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METHODOLOGIES AND APPLICATION

A novel recommendation approach based on users' weighted trust relations and the rating similarities

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Abstract With the growing popularity of open social networks, approaches incorporating social relationships into recommender systems are gaining momentum, especially matrix factorization-based ones. The experiments in previous literatures indicate that social information is very effective in improving the performance of traditional recommendation algorithms. However, most of existing social recommendation methods only take one kind of social relations—trust information into consideration, which is far from satisfactory. Furthermore, most of the existing trust networks are binary, which results in the equal treatment to different users who are trusted by the same user in these methods. In this paper, based on matrix factorization methods, we propose a new approach to make recommendation with social information. Its novelty can be summarized as follows: (1) it shows how to add different weights on the social trust relationships among users based on the trustee's competence and trustworthiness; (2) it incorporates the similarity relationships among users as a complement into the social trust relationships to enhance the computation of user's neighborhood; (3) it can balance the influence of these two kinds of relationships based on user's individuality adaptively. Experiments on Epinions and Ciao datasets demonstrate that our approach outperforms the state-of-the-art algorithms in terms of mean absolute error and root mean square error, in particular for the users who rated a few items.

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1 Introduction

With the proliferation of social media, recommender system has become an indispensable technique for filtering and recommending information to avoid users inundated with information flood. Examples of successful applications of recommender systems include product recommendations at Amazon, movie recommendations at Netfilx, etc.

Most of these commercial systems are based on collaborative filtering (CF), which has been proven to be very promising and gained a tremendous amount of popularity since it was come up (Resnick et al. 1994; de Campos et al. 2010). There are two general forms of successful approaches in CF: *memory-based* and *model-based*. Memory-based methods are centered on computing the relationships between items or users by exploring the user–item rating matrix, which are known as user-based approaches (Jin et al. 2004) and item-based ones (Sarwar et al. 2001). In the model-based approaches, probabilistic methods are used to develop a model of a user based on the user's history or profile. Examples of model-based approaches include the latent factor model (Canny 2002) and the clustering model (Demir 2010).

Despite their remarkable performance, traditional CF recommender systems have some inherent weaknesses. First, they only utilize the user–item rating matrix for recommendations, however the rating matrices are usually extremely sparse in the real world. Furthermore, these approaches perform poorly for users who rated a few items(known as cold start users). Hence, the traditional CF recommender systems which purely mine the user–item rating matrix may give somewhat impractical suggestions.

Recently, the increasing popularity of open social media allows people to contact with their families, friends and colleagues online, which produces rich social relations such as friendships in Facebook,[1] trust relations in Epinions,[2] and following relations in Twitter.[3] Social relations provide an independent information source about users beyond rating information. Therefore, social network information might be an important element that recommender algorithms can take advantage of. To date, a variety of social network-based methods have been proposed (Jamali and Ester 2010; Ma et al. 2008, 2009; Massa and Avesani 2007). Most of these social recommendation methods are based on the matrix factorization (MF) framework, which is both effective and efficient in generating recommendations. For example, in Ma et al. (2009), the authors put forward a framework, which is a linear combination of basic MF and social network-based approach wherein one user's final decision is interpreted as the balance between his/her own taste and his/her trusted friend's favors; in Jamali and Ester (2010), a model-based approach SocialMF is developed by incorporating the mechanism of trust propagation into MF techniques.

As mentioned above, most existing successful social recommendation methods use trust relations and then recommend items to a user from his/her trusted users. Although these social recommendation methods have moved a nice step forward in the research of recommender systems, these methods have several problems. First, only few online recommender systems, like Epinions, have the implementation of trust mechanism and the trust values between users are binary. Based on existing binary trust network, these methods assume that different friends trusted by the same user affect the user in the same degree. However in the real world, people may build different strength relationships with different persons. Some relationships may be strong, just like intimate friends; some may be weak, such as acquaintances. So we cannot consider them equally. Secondly, when modeling the relations among users, most of these methods only consider the explicit social trust information. However, the trust relation is only one of many types of social relations. Relying only on trust friends is far from enough. So some other kind of relationships can be exploited, such as the similarity relationship in traditional CF methods. It has been shown that there is little overlap between a user's social network and his/her similar users (Crandall et al. 2008). Due to the fact that the rating matrix provides an independent source of information about online users, exploiting the similarity relationships computed based on the user–item rating matrix can potentially improve social recommendation performance.

Aiming at solving the problems mentioned above, on the basis of the previous work, we synthesize the social network information and rating information to design a novel social recommendation model that combines the factors of preference similarity and social trust relation together to increase the performance of social recommendation. The main contributions of our work can be summarized as follows:

- We model the trust relationships from two aspects: the trustee's competence and trustworthiness, to assign different trust values to different users who are trusted by the same user.
- We incorporate the similarity relationships among users as a complement into the social trust relationships to enhance the computation of user's neighborhood.
- We propose a novel recommendation approach TS_MF which fuses the trust and similarity relations with adaptive weights based on users' individualities.

We conduct a lot of experiments on two large data sets from Epinions.com and Ciao.com. The experiment results show that our proposed method can better utilize user's social information and outperforms the state-of-the-art algorithms in terms of mean absolute error (MAE) and root mean square error (RMSE), in particular for the users who rated a few items.

