# AHP (Analytical Hyerarquical Processing)

## Linear Programing

MultiCriteria Problems

Applied to recommendation systems

Constantino Martins and Goreti Marreiros

# AHP (Analytical Hyerarquical Processing)

- AHP can be used to determine the relative importance of each criterion, allowing the recommendation system to adjust its recommendations based on these weights
- Users make informed decisions when choosing between different recommendation options
- AHP can be incorporated into recommendation systems to personalize recommendations based on individual user preferences
- Users can be asked to assess the importance of different criteria, and the recommendation system can then generate recommendations that best meet those preferences
- Considering multiple criteria and their weights, AHP can help improve the accuracy of recommendations

# AHP (Analytical Hyerarquical Processing)

- Multi-criteria decision making

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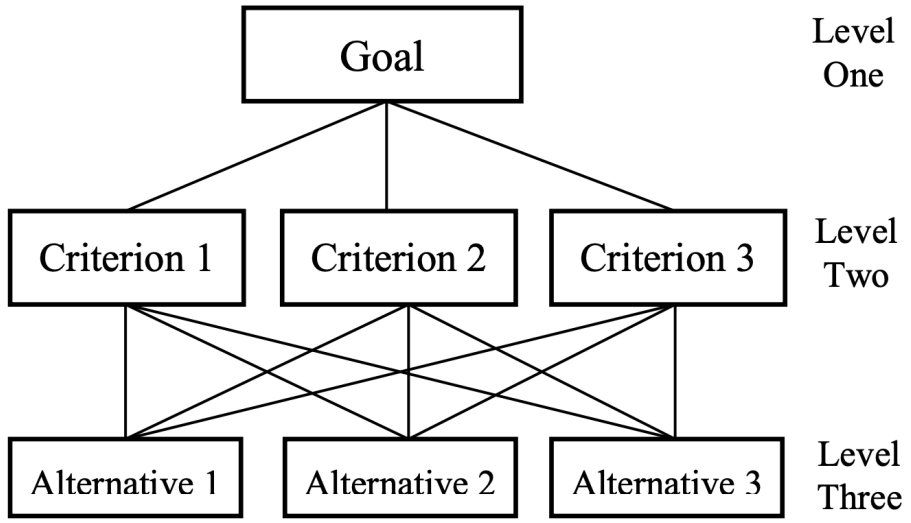
- Top of the hierarchy:  the objective

- Second level:  how the three criteria contribute to the objective

- Third level:  how each of the three alternatives contributes to each of the criteria

# AHP (Analytical Hyerarquical Processing)

- AHP (Analytical Hyerarquical Processing)
  - Proposed by Thomas Saaty, University of Pittsburgh, in the 70s
  - belongs to the family of normative methods
  - decompose the decision problem into a hierarchy of subproblems
  - elicitation of pairwise comparison judgments




Taherdoost 2018

# Multi-criteria decision making

- AHP (Analytical Hierarquical Processing)
  - Problem structuring and definition of the hierarchy
  - Elicitation of pairwise comparisons
    - normalizing the resulting matrix
  - Derivation of priority vectors
    - averaging the values in each row to get the corresponding rating
  - Check consistency
  - Calculate overall scores

