Problems

Chapter-1

(1) Find the interval in which the smallest positive root of the equation :- $\tan x + \tanh x = 0$. Determine the roots correct to two decimals using the bisection method?

(2) Show that the equation $\log_e x = x^2 - 1$ has exactly two real roots $\alpha_1 = 0.45$ and $\alpha_2 = 1$.

(3) Apply the Newton-Raphson method with $x_0 = 0.8$, the secant method with $x_0 = 0.8$, $x_1 = 1.2$ and the Mueller method with $x_0 = 0.6$, $x_1 = 0.8$, $x_2 = 1.2$ to the equation

$$f(x) = x^3 - x^2 - x + 1 = 0$$

(4) Find all the roots of $\cos x - x^2 - x = 0$ to five decimal places.

Chapter-2

① Calculate $f(A) = e^{A} - e^{-A}$ where $A = \begin{bmatrix} 2 & 4 & 0 \\ 6 & 0 & 8 \\ 0 & 3 & -2 \end{bmatrix}$

② Solve

$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 + 5x_3 &= -3 \\ 3x_1 + 2x_2 - 3x_3 &= 6 \end{aligned}$$

by (a) Gauss Elimination with partial pivoting

(b) by decomposition method.

③ $A = I + L + U$ where $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

L and U are strictly ~~upper~~ lower and ~~lower~~ upper triangular matrices

Decide whether (a) Jacobi and (b) Gauss-Seidel methods converge to the solution $Ax = b$.

④ $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 1 & 2 \\ -1 & 2 & -1 \end{bmatrix}$ Find A^{-1} using Cholesky method.

⑤

$$\begin{aligned} 3x + 2y &= 4.5 \\ 2x + 3y - z &= 5 \\ -y + 2z &= -0.5 \end{aligned}$$

(a) set up the SOR iterative scheme for the solution

(b) find the optimal relaxation factor

(c) Iterate '3' times.

(6)

$$A = \begin{bmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{bmatrix}, \quad |\varepsilon| \le 1$$

Show that the smallest Eigen value of the matrix is equal to $\lambda_1 = \varepsilon^2 + O(\varepsilon^{5/2})$

(7)

$$A = \frac{1}{9}\begin{bmatrix} 4 & 1 & -8 \\ 7 & 4 & 4 \\ 4 & -8 & 1 \end{bmatrix}$$

Compute A^{10}.

(x) Corresponding Eigen vector. Estimate the Error also.

(8) Find all the Eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ using Householder's method.

(9) Find the approximate Eigenvalue of $A = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$ using Rutihauser's method.

(10) Calculate an approximate least Eigen Value of $A = LL^T$ where $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

First approximation is $\begin{bmatrix} 6 \\ -7 \\ 3 \end{bmatrix}$ to the (x)

Interpolation & Differentiation.

(1) use Lagrangian and Newton-divided difference methods to calculate $f(3)$ from the table :-

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

(2) Obtain the unique polynomial $P(x)$ of degree 5 or less corresponding to a function $f(x)$

where $f(x_0)=1$, $f'(x_0)=2$ $f''(x_0)=1$

$f(x_1)=3$, $f'(x_1)=0$ $f''(x_1)=-2$

also find $P\left(\frac{(x_0+x_1)}{2}\right)$

(3) Fit the following four points by the cubic splines:

i	0	1	2	3
x_i	1	2	3	4
y_i	1	5	11	8

use end conditions

$y_0'' = y_3'' = 0$.

(4) Use the following data, find $f'(6.0)$ error $= o(h)$ and $f''(6.3)$ error $o(h^2)$

use both Lagrangian as well as Newton's methods

x	6.0	6.1	6.2	6.3	6.4
$f(x)$	0.1750	−0.1998	−0.2223	−0.2422	−0.2596

(5) $\int_0^1 (\cos 2x)(1-x^2)^{-1/2}\,dx$ Calculate the integral to 4 decimal places.

ODES

1. Find y_j from the difference Equation:

$$\Delta^2 y_{j+1} + \frac{1}{2}\Delta^2 y_j = 0, \quad j = 0, 1, 2 \ldots$$

when $y_0 = 0$, $y_1 = \frac{1}{2}$, $y_2 = \frac{1}{4}$. Is this numerical method stable?

2. Consider the initial value problem

$$y' = x(y+x) - 2, \quad y(0) = 2$$

Use Euler method with step sizes $h = 0.3$, $h = 0.2$ and $h = 0.15$ to compute approximations to $y(0.6)$ [5 decimals].

3. Use the Classical Runge-Kutta formula of fourth order to find the numerical solution at $x = 0.8$ for

$$\frac{dy}{dx} = \sqrt{x+y}, \quad y(0.4) = 0.41$$

Assume the step length $h = 0.2$

4. Find the solution at $t = 0.3$ for the differential Equation

$$y' = t - y^2 , \quad y(0) = 1$$

by the Adams-Bashforth method of order two with $h = 0.1$. Determine the starting values using a second Order Runge-Kutta method.

5. Given the Equation

$$y' = x + \sin y \qquad y(0) = 1$$

Show that it is sufficient to use Euler's method with the ~~[illegible]~~ step $h = 0.2$ to compute $y(0.2)$ with an Error less than 0.05.