LFA – P2

- ## Expressões Regulares

Notação algébrica

"Uma coleção de objetos é fechada sob alguma operação se, aplicando-se essa operação a membros da coleção, recebe-se um objeto ainda na coleção".

Operações Regulares

Linguagens regulares são fechadas sob operações regulares

Uniã́o: $A \cup B = \{x \mid x \in A \text{ ou } x \in B\}$ } 2-ária

Concatenação: $A \cdot B = \{xy \mid x \in A \text{ e } y \in B\}$

Fechamento: $A^* = \{x_1 x_2 \dots x_k \mid k \geq 0 \text{ e cada } x_i \in A\}$ } 1-ária
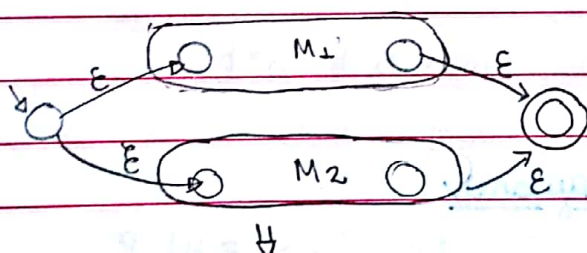
Ex: $L = \{001, 10, 111\}$ e $M = \{\varepsilon, 001\}$
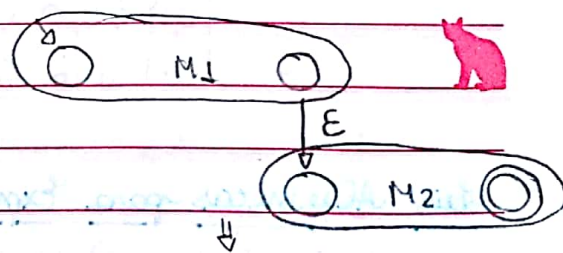
$L \cup M = \{001, 10, 111, \varepsilon\}$

$L \cdot M = \{001, 10, 111, 001001, 10001, 111001\}$

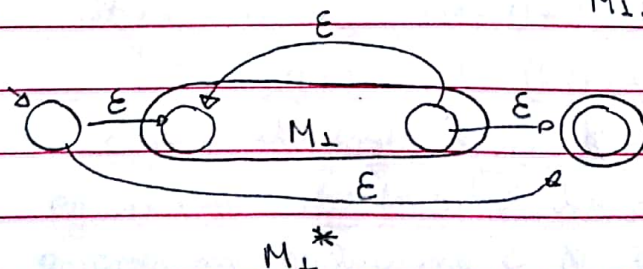$M \cdot L = \{001, 10, 111, 001001, 00110, 001111\}$

$L^* = \{\varepsilon, 001, 10, 111, 00110, 001111, 001001, 10001, 10111, 1010, \dots\}$

$M_1 \cup M_2$

$M_1 \cdot M_2$

M_1^*

$L(\varepsilon) = \{\varepsilon\}$

$L(\emptyset) = \{\emptyset\}$

$L(a) = \{a\}$

$L(E+F) = L(E) \cup L(F)$

$L(EF) = L(E) . L(F)$

$L(E^*) = (L(E))^*$

$L((E)) = L(E)$

Precedência: Estrela → Concatenação → União

Exemplos: 1) $0^* 1 0^*$

$\{\varepsilon, 0, 00, 000, ...\} . 1 . \{\varepsilon, 0, 00, 000, ...\} = \{w \mid w$ contém um único $1\}$

2) $01 + 10$

$\{01\} \cup \{10\} = \{01, 10\}$

3) $(\varepsilon + 0) 1^*$

$(\varepsilon + 0) \{\varepsilon, 1, 11, 111\} = \{\varepsilon, 0\} . \{\varepsilon, 1, 11, 111\} = \{\varepsilon, 0, 1, 01, 11, 011, ...\}$