# Multi-criteria decision making

- AHP – Saaty Scale

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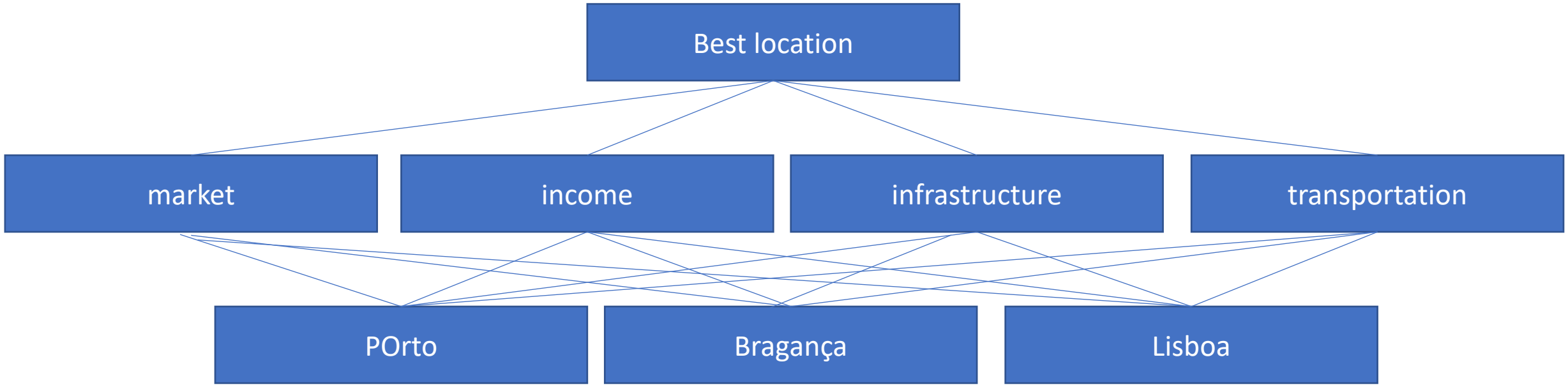
| Intensity of importance | Description |
|---|---|
| 9 | Extreme importance |
| 7 | Very strong or demonstrated importance |
| 5 | Strong importance |
| 3 | Moderate importance |
| 1 | Equal importance |
| 2,4,6,8 | Intermediate values |

# AHP

- A company wants to choose a location for a new delegation, and have 3 potential itens: Porto, Lisboa, Bragança. There are several criteria that influence the choice: market, income level, transportation network, infrastruture



# AHP

- Pairwise comparison for Alternatives regarding criteria **market**

| Location | Market | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 1 | 3 | 2 |
| Bragança | 1/3 | 1 | 1/5 |
| Lisboa | 1/2 | 5 | 1 |

The diagonal elements of the matrix are always 1.
We only need to fill up the upper triangular matrix.
How to fill up the upper triangular matrix is using the following rules:

- If the judgment value is on the left side of 1, we put the actual Judgment value.
- If the judgment value is on the right side of 1, we put the reciprocal Value

The lower triangular matrix: $a_{ij}=1/a_{ji}$

| Intensity of importance | Description |
|---|---|
| 9 | Extreme importance |
| 7 | Very strong or demonstrated importance |
| 5 | Strong importance |
| 3 | Moderate importance |
| 1 | Equal importance |
| 2,4,6,8 | Intermediate values |

Porto. Lisboa
9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9

Bragança. 1/5 Lisboa
9 8 7 6 5 4 3 2 1 2 3 4 5 6 7 8 9

# AHP

- Pairwise comparison for Alternatives regarding criteria **Income**

| Location | Income | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 1 | 6 | 1/3 |
| Bragança | 1/6 | 1 | 1/9 |
| Lisboa | 3 | 9 | 1 |

# AHP

- Pairwise comparison for Alternatives regarding criteria **Infrastructure**

| Location | Infrastructure | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 1 | 1/3 | 1 |
| Bragança | 3 | 1 | 7 |
| Lisboa | 1 | 1/7 | 1 |

# AHP

- Pairwise comparison for Alternatives regarding criteria **Transportation**

| Location | Transportation | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 1 | 1/3 | 1/2 |
| Bragança | 3 | 1 | 4 |
| Lisboa | 2 | 1/4 | 1 |

# AHP

- Pairwise comparison for criteria

| | Market | Income | Infrastructure | Transportation |
|---|---|---|---|---|
| Market | 1 | 1/5 | 3 | 4 |
| Income | 5 | 1 | 9 | 7 |
| Infrastructure | 1/3 | 1/9 | 1 | 2 |
| Trasnportation | 1/4 | 1/7 | ½ | 1 |

