# Prova III Cálculo

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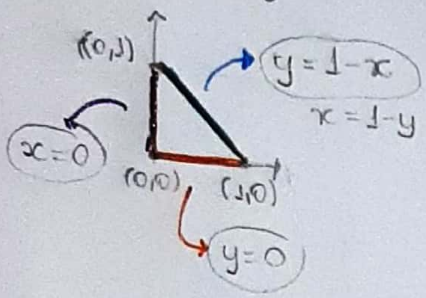
① Calcule:

a) $\iint_R x^2+xy+y^2\,dx\,dy$ onde R é o triangulo com vértices em (0,0); (0,1); (1,0)

esboçar região triangular:




integração em x:

$0 \le x \le 1$

$0 \le y \le 1-x$

então:

$$\int_0^1 \int_0^{1-x} x^2+xy+y^2 \, dy\, dx$$

$$\int_0^1 x^2y+\frac{y^2}{2}x+\frac{y^3}{3}\Big|_0^{1-x} dx$$

$$\int_0^1 x^2(1-x)+\frac{(1-x)^2x}{2}+\frac{(1-x)^3}{3}-0 \, dx$$

$$\int_0^1 x^2-x^3\,dx+\int_0^1 \frac{1-2x+x^2}{2}\cdot x\,dx+\frac{1}{3}\int_0^1 -x^3+3x^2-3x+1\,dx$$

$$\frac{x^3}{3}-\frac{x^4}{4}\Big|_0^1+\frac{1}{2}\int_0^1 x-2x^2+x^3\,dx+\frac{1}{3}\left(-\frac{x^4}{4}+\frac{\cancel{3}x^3}{\cancel{3}}-\frac{3x^2}{2}+x\Big|_0^1\right)$$

$$\frac{1^3}{3}-\frac{1^4}{4}-0+\frac{1}{2}\left(\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\Big|_0^1\right)+\frac{1}{3}\left(-\frac{1}{4}+1-\frac{3}{2}+1\right)$$

$$\frac{4-3}{12}+\frac{1}{2}\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)+\left(\frac{-1+4-6+4}{4}\right)\frac{1}{3}$$

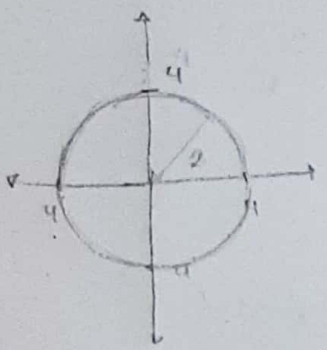
$$\frac{1}{12}+\frac{1}{2}\left(\frac{6-8+3}{12}\right)+\frac{1}{12}$$

$$\frac{1}{12}+\frac{1}{24}+\frac{1}{12}=\frac{4+1}{24}$$

$$=\boxed{\frac{5}{24}}$$

b) $\iint_R 25 - x^2 - y^2 \, dx\,dy$ onde $R = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 4\}$

esboçar região:




$f(x,y) = 25 - (x^2+y^2)$

$f(\theta, r) = 25 - r^2$

$0 \leq x+y^2 \leq 4$

$0 \leq r^2 \leq 4$

$0 \leq r \leq 2$

e

$0 \leq \theta \leq 2\pi$

então:

$$\iint_R f(x,y)\,dx\,dy = \iint_R f(\theta,r)\, r \, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^2 (25-r^2)\, r \, dr\,d\theta = \int_0^{2\pi}\left[\int_0^2 25r - r^3 \, dr\right] d\theta$$

$$\int_0^{2\pi} \frac{25r^2}{2} - \frac{r^4}{4}\Big|_0^2 \, d\theta = \int_0^{2\pi} \frac{25 \cdot 4}{2} - \frac{16}{4} - 0 \, d\theta$$

$$\int_0^{2\pi} 46 \, d\theta = 46\theta \Big|_0^{2\pi} = 46 \cdot 2\pi - 0 =$$

$\boxed{92\pi}$ //

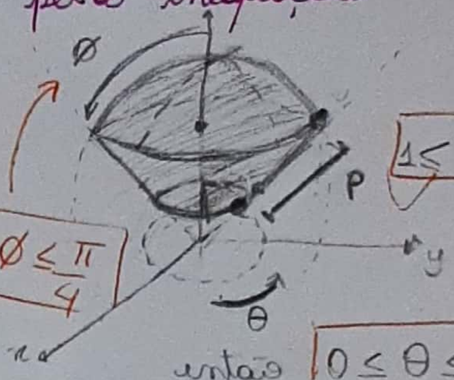
b)

c) O volume do sólido dados pelas inequações

1) $1 \leq x^2+y^2+z^2 \leq 9$ (esfera)

2) $z \geq \sqrt{x^2+y^2}$ (cone e)

3) $z \geq 0$ (e)




