CDI II - ÚLTIMA AVALIAÇÃO 13/04/2021

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Turma da noite

Exercícios - Aula 28

1. Calcule:

a) $\iint_R 2xy + x^2 \, dx\,dy$ onde R é o retângulo $[2,5] \times [0,2]$

$$\iint_R 2xy + x^2 \, dx\,dy = \int_2^5 \int_0^2 (2xy + x^2) \, dx\,dy =$$

$$= \int_2^5 \left[xy^2 + x^2 y \right]_0^2 dx = \int_2^5 (2x^2 + 4x) \, dx =$$

$$= \left[\frac{2x^3}{3} + 2x^2 \right]_2^5 = 120 \quad //$$

b) $\iint_R 1 \, dx\,dy$ onde R é a região delimitada pelo eixo x e a parábola $y = 4 - x^2$

* Pontos de intersecção:

$$4 - x^2 = 0$$
$$(x-2)(x+2) = 0$$
$$x = \pm 2$$

logo

$$\iint_R 1 \, dx\,dy = \int_{-2}^{2} \int^{4-x^2} 1 \, dx\,dy = \int_{-2}^{2} (4 - x^2) \, dx = \left[4x - \frac{x^3}{3} \right]_{-2}^{2} = \frac{32}{3} \quad //$$

c) $\iint_R xy\,dx\,dy$ onde $R = \{(x,y) \in \mathbb{R}^2;\ x^2+y^2 \leq 2,\ y \leq x$ e $x \geq 0\}$

Usando coordenadas polares

$$x = r\cos\theta$$
$$y = r\,\text{sen}\,\theta$$

Substituindo temos: $R' = \{(r,\theta): 0 \leq r \leq \sqrt{2},\ -\pi < \theta < \frac{\pi}{4}\}$

logo

$$\iint_R xy\,dx\,dy = \int_0^{\sqrt{2}} \int_{-\pi}^{\frac{\pi}{4}} (r^2\,\text{sen}\theta\cos\theta)\,r\,d\theta\,dr =$$

$$= \int_0^{\sqrt{2}} r^3 \int_{-\pi}^{\frac{\pi}{4}} \frac{\text{sen}\,2\theta}{2}\,d\theta\,dr = \int_0^{\sqrt{2}} \frac{r^3}{4}\left(\cos 2\pi - \cos\frac{\pi}{2}\right)dr =$$

$$= \int_0^{\sqrt{2}} \frac{r^3}{4}\,dr = \left[\frac{r^4}{16}\right]_0^{\sqrt{2}} = \frac{1}{4} \quad //$$

2. Calcule o volume do sólido cuja base é o triângulo no plano xoy com vértices nos pontos $(0,0)$; $(1,1)$ e $(2,0)$ e altura dada pela função $f(x,y) = x^2 + y$

Considerando a região $R = \{(x,y): 0 \leq y \leq 1,\ y \leq x \leq 2-y\}$

$$V = \iiint_S dV = \int_0^1 \int_y^{2-y} (x^2+y)\,dx\,dy = \int_0^1 \left[\frac{x^3}{3} + xy\right]_y^{2-y} dy$$

$$V = \int_0^1 \left(-\frac{2y^3}{3} - 2y + \frac{8}{3}\right)dy = \left[-\frac{y^4}{6} - y^2 + \frac{8y}{3}\right]_0^1 = \frac{3}{2} \quad //$$

Exercícios - Aula 29

1. Calcule as seguintes integrais em coordenadas polares:

a) $\iint_R y - 2x \, dx dy$ onde $R = \{(x,y); x^2 + y^2 \leq 3 \text{ e } x \geq 0\}$

