# Sensor Fusion

Robótica Móvel e Inteligente

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# Outline

- Introduction to Sensor Fusion
- Kalman Filter
- Particle Filter
- Conclusion

# Sensor Fusion

- **Act of combining sensory data** or data derived from sensory data from disparate sources
- **The resulting information is “better“** than it would be possible when the sources were used individually
- **“Better”** is defined according to the context. Can be **more accurate**, **more complete**, **a different view**, etc.
- Using a broader definition, we can speak of **Information Fusion**
- Sensor Fusion is usually considered a subset of information fusion, although the terms are often used with the same meaning
- Another very used term for this task is Multi-Sensor Data Fusion

# Sensor Fusion

- Usually based on **modeling the sensors** and **modeling the system** being measured
- Most common methodologies are **probability based**
- However, some methodologies try to overcome some limitations of probability models (complexity, inconsistency, precision of models, uncertainty about uncertainty)
- Some of these methodologies are:
  - Interval calculus
  - Fuzzy logic
  - Theory of evidence (Dempster-Shafer methods)

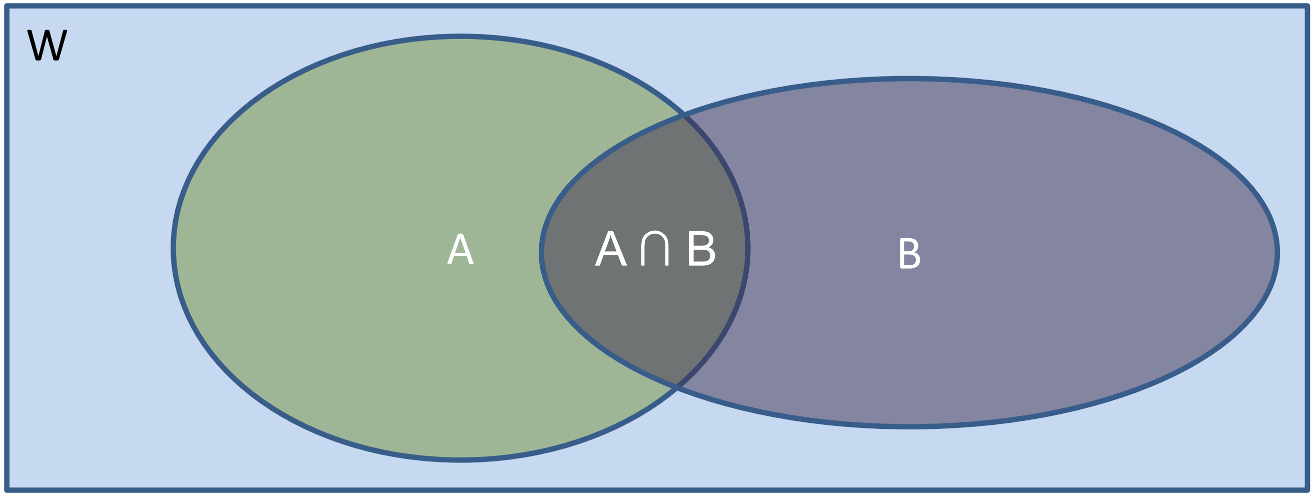
# Bayes Rule

- Determine the probability of a state/event, given the result of other states/events that are related.

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

- P(A | B) is the probability of A given that B is true, called **conditional probability**
- P(B | A) is the probability of B given that A is true, called conditional probability
- P(A) and P(B) are the **marginal probabilities** of A and B

# Bayes Rule




- Consider that the probability of a given event is related to its area in the above diagram, and that $area(W) = 1$
- $P(A) = area(A) \quad P(B) = area(B)$
- $P(A \mid B) = \dfrac{area(A \cap B)}{area(B)} = \dfrac{\frac{area(A \cap B)}{area(A)} . \frac{area(A)}{area(W)}}{area(B)} = \dfrac{P(B \mid A) . P(A)}{P(B)}$

# Bayes Rule

- Applying Bayes Rule to determine the probability of being sick if the test for the disease is positive (+).
  - Assumptions:
    - Tests are correct 99% (test are positive with 99% probability if the person is sick, and negative with 99% if the person is not sick)
    - Disease is rare, happening only 1 in every 10000 people
  - If a person is tested and result is positive which is the probability of being sick?

