1 (a) $\int_0^3 x^4 + 4\sqrt[3]{x} - \frac{4}{\sqrt{x}}\, dx$

$= \int_0^3 x^4 + 4x^{1/3} - 4x^{1/2}\, dx$

$= \int_0^3 x^4\, dx - \int_0^3 1x^{1/3}\, dx - \int_0^3 4x^{-1/2}\, dx$

$= \frac{x^5}{5}\Big|_0^3 + 4\left(\frac{3}{4} \cdot x^{4/3}\right)\Big|_0^3 - 4(2x^{1/2})\Big|_0^3$

$= \frac{3^5}{5} + 3(3)^{4/3} - 8(3)^{1/3}$

$= \boxed{\frac{243}{5} + 9\sqrt[3]{3} - 8\sqrt{3}}$

(b) $\int_0^{\pi/3} \sec^2 x \cdot e^{\tan x}\, dx$

$u = tg\, x$; $\frac{du}{\sec^2 x} = dx$

$du = \sec^2 x\, dx$

$= \int_0^{\pi/3} \sec^2 x \frac{e^u}{\sec^2 x}\, du$

$= \int_0^{\sqrt{3}} e^u\, du$

$= e^u\Big|_0^{\sqrt{3}} = e^{\sqrt{3}} - e^0 = e^{\sqrt{3}} - 1$

(c) $\int_0^3 x^2 \ln(x)\, dx$

$u = \ln(x)$; $v = x^3/3$

$du = \frac{1}{x}\, dx$; $dv = x^2$

$= \ln(x) \cdot \frac{x^3}{3}\Big|_0^3 - \int_0^3 \frac{x}{3} \cdot \frac{1}{3}\, dx$

$= \ln(x)\, \frac{x^3}{3}\Big|_0^3 - \int_0^3 \frac{x^2}{3}\, dx$

$= \left(\ln(x)\, \frac{x^3}{3} - \frac{x^3}{9}\right)\Big|_0^3$

$= \frac{x^3}{9}\left(3\ln(x) - 1\right)\Big|_0^3$

$= \frac{27}{9}(3\ln(3) - 1) - 0$

$\boxed{= +6{,}8875}$

(d) $\int_0^{\pi/3} tg\, x \cdot \frac{\ln(\sec^4 x)}{\sec^2 x}\, dx$

$= \int_0^{\pi/3} \frac{\operatorname{sen} x}{\cos x} \cos^2 x \ln(\sec^4 x)\, dx$

$= \int_0^{\pi/3} \operatorname{sen} x \cdot \cos x \ln(\sec^4 x)\, dx$

$u = \cos x$

$du = -\operatorname{sen} x\, dx$

$= \int_0^{\pi/3} -u \cdot \ln(u)^{-4}\, du$

$w = \ln(v)$; $v = \frac{u^2}{2}$

$dw = \frac{1}{u}\, du$

$dv = u \cdot du$

$= +4\left(\ln(u) \cdot \frac{u^2}{2} - \int_0^{\pi/3} \frac{u^2}{2} \cdot \frac{1}{u}\, du\right)$

$= +4\left(\frac{\ln(4) \cdot u^2}{2} - \frac{u^2}{4}\right)\Big|_0^{\pi/3}$

→

1d continuação:

$= + 2\ln(u) \cdot u^2 - u^2 \Big|_0^{\pi/3}$

$= u^2(2 \cdot \ln(u) - 1) \Big|_0^{\pi/3}$

$= \cos^2 x \, (2\ln(\cos x) - 1) \Big|_0^{\pi/3}$

$= \frac{1}{4}\left(2\ln\left(\frac{1}{2}\right) - 1\right) + 1$

$\boxed{= 0{,}903}$

2 ⓐ

1ª parte

$y_1 = y_2$

$5 - x = \frac{4}{x}$

$x^2 + 5x - 4 = 0$

$x^2 - 5x + 4 = 0$

$x' = 4$

$x'' = 1$

2ª parte

$= \int_1^4 \int_{y_2}^{y_1} dy\, dx$

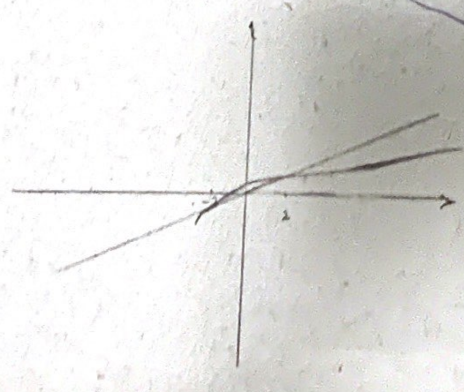
$= \int_1^4 5 - x - \frac{4}{x}\, dx$

$= \left(5x - \frac{x^2}{2} - 4\ln(x)\right)\Big|_1^4$

$= 5 \cdot 4 - \frac{4^2}{2} - 4\ln(4) - \left(5 - 1^2 - 4\ln(1)\right)$

$= 20 + 8 - 4\ln(4) - 4$

$\boxed{= 28 - 4\ln(4) - 4}$



2ⓑ $x - y^2 - 3y = 0$

$x = y^2 + 3y$

$x + \frac{9}{4} = y^2 + 3y + \frac{9}{4}$

$x + \frac{9}{4} = \left(y + \frac{3}{2}\right)^2$

$y_1 = \sqrt{x + 9/4} - \frac{3}{2}$

$y_2 = y_1$

$\frac{x}{2} = \sqrt{x + 9/4} - 3/2$

$\left(\frac{x}{2} + \frac{3}{2}\right)^2 = \left(\sqrt{x + 9/4}\right)^2$

$\frac{x^2}{4} + \frac{3x}{2} + \cancel{\frac{9}{4}} = x + \cancel{\frac{9}{4}}$

$\frac{x^2}{4} + \frac{x}{2} = 0$

$x^2 + 2x = 0$

$\Delta = 4$

$x_1 = \frac{-2 \pm 2}{2}$ → $x'' = -2$, $x' = 0$

$\int_{-2}^{0} \int_{y_2}^{y_1} dy\, dx$

$= \int_{-2}^{0} \sqrt{x + 9/4} - 3/2 - x/2 \; dx$

$= -\frac{3x}{2} - \frac{x^2}{4} + \int_{-2}^{0} \sqrt{x + 9/4}\, dx$

$u = x - 9/4$

$du = dx$

$\int u^{1/2} = \frac{2u^{3/2}}{3}$

$= \left(-\frac{3x}{2} - \frac{x^2}{4} + \frac{2(x + 9/4)^{3/2}}{3}\right)\Big|_{-2}^{0}$

$= \frac{9}{4} - 2 - \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$ //

2 c) $r_1 = 1 + \operatorname{sen}\theta$

$r_2 = 1 - \cos\theta$

$r_1 = r_2$

$1 + \operatorname{sen}\theta = 1 - \cos\theta$

$\operatorname{sen}\theta = -\cos\theta$

$\theta = 135° = 3\pi/4$

$\theta = 315° = 7\pi/4$

$\frac{1}{2}\int_{\theta_0}^{\theta_1} r^2 \, d\theta$

3) $c: y = x^2 - 2x + 2, \quad 1 \leq x \leq 3$

$x = 1 \therefore y = 1, \quad x = 3 \therefore y = 5 \quad y \in [1,5]$

$= \int_1^3 \pi f(x)^2 dx = \int_1^3 \pi (x^2 - 2x + 2)^2 dx$

$= \int_1^3 \pi (x^4 - 4x^3 + 8x^2 - 8x + 4) dx$

$= \pi \frac{x^5}{5} - x^4 + \frac{8x^3}{3} - 4x^2 + 4x) \Big|_1^3$

$= \pi (13,733)$

$y = x^2 - 2x + 2$

$y - 2 = x^2 - 2x$

$y - 1 = (x+1)^2$

$x = 1 + \sqrt{y-1},$

$\int_1^5 \pi f(y)^2 dy = \int_1^5 \pi (1 + \sqrt{y-1})^2 dy = \pi \int_1^5 1 + 2\sqrt{y-1} + y - \cancel{1} \, dy$

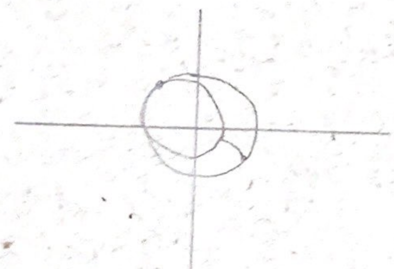
$= \pi \left(4/3 (y-1)^{3/2} + y^2/2 \right) \Big|_1^5 = \pi \left(\frac{32}{3} + \frac{24}{2} \right)$ //

$= \frac{1}{2}\left[\int_0^{3\pi/4} (1-\cos\theta)^2 d\theta + \int_{3\pi/4}^{7\pi/4} (1+\operatorname{sen}\theta)^2 d\theta + \int_{7\pi/4}^{2\pi} (1-\cos\theta)^2 d\theta \right]$

$= \frac{1}{2}\left[\theta + 2\operatorname{sen}\theta + \frac{\theta}{2} - \frac{\operatorname{sen}\theta\cos\theta}{2} \Big|_0^{3\pi/4} + \left(\theta - 2\cos\theta + \frac{\theta}{2} - \frac{\operatorname{sen}\theta\cos\theta}{2}\right) \Big|_{3\pi/4}^{7\pi/4} + \left(\theta + 2\operatorname{sen}\theta + \frac{\theta}{2} + \frac{\operatorname{sen}\theta\cos\theta}{2}\right) \Big|_{7\pi/4} \right.$