# AHP

- Normalize preference Matrix:
  - Reduce all the values to the same unit
  - Each value of the matrix is divided by the total of its specific column

| Location | Market | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 1 | 3 | 2 |
| Bragança | 1/3 | 1 | 1/5 |
| Lisboa | 1/2 | 5 | 1 |
| sum | 11/6 | 9 | 16/5 |

Normalize the columns so that the sum of all column values becomes 1

1. Sum each column of the matrix
2. Then divide each element of the matrix with the sum of its column, we have normalized relative weight

| Location | Market | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 6/11 | 1/3 | 5/8 |
| Bragança | 2/11 | 1/9 | 1/16 |
| Lisboa | 3/11 | 5/9 | 5/16 |

# AHP

- Preference Vector
  - Identify the order of importance of element on the hierarchy
  - Calculate the average of the values of each line of the normalized preference matrix

We can obtain the overall or final priorities by calculating the arithmetic mean of each row

The sum must be 1

| Location | Market | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 6/11 | 1/3 | 5/8 |
| Bragança | 2/11 | 1/9 | 1/16 |
| Lisboa | 3/11 | 5/9 | 5/16 |

| Location | Market | | | **Average** |
|---|---|---|---|---|
| | Porto | Bragança | Lisboa | |
| Porto | 0,54545 | 0,33333 | 0,625 | **0,50126** |
| Bragança | 0,18182 | 0,11111 | 0,0625 | **0,11848** |
| Lisboa | 0,27273 | 0,55556 | 0,3125 | **0,38026** |
| | | | **sum** | **1** |

# AHP

- Preference Vector for alternative (criteria Income)
  - identify the order of importance of element pf the hierarchy
  - calculate the average of the values of each line of the normalized preference matrix

| Location | Income | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 1 | 6 | 1/3 |
| Bragança | 1/6 | 1 | 1/9 |
| Lisboa | 3 | 9 | 1 |

| Location | Income | | | **Average** |
|---|---|---|---|---|
| | Porto | Bragança | Lisboa | |
| Porto | 0,24 | 0,375 | 0,23076923 | **0,28192308** |
| Bragança | 0,04 | 0,0625 | 0,07692308 | **0,05980769** |
| Lisboa | 0,72 | 0,5625 | 0,69230769 | **0,65826923** |

# AHP

- Preference Vector for alternative (criteria infrastructure)
  - identify the order of importance of element pf the hierarchy
  - calculate the average of the values of each line of the normalized preference matrix

| Location | infrastructure | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 1 | 1/3 | 1 |
| Bragança | 3 | 1 | 7 |
| Lisboa | 1 | 1/7 | 1 |

| Location | infrastructure | | | **Average** |
|---|---|---|---|---|
| | Porto | Bragança | Lisboa | |
| Porto | 0,2 | 0,22580645 | 0,11111111 | **0,17897252** |
| Bragança | 0,6 | 0,67741935 | 0,77777778 | **0,68506571** |
| Lisboa | 0,2 | 0,09677419 | 0,11111111 | **0,13596177** |

# AHP

- Preference Vector for alternative (criteria transportation)
  - identify the order of importance of element pf the hierarchy
  - calculate the average of the values of each line of the normalized preference matrix

| Location | transportation | | |
|---|---|---|---|
| | Porto | Bragança | Lisboa |
| Porto | 1 | 1/3 | 1/2 |
| Bragança | 3 | 1 | 4 |
| Lisboa | 2 | 1/4 | 1 |

| Location | transportation | | | **Average** |
|---|---|---|---|---|
| | Porto | Bragança | Lisboa | |
| Porto | 0,16666667 | 0,21052632 | 0,09090909 | **0,15603402** |
| Bragança | 0,5 | 0,63157895 | 0,72727273 | **0,61961722** |
| Lisboa | 0,33333333 | 0,15789474 | 0,18181818 | **0,22434875** |

