-

$$-\ddot{x} + 2t^{-10}x^3 = -10t^2 \tag{1}$$
$$x(1) = 1,\ x(2) = 16 \tag{2}$$

: $x^*(t) = t^4$

$$\ddot{x}(t) + f(t,\ x_0(t),\ \dot{x}_0(t)) = 0 \tag{3}$$

$$\begin{aligned}- \ddot{x}_{n+1}(t) + f'_x(t,\ x_0(t),\ \dot{x_0}(t))\ x_{n+1}(t) + f'_{\dot{x}}(t,\ x_0(t),\ \dot{x}_0(t))\ \dot{x}_{n+1}(t) = \\ f'_x(t,\ x_0(t),\ \dot{x_0}(t))\ x_n(t) + f'_{\dot{x}}(t,\ x_0(t),\ \dot{x}_0(t))\ \dot{x}_n(t) - f(t,\ x_n(t),\ \dot{x}_n(t))\end{aligned} \tag{4}$$

$$x_{n+1}(a) = \alpha_0,\ x_{n+1}(b) = \alpha_1 \tag{5}$$

(1)-(2) :

$$-\ddot{x} + 2t^{-10}x^3 + 10t^2 = 0$$
$$f(t,\ x(t),\ \dot{x}(t)) = 2t^{-10}x^3(t) + 10t^2$$
$$f(t,\ x_0(t),\ \dot{x}_0(t)) = 2t^{-10}x_0^3(t) + 10t^2$$
$$f'_x(t,\ x_0(t),\ \dot{x}_0(t)) = 6t^{-10}x_0^2(t)$$
$$f'_{\dot{x}}(t,\ x_0(t),\ \dot{x}_0(t)) = 0$$

$$-\ddot{x}_{n+1}(t) + 6t^{-10}x_0^2(t)x_{n+1}(t) = 6t^{-10}x_0^2(t)x_n(t) - 2t^{-10}x_0^3(t) - 10t^2 \tag{6}$$
$$x_{n+1}(1) = 1,\ x_{n+1}(2) = 32 \tag{7}$$

$x_{n+1}(t) = y_{n+1}(t) + G(t),\ \ G(t)$:

$$G(t) = \alpha\frac{b-t}{b-a} + \beta\frac{t-a}{b-a}$$
$$G(t) = 1(2-t) + 16(t-1)$$
$$G(t) = 2 - t + 16t - 16$$

$$G(t) = 15t - 14 \tag{8}$$

$$\begin{aligned}- \ddot{y}_{n+1}(t) + 6t^{-10}(y_0(t) + G(t))^2\ (y_{n+1}(t) + G(t)) = 6t^{-10}(y_0(t) + G(t))^2\ (y_n(t) + G(t)) \\ - 2t^{-10}(y_0(t) + G(t))^3 - 10t^2\end{aligned} \tag{9}$$

$$y_{n+1}(1) = 0,\ y_{n+1}(2) = 0 \tag{10}$$

(9)-(10) :

$$p(t)\ddot{y}_{n+1}(t) + q(t)\dot{y}_{n+1}(t) + r(t)y_{n+1}(t) = g_1(t, y_n(t), \dot{y}_n(t)) \tag{11}$$

$$y_{n+1}(1) = 0,\ \ y_{n+1}(2) = 0 \tag{12}$$

$$\begin{aligned} -\ddot{y}_{n+1}(t) + 6t^{-10}(y_0(t) + G(t))^2\ y_{n+1}(t) = 6t^{-10}(y_0(t) + G(t))^2\ y_n(t) \\ - 2t^{-10}(y_0(t) + G(t))^3 - 10t^2 \end{aligned} \tag{13}$$

$$y_{n+1}(1) = 0,\ \ y_{n+1}(2) = 0 \tag{14}$$

:

$$y_1^N(t) = \sum_{k=1}^{N} a_k e_k(t) \tag{15}$$

:

$$\begin{aligned} \sum_{k=1}^{N} a_k \int_1^2 [-\ddot{e}_k(t) + 6t^{-10}(y_0(t) + G(t))^2\ e_k(t)][-\ddot{e}_j(t) + 6t^{-10}(y_0(t) + G(t))^2\ e_j(t)]dt = \\ \int_1^2 [6t^{-10}(y_0(t) + G(t))^2\ y_n(t) - 2t^{-10}(y_0(t) + G(t))^3 - 10t^2] \\ [-\ddot{e}_j(t) + 6t^{-10}(y_0(t) + G(t))^2\ e_j(t)]dt, \quad j = \overline{1, N} \end{aligned} \tag{16}$$

:

$$\begin{aligned} e_k(t) &= \sin \pi k(t-1) \\ \ddot{e}_k(t) &= -\pi^2 k^2 \sin \pi k(t-1) \end{aligned} \tag{17}$$