$= \frac{1}{2}\left[3\pi + 2\operatorname{sen}\left(\frac{3\pi}{4}\right) - 2\cos\left(\frac{7\pi}{4}\right) - 2\cos\left(\frac{3\pi}{4}\right) + 2\operatorname{sen}\left(\frac{7\pi}{4}\right) \right]$

$= \frac{3\pi}{2}$ //



4) $f(x) = \ln(\cos x); \quad f [0, \pi/4] = \mathbb{R}$

$C = \int_a^b \sqrt{1 + f'(x)^2} \, dx \qquad f'(x) = -\operatorname{sen} x \frac{1}{\cos x} = -\operatorname{tg} x$

$C = \int_0^{\pi/4} \sqrt{1 + [\sqrt{-\operatorname{tg} x}]^2} \, dx = \int_0^{\pi/4} \sqrt{1 + \frac{\operatorname{sen}^2 x}{\cos^2 x}} \, dx = \int_0^{\pi/4} \sqrt{\frac{\cos^2 x + \operatorname{sen}^2 x}{\cos^2 x}} \, dx$

$= \int_0^{\pi/4} \sqrt{\frac{1}{\cos^2 x}} \, dx = \int_0^{\pi/4} \frac{1}{\cos x} dx = \int_0^{\pi/4} \frac{\operatorname{tg} x \sec x + \sec^2 x \, dx}{\operatorname{tg} x + \sec x}$

$u = \operatorname{tg} x + \sec x$

$du = \sec^2 x + \operatorname{tg} x \sec x \, dx \longrightarrow$

(4) continuação.

$= \int \frac{1}{u}\, du = \ln |u|$

$= \ln(tg\,x + sec\,x) \Big|_0^{\pi/4}$

$= \ln\left(tg\,\pi/4 + sec\,\pi/4\right) - \ln(tg\,0 + sec\,0)$ //

(5) $f(x) = x^2 + 1 \ : \ [-1,2]$

$I_1: [-1,0] \to \Delta x_1 = \frac{1}{n} \to x_0 = -1, \ \vdots \ x_{n-1} = -1 + n\Delta x$

$I_2: [0,2] \to \Delta x_2 = \frac{2}{n} \to x_0 = 0, \ x_1 = \Delta x, \ \vdots \ x_n = n\Delta x$

$\bar{S} = \bar{S_1} + \bar{S_2} = \sum_{i=0}^{n-1} f(x_i)\,\Delta x_1 + \sum_{i=1}^{n} f(x_i)\,\Delta x_2$

$\bar{S} = (f(x_0) + \dots + f(x_{n-1}))\Delta x_1 + (f(x_1) + \dots + f(x_n))\,\Delta x_2$

$\bar{S} = f(-1) + \dots + f(n\Delta x_1 - 1)\,\Delta x_1 + (f(\Delta x_2) + \dots + f(n\Delta x_2))\,\Delta x_2$

$\bar{S} = 2 + \dots + (n\Delta x_1 - 1)^2 + 1)\,\Delta x_1 + (\Delta x_2^2 + 1 + \dots + n^2\Delta x_2^2 + 1)\,\Delta x_2$

$\bar{S} = 2n + \frac{1}{n^2}\left(\frac{(n-1)n(2n-1)}{6}\right) - \frac{2}{n}\left(\frac{(n-1)n}{2}\right)\frac{1}{n} + \left(n + \frac{4}{n^2}\left(\frac{(n+1)n(2n+1)}{6}\right)\right)\frac{2}{n}$

$\bar{S} = \left(2n + \left(\frac{2n^3 - 3n^2 + n}{6n^2}\right) - n + 1\right)\frac{1}{n} + \left(n + \frac{2}{3}\left(\frac{2n^3 + 3n^2 + n}{n^2}\right)\right)\frac{2}{n}$

$\bar{S} = \left(n + 1 + \frac{n}{3} - \frac{1}{2} + \frac{1}{6n}\right)\frac{1}{n} + \left(n + \frac{2}{3}\left(2n + 3 + \frac{1}{n}\right)\right)\frac{2}{n}$

$\bar{S} = \left(\frac{4n}{3} + \frac{1}{2} + \frac{1}{6n}\right)\frac{1}{n} + \left(n + \frac{4n}{3} + 2 + \frac{2}{3n}\right)\frac{2}{n}$

$\bar{S} = \left(\frac{18}{3} + \frac{9}{2n} + \frac{9}{6n^2}\right)$

$\lim_{n \to \infty} \bar{S} = \frac{18}{3} + 0 + 0 = 6$