The remainder of this paper is organized as follows: Sect. 2 gives a review of the background knowledge of our work. Section 3 presents our novel social recommendation model. Section 4 describes the experiments and the performance of our proposed methods. Section 5 concludes our work.

2 Background

In a typical recommender system, there is a set of M users, $U = \{u_1, u_2, \ldots, u_M\}$, and a set of N items, $I = \{i_1, i_2, \ldots, i_N\}$. The rating of user u for item i is denoted by a real number $r_{u,i}$. All the ratings from users to items are denoted by a user–item rating matrix $R_{M\times N} = \{r_{u,i}\}$. For social recommender systems, each user also has a set N_u of direct neighbors and $t_{u,v}$ denotes the social trust value u has assigned on v. The trust network can be represented by a $M \times M$ matrix $T = \{t_{u,v}\}$. The task of a social recommender system is as follows: given a user $u \in U$ and an item $i \in I$, predict the missing rating for u on item i using R and T.

As mentioned above, MF is a widely adopted framework in generating recommendations. In this paper, we focus on MF-based social recommender systems. In this section, we first review the basic MF approach for recommendation using

[1] https://www.facebook.com.

[2] https://www.epinions.com.

[3] https://www.twitter.com.

only rating matrix. Then we introduce some state-of-the-art social recommendation methods.

2.1 Low-rank matrix factorization

An efficient and effective approach to recommender system is to factorize the user–item rating matrix into low-rank approximation based on low-dimensional hidden representations of users and items, then use them to predict the missing values in the rating matrix. Let $U \in \mathbf{R}^{D\times M}$ and $V \in \mathbf{R}^{D\times N}$ be latent user and item matrices with column vectors U_u and V_i representing the D-dimensional user-specific and item-specific latent vectors of user u and item i, respectively. The goal of MF is to learn these latent variables U and V. There are various ways to get the solutions of U and V, one of which is probabilistic matrix factorization (PMF) proposed in Salakhutdinov (2008). Several works based on PMF have proved its success. In view of its good performance, we employ PMF to factorize the user–item matrix in our work. In the PMF model (Salakhutdinov 2008), the author adopts probabilistic linear model with Gaussian observation noise and define the conditional distribution over the observed ratings as follow:

$$p(R|U, V, \sigma^2) = \prod_{r_{u,i}\in R} \mathcal{N}\left(r_{u,i}|U_u^T V_i, \sigma^2\right), \tag{1}$$

where $\mathcal{N}(x|\mu, \sigma^2)$ is a Gaussian distribution with mean μ and variance σ^2. The dot product of latent user and item vectors $U_u^T V_i$ is the expected mean of rating $r_{u,i}$. The zero-mean Gaussian distributions are also placed on user and item feature vectors:

$$\begin{aligned} p(U|\sigma_U^2) &= \prod_{u=1}^{M} \mathcal{N}(U_u|0, \sigma_U^2\mathbf{I}),\ p(V|\sigma_V^2) \\ &= \prod_{i=1}^{N} \mathcal{N}(V_i|0, \sigma_V^2\mathbf{I}) \end{aligned} \tag{2}$$

Now, through a Bayesian inference, the posterior distribution over the user and item latent vectors can be obtained as follows:

$$\begin{aligned} p(U, V|R, \sigma^2, \sigma_U^2, \sigma_V^2) &\propto p(R|U, V, \sigma^2)p(U|\sigma_U^2)p(V|\sigma_V^2) \\ &= \prod_{r_{u,i}\in R} \mathcal{N}(r_{u,i}|U_u^T V_i, \sigma^2) \\ &\times \prod_{u=1}^{M} \mathcal{N}(U_u|\sigma_U^2\mathbf{I}) \times \prod_{i=1}^{N} \mathcal{N}(V_i|\sigma_V^2\mathbf{I}) \end{aligned} \tag{3}$$

By maximizing the above posterior likelihood, we can learn the latent feature vectors of users and items. The rating of user u on item i, if not available, can be predicted by $U_u^T V_i$.

2.2 Recommendation with social network

In this part, we briefly introduce the SocialMF method (Jamali and Ester 2010), because it is known as the state-of-the-art framework in social recommendation. In socialMF, the mechanism of trust propagation is incorporated into the MF framework for recommendation in social networks. Based on the assumption that the behavior of a user u is affected by his direct neighbors N_u, the latent feature vector of u is dependent on the latent feature vectors of all his direct neighbors $v \in N_u$. It is formulated as follows:

$$\widehat{U}_u = \frac{\sum_{v\in N_u} T_{u,v}U_v}{\sum_{v\in N_u} T_{u,v}} = \frac{\sum_{v\in N_u} T_{u,v}U_v}{|N_u|}, \tag{4}$$

where $\widehat{U}_u$ is the estimated latent feature vector of u given the feature vectors of his direct neighbors. Since in binary social networks, all none-zero values of $T_{u,v}$ are 1, in this approach each row of trust matrix is normalized so that $\sum_{v=1}^{N} T_{u,v} = 1$. Then there are two factors for the user features:

$$\begin{aligned} &p(U|T, \sigma_U^2, \sigma_T^2) \propto p(U|\sigma_U^2) \times p(U|T, \sigma_T^2) \\ &= \prod_{u=1}^{M} \mathcal{N}(U_u|0, \sigma_U^2\mathbf{I}) \times \prod_{u=1}^{M} \mathcal{N}\left(U_u|\sum_{v\in N_u} T_{u,v}U_v, \sigma_T^2\mathbf{I}\right) \end{aligned} \tag{5}$$

