Aula 4 – Álgebra de Boole

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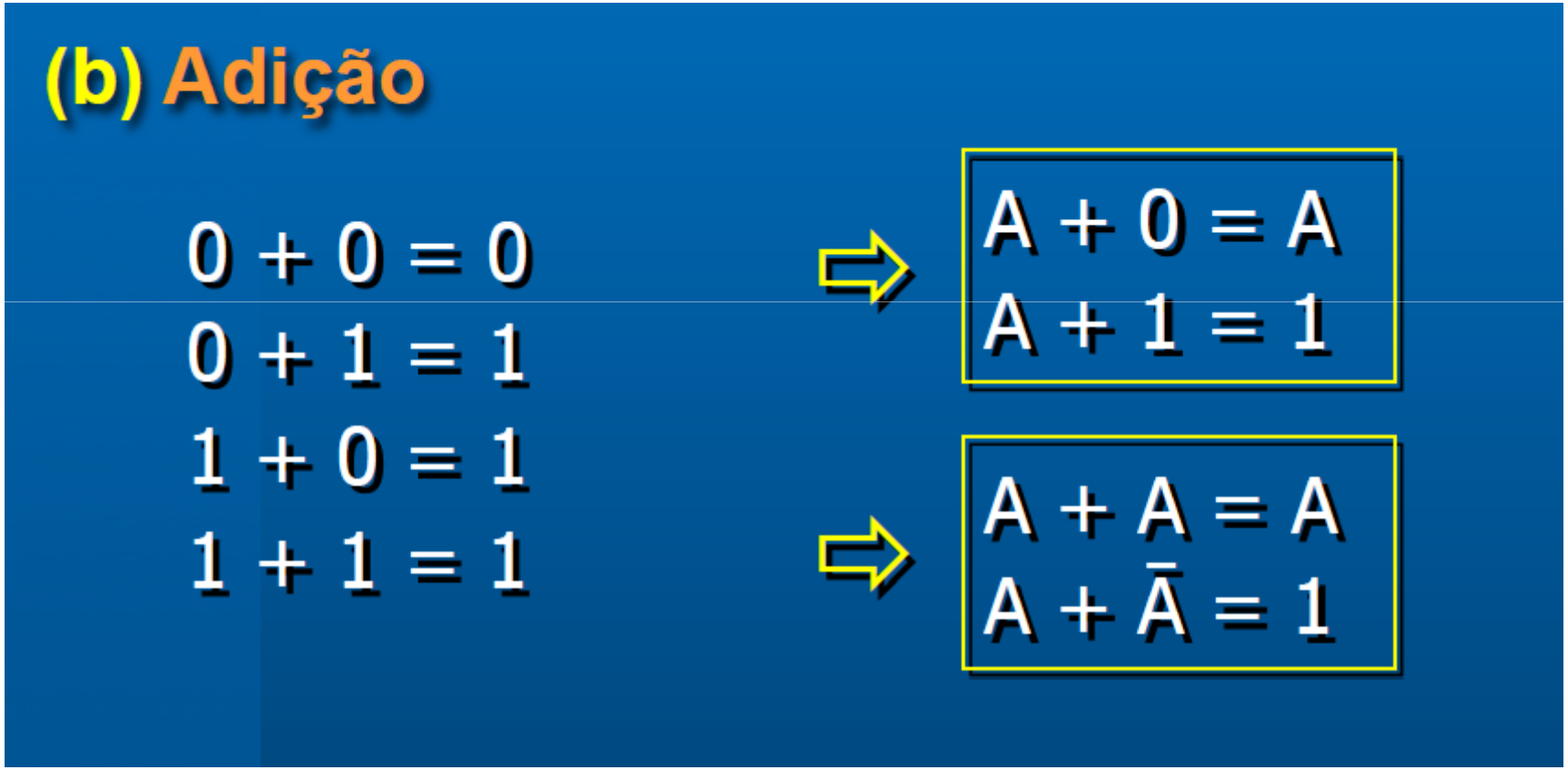
Postulados

