = 45\ \mu\text{C} .$$

62. (a) The potential across  $C_1$  is 10 V, so the charge on it is

$$q_1 = C_1 V_1 = (10.0 \mu\text{F})(10.0 \text{ V}) = 100 \mu\text{C}.$$

(b) Reducing the right portion of the circuit produces an equivalence equal to  $6.00 \mu\text{F}$ , with 10.0 V across it. Thus, a charge of  $60.0 \mu\text{C}$  is on it -- and consequently also on the bottom right capacitor. The bottom right capacitor has, as a result, a potential across it equal to

$$V = \frac{q}{C} = \frac{60 \mu\text{C}}{10 \mu\text{F}} = 6.00 \text{ V}$$

which leaves  $10.0 \text{ V} - 6.00 \text{ V} = 4.00 \text{ V}$  across the group of capacitors in the upper right portion of the circuit. Inspection of the arrangement (and capacitance values) of that group reveals that this 4.00 V must be equally divided by  $C_2$  and the capacitor directly below it (in series with it). Therefore, with 2.00 V across  $C_2$  we find

$$q_2 = C_2 V_2 = (10.0 \mu\text{F})(2.00 \text{ V}) = 20.0 \mu\text{C} .$$

63. The pair  $C_1$  and  $C_2$  are in parallel, as are the pair  $C_3$  and  $C_4$ ; they reduce to equivalent values  $6.0\ \mu\text{F}$  and  $3.0\ \mu\text{F}$ , respectively. These are now in series and reduce to  $2.0\ \mu\text{F}$ , across which we have the battery voltage. Consequently, the charge on the  $2.0\ \mu\text{F}$  equivalence is  $(2.0\ \mu\text{F})(12\ \text{V}) = 24\ \mu\text{C}$ . This charge on the  $3.0\ \mu\text{F}$  equivalence (of  $C_3$  and  $C_4$ ) has a voltage of

$$V = \frac{q}{C} = \frac{24\ \mu\text{C}}{3\ \mu\text{F}} = 8.0\ \text{V} .$$

Finally, this voltage on capacitor  $C_4$  produces a charge  $(2.0\ \mu\text{F})(8.0\ \text{V}) = 16\ \mu\text{C}$ .

64. (a) Here  $D$  is not attached to anything, so that the  $6C$  and  $4C$  capacitors are in series (equivalent to  $2.4C$ ). This is then in parallel with the  $2C$  capacitor, which produces an equivalence of  $4.4C$ . Finally the  $4.4C$  is in series with  $C$  and we obtain

$$C_{\text{eq}} = \frac{(C)(4.4C)}{C + 4.4C} = 0.82C = 0.82(50\mu\text{F}) = 41\mu\text{F}$$

where we have used the fact that  $C = 50\mu\text{F}$ .

(b) Now,  $B$  is the point which is not attached to anything, so that the  $6C$  and  $2C$  capacitors are now in series (equivalent to  $1.5C$ ), which is then in parallel with the  $4C$  capacitor (and thus equivalent to  $5.5C$ ). The  $5.5C$  is then in series with the  $C$  capacitor; consequently,

$$C_{\text{eq}} = \frac{(C)(5.5C)}{C + 5.5C} = 0.85C = 42\mu\text{F} .$$



65. (a) The equivalent capacitance is

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6.00 \mu\text{F})(4.00 \mu\text{F})}{6.00 \mu\text{F} + 4.00 \mu\text{F}} = 2.40 \mu\text{F} .$$

(b)  $q_1 = C_{\text{eq}} V = (2.40 \mu\text{F})(200 \text{ V}) = 4.80 \times 10^{-4} \text{ C} .$

(c)  $V_1 = q_1 / C_1 = 4.80 \times 10^{-4} \text{ C} / 6.00 \mu\text{F} = 80.0 \text{ V} .$

(d)  $q_2 = q_1 = 4.80 \times 10^{-4} \text{ C} .$

(e)  $V_2 = V - V_1 = 200 \text{ V} - 80.0 \text{ V} = 120 \text{ V} .$

66. (a) Now  $C_{\text{eq}} = C_1 + C_2 = 6.00 \mu\text{F} + 4.00 \mu\text{F} = 10.0 \mu\text{F}$ .