Considerando $x = r\cos\theta$ e $y = r \operatorname{sen}\theta$

logo $x^2 + y^2 = r^2 \leq 3 \to x \geq 0 \to -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

logo $dxdy = r\,drd\theta$ e $R' \{(r,\theta): 0 \leq r \leq \sqrt{3}, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\}$

então

$$\iint_R y - 2x \, dxdy = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\sqrt{3}} (r\operatorname{sen}\theta - 2r\cos\theta)(r\,dr\,d\theta) =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\sqrt{3}} r^2(\operatorname{sen}\theta - 2\cos\theta)\,dr\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\operatorname{sen}\theta - 2\cos\theta)\left(\int_0^{\sqrt{3}} r^2 dr\right) d\theta$$

$$= \left[\frac{r^3}{3}\right]_0^{\sqrt{3}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\operatorname{sen}\theta - 2\cos\theta)\,d\theta = \sqrt{3}\left[-\cos\theta - 2\operatorname{sen}\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$= \sqrt{3}(-2-2) = -4\sqrt{3} \quad //$$

b) $\iint_R e^{\sqrt{x^2+y^2}}\, dx\, dy$ onde $R = \{(x,y);\ 4 \leq x^2+y^2 \leq 16\}$

Considerando $x = r\cos\theta$ e $y = r\,\text{sen}\,\theta$

logo $4 \leq x^2+y^2 = r^2 \leq 16 \to 2 \leq r \leq 4 \to 0 \leq \theta \leq 2\pi$

logo $dx\,dy = r\,dr\,d\theta$ e $R' = \{(r,\theta): 2 \leq r \leq 4,\ 0 \leq \theta \leq 2\pi\}$

Assim

$$\iint_R e^{\sqrt{x^2+y^2}}\, dx\,dy = \int_0^{2\pi}\int_2^4 e^{\sqrt{r^2}}\,(r\,dr\,d\theta) =$$

$$= \int_0^{2\pi}\left(\int_2^4 r e^r\, dr\right) d\theta = \left[(r-1).e^r\right]_2^4 \int_0^{2\pi} d\theta =$$

$$= (3e^4 - e^2)(2\pi - 0) = 2\pi e^2(3e^2 - 1) \quad //$$

c) $\iint_R \sin(x^2+y^2)\,dx\,dy$ onde R é a região no primeiro quadrante delimitado pelas curvas $y=\sqrt{3}x$; $x=\sqrt{3}y$ e $x^2+y^2=9$

Considerando $x = r\cos\theta$
$y = r\,\text{sen}\,\theta$

assim

$$x^2+y^2=r^2 \leq 9 \rightarrow 0 \leq r \leq 3$$

$$tg\,\theta = \sqrt{3},\ tg\,\theta\ \frac{1}{\sqrt{3}} \rightarrow \frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$$

logo $dx\,dy = r\,dr\,d\theta$ e $R' = \{(r,\theta): 0 \leq r \leq 3, \frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}\}$

Portanto

$$\iint_R \sin(x^2+y^2)\,dx\,dy = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\int_0^3 \sin(r^2)(r\,dr\,d\theta) =$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\int_0^3 r\sin(r^2)\,dr\right)d\theta = \left(-\frac{1}{2}\cos(r^2)\right)_0^3 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d\theta =$$

$$= \frac{(1-\cos(9))}{2}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{\pi}{12} - \frac{\pi}{12}\cos(9)$$

//

2. Reescreva as integrais abaixo em coordenadas polares e calcule-as:

a) $\displaystyle\int_0^1 \int_y^{\sqrt{2-y^2}} x + y \, dx\, dy$

A região é $R = \{(x,y): 0 \leq y \leq 1,\ y \leq x \leq \sqrt{2-y^2}\}$

$R' = \{(r,\theta): 0 \leq r \leq \sqrt{2},\ 0 \leq \theta \leq \frac{\pi}{4}\}$

Considerando $x = r\cos\theta$ e $y = r\,\text{sen}\,\theta$ temos:

$$\int_0^1 \int_y^{\sqrt{2-y^2}} x + y \, dx\, dy = \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2}} (r\cos\theta + r\,\text{sen}\,\theta)(r\,dr\,d\theta) =$$

$$= \int_0^{\frac{\pi}{4}} (\cos\theta + \text{sen}\,\theta)\left(\int_0^{\sqrt{2}} r^2\, dr\right) d\theta =$$