(a) Complemento

$\bar{A}$ = complemento de A

- A = 0 ➡ $\bar{A}$ = 1
- A = 1 ➡ $\bar{A}$ = 0

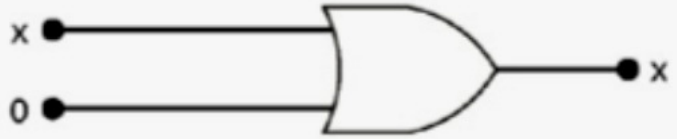
Postulados



Postulados

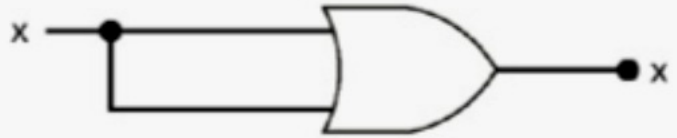
(b) Adição

(5) $x + 0 = x$

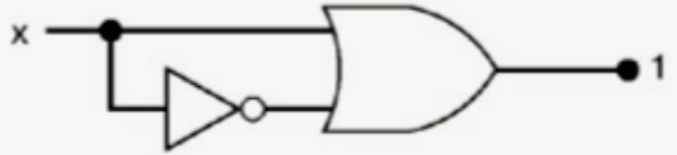


(6) $x + 1 = 1$

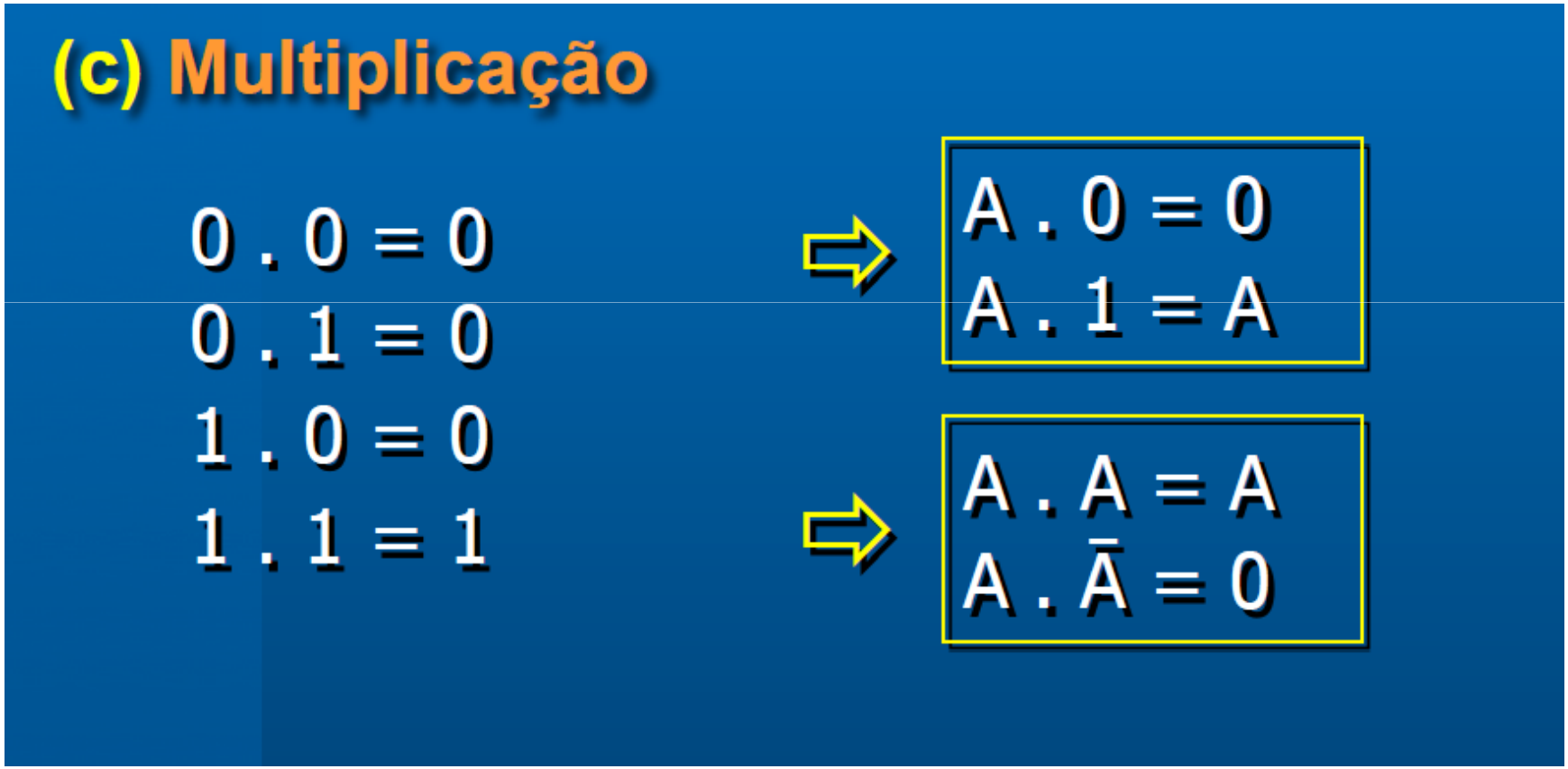
(7) $x + x = x$



(8) $x + \bar{x} = 1$



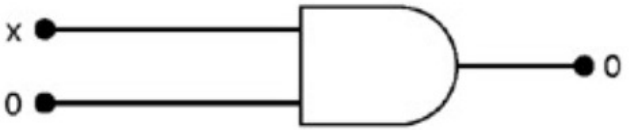
Postulados



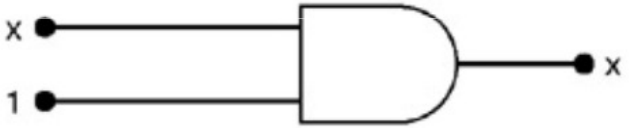
Postulados

(c) Multiplicação

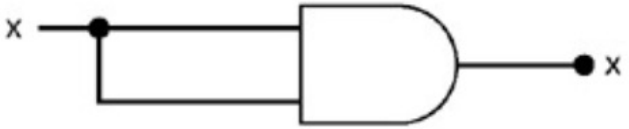
(1) $x \cdot 0 = 0$



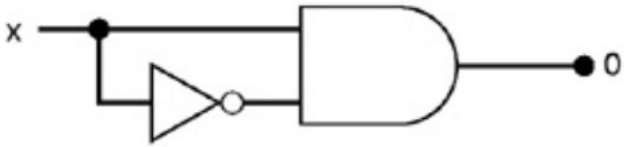
(2) $x \cdot 1 = x$



(3) $x \cdot x = x$



(4) $x \cdot \bar{x} = 0$



Propriedades

1.2. PROPRIEDADES

(a) Comutativa ⇨

- $A + B = B + A$