(b)  $q_1 = C_1 V = (6.00 \mu\text{F})(200 \text{ V}) = 1.20 \times 10^{-3} \text{ C}$ .

(c)  $V_1 = 200 \text{ V}$ .

(d)  $q_2 = C_2 V = (4.00 \mu\text{F})(200 \text{ V}) = 8.00 \times 10^{-4} \text{ C}$ .

(e)  $V_2 = V_1 = 200 \text{ V}$ .

67. We cannot expect simple energy conservation to hold since energy is presumably dissipated either as heat in the hookup wires or as radio waves while the charge oscillates in the course of the system “settling down” to its final state (of having 40 V across the parallel pair of capacitors  $C$  and  $60\ \mu\text{F}$ ). We do expect charge to be conserved. Thus, if  $Q$  is the charge originally stored on  $C$  and  $q_1, q_2$  are the charges on the parallel pair after “settling down,” then

$$Q = q_1 + q_2 \quad \Rightarrow \quad C(100\text{ V}) = C(40\text{ V}) + (60\ \mu\text{F})(40\text{ V})$$

which leads to the solution  $C = 40\ \mu\text{F}$ .

68. We first need to find an expression for the energy stored in a cylinder of radius  $R$  and length  $L$ , whose surface lies between the inner and outer cylinders of the capacitor ( $a < R < b$ ). The energy density at any point is given by  $u = \frac{1}{2} \epsilon_0 E^2$ , where  $E$  is the magnitude of the electric field at that point. If  $q$  is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance  $r$  from the cylinder axis is given by (see Eq. 25-12)

$$E = \frac{q}{2\pi\epsilon_0 Lr},$$

and the energy density at that point is

$$u = \frac{1}{2} \epsilon_0 E^2 = \frac{q^2}{8\pi^2 \epsilon_0 L^2 r^2}.$$

The corresponding energy in the cylinder is the volume integral

$$U_R = \int u d\mathcal{V}.$$

Now,  $d\mathcal{V} = 2\pi r L dr$ , so

$$U_R = \int_a^R \frac{q^2}{8\pi^2 \epsilon_0 L^2 r^2} 2\pi r L dr = \frac{q^2}{4\pi \epsilon_0 L} \int_a^R \frac{dr}{r} = \frac{q^2}{4\pi \epsilon_0 L} \ln \frac{R}{a}.$$

To find an expression for the total energy stored in the capacitor, we replace  $R$  with  $b$ :

$$U_b = \frac{q^2}{4\pi \epsilon_0 L} \ln \frac{b}{a}.$$

We want the ratio  $U_R/U_b$  to be  $1/2$ , so

$$\ln \frac{R}{a} = \frac{1}{2} \ln \frac{b}{a}$$

or, since  $\frac{1}{2} \ln(b/a) = \ln(\sqrt{b/a})$ ,  $\ln(R/a) = \ln(\sqrt{b/a})$ . This means  $R/a = \sqrt{b/a}$  or  $R = \sqrt{ab}$ .

69. (a) Since the field is constant and the capacitors are in parallel (each with 600 V across them) with identical distances ( $d = 0.00300$  m) between the plates, then the field in  $A$  is equal to the field in  $B$ :

$$|\vec{E}| = \frac{V}{d} = 2.00 \times 10^5 \text{ V/m} .$$

(b)  $|\vec{E}| = 2.00 \times 10^5 \text{ V/m}$  . See the note in part (a).