$\boxed{1 \leq p \leq 3}$

$\boxed{0 \leq \phi \leq \frac{\pi}{4}}$

então $\boxed{0 \leq \theta \leq 2\pi}$

1) $1 \leq x^2+y^2+z^2 \leq 9$

$1 \leq p^2 \leq 9$

$\boxed{1 \leq p \leq 3}$

2) $z \geq \sqrt{x^2+y^2}$

$p\cos\phi \geq \sqrt{p^2 sen^2\phi}$

$p\cos\phi \geq p\,sen\,\phi$

$\tan\phi \leq 1$

$\boxed{\phi \leq \frac{\pi}{4} \quad e \quad \phi \geq 0}$

3) $z \geq 0$

temos: 1) $1 \leq p \leq 3$

2) $0 \leq \theta \leq 2\pi$

3) $0 \leq \phi \leq \pi/4$

$$\int_0^{\pi/4}\int_0^{2\pi}\int_1^{3} p^2 sen\phi \; dp\, d\theta\, d\phi = \int_0^{\pi/4}\int_0^{2\pi} \frac{p^3}{3} sen\phi \Big|_1^3 \, d\theta\, d\phi =$$

$$\int_0^{\pi/4}\int_0^{2\pi} \frac{27}{3} sen\phi - \frac{1}{3} sen\phi \, d\theta\, d\phi = \int_0^{\pi/4}\int_0^{2\pi} \frac{26}{3} sen\phi \, d\theta\, d\phi$$

$$\int_0^{\pi/4} \frac{26}{3} sen\phi\, \theta \Big|_0^{2\pi} d\phi = \int_0^{\pi/4} \frac{26 \cdot 2\pi}{3} sen\phi = \frac{52\pi}{3} - \cos\phi \Big|_0^{\pi/4} =$$

$$\left(\frac{52\pi}{3}\right) \cdot \frac{-\sqrt{2}}{2} + 1 = \frac{52\pi}{3} \cdot \frac{-\sqrt{2}+2}{2} = \frac{-52\pi\sqrt{2} + 104\pi}{3 \cdot 2} = \boxed{\frac{52\pi - 26\sqrt{2}\pi}{3}}$$

② Converta a integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dx\,dy \qquad \frac{2}{5}$$

em uma integral equivalente em coordenadas cilíndricas e calcule-a.

$x = r\cos\theta$
$y = r\,\text{sen}\,\theta$
$z = z$
$x^2+y^2=r^2$
$y \neq 0 \Rightarrow \frac{y}{x} = \tan\theta$
$x \neq 0 \Rightarrow \frac{x}{y} = \cot\theta$

**para z:**

$z = x$

$\boxed{z = r\cos\theta}$

e

$\boxed{z = 0}$

**para x:**

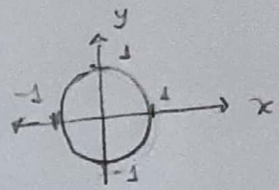
$x = \sqrt{1-y^2}$

$x^2 = 1-y^2$

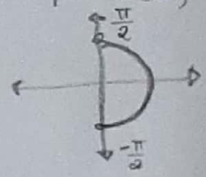
$x^2+y^2 = 1$

$r^2 = 1$

$\boxed{r = 1}$



porém, como $x = (+)\sqrt{1-y^2}$, temos a parte somente a parte positiva, então:



em que $\theta$ varia de $\frac{\pi}{2}$ a $-\frac{\pi}{2}$.

**para y:**

$y = 1$ e $y = -1$

o raio varia de 0 a 1.

logo, $\boxed{y = 0}$ $\boxed{y = 1}$

então,

$\boxed{\theta = \frac{\pi}{2}}$

$\boxed{\theta = -\frac{\pi}{2}}$

temos também que $x^2+y^2 = r^2$

Assim:

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{1}\int_{0}^{r\cos\theta} r^2\cdot r\,dz\,dr\,d\theta = \int_{-\pi/2}^{\pi/2}\int_{0}^{1}\int_{0}^{r\cos\theta} r^3\,dz\,dr\,d\theta$$

$$\int_{-\frac{\pi}{2}}^{\pi/2}\int_{0}^{1} r^3 z\Big|_{0}^{r\cos\theta} dr\,d\theta = \int_{-\pi/2}^{\pi/2}\int_{0}^{1} r^3 r\cos\theta - 0\,dr\,d\theta$$

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{1} r^4\cos\theta\,dr\,d\theta = \int_{-\pi/2}^{\pi/2}\frac{r^5}{5}\cos\theta\Big|_{0}^{1} d\theta = \int_{-\pi/2}^{\pi/2}\frac{\cos\theta}{5} - 0$$

$$\frac{1}{5}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{1}{5}\,\text{sen}\,\theta\Big|_{-\pi/2}^{\pi/2} = \frac{1}{5}\left(1-(-1)\right) = \frac{1}{5}\cdot 2 = \boxed{\frac{2}{5}}$$