$\{w \mid w$ é uma seq. de 0 ou + 1's, começando opcionalmente com 0$\}$

4) $(0+1)^*$

$\{0,1\}^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, ...\}$

5) $(0+1)(0+1)^*$

$\{0,1\} \{\varepsilon, 0, 1, 00, 01, 10, 11, ...\}$

$\{w \mid w$ contém 1 ou mais elementos de $w^*\}$

Leis Algébricas para Expressões Regulares

- $L + M = M + L$ ⇒ comutatividade $LM \neq ML$ ‼

- $(L+M) + N = L + (M+N)$ ⎫
- $(LM)N = L(MN)$ ⎬ ⇒ associatividade

- $\emptyset + L = L + \emptyset = L$ ⇒ identidade união

- $\varepsilon L = L\varepsilon = L$ ⇒ identidade concatenação

- $\emptyset L = L \emptyset = \emptyset$ ⇒ aniquilador concatenação

- $\lambda(M+N) = \lambda M + \lambda N$ ⎫ ⇒ distributiva
- $(M+N)\lambda = M\lambda + N\lambda$ ⎭
- $\lambda + \lambda = \lambda$ ⇒ idempotência
- $(\lambda^*)^* = \lambda^*$ ⎫
- $\varnothing^* = \epsilon$ ⎬ ⇒ leis sob fechamento
- $\epsilon^* = \epsilon$ ⎭

- $\lambda? = \epsilon + \lambda$
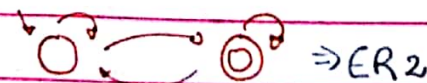- $\lambda^* = \lambda^+ + \epsilon$
- $\lambda^+ = \lambda \cdot \lambda^*$

Autômatos finitos e expressões regulares

Conversão DFA → ER:
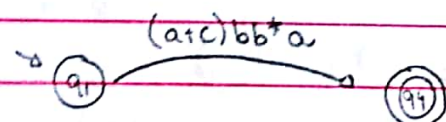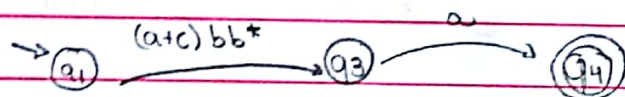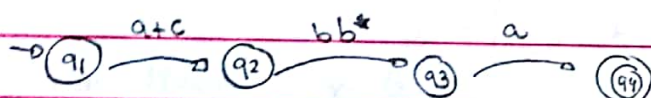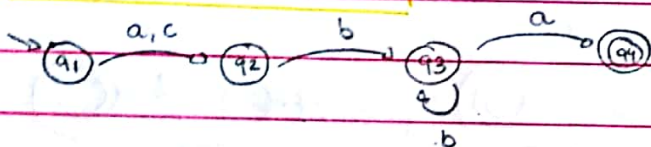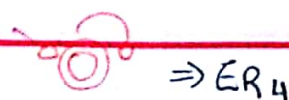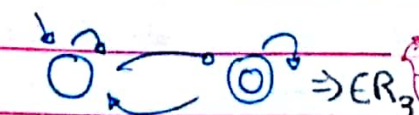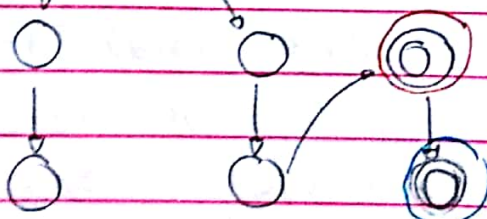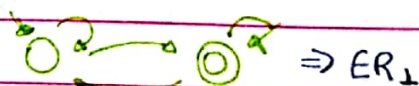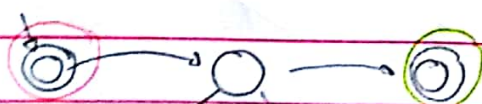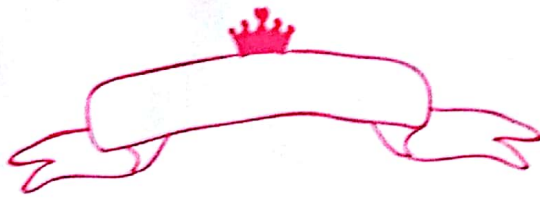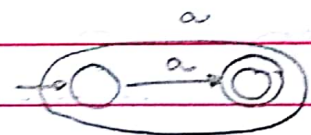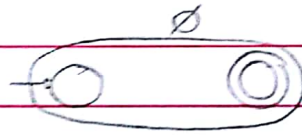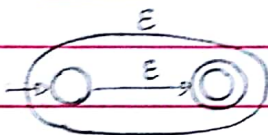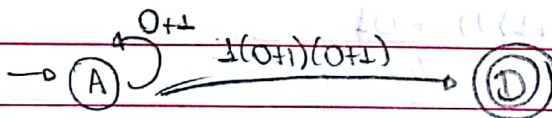
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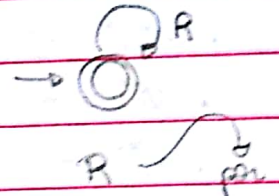Criar um autômato mínimo para cada combinação q_0 e q_{fi}