(c) For the air-filled capacitor, Eq. 25-4 leads to

$$\sigma = \frac{q}{A} = \epsilon_0 |\vec{E}| = 1.77 \times 10^{-6} \text{ C/m}^2 .$$

(d) For the dielectric-filled capacitor, we use Eq. 25-29:

$$\sigma = \kappa \epsilon_0 |\vec{E}| = 4.60 \times 10^{-6} \text{ C/m}^2 .$$

(e) Although the discussion in the textbook (§25-8) is in terms of the charge being held fixed (while a dielectric is inserted), it is readily adapted to this situation (where comparison is made of two capacitors which have the same *voltage* and are identical except for the fact that one has a dielectric). The fact that capacitor  $B$  has a relatively large charge but only produces the field that  $A$  produces (with its smaller charge) is in line with the point being made (in the text) with Eq. 25-34 and in the material that follows. Adapting Eq. 25-35 to this problem, we see that the difference in charge densities between parts (c) and (d) is due, in part, to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma' = (1.77 \times 10^{-6}) - (4.60 \times 10^{-6}) = -2.83 \times 10^{-6} \text{ C/m}^2 .$$

70. (a) The equivalent capacitance is  $C_{\text{eq}} = C_1 C_2 / (C_1 + C_2)$ . Thus the charge  $q$  on each capacitor is

$$q = q_1 = q_2 = C_{\text{eq}} V = \frac{C_1 C_2 V}{C_1 + C_2} = \frac{(2.00 \mu\text{F})(8.00 \mu\text{F})(300 \text{V})}{2.00 \mu\text{F} + 8.00 \mu\text{F}} = 4.80 \times 10^{-4} \text{C}.$$

(b) The potential difference is  $V_1 = q/C_1 = 4.80 \times 10^{-4} \text{C} / 2.0 \mu\text{F} = 240 \text{V}$ .

(c) As noted in part (a),  $q_2 = q_1 = 4.80 \times 10^{-4} \text{C}$ .

(d)  $V_2 = V - V_1 = 300 \text{V} - 240 \text{V} = 60.0 \text{V}$ .

Now we have  $q'_1/C_1 = q'_2/C_2 = V'$  ( $V'$  being the new potential difference across each capacitor) and  $q'_1 + q'_2 = 2q$ . We solve for  $q'_1$ ,  $q'_2$  and  $V$ :

$$(e) \quad q'_1 = \frac{2C_1 q}{C_1 + C_2} = \frac{2(2.00 \mu\text{F})(4.80 \times 10^{-4} \text{C})}{2.00 \mu\text{F} + 8.00 \mu\text{F}} = 1.92 \times 10^{-4} \text{C}.$$

$$(f) \quad V'_1 = \frac{q'_1}{C_1} = \frac{1.92 \times 10^{-4} \text{C}}{2.00 \mu\text{F}} = 96.0 \text{V}.$$

$$(g) \quad q'_2 = 2q - q'_1 = 7.68 \times 10^{-4} \text{C}.$$

$$(h) \quad V'_2 = V'_1 = 96.0 \text{V}.$$

(i) In this circumstance, the capacitors will simply discharge themselves, leaving  $q_1 = 0$ ,

$$(j) \quad V_1 = 0,$$

$$(k) \quad q_2 = 0,$$

$$(l) \quad \text{and } V_2 = V_1 = 0.$$

71. We use  $U = \frac{1}{2}CV^2$ . As  $V$  is increased by  $\Delta V$ , the energy stored in the capacitor increases correspondingly from  $U$  to  $U + \Delta U$ :  $U + \Delta U = \frac{1}{2}C(V + \Delta V)^2$ . Thus,  $(1 + \Delta V/V)^2 = 1 + \Delta U/U$ , or

$$\frac{\Delta V}{V} = \sqrt{1 + \frac{\Delta U}{U}} - 1 = \sqrt{1 + 10\%} - 1 = 4.9\% .$$

72. We use  $C = \varepsilon_0 \kappa A/d \propto \kappa/d$ . To maximize  $C$  we need to choose the material with the greatest value of  $\kappa/d$ . It follows that the mica sheet should be chosen.



73. We may think of this as two capacitors in series  $C_1$  and  $C_2$ , the former with the  $\kappa_1 = 3.00$  material and the latter with the  $\kappa_2 = 4.00$  material. Upon using Eq. 25-9, Eq. 25-27 and then reducing  $C_1$  and  $C_2$  to an equivalent capacitance (connected directly to the battery) with Eq. 25-20, we obtain

$$C_{\text{eq}} = \left( \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} \right) \frac{\epsilon_0 A}{d} = 1.52 \times 10^{-10} \text{ F} .$$

Therefore,  $q = C_{\text{eq}} V = 1.06 \times 10^{-9} \text{ C}$ .