- $A \cdot B = B \cdot A$

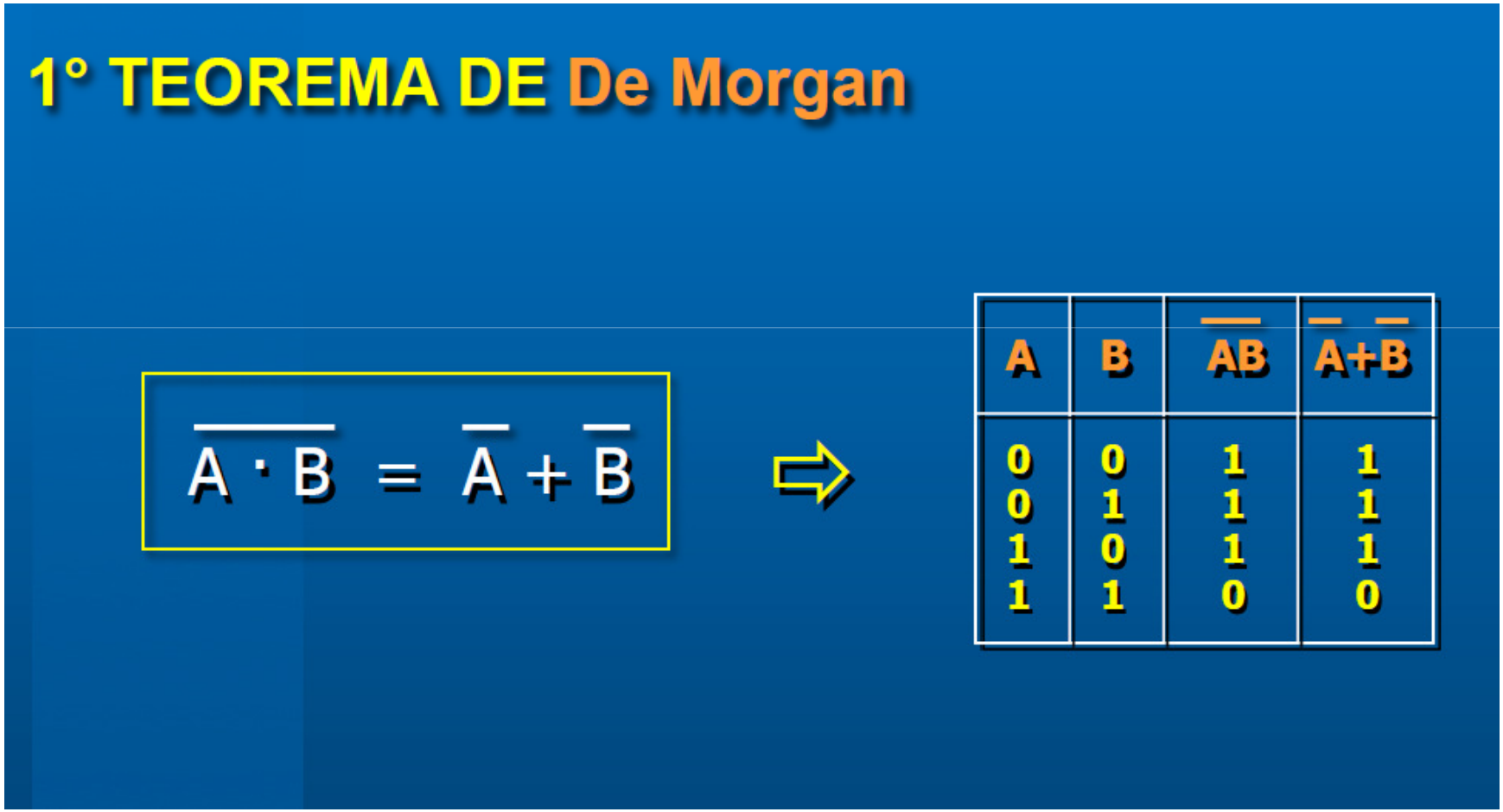
(b) Associativa ⇨

- $A + (B+C) = (A+B) + C = A + B + C$
- $A \cdot (BC) = (AB) \cdot C = ABC$

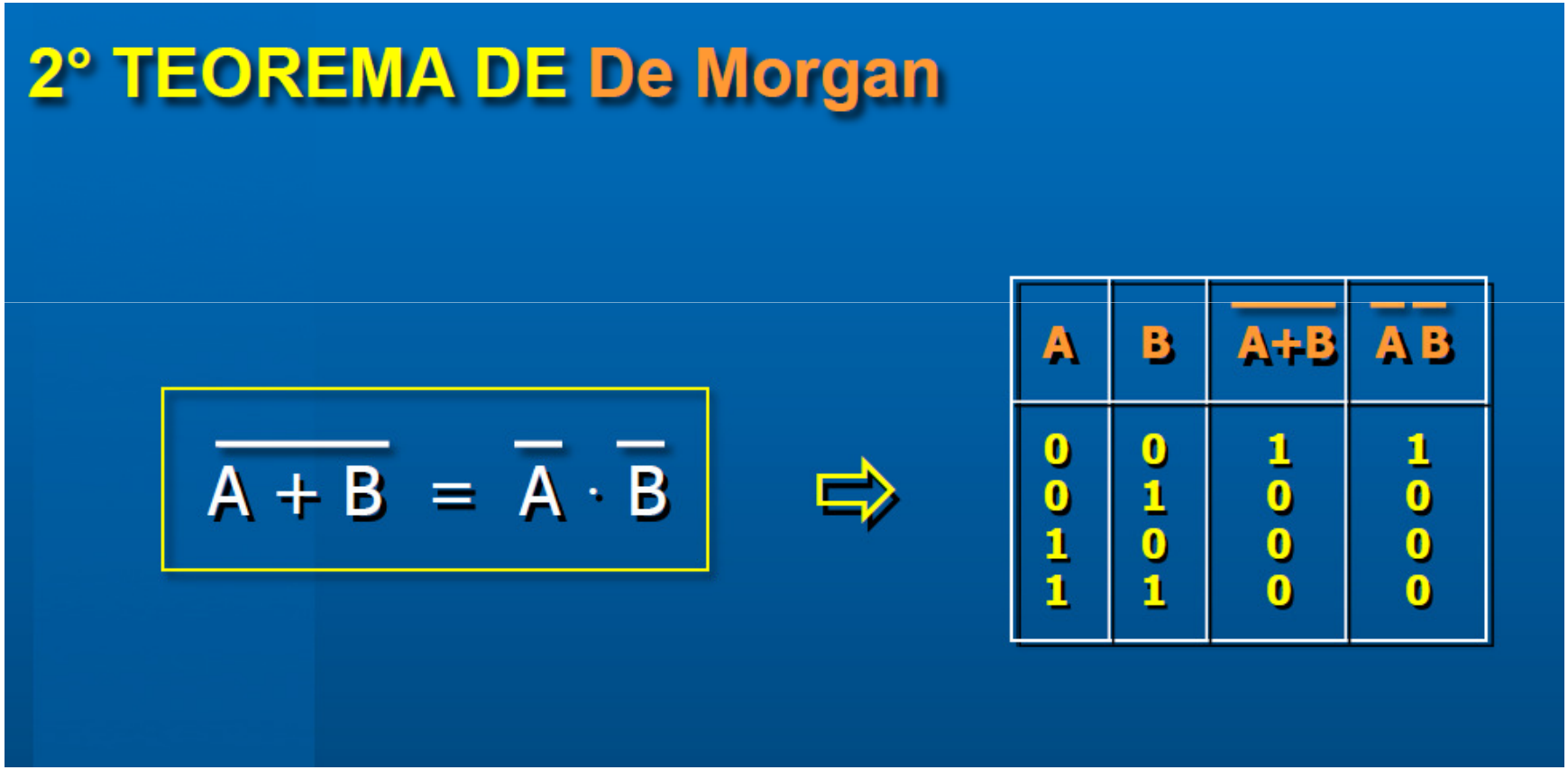
(c) Distributiva ⇨

$A \cdot (B+C) = AB + AC$

Teoremas de De Morgan

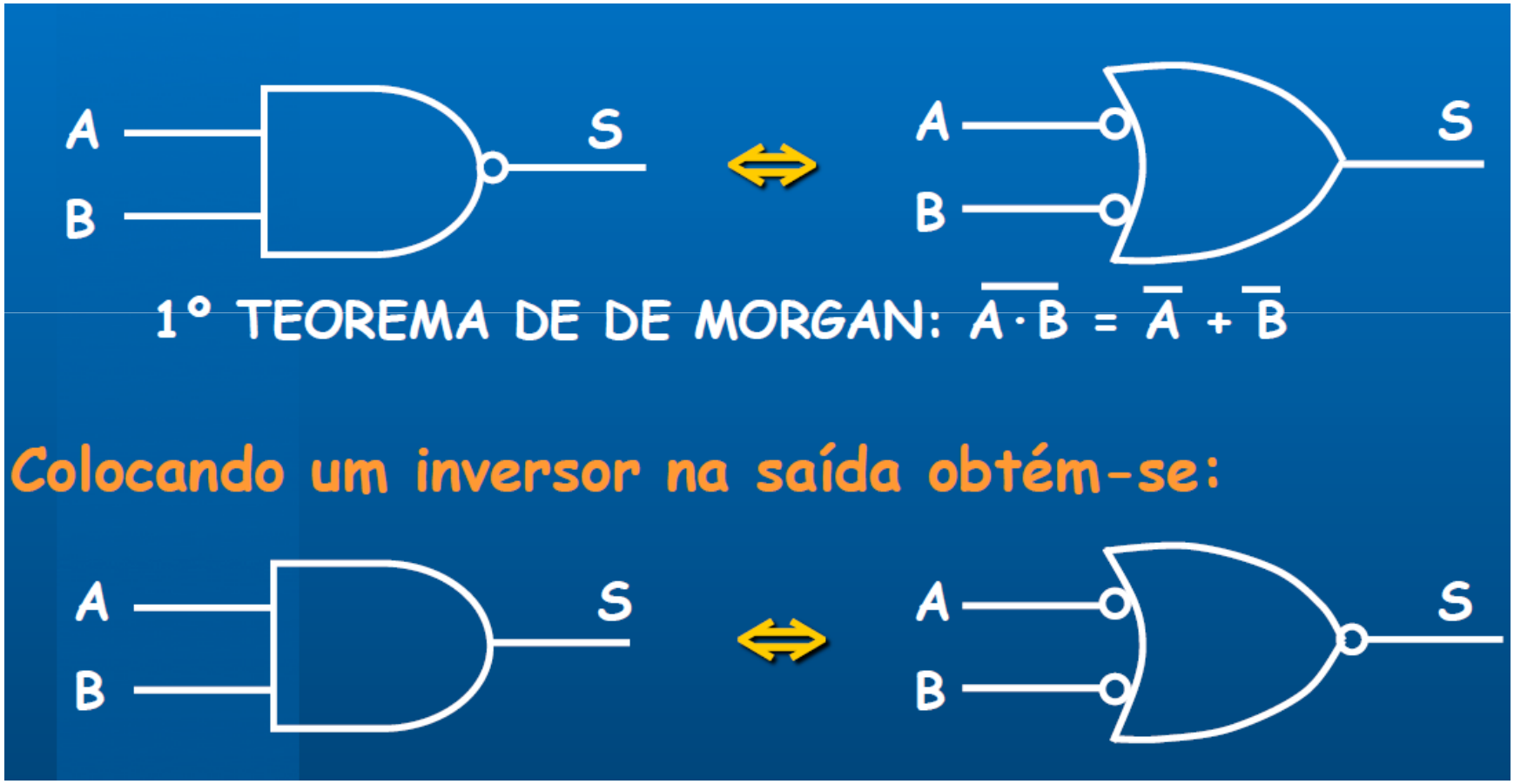


Teoremas de De Morgan

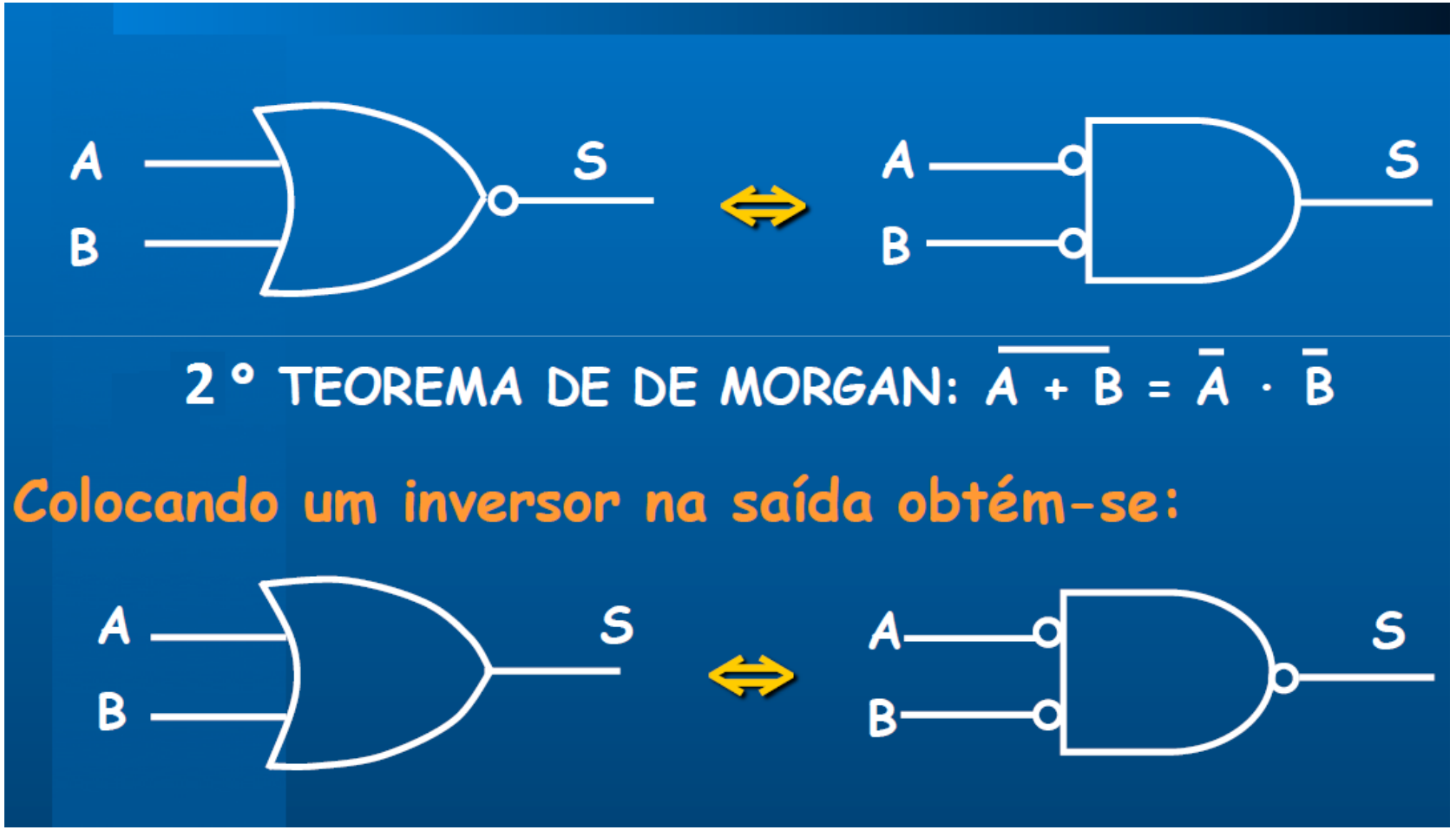


A	B	$\overline{A+B}$	$\overline{A}\,\overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Equivalência entre Blocos Lógicos



Equivalência entre Blocos Lógicos



Outras Identidades

(a) $\overline{\overline{A}} = A$

(b) $A + A \cdot B = A$

(c) $A + \overline{A}\,B = A + B$

(d) $(A + B)(A + C) = A + B \cdot C$

Exercícios

Simplificar as expressões:

1. $S = A\bar{B}C + A\bar{B}\bar{C}$

2. $S = (\bar{A} + B) \cdot (A + B)$

3. $S = ABC + A\bar{C} + A\bar{B}$

4. $S = \overline{(\bar{A} + C) \cdot (\bar{A} + D)}$

Exercícios Extras de Simplificação*☺

(a) $x = ABC + \overline{A}C$

(b) $y = (Q + R)(\overline{Q} + \overline{R})$

(c) $w = ABC + A\overline{B}C + \overline{A}$

(d) $q = \overline{RST}\overline{(R + S + T)}$

(e) $x = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}BC + ABC + A\,\overline{B}\,\overline{C} + A\overline{B}C$

(f) $z = (B + \overline{C})(\overline{B} + C) + \overline{\overline{A} + B + \overline{C}}$

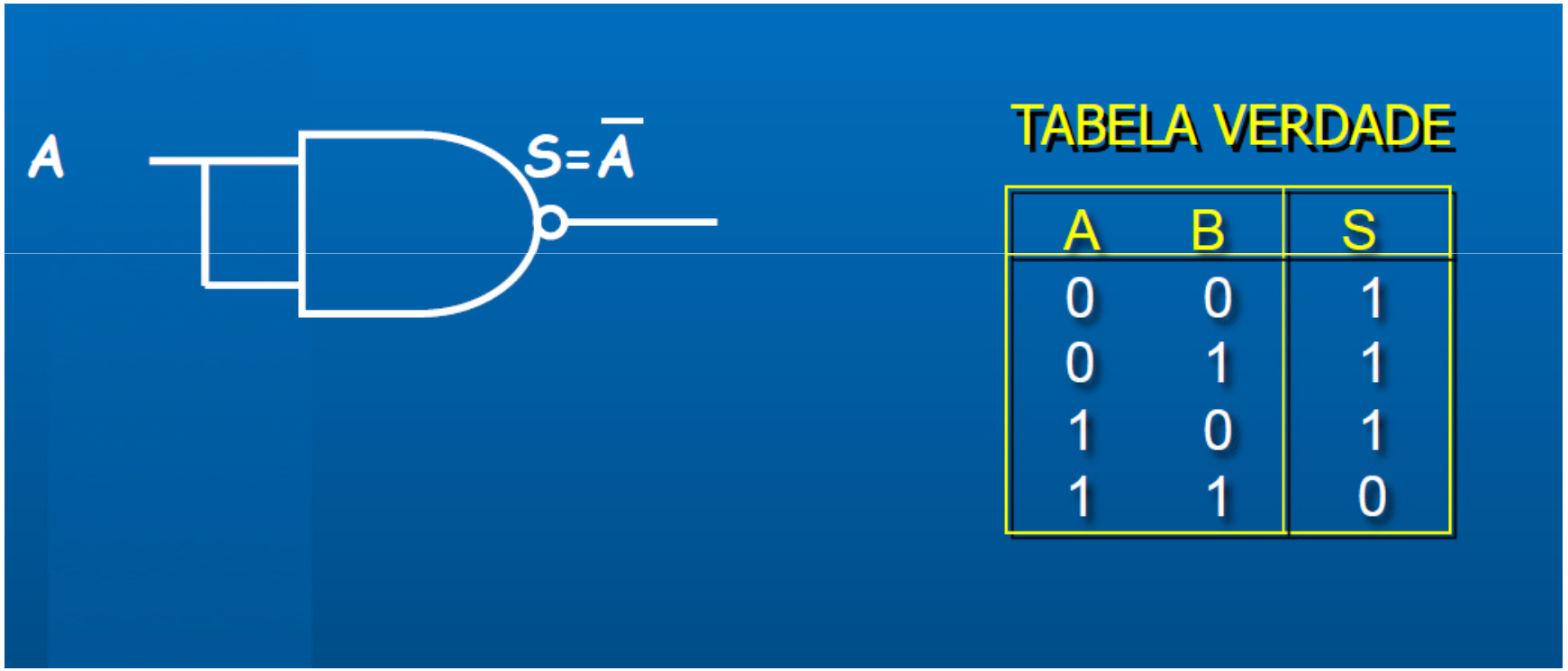
(g) $y = \overline{(C + D)} + \overline{A}C\overline{D} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}CD + AC\overline{D}$