ex:

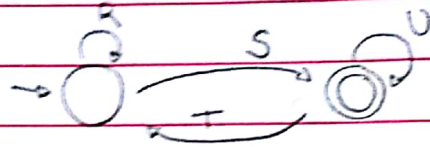⇒ ER_1

⇒ ER_2

⇒ ER_3

⇒ ER_4

$$ER_{final} = ER_1 + ER_2 + ER_3 + ER_4$$
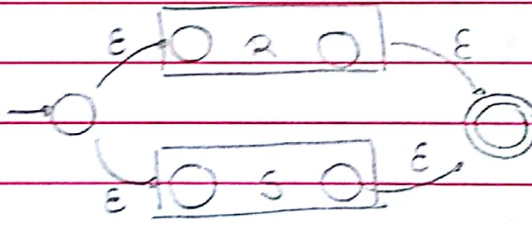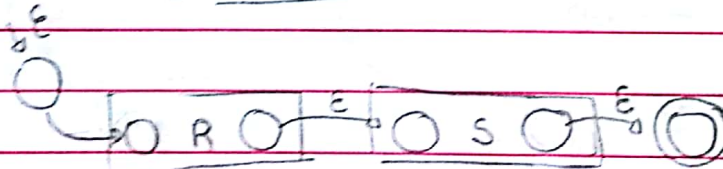
. Expressão

$$(R + SU^*T)^* SU^*$$

que vão

Conversão ER → E-NFA :
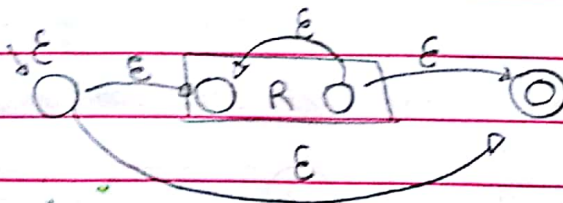

R+S

RS

R*

• Exercícios

① Escreva as expressões regulares correspondentes $(\Sigma = \{0,1\})$:

a) $\{w \mid w$ começa com um 1 e termina com um 0$\}$

$\{10, 101, 110, 1000, 1100, 1110, \ldots\}$

$1 \cdot (0+1)^* \cdot 0$

b) $\{w \mid w$ contém pelo menos três 1s$\}$

$0^* 1 0^* 1 0^* 1 (0+1)^*$

c) $\{w \mid$ o comprimento de w é no máximo 5$\}$

$(0+1+\varepsilon)(0+1+\varepsilon)(0+1+\varepsilon)(0+1+\varepsilon)(0+1+\varepsilon)$

$\hookrightarrow$ se fosse "igual a 5", não teria os ε's

d) $\{w \mid$ toda posição ímpar de w é 1$\}$

$(1(0+1))^* (1+\varepsilon)$

② Simplifique as expressões:

a) $\varepsilon a (b+c) + a a \varepsilon$

$a(b+c) + aa = a((b+c)+a) = a(b+c+a)$

b) $\emptyset (\varepsilon+1)^* (1+0(01^* 10(0+1))) + 01$

$\emptyset + 01 = 01$

c) $(0+1+\varepsilon) + 01^* 0 + (\varepsilon+0+1) + (\varepsilon+0+1)$

$(\varepsilon+0+1) + (\varepsilon+0+1) + (\varepsilon+0+1) + 01^* 0 = (\varepsilon+0+1) + 01^* 0$