Therefore, given the rating and social trust matrices, the posterior distribution of latent vectors can be obtained as follows:

$$\begin{aligned} &p(U, V|R, T, \sigma_R^2, \sigma_T^2, \sigma_U^2, \sigma_V^2) \\ &\propto p(R|U, V, \sigma_R^2)p(U|T, \sigma_U^2, \sigma_T^2)p(V|\sigma_V^2) \\ &= \prod_{r_{u,i}\in R} \mathcal{N}(r_{u,i}|U_u^T V_i, \sigma_R^2) \\ &\quad \times \prod_{u=1}^{M} \mathcal{N}\left(U_u|\sum_{v\in N_u} T_{u,v}U_v, \sigma_T^2\mathbf{I}\right) \times \prod_{u=1}^{M} \mathcal{N}(U_u|\sigma_U^2\mathbf{I}) \\ &\quad \times \prod_{i=1}^{N} \mathcal{N}(V_i|\sigma_V^2\mathbf{I}) \end{aligned} \tag{6}$$

In general, the existing social recommendation methods only consider social trust neighbors and roughly treat each neighbor equally. We will discuss these issues and propose our solution later.

3 Proposed approach

To solve the problems mentioned previously, we propose the following suggestions.

3.1 Modeling social trust relation

Trust is a frequently used concept in many disciplines. In peer-to-peer networks, mechanisms for trust and reputation are used to help peers distinguish good from bad partners to reduce the effect of malicious activities (Achim et al. 2011; Ghit et al. 2010; Visan et al. 2011; Wang and Vassileva 2003). In social sciences, the emphasis is on understanding how trust impacts the way people make decisions (Golbeck and Hendler 2006; Massa and Avesani 2007; Jamali and Ester 2010; Ma et al. 2008, 2009). In this paper, we concentrate on the role social networks play in how users trust each other in recommender systems. In particular, we concentrate on using trust relationships between users to improve the performance of social recommender systems.

In the existing social-based recommendation methods (Jamali and Ester 2010; Ma et al. 2008, 2009), the trust network of users is represented by a matrix $T \in \mathbf{R}^{M\times M}$, where M is the number of users. The directed social trust relationship of user u with user v is represented by a binary value: if u trusts v, the trust score $T_{u,v}$ is equal to 1 and 0 otherwise. However, in reality, we may trust different friends to different degrees. To alleviate this problem, we model the trust relation from following factors by a real number in [0, 1].

It is revealed that in relationships with other people, two distinct attributes play a role in trust evaluation: the competence and the trustworthiness of the trustee (Adali 2013). Based on this intuition, we evaluate social trust relation using these two dimensions and define them as follows:

- Competence: the extent to which the trustee is perceived as being capable of providing correct suggestions. It reflects opinions about the trustee's ability or expertise.
- Trustworthiness: the degree to which the trustee is considered credible as being a close friend of his/her direct neighbors.

The concrete computations of competence and trustworthiness are discussed below.

3.1.1 Competence

In our approach, if a trustee can always provide accurate predictions and exhibit high activity, we think s/he is a person with high competence. Based on this assumption, a trustee's recommendation competence is evaluated by her/his correct rate of product recommendations with activity weighting.

For a trustee v, the recommendation s/he made to his trustor u is represented by a triple $\langle u, v, i\rangle$ where i is the item rated by trustor u and trustee v. The set of recommendations made by v $\mathrm{RecSet}(v)$ to all his trustors in the trust network is defined as follows:

$$\mathrm{RecSet}(v) = \{\langle u, v, i\rangle | \forall u \in T, v \in N_u, r_{u,i} > 0, r_{v,i} > 0\} \tag{7}$$

In O'Donovan and Smyth (2005), it is assumed that a recommendation is correct only if the rating of the recommender user v on item i, $r_{v,i}$, is very close to the target user u's rating on i, $r_{u,i}$. We employ the equation used in it to define the correct value:

$$\mathrm{Correct}_{u,v,i} = \begin{cases} 0, & \text{if } |r_{u,i} - r_{v,i}| > \varepsilon \\ 1, & \text{if } |r_{u,i} - r_{v,i}| \le \varepsilon \end{cases} \tag{8}$$

where ε is the threshold represented by a small real number. The set of correct recommendations made by user v $\mathrm{CorSet}(v)$ can be defined as:

$$\mathrm{CorSet}(v) = \{\langle u, v, i\rangle \in \mathrm{RecSet}(v) |\ \mathrm{Correct}_{u,v,i} = 1\} \tag{9}$$

In order to give a higher value of competence to a user who does more rating activities, we introduce an activity weighting to the measure in previous work. Denote $|I_v|$ as the number of items rated by user v, and $\overline{\mathrm{avg}}$ as the average rating number of all users in the recommender system. For a trustee v, the activity weighting α_v is defined in this way:

$$\alpha_v = \begin{cases} \frac{|I_v|}{\overline{\mathrm{avg}}}, & \text{if } |I_v| < \overline{\mathrm{avg}} \\ 1, & \text{otherwise} \end{cases} \tag{10}$$