(h) $x = AB\overline{(\overline{C}D)} + \overline{A}BD + \overline{B}\,\overline{C}\,\overline{D}$

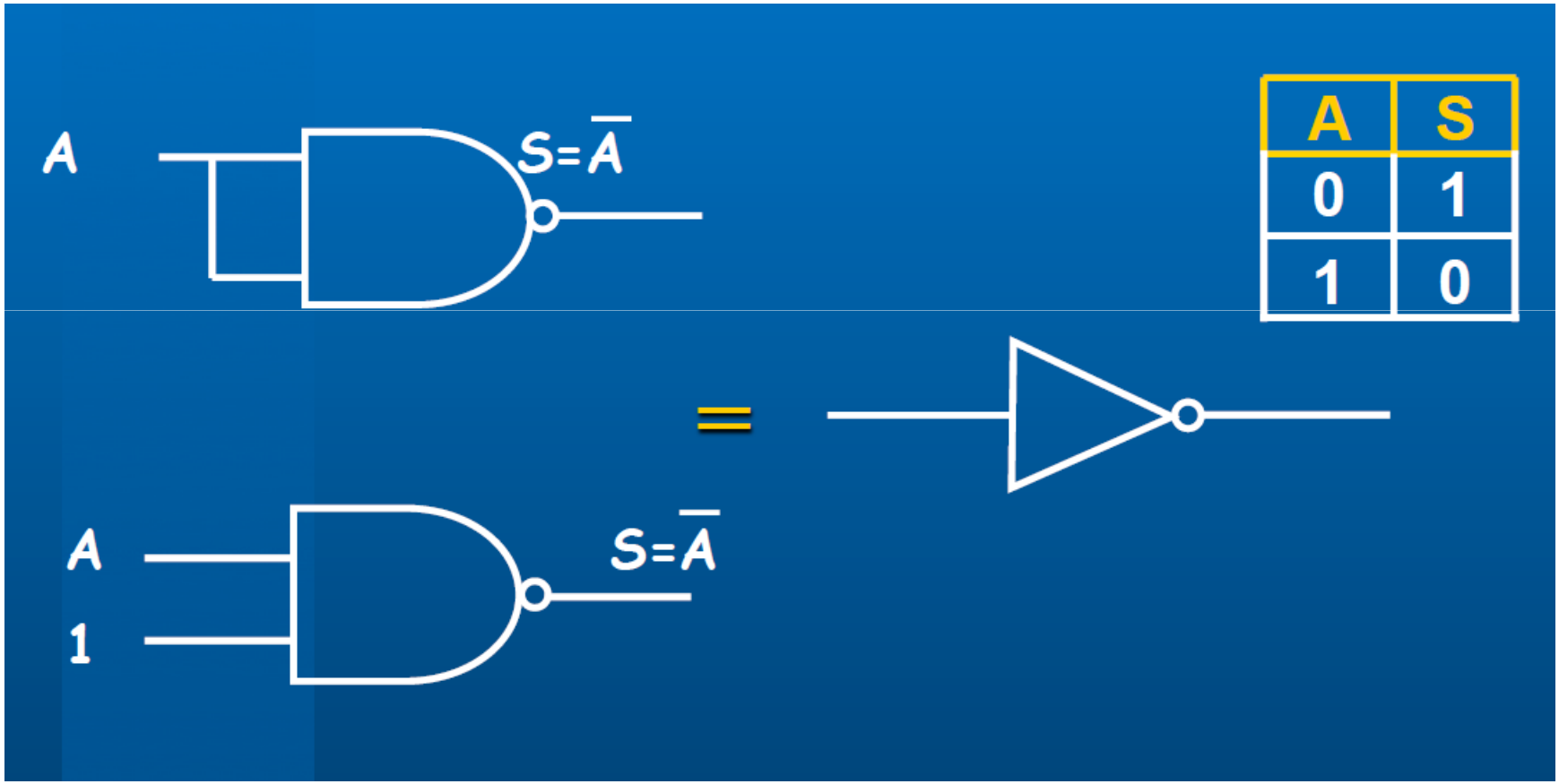
Universalidade das Portas NAND e NOR

- Todas as expressões Booleanas consistem de combinações de funções OR, AND e NOT;
- Portas NAND e NOR são universais, ou seja, podem se "transformar" em qualquer outra porta lógica e podem, portanto, ser usadas para representar qualquer expressão Booleana;

