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Prova 02 - CDI II

1 (a) $u_{n+1} = \dfrac{1}{(n+1)\ln(n+1)} \leq u_n = \dfrac{1}{n\ln(n)}$ $\quad u_n > 0$, $n \in (2, \infty)$

$$1 \leq \frac{(n+1)\ln(n+1)}{n\ln(n)}$$

$0 < n\ln(n) \leq (n+1)\ln(n+1) \rightarrow$ ok!

$n < n+1 \quad \ln(n) < \ln(n+1) \rightarrow$ pois $n \geq 2$

→ usando o critério da integral

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)} \qquad f(x) = \frac{1}{x\ln(x)}$$

$$\int_2^{\infty} \frac{1}{x\ln(x)}\,dx \quad \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array} \right.$$

$$\int_2^{\infty} \frac{1}{u}\,du = \ln u \Big|_2^{\infty} = \lim_{b \to \infty} \ln(b) - \ln(2) = \infty$$

→ integral diverge então a série diverge!

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n - 1}{n^n} < \sum_{n=1}^{\infty} \frac{2^n}{n^n}$

→ verificando a convergência

$\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^n}$ → critério da raiz

$$\lim_{n \to \infty} \sqrt[n]{\left(\frac{2}{n}\right)^n} = \lim_{n \to \infty} \frac{2}{n} = 0 \rightarrow L < 1$$

logo $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^n}$ converge e $\displaystyle\sum_{n=1}^{\infty} \frac{2^n - 1}{n^n}$ também converge por comparação!

2 ⓐ $\sum_{n=0}^{\infty} \frac{2\,2^n}{7^n} - \frac{5^n}{8^n} = 2\sum_{n=1}^{\infty}\left(\frac{2}{7}\right)^{n-1} - \sum_{n=1}^{\infty}\left(\frac{5}{8}\right)^{n-1}$

→ as séries são geométricas, sendo:

$q_1 = \frac{2}{7} < 1$ → série convergente $S_1 = \frac{-a_1}{(q_1-1)}$

$q_2 = \frac{5}{8} < 1$ → série convergente $S_2 = -\frac{a_2}{(q_2-1)}$

→ a soma das séries $\sum_{n=0}^{\infty} \frac{2\,2^n}{7^n} - \frac{5^n}{8^n} = S$

$S = S_1 + S_2 = \frac{-2}{2/7 - 1} - \frac{1}{5/8 - 1} = \frac{-2.7}{2-7} - \frac{8}{5-8} = \frac{14}{5} + \frac{8}{3} = \frac{42+40}{15} = \frac{82}{15}$,

ⓑ $\sum_{n=0}^{\infty} \frac{(-1)^n(\pi)^{2n+1}}{(2n+1)!}$ verificando se $\sum_{n=0}^{\infty}\left|\frac{(-1)^n(\pi)^{2n+1}}{(2n+1)!}\right|$ é convergente, logo.

$\sum_{n=0}^{\infty}\left|\frac{(\pi)^{2n+1}(-1)^n}{(2n+1)!}\right| = \sum_{n=0}^{\infty}\frac{(\pi)^{2n+1}}{(2n+1)!}$

→ usando o critério da razão, pois $u_n > 0$

$u_{n+1} = \frac{(\pi)^{2(n+1)+1}}{(2(n+1)+1)!} = \frac{(\pi)^{2n+3}}{(2(n+1)+1)!}$

$u_n = \frac{(\pi)^{2n+1}}{(2n+1)!}$

$\lim_{n\to\infty}\frac{u_{n+1}}{u_n} = \lim_{n\to\infty}\frac{(\pi)^{2n+3}}{(2(n+1)+1)!}\cdot\frac{(2n+1)!}{(\pi)^{2n+1}} = \lim_{n\to\infty}(\pi)^{2n+3-2n-1}\cdot\frac{\cancel{(2n+1)!}}{(2(n+1)+1)\cancel{(2n+1)!}}$

$\lim_{n\to\infty}\frac{\pi^2}{2n+3} = 0 = L < 1$ → convergente!

→ logo se o módulo da série converge, a série também é convergente!

3 $\sum_{n=1}^{\infty} \frac{2^n x^n}{n5^n}$ $c_n = \frac{2^n}{n5^n}$

→ verificando se o limite existe

$$\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n\to\infty} \left(\frac{2}{5}\right)^{n+1}_{(n+1)} |x^{n+1}| \cdot \frac{n}{x^n} \left(\frac{5}{2}\right)^n$$

$$\lim_{n\to\infty} \left(\frac{2}{5}\right)^{n+1-n} \frac{n}{n+1} |x^{n+1-n}| = \lim_{n\to\infty} \frac{2}{5} \frac{n}{(n+1)} |x| \quad \frac{2}{5}|x|$$

$$\lim_{x\to\infty} \frac{n}{n+1} \Rightarrow \frac{2}{5} |x| \lim_{n\to\infty} \frac{1}{1} = |x| \frac{2}{5}$$

$|x| \in \mathbb{R}$, logo: $L = \frac{2}{5}$ e o raio da convergência é

$$R = \frac{1}{L} = \frac{5}{2}$$

o intervalo da convergência

$$|x| L < 1$$

$$|x| < \frac{5}{2} \rightarrow -\frac{5}{2} < x < \frac{5}{2} \qquad x \in (-5/2, 5/2)$$

4 a) $f(x) = \frac{x}{8 - x^3}$ $-2 < x < 2$

→ utilizando série de Maclaurin

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(0) x^n}{n!}$$

$f^0(0) = 0$

$$f'(0) = \frac{(8 - x^3) - x(-3x^2)}{(8 - x^1)^2} = \frac{8 + 2x^1}{(8 - x^1)^2} = \frac{8}{8^2} = \frac{1}{8}$$

$$f''(0) = \frac{6x^2 (8 - x^1)^8 - (8 + 2x^3) . 2(8x^8) - 3x^2}{(8 - x^3)^4} = \frac{18x^2 - 6x^5 + 18x^2 + 2 \cdot 6x^5}{(8 - x^3)^3} = \frac{96x^2 + 2x^5}{(8 - x^3)^3}$$

$f^n(0) = 0$

$$f'''(0) = \frac{(192x + 10x^4)(8-x^3)^3 - (96x^2+2x^5)\,3(8-x^3)^2(3x^2)}{(8-x^3)^6}$$

$$f'''(0) = \frac{1536x + 80x^4 - 192x^4 - 10x^7 + 864x^4 + 18x^7}{(8-x^3)^4} = \frac{1536x + 752x^4 + 8x^7}{(8-x^3)^4} = \frac{A}{(8-x^3)^4} = 0$$

$$f^{4}(0) = \frac{(1536 + 752 \cdot 4x^3 + 56x^6)(8-x^3)^4 - 4(8-x^3)^3 \cdot \frac{dA}{dx} \cdot 3x^2}{(8-x^3)^8} = \frac{1536 \cdot 8}{8^3} = \frac{1536}{8^4} = \frac{3}{8}"$$

logo a cada derivada o resultado é não nula da forma $\frac{(2i-1)}{8}$