Algebraic topology 1

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Contents

Introduction

These are my lecture notes on the course Algebraic topology 1 in the year 2023/24. The lecturer that year was prof. dr. Petar Pavešić.

The notes are not perfect. I did not write down most of the examples that help with understanding the course material. I also did not formally prove every theorem and may have labeled some as trivial or only wrote down the main ideas.

I have most likely made some mistakes when writing these notes – feel free to correct them.

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1 Basic homotopy theory

1.1 Definition

Definition 1.1.1. Continuous maps $f, g\colon X \to Y$ of topological spaces are *homotopic*, if there is a continuous map $H\colon X \times I \to Y$, such that $H(x, 0) = f(x)$ and $H(x, 1) = g(x)$. Such H is called a *homotopy*. We write $H\colon f \simeq g$.

Remark 1.1.1.1. If X is a locally compact and Hausdorff space, homotopies coincide with paths in the space $\mathcal{C}(X, Y)$.

Proposition 1.1.2. Homotopy is an equivalence relation on $\mathcal{C}(X, Y)$.

Proof. The proof is obvious and need not be mentioned. □

Definition 1.1.3. We denote the set of equivalence classes of the homotopy relation on $\mathcal{C}(X, Y)$ by $[X, Y]$.

Remark 1.1.3.1. If X is a locally compact and Hausdorff space, $[X, Y]$ is the set of path components of $\mathcal{C}(X, Y)$.

Definition 1.1.4. With $f\colon (X, A) \to (Y, B)$ we denote maps $f\colon X \to Y$ such that $f(A) \subseteq f(B)$. Similarly, we define $\mathcal{C}((X, A), (Y, B))$ and $[(X, A), (Y, B)]$.

Definition 1.1.5. Let $A \subseteq X$ and $f, g\colon X \to Y$ be maps, such that $f|_A = g|_A$. $G\colon X \times I \to Y$ is a *homotopy relative to A* if $H\colon f \simeq g$ and $H_t|_A = f|_A$ for all $t \in I$.

Index