https://math.hmc.edu/funfacts/medical-tests-and-bayes-theorem/

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$P(\text{sick} \mid +) = P(+ \mid \text{sick}).P(\text{sick}) / P(+)$

$P(+ \mid \text{sick}) = 0.99 \qquad P(\text{sick}) = 1/10000$

$P(+) = P(+ \mid \text{sick}).P(\text{sick}) + P(+ \mid \text{not sick}).P(\text{not sick})$

$\quad = 0.99 \times 1/10000 + 0.01 \times 9999/10000 = 0.010098$

$P(\text{sick} \mid +) = 0.99 \times 1/10000 / 0.010098 = 0.009804 \approx \mathbf{1\%}$

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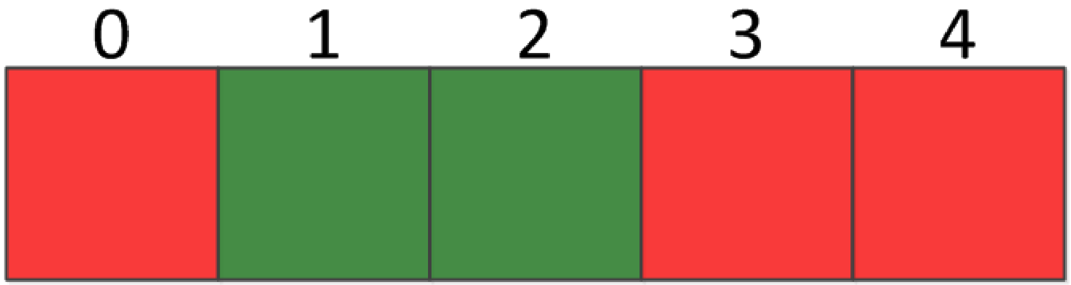
# Bayesian Filter

- Application of the **Bayes' Rule** when:
  - $X_t$ is the state vector at time ***t***.
  - $U_t$ is the control vector used to drive from state $X_{t-1}$ to $X_t$ .
  - $Z_t$ is the observation of the state at time ***t***
- At an instant ***t*** an estimation of the current state can be achieved by application of Bayes' Rule:

$$P(X_t|Z_t, U_t) = \frac{P(Z_t|X_t)P(X_t|Z_{t-1}, U_t)}{P(Z_t|Z_{t-1}, U_t)}$$

# Grid World

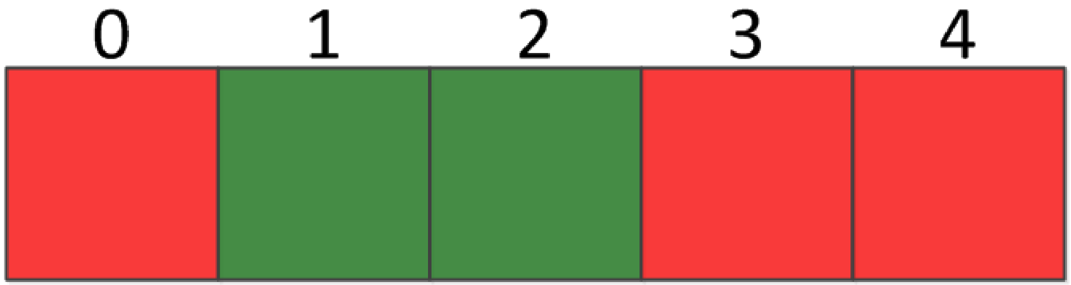
- Consider a grid world:



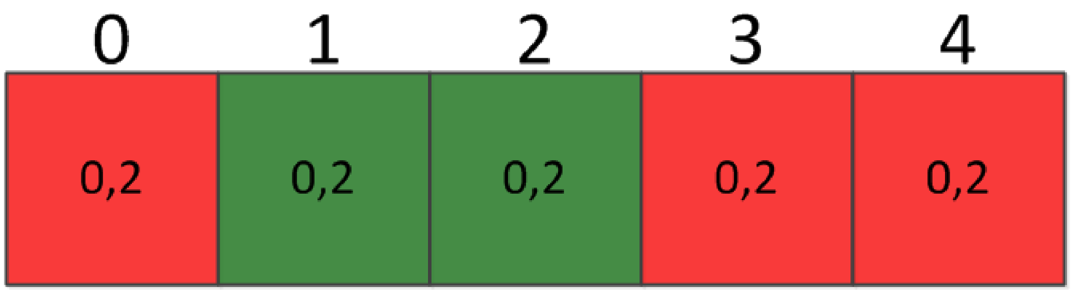

- Where is the robot?

