Respostas - Lista 01 de Exercícios - Geometria Analítica

1. Considere as matrizes

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 2 \\ 6 & 4 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) Calcule $3(A - \frac{1}{2}B) + C$

RESPOSTA:

$$3(A - \frac{1}{2}B) + C = \begin{bmatrix} 6 & 3 & 0 \\ 3 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -3 \\ -9 & -6 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 5 & -3 \\ -6 & 1 & 0 \end{bmatrix}$$

(b) Encontre uma matriz X tal que $\frac{1}{2}(X + A) = 3(X + (B - A)) - C$

RESPOSTA:

$$\frac{1}{2}(X + A) = 3(X + (B - A)) - C \Leftrightarrow X + A = 6X + 6B - 6A - 2C$$

$$\Leftrightarrow 5X = 7A - 6B + 2C \Leftrightarrow X = \frac{1}{5}(7A - 6B + 2C) = \cdots = \begin{bmatrix} 4 & \frac{11}{5} & -\frac{12}{5} \\ -\frac{29}{5} & -\frac{8}{5} & -1 \end{bmatrix}$$

2. Sejam A e B matrizes tais que

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -4 & -4 \\ 5 & 6 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -5 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & -4 \end{bmatrix}$$

(a) Calcule AB, BA e $A^2 = AA$.

RESPOSTA:

$$AB = \begin{bmatrix} 5 & 7 & -15 \\ -12 & 0 & 20 \\ 17 & 7 & -35 \end{bmatrix} \quad BA = \begin{bmatrix} 27 & 30 & 33 \\ -22 & -24 & -26 \\ -27 & -30 & -33 \end{bmatrix} \quad A^2 = \begin{bmatrix} 8 & 12 & 16 \\ -8 & -16 & -24 \\ 16 & 28 & 40 \end{bmatrix}$$

(b) Calcule $(AB)^t$.

RESPOSTA:

$$(AB)^t = B^t A^t = \begin{bmatrix} 2 & 0 & 1 \\ -5 & 3 & 2 \\ 1 & -2 & -4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 5 \\ 2 & -4 & 6 \\ 3 & -4 & 7 \end{bmatrix} = \begin{bmatrix} 5 & -12 & 17 \\ 7 & 0 & 7 \\ -15 & 20 & -35 \end{bmatrix}$$

3. Nas matrizes A e B abaixo complete as entradas ausentes para fazer sentido o produto das matrizes AB.

$$\begin{bmatrix} 1 & 2 & -3 \\ \dots & -1 & \dots \\ 9 & -13 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 & 4 \\ 1 & -3 & \dots \\ 5 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -11 & \dots & 12 \\ 28 & 4 & 11 \\ 15 & 23 & -51 \end{bmatrix}$$

RESPOSTA:

Denotemos estas entradas por x, y, z e w.

$$\begin{bmatrix} 1 & 2 & -3 \\ x & -1 & y \\ 9 & -13 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 & 4 \\ 1 & -3 & z \\ 5 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -11 & w & 12 \\ 28 & 4 & 11 \\ 15 & 23 & -51 \end{bmatrix}$$

Assim efetuando-se o produto obtemos

$$\begin{bmatrix} 1 & 2 & -3 \\ x & -1 & y \\ 9 & -13 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 & 4 \\ 1 & -3 & z \\ 5 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -11 & -11 & -2+2z \\ 2x-1+5y & -2x+3+y & 4x-z+2y \\ 15 & 23 & 40-13z \end{bmatrix}$$

Comparando as duas matrizes

$$\begin{bmatrix} -11 & w & 12 \\ 28 & 4 & 11 \\ 15 & 23 & -51 \end{bmatrix} = \begin{bmatrix} -11 & -11 & -2+2z \\ 2x-1+5y & -2x+3+y & 4x-z+2y \\ 15 & 23 & 40-13z \end{bmatrix}$$

obtemos que $x = 2$, $y = 5$, $z = 7$ e $w = -11$.

4. Seja α um número real e defina a matriz T_α por

$$T_\alpha = \begin{bmatrix} \cos(\alpha) & -\text{sen}\,(\alpha) \\ \text{sen}\,(\alpha) & \cos(\alpha) \end{bmatrix}.$$

Mostre que

(a) $T_\alpha \cdot T_\beta = T_{\alpha+\beta}$.

RESPOSTA:

$$\begin{aligned} T_\alpha \cdot T_\beta &= \begin{bmatrix} \cos(\alpha) & -\text{sen}\,(\alpha) \\ \text{sen}\,(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} \cos(\beta) & -\text{sen}\,(\beta) \\ \text{sen}\,(\beta) & \cos(\beta) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\alpha)\cos(\beta) - \text{sen}\,(\alpha)\text{sen}\,(\beta) & -\cos(\alpha)\text{sen}\,(\beta) - \text{sen}\,(\alpha)\cos(\beta) \\ \text{sen}\,(\alpha)\cos(\beta) + \text{sen}\,(\beta)\cos(\alpha) & \cos(\alpha)\cos(\beta) - \text{sen}\,(\alpha)\text{sen}\,(\beta) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\alpha+\beta) & -\text{sen}\,(\alpha+\beta) \\ \text{sen}\,(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = T_{\alpha+\beta}, \end{aligned}$$

lembrando que

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \text{sen}\,(\alpha)\text{sen}\,(\beta)$$
$$\text{sen}\,(\alpha \pm \beta) = \text{sen}\,(\alpha)\cos(\beta) \pm \text{sen}\,(\beta)\cos(\alpha).$$

(b) $T_{-\alpha} = T_\alpha{}^t$.

RESPOSTA:

$$T_{-\alpha} = \begin{bmatrix} \cos(-\alpha) & -\text{sen}\,(-\alpha) \\ \text{sen}\,(-\alpha) & \cos(-\alpha) \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \text{sen}\,(\alpha) \\ -\text{sen}\,(\alpha) & \cos(\alpha) \end{bmatrix} = T_\alpha^t,$$

lembrando que

$$\cos(-\alpha) = \cos(\alpha) \text{ e } \text{sen}\,(-\alpha) = -\text{sen}\,(\alpha).$$

5. Verifique se cada uma das proposições seguintes é verdadeira ou falsa, justificando sua resposta.

(a) Sejam $A_{n\times n}$ e $B_{n\times n}$ matrizes quadradas. Então $(A+B)^2 = A^2 + 2AB + B^2$.

RESPOSTA: FALSO (vide exemplos dados na aula)

(b) Se $AB = \mathbf{0}$ então ou a matriz $A = \mathbf{0}$ ou $B = \mathbf{0}$ (aqui $\mathbf{0}$ significa a matriz nula).

RESPOSTA: FALSO (vide exemplo dado na aula)