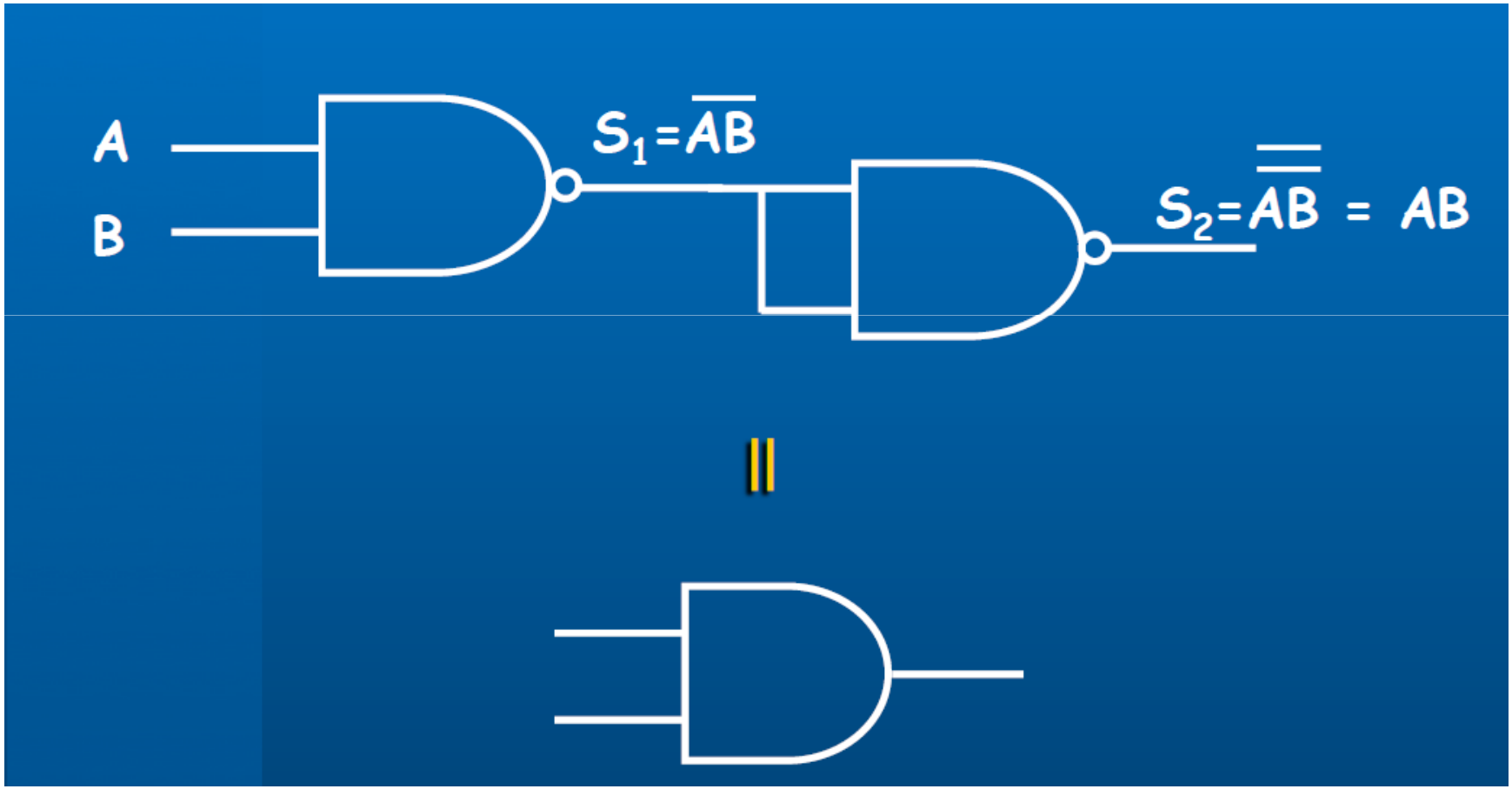
Inversor usando Porta NAND



Inversor usando Porta NAND



Porta AND usando 2 Portas NAND

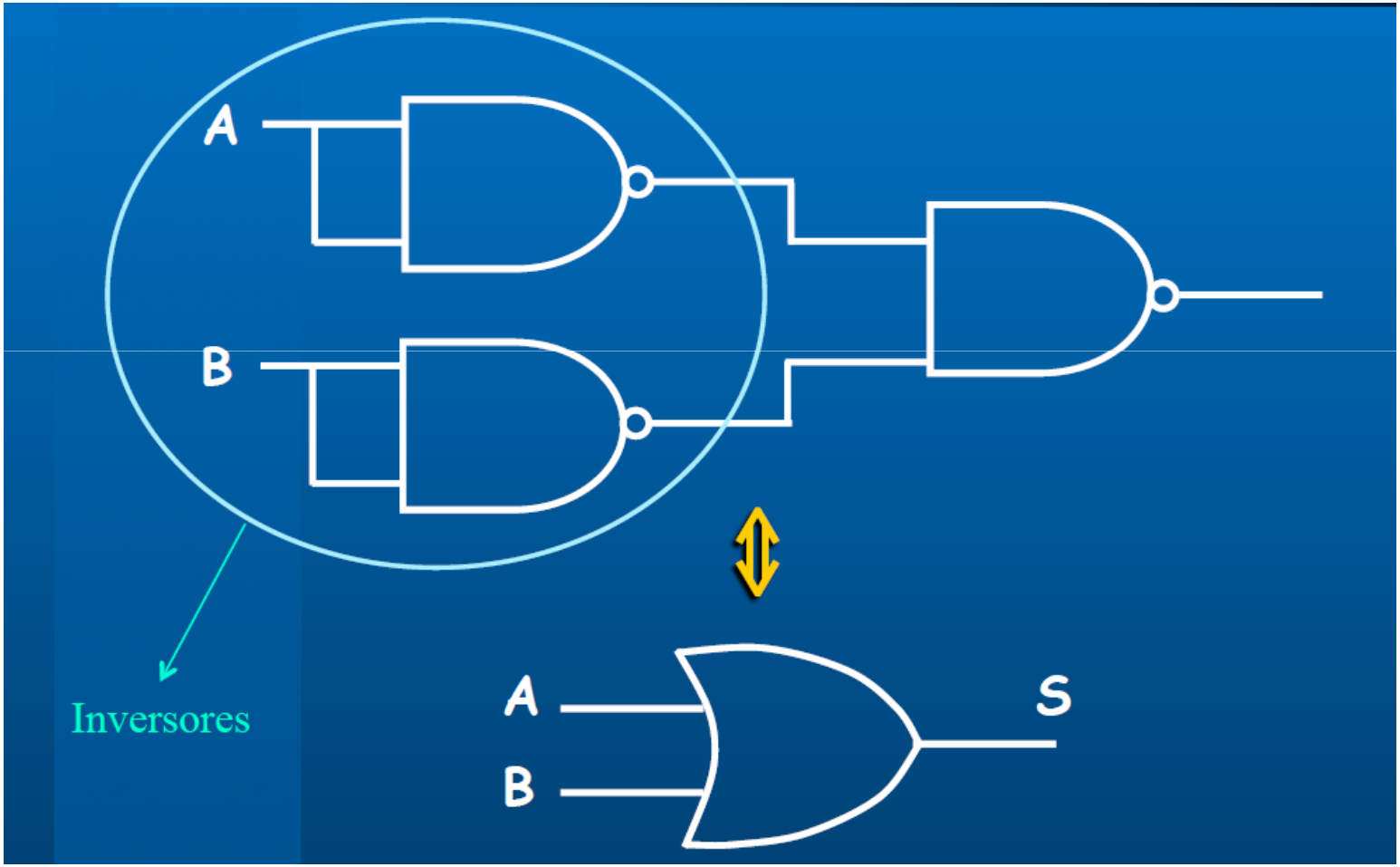


Porta OR usando 3 Portas NAND

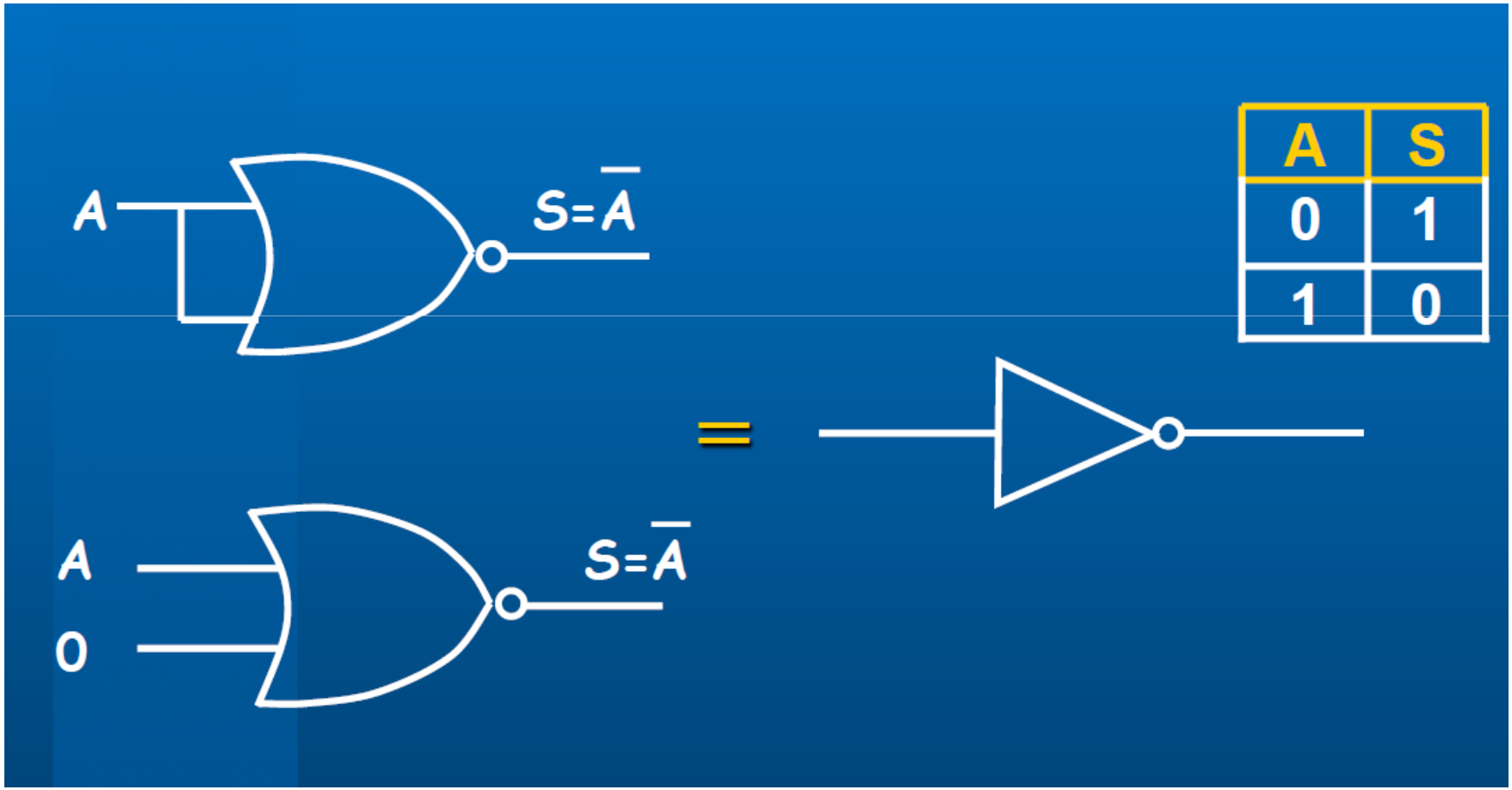
Pelo Teorema de De Morgan temos:

$$\overline{(\overline{A} \cdot \overline{B})} = \overline{\overline{(A + B)}} = A + B$$