# AHP

- Preference Vector for criteria

| | Market | Income | Infrastructure | Transportation |
|---|---|---|---|---|
| Market | 1 | 1/5 | 3 | 4 |
| Income | 5 | 1 | 9 | 7 |
| Infrastructure | 1/3 | 1/9 | 1 | 2 |
| Trasnportation | 1/4 | 1/7 | ½ | 1 |

| | Market | Income | Infrastructure | Transportation | Average |
|---|---|---|---|---|---|
| Market | 0,15189873 | 0,13755459 | 0,222222222 | 0,28571429 | **0,19934746** |
| Income | 0,75949367 | 0,68777293 | 0,666666667 | 0,5 | **0,65348332** |
| Infrastructure | 0,05063291 | 0,07641921 | 0,074074074 | 0,14285714 | **0,08599584** |
| Trasnportation | 0,03797468 | 0,09825328 | 0,037037037 | 0,07142857 | **0,06117339** |

# AHP

- Overal score

| | Market | Income | Infrastructure | Transportation |
|---|---|---|---|---|
| Porto | 0,50126263 | 0,28192308 | 0,178972521 | 0,15603402 |
| Bragança | 0,11847643 | 0,05980769 | 0,685065711 | 0,61961722 |
| Lisboa | 0,38026094 | 0,65826923 | 0,135961768 | 0,22434875 |

| | **Criteria weigth** |
|---|---|
| Market | 0,19934746 |
| Income | 0,65348332 |
| Infrastructure | 0,08599584 |
| Transportation | 0,06117339 |

| | Market | Income | Infrastructure | Transportation | Rank |
|---|---|---|---|---|---|
| Porto | 0,50126263 | 0,28192308 | 0,178972521 | 0,15603402 | 0,30909348 |
| Bragança | 0,11847643 | 0,05980769 | 0,685065711 | 0,61961722 | 0,15951819 |
| Lisboa | 0,38026094 | 0,65826923 | 0,135961768 | 0,22434875 | 0,53138833 |

# AHP – Consistency analysis

- Measure the consistency of the judgments
  - Calculate the consistency measure
  - Calculate the consistency index (CI)
    - (λmax-n)/(n-1)
  - Calculate the consistency ratio
    - CR=CI/RI where RI is a random index
    - If CR is greater than 0.1 the set of judgments may be too inconsistent to be reliable
  - Derived from Saaty's book, Random Index Values for n Items Being Compared

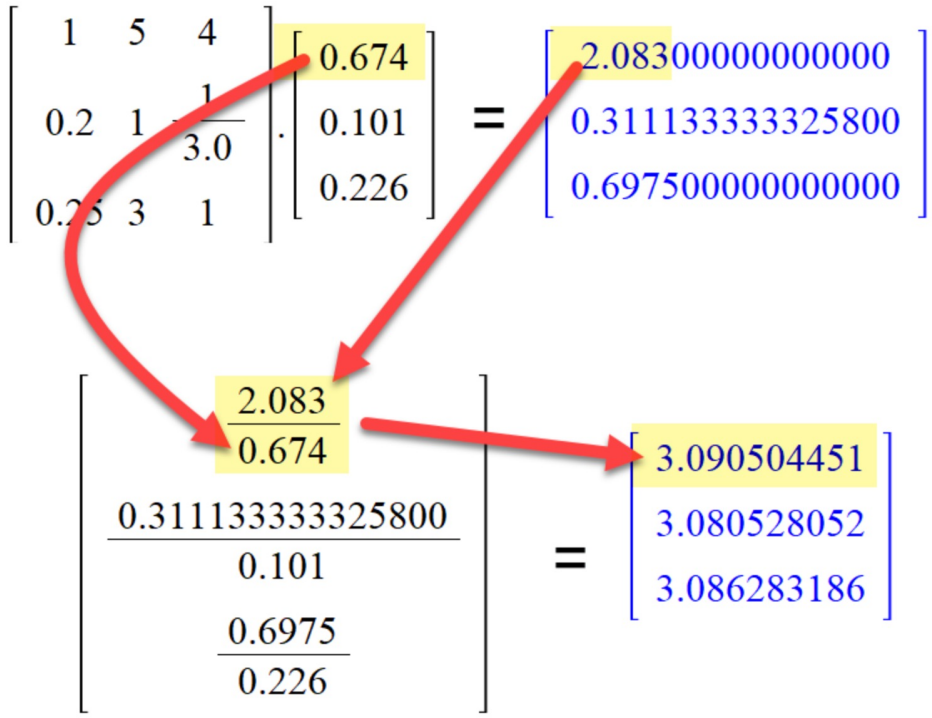
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.00 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 | 1.48 | 1.56 | 1.57 | 1.59 |