Now, a user's recommendation competence is calculated as:

$$\mathrm{Comp}(v) = \alpha_v \frac{|\mathrm{CorSet}(v)|}{|\mathrm{RecSet}(v)|} \tag{11}$$

3.1.2 Trustworthiness

Trustworthiness is defined as the degree to which the trustee is considered to be credible as being a close friend of his/her direct neighbors. We consider from the structure of the social trust network. In the academic field, the papers cited many times have relatively high quality. Based on this intuition, we think that the users trusted by many users in the trust network are more trustworthy than the users trusted by few ones. The value of trustworthiness the trustor u has on the trustee v is estimated as:

$$\mathrm{TW}(u, v) = \frac{\mathrm{Ind}(v)}{\sum_{w\in N_u} \mathrm{Ind}(w)}, \tag{12}$$

where Ind_v is the indegree of user v in social trust network and N_u is the set of users who are trusted by user u.

3.1.3 Final trust value

After we obtain the values of users' competence and trustworthiness, we use them to get the final trust values. In social trust network, the trust value assigned by the trustor u on the trustee v is:

$$T_{u,v} = \begin{cases} 0.5 * \mathrm{TW}(u,v), & \text{if } \mathrm{Comp}(v) = 0 \\ (1-\mathrm{Comp}(v))(1-\mathrm{TW}(u,v)), & \\ & \text{if } \mathrm{Comp}(v) > 0 \text{ and } \mathrm{TW}(u,v) > 0 \end{cases} \tag{13}$$

A higher value means more trust. If the user is considered credible but incompetent, the final trust value will be the value of his/her trustworthiness weakened by half. For users who add no users to their social list, we select some users who are competent in recommendation to be their neighbors and the trust values set on neighbors are half of the values of their competence.

3.2 Similarity based on rating information

The essence of most social-based methods (Jamali and Ester 2010; Ma et al. 2008, 2009) is to utilize users explicit social trust information to improve recommendation quality. They ignore some kind of relations that may exist between two users who have no explicit social connections with each other, just like the users' similarities in traditional CF. In this paper, we exploit the similarity relation information among users as the complement to users' social trust information. There are many techniques proposed to measure similarities among users, among which Pearson correlation coefficient (PCC) (Resnick et al. 1994) and Cosine similarity (Chowdhury 2010) are the most widely used ones. Since PCC can generally achieve higher performance than other approaches, we chose it to calculate the similarities among users. However, if the number of items that user u and user v have both rated is small, the similarity between them should be underestimated. Thus, we introduce a significance weight parameter β, as $n/5$ if $2 < n < 5$, where n is the number of common items the two users have both rated and at least 2; otherwise, let $\beta = 1$. So the metric equation of similarity between u and v could be expressed as:

$$S_{u,v} = \beta \frac{\sum_{i \in I_{u,v}} (r_{u,i} - \overline{r}_u)(r_{v,i} - \overline{r}_v)}{\sqrt{\sum_{i \in I_{u,v}} (r_{u,i} - \overline{r}_u)^2} \sqrt{\sum_{i \in I_{u,v}} (r_{v,i} - \overline{r}_v)^2}}, \tag{14}$$

where $I_{u,v}$ is the set of items rated by both u and v; $\overline{r}_u$ is the average rating of user u; $\overline{r}_v$ is the average rating of user v. From the equation, $S_{u,v}$ is in $[-1, 1]$ and the higher the value is, the more similar the two users are. We bound the range of PCC values into [0, 1] by using a sample mapping function $f(x) = (x+1)/2$. Then we normalize each row of the similarity matrix so that $\sum_{v \in B_u} S_{u,v} = 1$, where B_u is the set of users who are similar to user u, i.e., $B_u = \{v | v \in U, S_{u,v} > 0\}$.

3.3 Fusion of relation information with adaptive weights

As mentioned in Sect. 2, in socialMF, the feature vectors of users are only influenced by social trust direct neighbors. In our work, both social trust neighbors and similar neighbors are taken into consideration. Based on this intuition, the feature vector of a given user is formulated as:

$$\widehat{U}_u = \omega_t(u) \sum_{v \in N_u} T_{u,v} U_v + \omega_s(u) \sum_{k \in B_u} S_{u,k} U_k, \tag{15}$$

where N_u is the set of direct neighbors of user u in social trust network, and B_u is the set of similar neighbors of user u. From the equation, a user's feature is a weighted average of features of his/her social trust neighbors and similar neighbors, which is smoothed by ω_t and ω_s, where ω_t and ω_s are adaptive weights set based on the user's activity characteristics. Because we think that if the number of a user's social trust neighbors is larger than the number of his/her similar neighbors, it indicates that this user likes to add users to be his/her neighbors through the social trust network and prefers to seek suggestions from his/her social trust network rather than do more rating actions to form his/her similarity neighborhood, and vice versa. So for each given user u, we set $\omega_t(u) = |N_u|/(|N_u| + |S_u|)$ and $\omega_s(u) = |S_u|/(|N_u| + |S_u|)$, respectively.