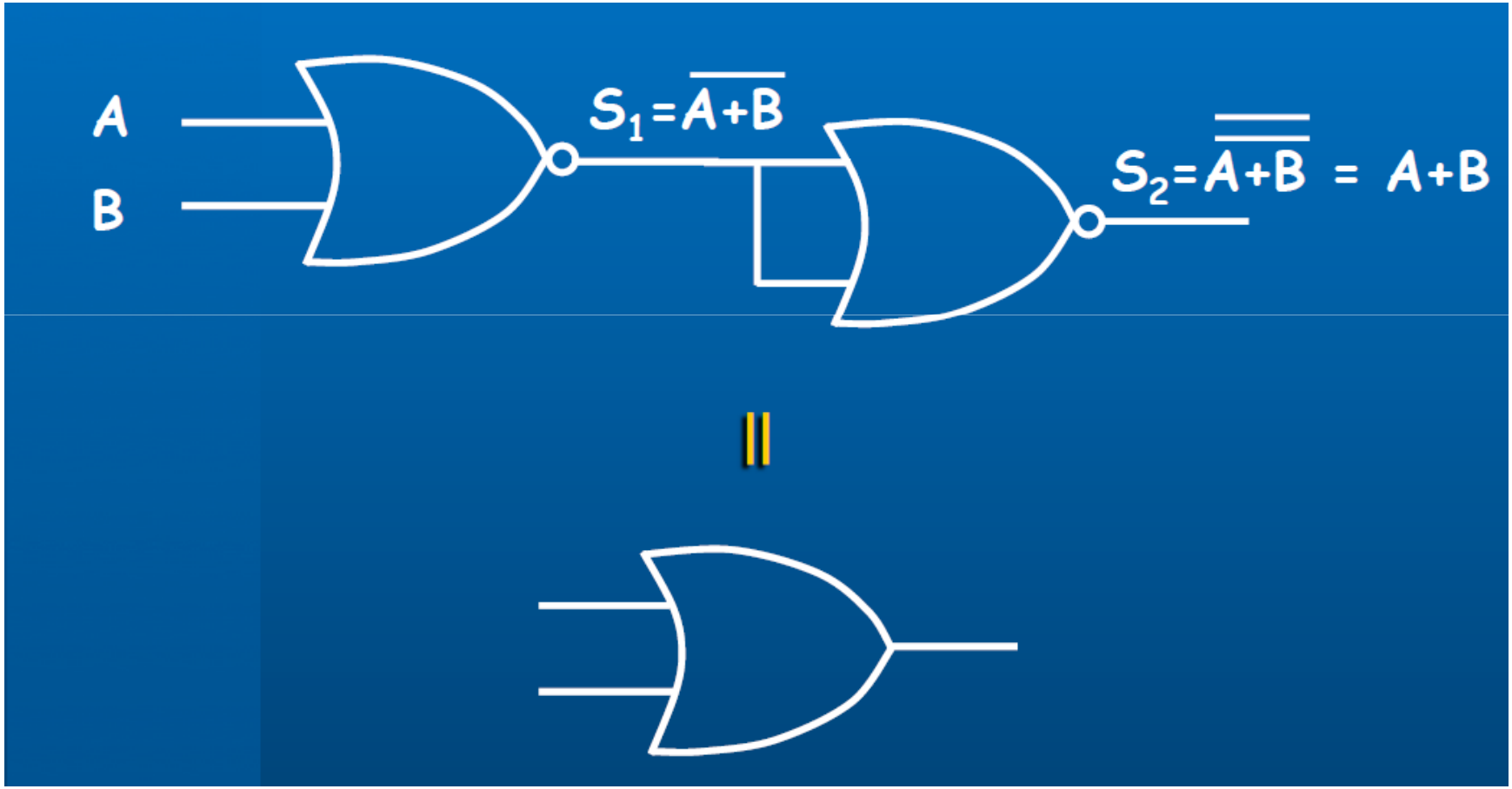
Porta OR usando 3 Portas NAND



Inversor usando Porta NOR



Porta OR usando 2 Portas NOR

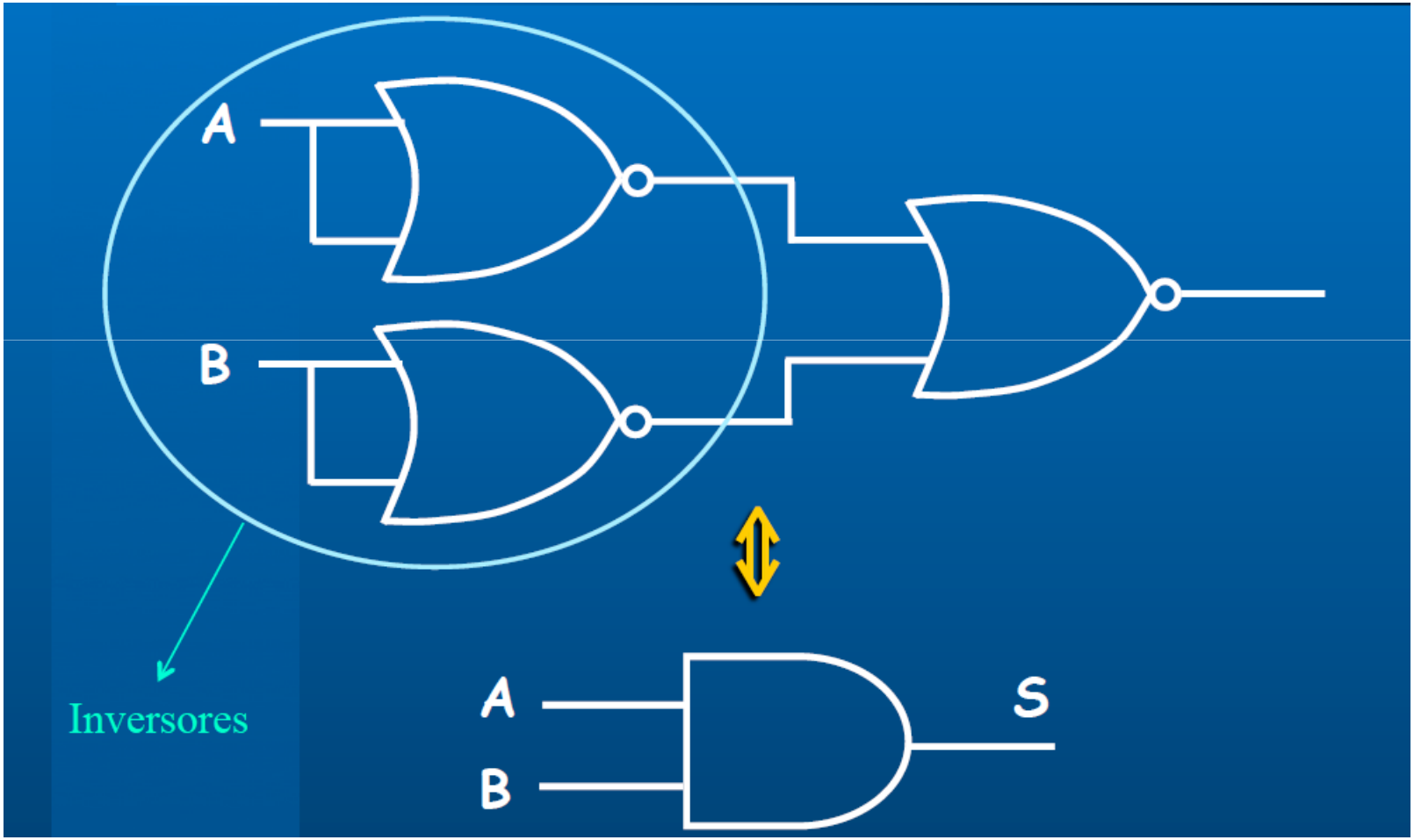


Porta AND usando 3 Portas NOR

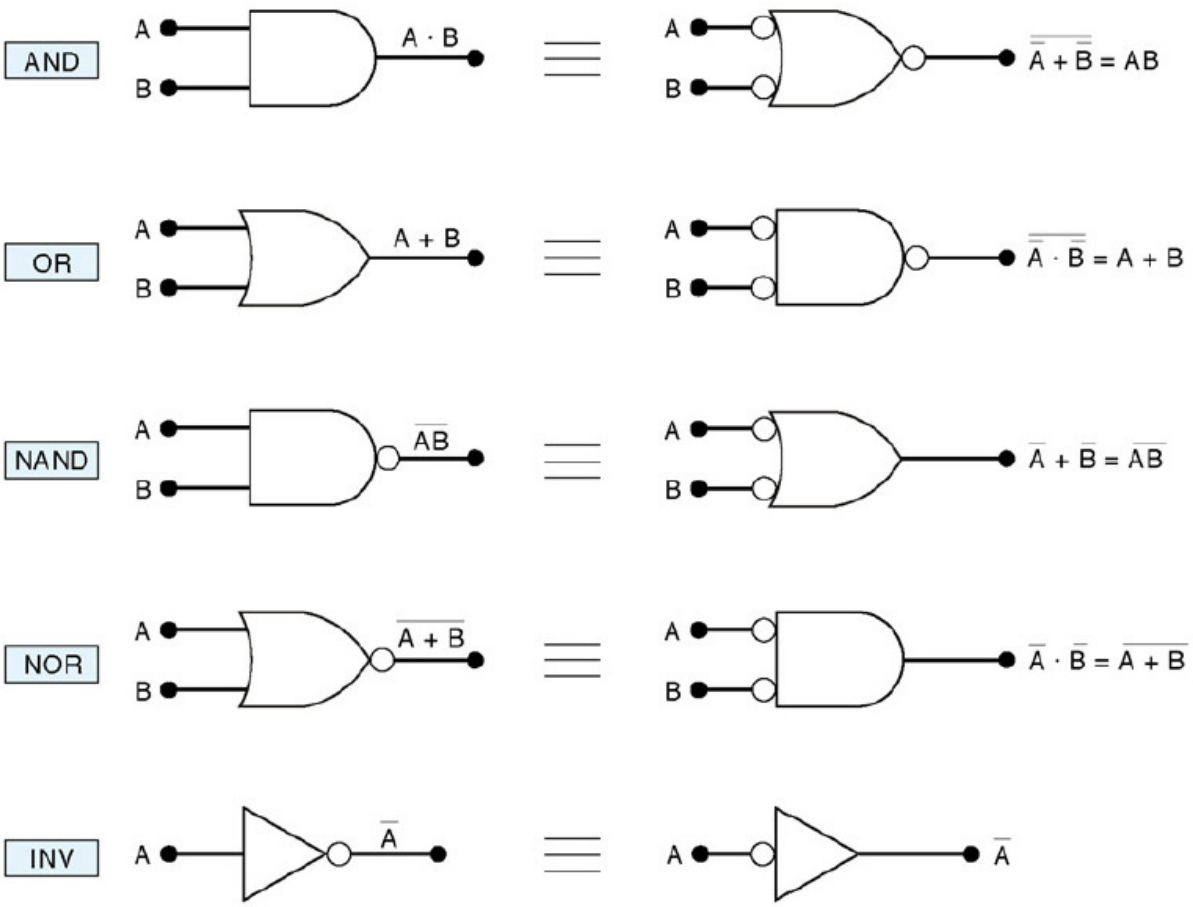
Pelo Teorema de De Morgan temos:

$$\overline{(\overline{A} + \overline{B})} = \overline{\overline{(A \cdot B)}} = A \cdot B$$

Porta AND usando 3 Portas NOR



Resumo


AND
A
B
A · B
A
B
$\overline{\overline{A} + \overline{B}} = AB$
OR
A
B
A + B
A
B
$\overline{\overline{A} \cdot \overline{B}} = A + B$
NAND
A
B
$\overline{AB}$
A
B
$\overline{A} + \overline{B} = \overline{AB}$
NOR
A
B
$\overline{A + B}$
A
B
$\overline{A} \cdot \overline{B} = \overline{A + B}$
INV
A
$\overline{A}$
A
$\overline{A}$

Exercícios*☺

- Fazer todos os exercícios do Cap. 3 do livro texto, menos os de HDL.

Referências

- Tocci, R. J. et al. Sistemas Digitais (princípios e aplicações), 10a Edição. Pearson, 2007.
- Vieira, M. A. C. SEL-0414-Sistemas Digitais, EESC-USP.