Topics

- Canonical Forms
- Minimization of Boolean functions using the Karnaugh map method

Preliminary Definitions

- A **literal** is a variable or the complement of a variable. Examples: x, y, $\bar{x}$.
- A **product term** is a single literal or a logical product of two or more literals. Examples: $\bar{z}$, $x \cdot y$, $x \cdot \bar{y} \cdot z$.
- A **sum term** is a single literal or a logical sum of two or more literals. Examples: $\bar{z}$, $x + y$, $x + \bar{y} + z$.
- A **normal term** is a product or sum term in which no variable appears more than once.
- An *n*-variable **minterm** is a normal product term with *n* literals. There are 2^n such product terms.
  - A minterm m_i corresponds to row *i* of the truth table.
  - In minterm m_i, a particular variable appears complemented if the corresponding bit in the binary representation of *i* is 0; otherwise, it is uncomplemented.
- An *n*-variable **maxterm** is a normal sum term with *n* literals. There are 2^n such sum terms.
  - A maxterm M_i corresponds to row *i* of the truth table.
  - In maxterm M_i, a particular variable appears complemented if the corresponding bit in the binary representation of *i* is 1; otherwise, it is uncomplemented.

Canonical Forms:

1st canonical form: sum of products, SOP:

$$f(x_0, x_1, \ldots, x_{n-1}) = \sum_{i=0}^{2^n-1} m_i \cdot f_i$$

2nd canonical form: Product of sums, POS:

$$f(x_0, x_1, \ldots, x_{n-1}) = \prod_{i=0}^{2^n-1} (f_i + M_i)$$

3rd canonical form:

$$f(x_0, x_1, \ldots, x_{n-1}) = \overline{\prod_{i=0}^{2^n-1} \overline{f_i \cdot m_i}}$$

4th canonical form :

$$f(x_0, x_1, \ldots, x_{n-1}) = \overline{\sum_{i=0}^{2^n-1} \overline{f_i + M_i}}$$

Problems

1. Consider the following Boolean function $f(x, y, z) = x' \cdot y + z' + x \cdot y' \cdot z$.
   a. Draw the logic circuit
   b. Construct the truth table for the function *f(x,y,z)*.
   c. From the truth table write all the canonical forms.

2. Write all the canonical forms for the Boolean functions *f*,*g*,*h*,*w* of (*x*,*y*,*z*) defined in the following truth table:

x	y	z	f	g	h	w
0	0	0	0	1	0	1
0	0	1	1	0	1	0
0	1	0	1	1	1	0
0	1	1	0	0	1	0
1	0	0	1	1	1	1
1	0	1	0	0	1	1
1	1	0	0	1	1	0
1	1	1	1	0	0	1

3. Find the minimal SOP and POS algebraic expressions for the functions defined by the following Karnaugh maps:

K1

cd \ ab	00	01	11	10
00	1	1		
01				
11		1	1	
10		1	1	

K2

cd \ ab	00	01	11	10
00	1	1	1	1
01				
11				
10				

K3

cd \ ab	00	01	11	10
00	1			1
01				
11				
10	1			1

K4

cd \ ab	00	01	11	10
00		1	1	
01			1	
11		1	1	
10		1	1	

4. Propose a 4'variable Boolean function, that has more than one minimal expression (in either POS or SOP form). Locate the essential prime implicants (or essential prime implicates) if they exist.

5. Let $f(a,b,c,d) = a' \cdot c' + b' \cdot c' + a \cdot c \cdot d + a' \cdot b \cdot c'$.
   a. Write the Karnaugh map directly from the expression
   b. Find the minimal SOP expression for $f(a,b,c,d)$.

6. Let $f(a,b,c,d) = (a + b')$.(c'+d).(b'+d')
   a. Write the Karnaugh map directly from the expression
   b. Find the minimal SOP expression for $f(a,b,c,d)$.

7. Find the minimal SOP expressions for the two following Boolean functions. Compare the results.
   a. $f(x_3, x_2, x_1, x_0) = \sum m_{x_3,x_2,x_1,x_0}\ (0,1,4,5,12,13)$
   b. $g(x_3, x_2, x_1, x_0) = \prod M_{x_3,x_2,x_1,x_0}\ (2,3,6,7,8,9,10,11,14,15)$

8. Find the minimal SOP and POS expressions for the following Boolean functions

   a. $(w, x, y, z) = \sum m_{w,x,y,z}\ (0,1,2,4,6,9,11)$

   b. $f(x_3, x_2, x_1, x_0) = \sum m_{x_3,x_2,x_1,x_0}\ (0,1,4,5,8,9,10,11,12,14)$

   c. $f(x_4, x_3, x_2, x_1, x_0) =$
   $\sum m_{x_4,x_3,x_2,x_1,x_0}\ (0,2,4,5,6,8,9,10,12,14,17,18,20,24,25,28,30)$

9. Sometimes the output for a given combination of the input variables is not possible to define and/or is irrelevant. The existence of these don't care conditions may facilitate the minimization process. Find the minimal SOP expressions for the following Karnaugh maps:

K1

c \ ab	00	01	11	10
0		x	x	1
1	1	x	1	

K2

cd \ ab	00	01	11	10
00		1	1	
01	1	x		x
11	x		x	1
10		1	1	

10. Obtain the minimal SOP and POS expressions for the following functions with don't care conditions:

    a. $f(x_3, x_2, x_1, x_0) = \sum m_{x_3,x_2,x_1,x_0}\ (4,5,6,8,9,10,13) + \sum d_{x_3,x_2,x_1,x_0}\ (0,7,15)$

    b. $f(x_3, x_2, x_1, x_0) = \sum m_{x_3,x_2,x_1,x_0}\ (1,3,5,7,9) + \sum d_{x_3,x_2,x_1,x_0}\ (6,12,13)$