# Grid World

- Consider a grid world:




- Position probability distribution:



# Grid World

- The robot has a color sensor
- Sensor model

  $G_M$: green measure; $R_M$: red measure

  $G_C$: green cell; $R_C$: red cell

  $P(G_M|G_C) = 0.6$; $P(R_M|G_C) = 1\text{-}P(G_M|G_C) = 0.4$

  $P(G_M|R_C) = 0.2$; $P(R_M|R_C) = 1\text{-}P(G_M|R_C) = 0.8$

- The robot measures green
- Which is the new position estimate?

# Grid World

- Multiply cell values by $P(G_M|G_C)$ and $P(G_M|R_C)$

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 0,04 | 0,12 | 0,12 | 0,04 | 0,04 |

$$\sum P = 0{,}36$$

- Normalize

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 0,11 | 0,33 | 0,33 | 0,11 | 0,11 |

- **We have just applied Bayes' Rule!**

# Grid World – Measurement Integration

- Bayes' Rule

$$P(Xp|M) = \frac{P(M|Xp) * P(Xp)}{P(M)}$$

- $P(X_p)$ is the estimate before measurement integ.
- $P(X_p|M)$ is the posterior estimate
- $P(M)$ does not depend on $X_p$, so it can be considered as a constant that performs normalization
- **Measurement integration** is performed using the **product** of sensor model and previous estimate

# Grid World - Moving

- If the robot (tries) to move
- Action model

  $A_R$: Move Right action;

  $P(X{+}1 \mid A_R, X) = 0.8$; $P(X \mid A_R, X) = 0.2$
- Initial belief

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 0,5 | 0,5 | 0,0 | 0,0 | 0,0 |

# Grid World - Moving

- $P(1) = P(1 \mid A_R,0)*P(0) + P(1 \mid A_R,1)*P(1)$
- Predicted belief

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 0,1 | 0,5 | 0,4 | 0,0 | 0,0 |

- When the **robot moves** belief is updated through **convolution**

# Sensor Fusion

- Continuous environments
  - Measurement Integration (Product)

$$bel(X)_t = \eta \; p(Z_t|X) \; \hat{bel}(X)_t$$

  - Motion update (Convolution)

$$\hat{bel}(X)_t = \int p(X|U_t, X')bel(X')_{t-1}dX'$$

# Kalman Filter

- Assumptions
  - **Linear model** (transition, action and sensor)
  - **Every estimate is a gaussian**
    - Can be characterized by **mean** and **variance**
  - **Noise is gaussian** (mean=0)
- Integration of measures over time
- **Markovian assumption**
  - The next estimate only depends on the previous estimate
- Considers **action model** and **physics/observation model**

# Kalman Filter

- **Product** of gaussians distributions (**measurement integration**)

$$\mu_P = \frac{\mu_1.{\sigma_2}^2+\mu_2.{\sigma_1}^2}{{\sigma_1}^2+{\sigma_2}^2} \qquad {\sigma_P}^2 = \frac{{\sigma_1}^2.{\sigma_2}^2}{{\sigma_1}^2+{\sigma_2}^2}$$

