1.

a) There are two states, one when the princess is at tower A and one when she's at tower B. State Space:

$$X = \{A, B\}$$

The actions is to invade tower A (Ia), tower B (Ib) and observe the towers (O). Action Space:

$$A = \{Ia, Ib, O\}$$

The observations are to see the princess at tower A, see her at tower B or see nothing. Observation Space:

$$Z = \{A, B, N\}$$

For the following answers the number identifying the row and column, corresponds to the index from the list of the represating Space.

b) The transition probabilities for each action are:

Probability for Ia action:

Probability for Ib action:

Probability for O action:

	0	1
0	1	0
1	0	1

The observation probabilities for each action are:

Observation probability for Ia action:

	0	1	2
0	0	0	1
1	0	0	1

Observation probability for Ib action:

Observation probability for O action:

For the cost function we assume that when invading the tower where there's no princess, we give a maximum cost of 1, a small cost of 0.1 to peer and no cost if the knight invade the tower with the princess in it. Resulting in:

c) To compute the belief we used:

$$\boldsymbol{b}_{t+1} = \frac{\boldsymbol{b}_t \mathsf{P}_a \mathrm{diag}(\mathsf{O}_{a,z})}{\|\boldsymbol{b}_t \mathsf{P}_a \mathrm{diag}(\mathsf{O}_{a,z})\|_1}$$

Where bt is the belief that the princess is at tower A ([0.7, 0.3]), Pa is the transition probability of the action observe (O), and Oa,z is the selecting the column of observing the princess at tower B from the observation probability matrix for action observe (O).

Resulting in the new belief:

[0.20588235 0.79411765]

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