# Calculate the consistency measure λmax

- Example

$$\begin{bmatrix} 1 & 5 & 4 \\ 0.2 & 1 & \frac{1}{3.0} \\ 0.25 & 3 & 1 \end{bmatrix} . \begin{bmatrix} 0.674 \\ 0.101 \\ 0.226 \end{bmatrix} = \begin{bmatrix} 2.08300000000000 \\ 0.311133333325800 \\ 0.697500000000000 \end{bmatrix}$$

1. Perform a Matrix multiplication, the Pairwise comparison matrix X priority vector
2. Divide a cell value of the matrix multiplication result vector by the corresponding priority vector cell
3. Then, λmax is obtained by the average of this resulting vector, which is (3.090504451 + 3.080528052 + 3.086283186)/3 = 3.085771896.



# AHP – Consistency analysis

- Measure the consistency of the judgments
  - Calculate the consistency measure
    - λmax

| | Market | Income | Infrastructure | Transportation | **Criteria weigth** | |
|---|---|---|---|---|---|---|
| Market | 1 | 1/5 | 3 | 4 | 0,19934746 | 4,17725517 |
| Income | 5 | 1 | 9 | 7 | 0,65348332 | 4,36491153 |
| Infrastructure | 1/3 | 1/9 | 1 | 2 | 0,08599584 | 4,03974246 |
| Trasnportation | 1/4 | 1/7 | ½ | 1 | 0,06117339 | 4,04363606 |
| | | | | | λmax | **4,15638631** |

Média dos valores anteriores

# AHP – Consistency analysis

- Calculate the consistency index (CI)
  - (λmax-n)/(n-1) =(4,15638631-4)/3 = 0.0521
- Calculate the consistency ratio
  - CR=CI/RI where RI is a random index
  - CR=0.0521/0.9=0,057920855 <0.1
    - Degree of consistency is satisfactory

In our case, n = 4, and the λmax is 4,15638631. So, we get the consistency index as 0.0521

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.00 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 | 1.48 | 1.56 | 1.57 | 1.59 |

# Conlusions

- Calculating Relative Priorities:
- Based on the comparisons made by users, the relative priorities of each criterion and sub-criterion are calculated using the normalization method and weighted averages calculation of AHP.

- Personalized Recommendations:
- The recommendation system utilizes the calculated priorities to generate personalized recommendations for each user. For example, if a user assigns greater importance to the ”Market" criterion, the system may prioritize cities' with higher ratings in this aspect

# Bibliography


- Saaty, T.L. (1980). The analytic hierarchy process. McGraw-Hill, New York
- Introduction to Management Science. Pearson, 2013
- Russel and Norvig, Artificial Intelligence: A Modern Approach
- Efraim Turban, Ramesh Sharda; Dursun Delen (2010). Decision Support and Business Intelligence Systems (9th Edition) Prentice-Hall, 9th edition
- Sharda, Delen & Turban (2015) Business Intelligence and Analytics: Systems for Decision Support, 10th Edition, Pearson, 2015
- V.L. Sauter (2011). Decision Support Systems For Business Intelligence, 2nd Edition, New York: John Wiley & Sons