- **Convolution** of gaussians distributions (**motion forecast**)

$$\mu_C = \mu_1 + \mu_2 \qquad {\sigma_C}^2 = {\sigma_1}^2 + {\sigma_2}^2$$

# Kalman Filter – Motion Model

$$\hat{X}_t = F_t \hat{X}_{t-1} + B_t U_t + \omega_t$$

- $\hat{X}_t$ is the **estimated state**
- $F_t$ is the **state transition model**
- $B_t$ is the **control-input model**
- $U_t$ is the **control vector**
- $w_t$ is the **process noise** with covariance

$$Q_t : \omega_t \sim N(0, Q_t)$$

# Kalman Filter – Observation Model

$$\hat{Z}_t = H_t X_t + \upsilon_t$$

- $\hat{z}_t$ is the **measurement** taken at time t
- $H_t$ is the **observation model** of the state/event
- $v_t$ is the **observation noise** with covariance:

$$R_t : \upsilon_t \sim N(0, R_t)$$

# Kalman Filter – Implementation

- The filter state is represented by two variables:
  - $X_t$ is the **estimate of the state** at time t
  - $P_t$ is the **measure of estimated accuracy** of the process
- The filter works in **two steps**:

**Forecast**

$$\overline{X}_t = F_t X_{t-1} + B_t U_t$$

$$\overline{P}_t = F_t P_{t-1} F_t^T + Q_t$$

**Measurement integration**

$$K_t = \frac{\overline{P}_t H_t^T}{H_t \overline{P}_t H_t^T + R_t}$$

$$X_t = \overline{X}_t + K_t (Z_t - H_t \overline{X}_t)$$

$$P_t = (I - K_t H_t) \overline{P}_t$$

# Particle Filter (Monte Carlo)

- Integration of measures over time
- Is **non-parametric** and thus can cope with a several types of distributions, rather than just Gaussian
- The samples of the state are called **particles**
- **Each particle** is a **concrete instantiation of the state** at time t
- This filter also works in **two main steps**, applied to every single particle:
  - **Evolution** of the current state, **weighing** and **creation** of a dataset
  - **Resampling** of the dataset for the next evaluation

# Particle Filter (Monte Carlo)

- Consider M the number of particles used and $X_t$ the particle set at time t

**Evolution and weighing**

for $m = 0$ to $M$

$x_t^{[m]} \sim p(x_t | u_t, x_{t-1}^{[m]})$

$w_t^{[m]} = p(z_t | x_t^{[m]})$

$\overline{X}_t = \overline{X}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

**Resampling**

for $m = 0$ to $M$

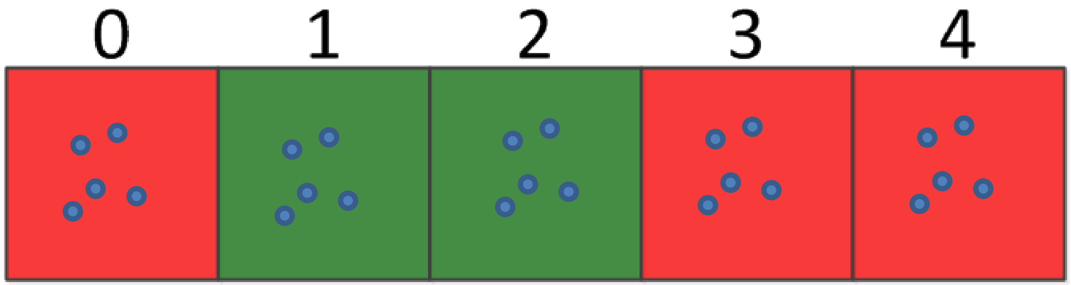
draw $i$ with prob $\propto w_t^{[i]}$

add $x_t^{[i]}$ to $X_t$

endfor

# Grid World - Weighing

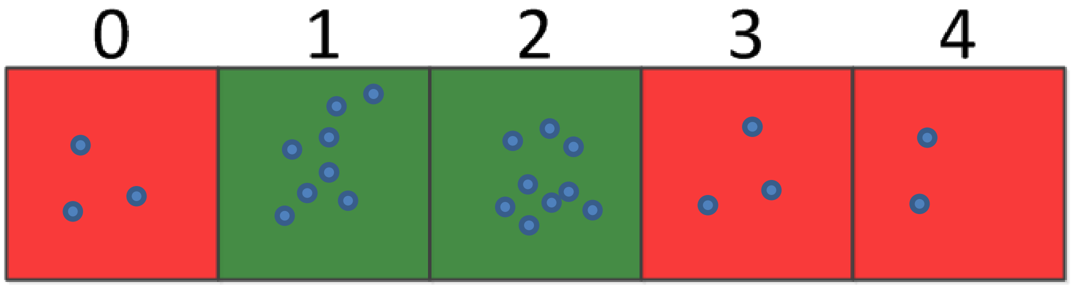
- Initial particle distribution:




- If green is seen, particles in green cells have higher weights
- They will have higher chances of getting into the new particle set

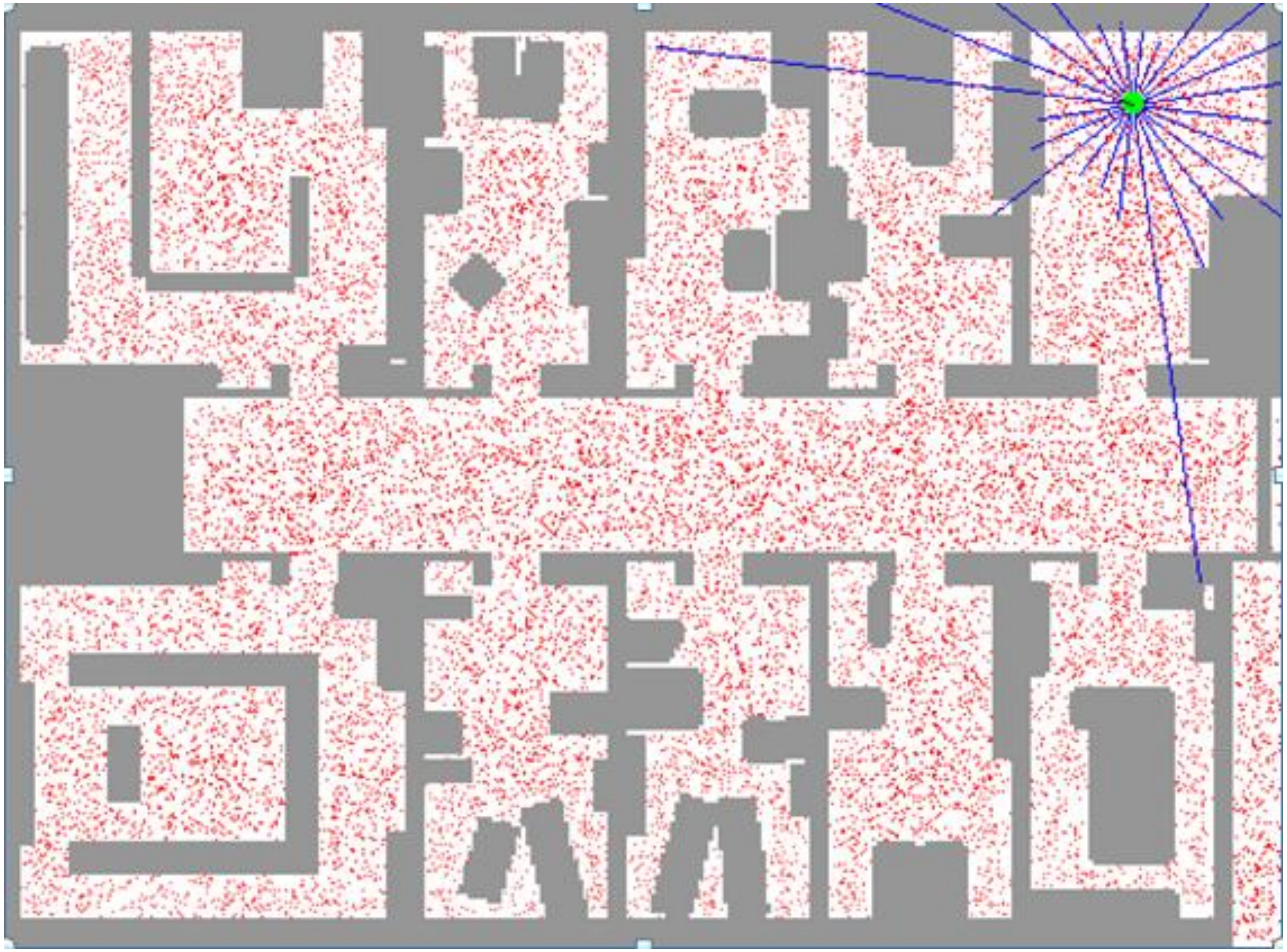
# Grid World - Resampling

- After green is seen:




- The motion model may be applied directly to each particle

# Particle Filter (Monte Carlo)



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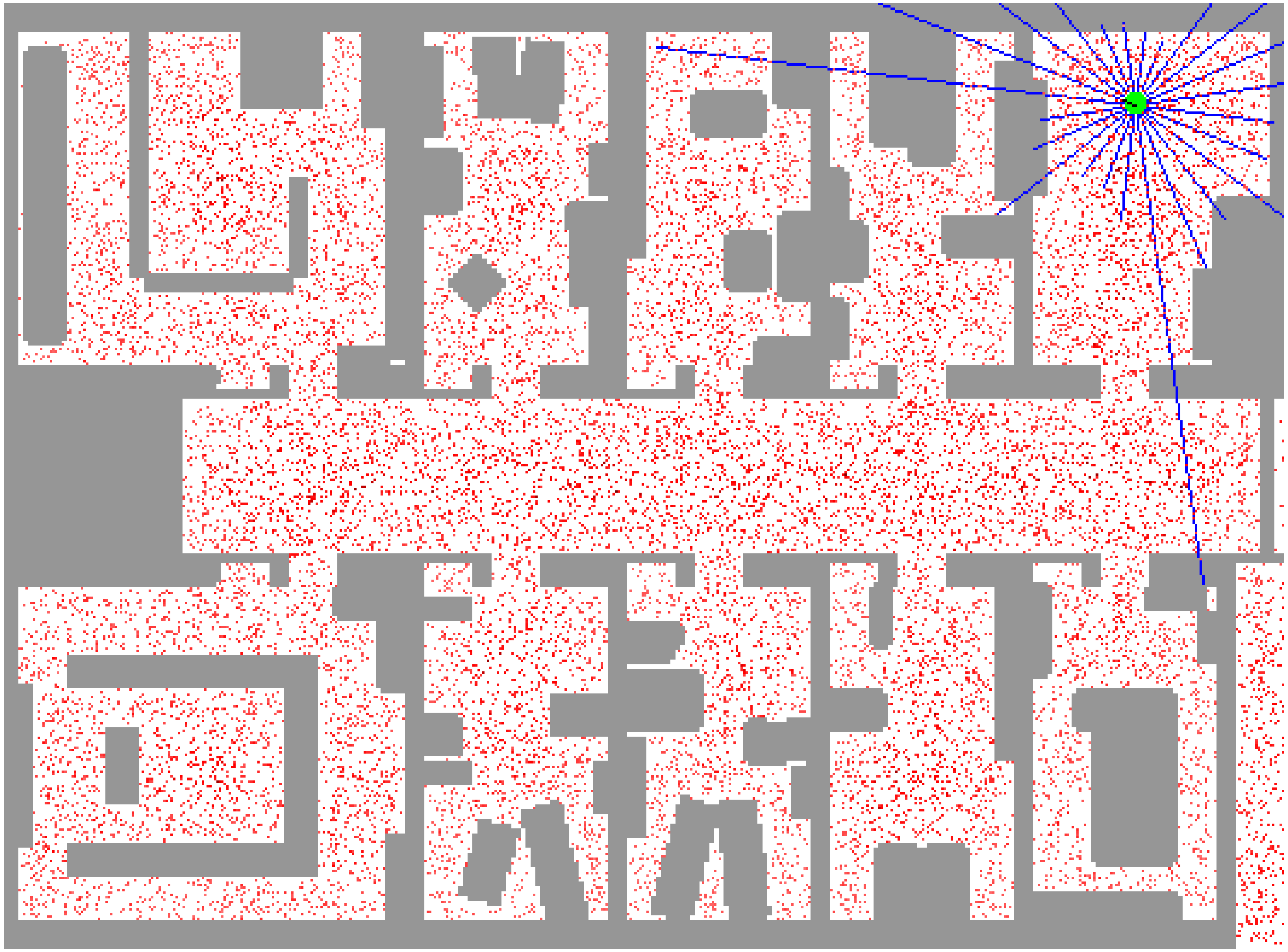

https://rse-lab.cs.washington.edu/projects/mcl/animations/global-floor.gif

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