$$= \left[\frac{r^3}{3}\right]_0^{\sqrt{2}} \int_0^{\frac{\pi}{4}} (\cos\theta + \text{sen}\,\theta)\, d\theta = \frac{2}{3}\sqrt{2}\,(\text{sen}\,\theta - \cos\theta)\Big|_0^{\frac{\pi}{4}} =$$

$$= \frac{2}{3}\sqrt{2} \quad //$$

b) $\displaystyle\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

A região é $R=\{(x,y): 0\leq x\leq 2,\ 0\leq y\leq \sqrt{2x-x^2}\}$
$R'=\{(r,\theta): 0\leq r\leq 2,\ 0\leq \theta\leq \arccos\left(\frac{r}{2}\right)\}$

Considerando $x=r\cos\theta$ e $y=r\operatorname{sen}\theta$ temos:

$$\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dx\, dy = \int_0^2\int_0^{\arccos\left(\frac{r}{2}\right)} r(r\, d\theta\, dr) =$$

$$= \int_0^2 r^2\left(\int_0^{\arccos\left(\frac{r}{2}\right)} d\theta\right) dr = \int_0^2 r^2 \arccos\left(\frac{r}{2}\right) dr =$$

$$= \left[\frac{r^3}{3}\arccos\left(\frac{r}{2}\right) - \frac{r^2\sqrt{4-r^2}}{9} - \frac{8\sqrt{4-r^2}}{9}\right]_0^2$$

$$= 0-\left(-\frac{16}{9}\right) = \frac{16}{9} \quad //$$

3. Utilize a integral dupla para calcular a área das regiões R dadas abaixo:

a) R é a região delimitada pela parábola $y=x^2$ e a reta $y=4x$;
Pontos de intersecção: $x^2=4x \rightarrow x^2-4x=0 \rightarrow x=0$
$x=4$
$R=\{(x,y): 0\leq x\leq 4,\ x^2\leq y\leq 4x\}$

$$A=\iint_R dR = \int_0^4\int_{x^2}^{4x} dy\, dx = \int_0^4 (4x-x^2)\, dx = \left[2x^2-\frac{x^3}{3}\right]_0^4$$

$$= 32-\frac{64}{3} = \frac{32}{3} \quad //$$

b) $R = \{(x,y);\ r_1 \leq x^2 + y^2 \leq r_2 \text{ onde } 0 < r_1 < r_2\}$

Com as coordenadas polares $x = r\cos\theta$

$y = r\,\text{sen}\,\theta$

Considerando a região

$R = \{(x,y): r_1 \leq x^2 + y^2 \leq r_2,\ \text{onde } 0 < r_1 < r_2\}$

$R' = \{(x,y): \sqrt{r_1} < r < \sqrt{r_2},\ 0 \leq \theta \leq 2\pi\}$

Assim $A = \iint_R dR = \int_0^{2\pi}\int_{\sqrt{r_1}}^{\sqrt{r_2}} r\,dr\,d\theta =$

$$= \int_0^{2\pi} \left[\frac{r^2}{2}\right]_{\sqrt{r_1}}^{\sqrt{r_2}} dx = \int_0^{2\pi} \frac{(r_2 - r_1)}{2}\, dx =$$

$$A = \frac{r_2 - r_1}{2}(2\pi - 0) = (r^2 - r_1)\pi \quad //$$

Exercícios - Aula 30

1. Calcule:

a) $\iiint_S xyz\,dx\,dy\,dz$ onde S é o paralelepípedo $[0,1] \times [1,3] \times [0,2]$