Now we reformulate Eq. 5 in Sect. 2.2 as follows:

$$\begin{aligned} p(U|T, S, \sigma_U^2, \sigma_W^2) &\propto p(U|\sigma_U^2) \times p(U|T, S, \sigma_W^2) \\ &= \prod_{u=1}^{M} \mathcal{N}(U_u | 0, \sigma_U^2 \mathbf{I}) \\ &\quad \times \prod_{u=1}^{M} \mathcal{N}\left(U_u | \omega_t(u) \sum_{v \in N_u} T_{u,v} U_v \right. \\ &\quad \left. + \, \omega_s(u) \sum_{k \in B_u} S_{u,k} U_k, \sigma_W^2 \mathbf{I}\right) \end{aligned} \tag{16}$$

where σ_W^2 is a variance parameter to control the divergence of each user's feature vector from his/her neighbors. And accordingly, the posterior probability of the latent user and item factors can be redefined as:

$$\begin{aligned} &p(U, V | R, T, \sigma_R^2, \sigma_W^2, \sigma_U^2, \sigma_V^2) \\ &\propto p(R|U, V, \sigma_R^2) p(U|T, S, \sigma_U^2, \sigma_W^2) p(V|\sigma_V^2) \end{aligned}$$

$$= \prod_{r_{u,i} \in R} \mathcal{N}(r_{u,i}|U_u^T V_i, \sigma_R^2)$$
$$\times \prod_{u=1}^{M} \mathcal{N}\left(U_u|\omega_t(u) \sum_{v \in N_u} T_{u,v} U_v + \omega_s(u) \sum_{k \in B_u} S_{u,k} U_k, \sigma_W^2 \mathbf{I}\right)$$
$$\times \prod_{u=1}^{M} \mathcal{N}(U_u|\sigma_U^2 \mathbf{I}) \times \prod_{i=1}^{N} \mathcal{N}(V_i|\sigma_V^2 \mathbf{I}) \tag{17}$$

Thus, the maximum likelihood estimation of the parameters can be learned by minimizing the following loss function:

$$\mathcal{L}(R, U, V, T, S) = \frac{1}{2} \sum_{r_{u,i} \in R} (r_{u,i} - U_u^T V_i)^2$$
$$+ \frac{\lambda_U}{2} \sum_{u=1}^{M} \|U_u\|_F^2 + \frac{\lambda_V}{2} \sum_{i=1}^{N} ||V_i||_F^2$$
$$+ \frac{\lambda_W}{2} \sum_{u=1}^{M} \|U_u - \widehat{U}_u\|_F^2 \tag{18}$$

where $\lambda_U = \sigma_R^2/\sigma_U^2$, $\lambda_V = \sigma_R^2/\sigma_V^2$, $\lambda_W = \sigma_R^2/\sigma_W^2$. The parameters λ_U and λ_V control the strength of fitting on the training data and λ_W controls how much the user's neighbors influence his/her feature vectors.

We can find a local minimum of the above objective function by performing gradient descent on U_u and V_i for all users and all items.

$$\frac{\partial \mathcal{L}}{\partial U_u} = \sum_{r_{u,i} \in R} (r_{u,i} - U_u^T V_i) V_i + \lambda_U U_u + \lambda_W (U_u - \widehat{U}_u)$$
$$- \omega_t(u) \lambda_W \sum_{\{v|u \in N_v\}} T_{u,v} (U_v - \widehat{U}_v)$$
$$- \omega_s(u) \lambda_W \sum_{\{v|u \in B_v\}} S_{u,v} (U_v - \widehat{U}_v) \tag{19}$$
$$\frac{\partial \mathcal{L}}{\partial V_i} = \sum_{r_{u,i} \in R} (r_{u,i} - U_u^T V_i) U_u + \lambda_v V_i \tag{20}$$

To reduce the model complexity, we set $\lambda_U = \lambda_V$ in all our experiments. The initial values of U and V are samples from normal noised with zero mean. In each iteration, U and V are updated based on the latent variables from the previous iteration.

3.4 Complexity analysis

In our work, the degree of trust of target user u to each user in his/her trust network and the rating similarities among users are computed off-line, so the main cost of our recommendation method in TS_MF is computing the objective function $\mathcal{L}$ and its gradients against variables. The computation complexity of evaluating the objective function $\mathcal{L}$ is $\mathcal{O}(\rho_r l + \rho_t l + \rho_s l)$, where ρ_r, ρ_t and ρ_s are the number of nonzero entries in matrices R, T and S, respectively, and l is the dimension of latent feature vectors. Since in the actual recommender system we often have $\rho_r \gg \rho_t$ and $\rho_t \approx \rho_s$ and the matrices R, T and S are very sparse, the increase of the time complexity of our method is considered in an acceptable range (compared with socialMF). The computational complexities for gradients $\frac{\partial \mathcal{L}}{\partial U_u}$ and $\frac{\partial \mathcal{L}}{\partial V_i}$ are $\mathcal{O}(\rho_r l + \rho_t l + \rho_s l)$ and $\mathcal{O}(\rho_r l)$, respectively. Therefore, the total computational complexity for gradients is $\mathcal{O}(\rho_r l+\rho_t l+\rho_s l)$, which is linear with respect to the number of observations in matrices R, T and S.

4 Performance evaluation

We performed experiments on two real-world datasets to compare the recommendation qualities of different recommendation methods. In this section, we first introduce the two public datasets used in our experiments and then report our experimental results and compare the results with existing methods. We also present the results for different settings of model parameters.