d) $0 + 010$

$0\varepsilon + 010 = 0(\varepsilon+10)$

e) $ab^* b (a+\varepsilon)$

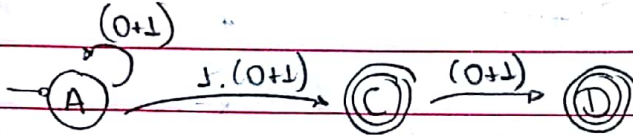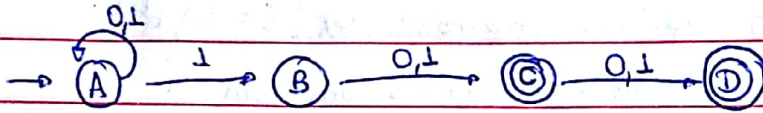$ab^* b (a+\varepsilon) = ab^+ (a+\varepsilon)$

f) $a + (b+c+\varepsilon)a(b+c) + ca + ba$

$\varepsilon a + (b+c+\varepsilon)a(b+c) + (c+b)a = (\varepsilon+c+b)a + (b+c+\varepsilon)a(b+c)$
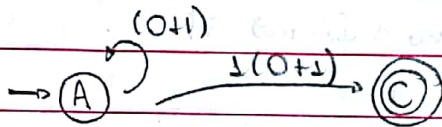
$= (b+c+\varepsilon)a\varepsilon + (b+c+\varepsilon)a(b+c) = (b+c+\varepsilon)a(\varepsilon+b+c)$

$= (b+c+\varepsilon)a(b+c+\varepsilon)$

FORONI

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③ Ache a ER equivalente:

$(0+1)$

• Fechando em C:

$(0+1)$

$$((0+1) + 1(0+1) \varnothing^* \varnothing)^* 1(0+1) \varnothing^*$$
$$((0+1) + 1(0+1) . \varepsilon . \varnothing)^* 1(0+1). \varepsilon$$
$$((0+1))^* 1(0+1)$$
$$(0+1)^* 1(0+1)$$

• Fechando em D:

$0+1$

$$((0+1) + 1(0+1)(0+1) \varnothing^* \varnothing)^* 1(0+1)(0+1) \varnothing^*$$
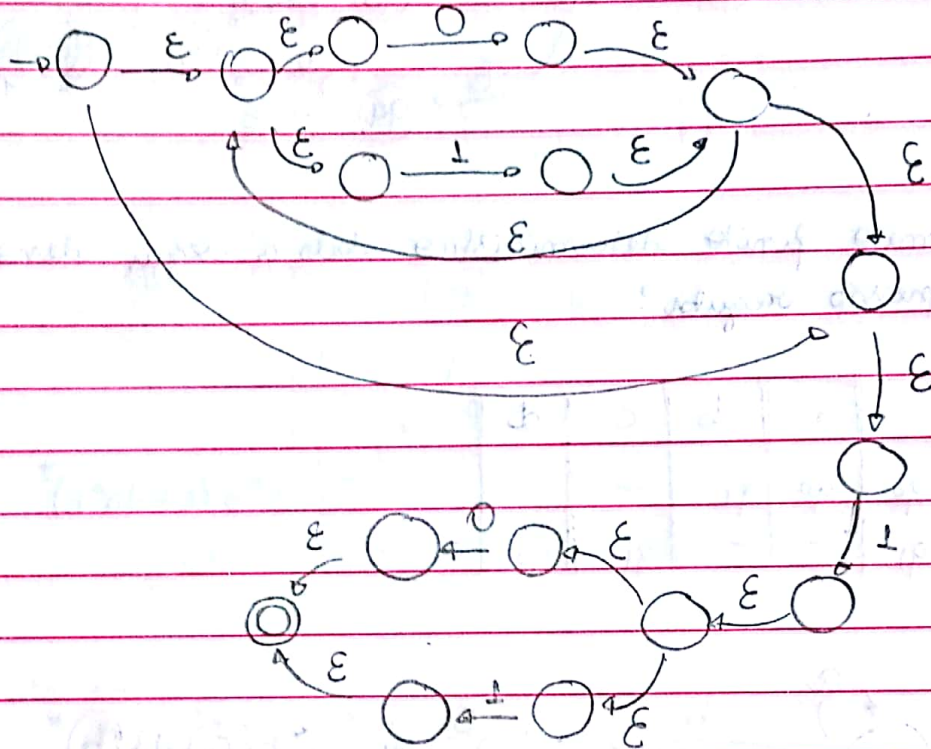$$((0+1))^* 1(0+1)(0+1) \varepsilon$$
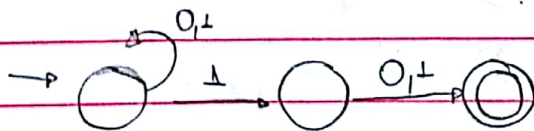$$(0+1)^* 1(0+1)(0+1)$$