$\Downarrow$

$$\int_0^1\int_1^3\int_0^2 xyz\,dx\,dy\,dz = \int_0^1\int_1^3 \left[\frac{xyz^2}{2}\right]_0^2 dy\,dx =$$

$$= \int_0^1\int_1^3 2xy\,dy\,dx = 2\int_0^1 \left[\frac{xy^2}{2}\right]_1^3 dx =$$

$$= \int_0^1 2\left(\frac{9x - x}{2}\right)dx = 8\int_0^1 x\,dx = 8\left[\frac{x^2}{2}\right]_0^1 = 4 \quad //$$

b) $\iiint_S x^2+y^2 \, dx dy dz$ onde $S = \{(x,y) \in \mathbb{R}^3;\ x^2+y^2 \leq 4$
e $0 \leq z \leq 2\}$

Considerando $x = r\cos\theta$, $y = r\,\text{sen}\,\theta$ e $z = z$

Assim $S' = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq r \leq 2$ e
$0 \leq \theta \leq 2\pi$ e $0 \leq z \leq 2\}$

$$\iiint_S x^2+y^2 \, dx dy dz = \int_0^{2\pi}\int_0^{2}\int_0^{2} (r^2)(r\,dr)\,dz\,d\theta =$$

$$= \int_0^{2\pi}\int_0^{2}\int_0^{2} r^3 \, dr\,dz\,d\theta = \int_0^{2\pi}\int_0^{2} \left[\frac{r^4}{4}\right]_0^2 dz\,d\theta =$$

$$= \int_0^{2\pi}\int_0^{2} 4\,dz\,d\theta = 4\int_0^{2\pi} (2-0)\,d\theta = 8\int_0^{2\pi} d\theta =$$

$$= 8(2\pi - 0) = 16\pi \quad //$$

c) O volume do sólido S delimitado pelos paraboloides $z = x^2 + y^2$ e $z = 18 - x^2 - y^2$

O sólido é delimitado por $x^2+y^2 \leq z \leq 18-x^2-y^2$

Pontos de intersecção

$$x^2+y^2 = 18-x^2-y^2$$
$$2(x^2+y^2) = 18$$
$$x^2+y^2 = 9$$

Considerando

$$x = r\cos\theta,\ y = r\,\text{sen}\,\theta \text{ e } z = z$$

Assim

$$S = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq r \leq 3 \text{ e } 0 \leq \theta \leq 2\pi \text{ e } r^2 \leq z \leq 18 - r^2\}$$

$$V = \iiint_S dV = \int_0^{2\pi}\int_0^{3}\int_{r^2}^{18-r^2} r\,dz\,dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^{3} r(18-r^2-r^2)\,dr\,d\theta = \int_0^{2\pi}\int_0^{3} 2(9r-r^3)\,dr\,d\theta =$$

$$= 2\int_0^{2\pi}\left[\frac{9r^2}{2}-\frac{r^4}{4}\right]_0^3 d\theta = 2\int_0^{2\pi}\frac{81}{4}\,d\theta =$$

$$= \frac{81}{2}\int_0^{2\pi} d\theta = \frac{81}{2}(2\pi - 0) = 81\pi \quad //$$

d) O volume do sólido S delimitado pelas superfícies $x^2 + z^2 = 4$; $y = -1$ e $y + z = 4$

O sólido é delimitado por $x^2 + z^2 = 4$

$$-1 \leq y \leq 4 - z$$

Considerando

$$x = r\cos\theta, \ z = r\,\text{sen}\,\theta \ \text{e} \ y = y$$

Assim

$$S = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq r \leq 2 \ \text{e} \ 0 \leq \theta \leq 2\pi \ \text{e} \ -1 \leq y \leq 4 - r\,\text{sen}\,\theta\}$$

$$V = \iiint_S dV = \int_0^{2\pi}\int_0^{2}\int_{-1}^{4-r\,\text{sen}\,\theta} r\,dy\,dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^{2} r(5 - r\,\text{sen}\,\theta)\,dr\,d\theta =$$

$$= 5\int_0^{2\pi}\left(\int_0^2 r\,dr\right)d\theta - \int_0^{2\pi}\left(\int_0^2 r^2\,\text{sen}\,\theta\,dr\right)d\theta =$$

$$= 5\int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^2 d\theta - \int_0^{2\pi}\text{sen}\,\theta\left[\frac{r^3}{3}\right]_0^2 d\theta$$

$$= 5\int_0^{2\pi} 2\,d\theta - \int_0^{2\pi}\frac{8}{3}\,\text{sen}\,\theta\,d\theta = 10\int_0^{2\pi} d\theta - \frac{8}{3}\int_0^{2\pi}\text{sen}\,\theta\,d\theta$$

$$V = 10(2\pi - 0) + 9(\cos 2\pi - \cos 0) = 20\pi \quad //$$

Exercícios - Aula 31

1. Calcule utilizando coordenadas cilíndricas ou esféricas:

a) $\iiint_S z\,dx\,dy\,dz$ onde S é o sólido limitado pelas superfícies $z = x^2 + y^2$ e $z = 4$

Limites de integração:

$0 \leq x^2 + y^2 \leq z \leq 4$

$0 \leq x^2 + y^2 \leq 4$

Usando as coordenadas cilíndricas: $x = r\cos\theta$, $y = r\,\text{sen}\,\theta$, $z = z$

Os novos limites de integração são:

$0 \leq r \leq 2$

$0 \leq \theta \leq 2\pi$

$r^2 \leq z \leq 4$

Assim

$$\iiint_S z\,dx\,dy\,dz = \int_0^{2\pi}\int_0^{2}\int_{r^2}^{4} z\,(r\,dz\,dr\,d\theta) =$$

$$= \int_0^{2\pi}\int_0^{2} r\left(\int_{r^2}^{4} z\,dz\right)dr\,d\theta = \int_0^{2\pi}\int_0^{2} r\left[\frac{z^2}{2}\right]_{r^2}^{4} dr\,d\theta =$$

$$= \int_0^{2\pi}\left(\int_0^{2} 8r - \frac{r^5}{2}\,dr\right)d\theta = \int_0^{2\pi}\left[4r^2 - \frac{r^6}{12}\right]_0^{2} d\theta =$$

$$= \frac{32}{3}\int_0^{2\pi} d\theta = \frac{32}{3}\,2\pi = \frac{64\pi}{3} \quad //$$

b) $\iiint_S 9 - x^2 - y^2 \, dx dy dz$ onde $S = \{(x,y) \in \mathbb{R}^3 ; \; x^2 + y^2 + z^2 \leq 9 \text{ e } z \geq 0\}$

Limites de integração: $0 \leq x^2 + y^2 + z^2 \leq 9$

$0 \leq z$

Coordenadas esféricas: $x = \rho \cos\theta \, \text{sen}\, \phi$

$y = \rho \, \text{sen}\, \theta \, \text{sen}\, \phi$

$z = \rho \cos \phi$

Novos limites: $0 \leq \rho \leq 3$

$0 \leq \phi \leq \frac{\pi}{2}$

$0 \leq \theta \leq 2\pi$

Dessa forma

$$\iiint_S 9 - x^2 - y^2 \, dx dy dz = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{3} (9 - \rho^2 \text{sen}^2 \phi)(\rho^2 \text{sen}\, \phi \, d\rho d\phi d\theta)$$

$$= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \left(\int_0^{3} (9 - \rho^2 \text{sen}^2 \phi)(\rho^2 \text{sen}\, \theta) d\rho \right) d\phi d\theta =$$

$$= \int_0^{2\pi} \left(\int_0^{\frac{\pi}{2}} \left(81 \, \text{sen}\, \phi - \frac{243}{5} \text{sen}^3 \phi \right) d\phi \right) d\theta =$$

$$= \int_0^{2\pi} \left(\frac{243}{5} \right) d\theta = \left(\frac{243}{5} \right) \int_0^{2\pi} d\theta = \left(\frac{243}{5} \right) (2\pi) =$$

$$= \frac{486}{5} \pi \quad //$$

c) $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x} (x^2+y^2)\, dz dx dy$

Limites de integração: $-1 \leq y \leq 1$

$0 \leq x \leq \sqrt{1-y^2} \rightarrow$

Coordenadas cilíndricas: $\rightarrow 0 \leq x^2+y^2 \leq 1$

$x = r\cos\theta$ $\qquad 0 \leq z \leq x$

$y = r\,\mathrm{sen}\,\theta$

$z = z$

Novos limites: $0 \leq r \leq 1$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$0 \leq z \leq r\cos\theta$

Então

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x} (x^2+y^2)\, dz dx dy =$$

$$= \int_{0}^{1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{r\cos\theta} r^2 (r\, dz d\theta dr) = \int_{0}^{1} r^3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\int_{0}^{r\cos\theta} dz\right) d\theta dr =$$

$$= \int_{0}^{1} r^3 \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r\cos\theta\, d\theta\right) dr = \int_{0}^{1} r^4 \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta\right) dr =$$

$$= \int_{0}^{1} r^4\, 2\, dr = 2\int_{0}^{1} r^4\, dr = 2\left[\frac{r^5}{5}\right]_{0}^{1} =$$

$$= \frac{2}{5} \quad //$$

d) O volume do sólido S delimitado pelos paraboloides $z = x^2 + y^2$ e $z = 18 - x^2 - y^2$.

Pontos de intersecção:

$$x^2 + y^2 \leq z \leq 18 - x^2 - y^2 \Rightarrow x^2 + y^2 = 9$$

Coordenadas cilíndricas: $x = r\cos\theta$

$y = r\,\mathrm{sen}\,\theta$

$z = z$

Novos limites:

$$0 \leq r \leq 3$$
$$0 \leq \theta \leq 2\pi$$
$$r^2 \leq z \leq 18 - r^2$$

Assim

$$V = \int_0^{2\pi}\int_0^3\int_{r^2}^{18-r^2} (r\,dz\,dr\,d\theta) = \int_0^{2\pi}\int_0^3 r\left(\int_{r^2}^{18-r^2} dz\right)dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^3 r(18-2r^2)\,dr\,d\theta = 2\int_0^{2\pi}\left(\int_0^3 (9r - r^3)\,dr\right)d\theta =$$

$$= 2\int_0^{2\pi} \frac{81}{4}\,d\theta = \frac{81}{2}\int_0^{2\pi} d\theta = \frac{81}{2}\cdot 2\pi$$

$V = 81\pi$ //

e) O volume do sólido S limitado inferiormente pelo cone $z = \sqrt{3x^2 + 3y^2}$, limitado superiormente pelo cone $\sqrt{3}z = \sqrt{x^2 + y^2}$, ambos contidos na semi bola $B_{z \geq 0}(0,4) = \{(x,y,z) \in \mathbb{R}^3 ; x^2 + y^2 + z^2 \leq 4 \text{ e } z \geq 0\}$

Limites de integração: $0 \leq x^2 + y^2 + z^2 \leq 4, \ 0 \leq z$

Coordenadas esféricas: $x = \rho \cos\theta \operatorname{sen}\phi$

$y = \rho \operatorname{sen}\theta \operatorname{sen}\phi$

$z = \rho \cos\phi$

Observando

$$z = \sqrt{3x^2 + 3y^2} \rightarrow \phi = \frac{\pi}{6}$$

$$z = \frac{\sqrt{x^2 + y^2}}{\sqrt{3}} \rightarrow \phi = \frac{\pi}{3}$$

Novos limites:

$$0 \leq \rho \leq 2$$

$$\frac{\pi}{6} \leq \phi \leq \frac{\pi}{3}$$

$$0 \leq \theta \leq 2\pi$$

Assim

$$V = \int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \int_0^2 (\rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta) =$$

$$= \int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{sen}\phi \left(\int_0^2 \rho^2 \, d\rho \right) d\phi \, d\theta =$$

$$= \int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{sen}\phi \left(\frac{\rho^3}{3} \right)_0^2 d\phi \, d\theta = \int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{sen}\phi \left(\frac{8}{3} \right) d\phi \, d\theta =$$

$$= \frac{8}{3} \int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{sen} \phi \, d\phi \, d\theta =$$

$$= \frac{8}{3} \int_0^{2\pi} \left(\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{sen} \phi \, d\phi \right) d\theta =$$

$$= \frac{8}{3} \int^{2\pi} \left(\frac{\sqrt{3}-1}{2} \right) d\theta = \frac{4(\sqrt{3}-1)}{3} \int_0^{2\pi} d\theta =$$

$$= \frac{8\sqrt{2}}{3} \pi \quad //$$

f) $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dz\,dy\,dx$

Limites de integração: $0 \leq x \leq 1$

$0 \leq y \leq \sqrt{1-x^2} \rightarrow$

$\rightarrow 0 \leq x^2 + y^2 \leq 1$

$\sqrt{x^2+y^2} \leq z \leq \sqrt{2-x^2-y^2}$

Coordenadas Cilíndricas:

$x = r\cos\theta$

$y = r\,\text{sen}\,\theta$

$z = z$

Novos limites:

$0 \leq r \leq 1$

$0 \leq \theta \leq \frac{\pi}{2}$

$r \leq z \leq \sqrt{2-r^2}$

Assim

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dz\,dy\,dx =$$

$$= \int_0^{\frac{\pi}{2}}\int_0^1\int_r^{\sqrt{2-r^2}} (r^2\cos\theta\,\text{sen}\,\theta)(r\,dz\,dr\,d\theta) =$$

$$= \int_0^{\frac{\pi}{2}} \cos\theta\,\text{sen}\,\theta \int_0^1 r^3 \left(\int_r^{\sqrt{2-r^2}} dz\right) dr\,d\theta =$$

$$= \int_0^{\frac{\pi}{2}} \cos\theta\,\text{sen}\,\theta \left(\int_0^1 r^3\left(\sqrt{2-r^2}-r\right)dr\right) d\theta =$$

$$= \int_0^{\frac{\pi}{2}} 2\cos\theta\,\text{sen}\,\theta \left(\frac{4\sqrt{2}-5}{15}\right) d\theta =$$

$$= \frac{4\sqrt{2}-5}{30} \int_0^{\frac{\pi}{2}} \operatorname{sen} 2\theta \, (2d\theta)$$

$$= \frac{4\sqrt{2}-5}{30} (\cos 0 - \cos \pi) =$$

$$= \frac{4\sqrt{2}-5}{15} \quad //$$

g) $$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} xy\, dz\, dx\, dy$$

Limites de integração: $-2 \leq x \leq 2$

$-\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2} \rightarrow$

$\rightarrow 0 \leq x^2+y^2 \leq 4$

$\sqrt{x^2+y^2} \leq z \leq 2$

Coordenadas cilíndricas:

$x = r\cos\theta$

$y = r\,\text{sen}\,\theta$

$z = z$

Novos limites:

$0 \leq r \leq 2$

$0 \leq \theta \leq 2\pi$

$r \leq z \leq 2$

Assim

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} xy\, dz\, dx\, dy =$$

$$= \int_{0}^{2\pi}\int_{0}^{2}\int_{r}^{2} r^2\cos\theta\,\text{sen}\,\theta\,(r\,dz\,dr\,d\theta) =$$

$$= \int_{0}^{2\pi}\cos\theta\,\text{sen}\,\theta\int_{0}^{2} r^3\left(\int_{r}^{2} dz\right) dr\, d\theta =$$

$$= \int_{0}^{2\pi}\cos\theta\,\text{sen}\,\theta\left(\int_{0}^{2} r^3(2-r)\,dr\right) d\theta =$$

$$= \int_{0}^{2\pi}\cos\theta\,\text{sen}\,\theta\left(\frac{8}{5}\right) d\theta = \frac{2}{5}\int_{0}^{2\pi}\text{sen}\,2\theta\,(2d\theta) =$$

$$= \frac{2}{5}(\cos 0 - \cos 4\pi) =$$

$$= \frac{2}{5} \cdot 0$$

$$= 0 \quad //$$