Topics

- Elementary Boolean operators
- Duality
- Basic theorems
- Algebraic minimization

Problems

1 Recall the truth table of the elementary Boolean operators. Solve the following system of equations for the variables A, B, C and D

$$\begin{cases} A' + A.B & = & 0 \\ A.C & = & A.B \\ A.B + A.C' + C.D & = & C'.D \end{cases}$$

2 Verify, by perfect induction, the following simplification theorems. Write the dual counterpart of each theorem.

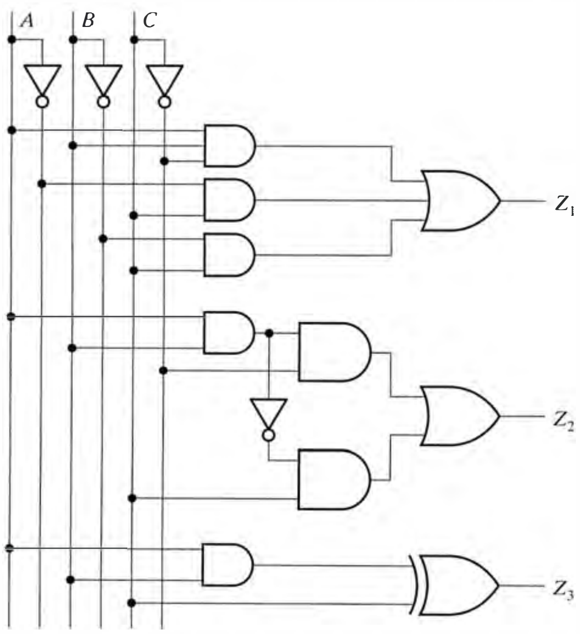
a) $x + x.y = x$ b) $x + x'.y = x + y$ c) $x.y + x.y' = x$

3 Show that $x'.y'.z' + x'.y'.z + x'.y.z' + x'.y.z + x.y.z' + x.y.z = x' + y$

4 Show that $x.y + x'.z + y.z = x.y + x'.z$. Write the dual version of the previous expression.

5 Write the truth table of the XOR operation $x \oplus y$. Express this operator as an elementary sum of logic products.

6 Consider the logic circuits in the figure and show, by algebraic methods, that $Z_1 = Z_2 = Z_3$.



7 Use DeMorgan's laws to obtain the complement of:

a) $(x.y' + x'.y)$ b) $(x.y + z.(x + y') + z.y)$

8 Show that $(a'.b + a.c).(a + b').(a' + c') = 0$

9 Show that the dual of an XOR is an XNOR, that is $(x \oplus y)^D = (x \oplus y)'$.

10 Implement the XOR operation with NAND gates. Assume that both uncomplemented and complemented inputs are available.

11 Consider the following Boolean functions:

$$S = x \oplus y \oplus c_i$$
$$C_o = x.y + c_i.(x + y)$$

a) Draw the logic circuit.

b) Redraw the circuit using only NAND gates.

12 The Majority function $M(x, y, z)$, is 1 whenever there are at least two inputs equal to 1.

a) Write the truth table for $M(x, y, z)$.

b) From the truth table propose a Boolean expression for $M(x, y, z)$.

c) Draw the corresponding logic circuit.

d) Show that using the set $S = \{M(x, y, z), NOT, \text{"0"}\}$ we can express any logic function. Suggestion: show how to implement the fundamental Boolean operators {"+","."} using the elements of *S*.