74. (a) The length  $d$  is effectively shortened by  $b$  so  $C' = \epsilon_0 A / (d - b) = 0.708 \text{ pF}$ .

(b) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'} = \frac{q^2 / 2C}{q^2 / 2C'} = \frac{C'}{C} = \frac{\epsilon_0 A / (d - b)}{\epsilon_0 A / d} = \frac{d}{d - b} = \frac{5.00}{5.00 - 2.00} = 1.67.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2} \left( \frac{1}{C'} - \frac{1}{C} \right) = \frac{q^2}{2\epsilon_0 A} (d - b - d) = -\frac{q^2 b}{2\epsilon_0 A} = -5.44 \text{ J}.$$

(d) Since  $W < 0$  the slab is sucked in.

75. (a)  $C' = \epsilon_0 A / (d - b) = 0.708 \text{ pF}$ , the same as part (a) in Problem 25-74.

(b) The ratio of the stored energy is now

$$\frac{U}{U'} = \frac{\frac{1}{2} C V^2}{\frac{1}{2} C' V^2} = \frac{C}{C'} = \frac{\epsilon_0 A / d}{\epsilon_0 A / (d - b)} = \frac{d - b}{d} = \frac{5.00 - 2.00}{5.00} = 0.600.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{1}{2} (C' - C) V^2 = \frac{\epsilon_0 A}{2} \left( \frac{1}{d - b} - \frac{1}{d} \right) V^2 = \frac{\epsilon_0 A b V^2}{2d(d - b)} = 1.02 \times 10^{-9} \text{ J}.$$

(d) In Problem 25-74 where the capacitor is disconnected from the battery and the slab is sucked in,  $F$  is certainly given by  $-dU/dx$ . However, that relation does not hold when the battery is left attached because the force on the slab is not conservative. The charge distribution in the slab causes the slab to be sucked into the gap by the charge distribution on the plates. This action causes an increase in the potential energy stored by the battery in the capacitor.

76. (a) Put five such capacitors in series. Then, the equivalent capacitance is  $2.0 \mu\text{F}/5 = 0.40 \mu\text{F}$ . With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now  $C_{\text{eq}} = 3(0.40 \mu\text{F}) = 1.2 \mu\text{F}$ . With each capacitor taking a 200-V potential difference the equivalent capacitor can withstand 1000 V.

77. The voltage across capacitor 1 is

$$V_1 = \frac{q_1}{C_1} = \frac{30 \mu\text{C}}{10 \mu\text{F}} = 3.0 \text{ V} .$$

Since  $V_1 = V_2$ , the total charge on capacitor 2 is

$$q_2 = C_2 V_2 = (20 \mu\text{F})(2 \text{ V}) = 60 \mu\text{C} ,$$

which means a total of  $90 \mu\text{C}$  of charge is on the pair of capacitors  $C_1$  and  $C_2$ . This implies there is a total of  $90 \mu\text{C}$  of charge also on the  $C_3$  and  $C_4$  pair. Since  $C_3 = C_4$ , the charge divides equally between them, so  $q_3 = q_4 = 45 \mu\text{C}$ . Thus, the voltage across capacitor 3 is

$$V_3 = \frac{q_3}{C_3} = \frac{45 \mu\text{C}}{20 \mu\text{F}} = 2.3 \text{ V} .$$

Therefore,  $|V_A - V_B| = V_1 + V_3 = 5.3 \text{ V}$ .

78. One way to approach this is to note that – since they are identical – the voltage is evenly divided between them. That is, the voltage across each capacitor is  $V = (10/n)$  volt. With  $C = 2.0 \times 10^{-6}$  F, the electric energy stored by each capacitor is  $\frac{1}{2}CV^2$ . The total energy stored by the capacitors is  $n$  times that value, and the problem requires the total be equal to  $25 \times 10^{-6}$  J. Thus,

$$\frac{n}{2}(2.0 \times 10^{-6})\left(\frac{10}{n}\right)^2 = 25 \times 10^{-6}$$

leads to  $n = 4$ .