8ª lista de exercícios

① Usando a definição calcule a derivada das funções:

a) $f(x) = 1 - 4x^2$ R: $f'(x) = -8x$

b) $f(x) = \dfrac{1}{\sqrt{2x-1}}$ R: $f'(x) = \dfrac{-1}{\sqrt{(2x-1)^3}}$

c) $f(x) = \dfrac{1}{x+2}$ R: $f'(x) = \dfrac{-1}{(x+2)^2}$

② Dada a função $f(x) = \begin{cases} x-1 \; ; & x \geq 0 \\ x \; ; & x < 0 \end{cases}$

Verificar se existe $f'(0)$. R: não.

③ Dada a função $f(x) = 2x^2 - 3x - 2$, determinar os intervalos em que:

a) $f'(x) > 0$ R: $x > \frac{3}{4}$

b) $f'(x) \geq 0$ R: $x < \frac{3}{4}$

④ Seja $f(x) = \begin{cases} x^2 - 1 \; ; & \text{se } |x| \leq 1 \\ 1 - x^2 \; ; & \text{se } |x| > 1 \end{cases}$

a) Verificar se f é contínua nos pontos -1 e 1
R: sim

b) Calcular:

$f'(-1-)$ R: 2

$f'(-1+)$ R: -2

$f'(1-)$ R: 2

$f'(1+)$ R: -2

5) Calcule a derivada das funções dadas:

1) $f(r) = \pi r^2$ R: $f'(r) = 2\pi r$

2) $f(x) = 3x^2 + 6x - 10$ R: $f'(x) = 6x + 6$

3) $f(x) = 14 - \frac{1}{2} x^{-3}$ R: $f'(x) = \frac{3}{2} x^{-4}$

4) $f(x) = (2x+1)(3x^2+6)$ R: $f'(x) = 18x^2 + 6x + 12$

5) $f(x) = (x-1)(x+1)$ R: $f'(x) = 2x$

6) $f(x) = \dfrac{x-1}{x+1}$ R: $f'(x) = \dfrac{2}{(x+1)^2}$

7) $f(x) = \dfrac{3}{x^4} + \dfrac{5}{x^5}$ R: $-\dfrac{12}{x^5} - \dfrac{25}{x^6} = f'(x)$

8) $f(x) = 10(3x^2+7x-3)^{10}$ R: $f(x) = 100(3x^2+7x-3)^9 . (6x+7)$

9) $f(x) = \frac{1}{a}(bx^2+ax)^3$ R: $f'(x) = \frac{3}{a}(bx^2+ax)^2(2bx+a)$

10) $f(x) = (4x^2-5x+2)^{-\frac{1}{3}}$ R: $f'(x) = -\frac{1}{3}(4x^2-5x+2)^{-\frac{4}{3}}(8x-5)$

11) $f(t) = \sqrt{\dfrac{2t+1}{t-1}}$ R: $f'(t) = -\dfrac{3}{2} \cdot \dfrac{1}{\sqrt{2t+1}\,\sqrt{(t-1)^3}}$

12) $f(x) = 2e^{(3x^2+3x)}$ R: $f'(x) = 2e^{(3x^2+3x)} . (6x+3)$

13) $f(x) = \frac{1}{3} e^{(3-x)}$ R: $f'(x) = -\frac{1}{3} e^{(3-x)}$

14) $f(x) = \left(\frac{1}{2}\right)^{-\ln(2x)}$ R: $f'(x) = \ln 2 \cdot \left(\frac{1}{2}\right)^{-\ln(2x)}$

15) $f(t) = \frac{e^{-t^2}+1}{t}$ R: $f'(t) = \frac{-e^{-t^2}(1+2t^2) - 1}{t^2}$

16) $f(t) = \left(\frac{a}{b}\right)^{\sqrt{t}}$ R: $f'(t) = \left(\frac{a}{b}\right)^{\sqrt{t}} \ln\left(\frac{a}{b}\right) \frac{1}{2\sqrt{t}}$

17) $f(x) = \left(e^{x^2}+4\right)^{\sqrt{x}}$ R: $f'(x) = \left(e^{x^2}+4\right)^{\sqrt{x}} \cdot \ln\left(e^{x^2}+4\right) \frac{1}{2\sqrt{x}} + 2x\sqrt{x}\left(e^{x^2}+4\right)^{(\sqrt{x}-1)} e^{x^2}$

18) $f(x) = \log_2(2x+4)$ R: $f'(x) = \frac{2}{2x+4} \log_2 e$

19) $f(x) = \ln\left(\frac{1}{x} + \frac{1}{x^2}\right)$ R: $f'(x) = \frac{-(x+3)}{(x+1)x}$

20) $f(x) = x^a \; ; \; a \in \mathbb{R}$ R: $f'(x) = a\, x^{a-1}$

21) $f(x) = e^x \ln x$ R: $f'(x) = e^x\left(\ln x + \frac{1}{x}\right)$

22) $f(x) = e^{-x}$ R: $f'(x) = -e^{-x}$