4.1 Datasets

The type of trust relation we studied in this work is directed. So we chose two publicly product review datasets in our experiments, i.e., the Epinions and the Ciao[4] datasets. On both sites, people not only assign ratings on various products but also write reviews about products. Furthermore, people can evaluate other users by adding them to their trust networks or "Circle of Trust", if they find their reviews consistently interesting and helpful. Both sites employ a 5-star rating system and the social networks are directed. The statistics of the two datasets are listed in Table 1. We removed users who rate less than 5 times, and items with less than 5 rating scores. Then we left only the corresponding relation data of these users in the social relation data.

4.2 Experimental setup

We performed fivefold cross-validation in our experiments. In each fold, we used 80 % of data as the training set and the remaining 20 % as the test set.

The metrics we adopted to measure the prediction quality of all mentioned algorithms are the MAE and the RMSE. The metric MAE is defined as:

$$\text{MAE} = \frac{\sum_{(u,i) \in R_{\text{test}}} |r_{u,i} - \widehat{r}_{u,i}|}{|R_{\text{test}}|} \tag{21}$$

[4] http://www.public.asu.edu/jtang20/datasetcode/truststudy.htm.

Table 1 Statistics of the datasets

Datasets	Epinions	Ciao
Users	22166	7375
Items	296277	106797
Ratings	922267	284086
Social relations	300548	111781
Average ratings per user	41.61	38.52
Average ratings per item	3.11	2.66
Trust network density	0.0012	0.0041

where $r_{u,i}$ is the real rating value of user u on item i, $\widehat{r}_{u,i}$ is the corresponding predicted rating value, and R_{test} is the set of all pairs (u, i) in the test data. The RMSE is defined as:

$$\text{RMSE} = \sqrt{\frac{\sum_{(u,i)\in R_{\text{test}}}(r_{u,i} - \widehat{r}_{u,i})^2}{|R_{\text{test}}|}} \tag{22}$$

From the definitions, we can see that a smaller MAE or RMSE value means a better performance.

We compared the recommendation results of the following approaches in our work:

1. CF: this is the well-known user-based CF method which recommends items based on user similarity relationship. The similarity computation method used is PCC.
2. CF_inf: the similarity in it is calculated with a significance weight parameter as Eq. 11.
3. PMF: This is the baseline matrix factorization approach proposed in Salakhutdinov (2008). It only uses the user–item matrix for recommendation.
4. SocialMF: it is a model-based approach which is developed in Jamali and Ester (2010) by incorporating the mechanism of trust propagation into matrix factorization techniques.
5. SMF_Tw: it is the SocialMF using the weighted trust network modeled in Sect. 3.1.
6. TS_MF: it is our model proposed in Sect. 3.3.

In our experiments, we set $\lambda_U = \lambda_V = 0.01$.

4.3 Experimental result

Table 2 summarizes the results of all comparison methods on the Epinions and Ciao datasets. We conducted experiments

Table 2 Performance comparisons on two datasets

Datasets	Dimensionalities	Metrics	CF	CF_inf	PMF	SocialMF	SMF_Tw	TS_MF
Epinions	5	MAE	0.9372	0.9246	0.9112	0.8869	0.8615	**0.8467**
		Improve	9.66 %		7.08 %	4.53 %		
		RMSE	1.2438	1.2291	1.1532	1.1231	1.0944	**1.0811**
		Improve	13.08 %		6.25 %	3.74 %		
	10	MAE	0.9372	0.9246	0.9098	0.8842	0.8604	**0.8463**
		Improve	9.70 %		6.98 %	4.29 %		
		RMSE	1.2438	1.2291	1.1491	1.1196	1.0932	**1.0804**
		Improve	13.14 %		5.98 %	3.50 %		
	20	MAE	0.9372	0.9246	0.9164	0.8834	0.8588	**0.8461**
		Improve	9.72 %		7.67 %	4.22 %		
		RMSE	1.2438	1.2291	1.1581	1.1185	1.0917	**1.0806**
		Improve	13.12 %		6.69 %	3.39 %		
Ciao	5	MAE	0.8332	0.8052	0.8214	0.7831	0.7687	**0.7555**
		Improve	9.33 %		8.02 %	3.52 %		
		RMSE	1.13	1.1024	1.0483	1.0087	0.9936	**0.9819**
		Improve	13.10 %		6.33 %	2.66 %		
	10	MAE	0.8332	0.8052	0.8179	0.7849	0.7694	**0.7559**
		Improve	9.28 %		7.58 %	3.69 %		
		RMSE	1.13	1.1024	1.0438	1.01	0.9937	**0.9823**
		Improve	13.07 %		5.89 %	2.74 %		
	20	MAE	0.8332	0.8052	0.8196	0.7815	0.7671	**0.755**
		Improve	9.39 %		7.74 %	3.34 %		
		RMSE	1.13	1.1024	1.0455	1.006	0.9913	**0.9812**
		Improve	13.20 %		6.15 %	2.47 %		

Bold values highlight the best results of the experiments we conducted under different latent vector dimensions settings

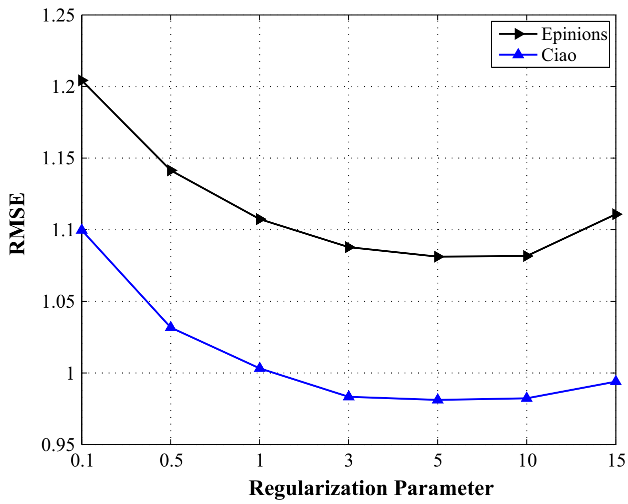

Fig. 1 Impact of different values of λ_W on two different datasets

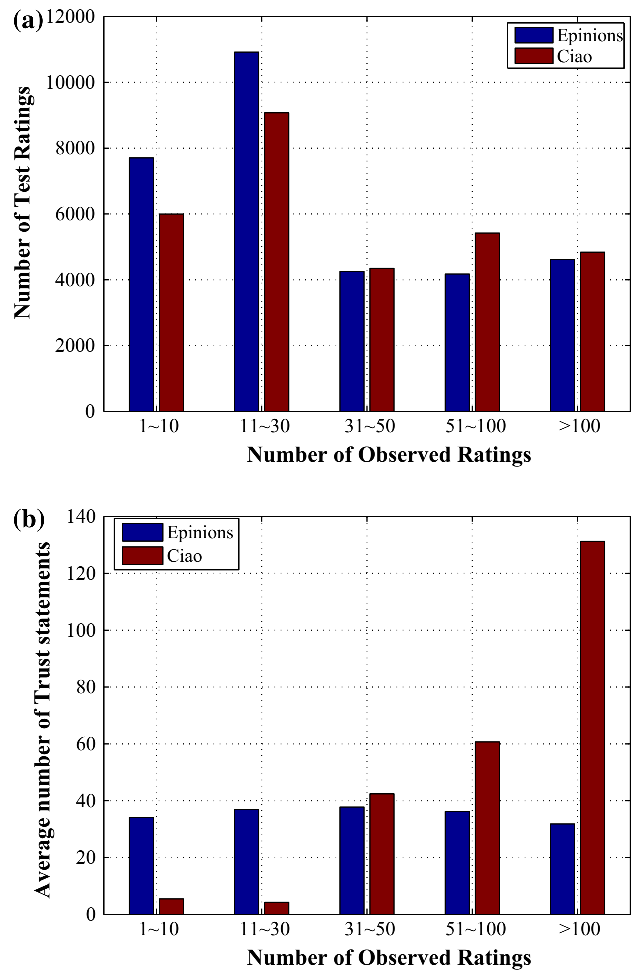

Fig. 2 Distributions of data in two different datasets. **a** Distribution of testing data in Epinions and Ciao. **b** Distribution of trust statements in Epinions and Ciao

on three latent vector dimensions: 5, 10, 20. The parameter λ_W was set to 5 for experiments on both datasets.

From the comparison, we can see that TS_MF outperforms other methods in all settings of both datasets. The percentages in Table 2 are the improvements of our proposed method TS_MF over the corresponding approaches. Our model TS_MF can generate better results than the state-of-the-art social recommendation method SocialMF. We can also find that all the model-based methods perform better than memory-based ones, which is consistent with previously published studies. And social recommendation approaches, such as STE, SocialMF and TS_MF, outperform PMF which only utilizes the user–item rating matrix, proving that social information is very effective in improving the performance of traditional recommendation algorithms once again. Besides, comparing the two memory-based methods, it shows that CF_inf has a better performance than traditional CF, meaning that introducing a significance weight parameter is helpful when dealing with two similar users who have rated only a few items in common. SocialMF_Tw shows a significant improvement over SocialMF, which indicates that modeling users' trust relation from trustworthiness and competence two dimensions is useful and a user who both has a higher competence in recommendation and more trustworthy should be more trusted than the other users. Among these four social recommendation methods, our TS_MF generates better results than the rest, which demonstrates that combining the trust relation with some other kind of relationships can help to improve the recommendation quality.

4.4 Impact of parameter λ_W

Similar to SocialMF, parameter λ_W plays an important role in our method. It controls the influence of social relationship on the behavior of users. The larger the value of λ_W is, the more impact of social relationship have on users' behavior. If we employ a very small value of λ_W, the social relationship information becomes negligible and only user–item rating matrix is mined for matrix factorization. On the other hand, if it is set to be very large, the social relationship information will dominate the learning process. In this part, we analyze how the change of λ_W can affect the final recommendation accuracy.

Figure 1 compares RMSE of our model with different range of values for λ_W on both datasets. The impacts of λ_W on both datasets share the same changing trend. We set λ_W to be $0.1, 0.5, 1, 3, 5, 10, 15$, respectively, with $D = 20$. It is easy to observe that the value of λ_W impacts the recommendation results significantly which demonstrates that incorporating social network information can help to improve the recommendation accuracy. From Fig. 1, we can find that no matter which dataset is used, the RMSE decreases as λ_W increases at first, but when λ_W surpasses a certain threshold, the RMSE increases, indicating that the prediction accuracy decreases. The existence of yielding point proves the intuition that only using the user–item rating matrix or only using the social network information for recommendations cannot generate better results than integrating these two sources appropriately. Clearly in Fig. 1, the best setting of λ_W on both data sets is $\lambda_W = 5$.

4.5 Performance on different users

In this part, we analyze how the size of training data per user affects the performance of different methods. We first classified all the users based on the number of observed ratings in the training set and then evaluated prediction accuracies of different user groups. Figures 2 and 3 show the results on different user groups on two datasets. Users were grouped in to 5 classes:"1–10", "11–30", "31–50", "51–100", "> 100", describing how many training ratings are available for a user in that class.

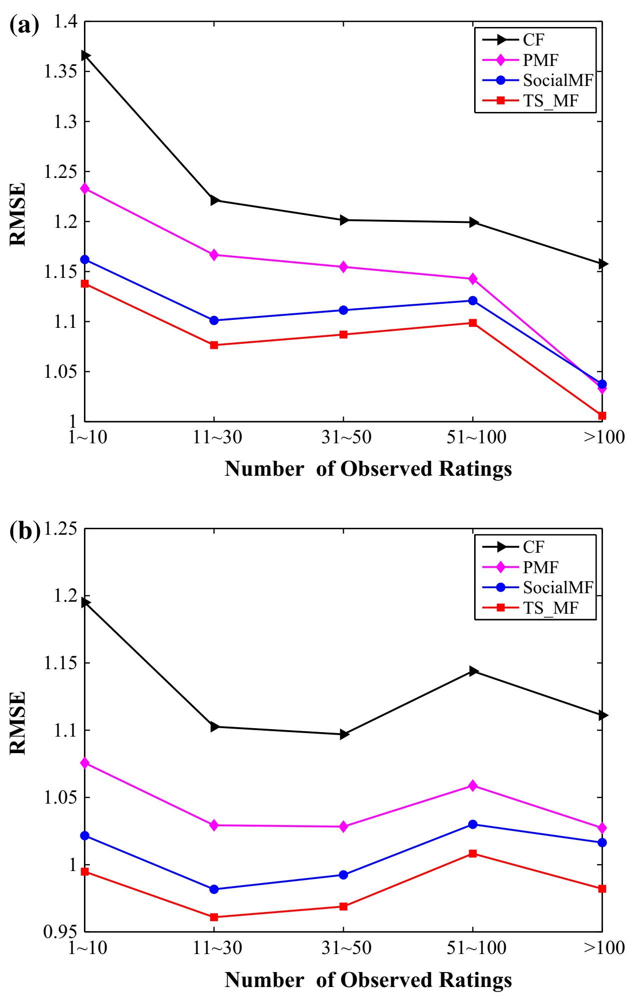

Fig. 3 RMSE performance comparisons on different users. **a** Epinions Dataset. **b** Ciao Dataset

Figure 2a shows the distribution of testing data in each class. As shown in the figure, in both data sets, there are more than 20 % of total number of user–item pairs to be predicted in which the related users have observed rating numbers from 1 to 10 in the training sets. Figure 2b is the distribution of social trust statements. From the figure, we can find that the distributions of social trusts of the two datasets are different. In Epinions, the users in different classes nearly have same number of trust neighbors, but in Ciao, the users who have rated few items also add few users to their trust network. Figure 3 shows the RMSE results of different methods on different user classes with $D = 20$. Given different size of training data and social network per user, our method TS_MF still outperforms other methods in both datasets, especially for users who have few observed ratings, owing to the combination of social trust relation and similarity relationship.

5 Conclusion

Social recommendation has attracted a lot of attention from both academic and industry. And many online social recommender system have been proposed in recent years. With the advent of online social networks, exploiting the useful information hidden in the social recommender system to improve the recommendation quality has becomes more and more important.

In this paper, we present a novel approach for social recommendation. First, based on the results of social cognition research, we model the binary trust relationship between two users and assign high values on trustees who have high recommendation competence and are much trustworthy. Then we use the similarity computation method in traditional memory-based CF to find similar neighbors by mining the rating actions. Lastly, we fuse these two kinds of relations based on users' own action characteristics to make predictions. Experiments on the public real-world datasets of Epinions and Ciao demonstrate that TS_MF outperforms the existing methods for social recommendation and can also handle the cold start users effectively.

This work suggests several interesting directions for future work. In this paper, the social relationship among users we considered is context independent. However, for trust relationship, in real-world people may trust some other people in some context while they may not trust these people in other contexts. We plan to extend our model TS_MF to be a context-based model for recommendations. Although we explicitly and implicitly utilize social relationships among users simultaneously, we ignore some other information existing in recommender systems, such as the correlations between the items. So incorporating item relationships is also a promising direction for future work. Besides, the conditional distributions of observed ratings in our work are defined based on Gaussian distributions following the models proposed previously, but there may be some other applicable distributions that can be used to produce some good results. Some related tests will be done in the future. Except the problem of predicting missing ratings, generating a recommendation list of items for a user is also a natural task for recommender systems. We plan to extend our model TS_MF to handle this task.

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