• Juntando:

$$(0+1)^* 1(0+1) + (0+1)^* 1(0+1)(0+1)$$
$$(0+1)^* 1(0+1) (\varepsilon + 0+1)$$

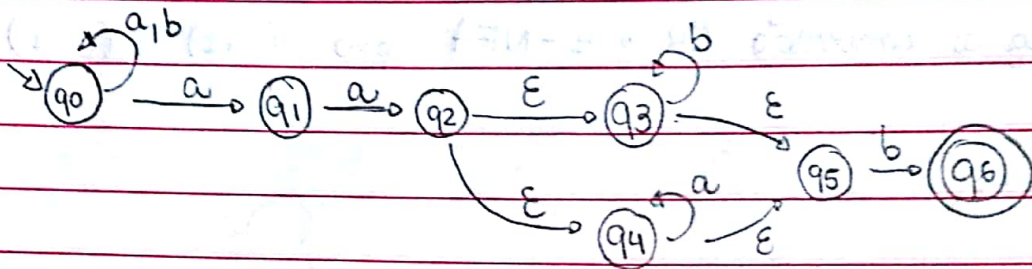④ Faça a conversão ER → ε-NFA para $(0+1)^* 1 (0+1)$

ou

⑤ Considere a expressão regular $(a+b)^*$ aa $(b^* + a^+) b$ que denota o conjunto L6

a) Enumere o conjunto L6 com os 10 primeiros elementos

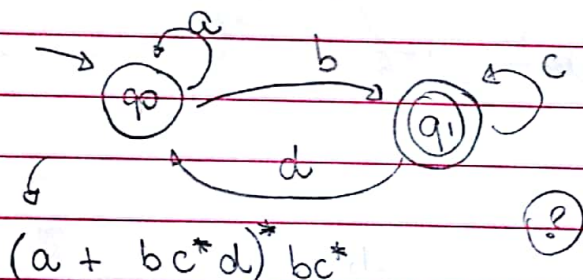L6 = { aab, aaab, baab, aabb, aaaab, baaab, aaabb, aabbb, aaaaab, aaaaab }

b) Construa um autômato finito que reconheça as cadeias de L6

⑥ O autômato finito determinístico dado a seguir denota igual expressão regular?

	a	b	c	d
→ q_0	q_0	q_1	−	−
* q_1	−	−	q_1	q_0

$$a^* b (c + d a^* b)^*$$

ou $a^* b (\overset{*}{c} + d a^* b)^*$

$$(a + b c^* d)^* b c^*$$

(?)

⑦ Dê o autômato finito e a expressão regular correspondentes.

	ε	a	b	c
→ {q_0}	{q_1}	{q_0}	∅	∅
{q_1}	{q_2}	∅	{q_1}	∅
* {q_2}	{q_1}	∅	∅	{q_2}

ECLOSE (q_0) = {q_0, q_1, q_2}
ECLOSE (q_1) = {q_1, q_2}
ECLOSE (q_2) = {q_1, q_2}

	a	b	c
→{q0, q1, q2}*	{q0, q1, q2}	{q1, q2}	{q1, q2}
{q1, q2}*	{ }	{q1, q2}	{q1, q2}
{ }	{ }	{ }	{ }

Fecha em A:

Fecha em B:

$$(a + (b+c)(b+c)^* \emptyset)^* (b+c)(b+c)^*$$
$$(a)^* (b+c)^+$$
$$a^* (b+c)^+$$

juntando:

$$a + a^* (b+c)^*$$

⑧ Dê o autômato finito e a expressão regular correspondentes: