Непрерывные математические модели

Модифицированным методом Ньютона-Канторовича найти первые два приближения к решению краевой задачи

$$-\ddot{x} + 2t^{-10} * x^3 = -10t^2 \tag{1}$$
$$x(1) = 1,\ x(2) = 32 \tag{2}$$

Точное решение: $x^*(t) = t^5$

Расчетные формулы

$$\ddot{x}(t) + f(t,\ x_0(t),\ \dot{x}_0(t)) = 0 \tag{3}$$

$$\begin{aligned} &-\ddot{x}_{n+1}(t) + f'_x(t,\ x_0(t),\ \dot{x_0}(t))\ x_{n+1}(t) + f'_{\dot{x}}(t,\ x_0(t),\ \dot{x}_0(t))\ \dot{x}_{n+1}(t) = \\ &\qquad f'_x(t,\ x_0(t),\ \dot{x_0}(t))\ x_n(t) + f'_{\dot{x}}(t,\ x_0(t),\ \dot{x}_0(t))\ \dot{x}_n(t) - f(t,\ x_n(t),\ \dot{x}_n(t)) \end{aligned} \tag{4}$$

$$x_{n+1}(a) = \alpha_0,\ xn + 1(b) = \alpha_1 \tag{5}$$

Для задачи (1)-(2) имеем:

$$f(t,\ x_0(t),\ \dot{x}_0(t)) = 2t^{-10}x_0^3(t) + 10t^2$$
$$f'_x(t,\ x_0(t),\ \dot{x}_0(t)) = 6t^{-10}x_0^2(t)$$
$$f'_{\dot{x}}(t,\ x_0(t),\ \dot{x}_0(t)) = 0$$

$$\begin{aligned} &-\ddot{x}_{n+1}(t) + 6t^{-10}x_0^2(t)x_{n+1}(t) = \\ &\qquad 6t^{-10}x_0^2(t)x_n(t) - 2t^{-10}x_0^3(t) - 10t^2 \end{aligned} \tag{6}$$

$$x_{n+1}(1) = 1,\ x_{n+1}(2) = 32 \tag{7}$$

Введем замену $x_{n+1}(t) = y_{n+1}(t) + G(t)$, где $G(t)$ имеет вид:

$$G(t) = 31t - 30 \tag{8}$$

$$\begin{aligned} &-\ddot{y}_{n+1}(t) + 6t^{-10}(y_0(t) + G(t))^2\ (y_{n+1}(t) + G(t)) = \\ &\qquad 6t^{-10}(y_0(t) + G(t))^2\ (y_n(t) + G(t)) \\ &\qquad\qquad - 2t^{-10}(y_0(t) + G(t))^3 - 10t^2 \end{aligned} \tag{9}$$

$$y_{n+1}(1) = 0,\ y_{n+1}(2) = 0 \tag{10}$$

Приведем (9)-(10) к следующему виду:

$$p(t)\ddot{y}_{n+1}(t) + q(t)\dot{y}_{n+1}(t) + r(t)y_{n+1}(t) = g_1(t, y_n(t), \dot{y}_n(t)) \tag{11}$$
$$y_{n+1}(1) = 0,\ y_{n+1}(2) = 0 \tag{12}$$

$$
\begin{aligned}
-\ddot{y}_{n+1}(t) + 6t^{-10}(y_0(t)+G(t))^2\ y_{n+1}(t) = \\
6t^{-10}(y_0(t)+G(t))^2\ y_n(t) \\
-2t^{-10}(y_0(t)+G(t))^3 - 10t^2
\end{aligned} \tag{13}
$$

$$
y_{n+1}(1) = 0,\ y_{n+1}(2) = 0 \tag{14}
$$

Первое приближение:

$$
y_1^N(t) = \sum_{k=1}^{N} a_k e_k(t) \tag{15}
$$

Метод наименьших квадратов:

$$
\begin{aligned}
\sum_{k=1}^{N} a_k \int_1^2 [-\ddot{e}_k(t) + 6t^{-10}(y_0(t)+G(t))^2\ e_k(t)][-\ddot{e}_j(t) + 6t^{-10}(y_0(t)+G(t))^2\ e_j(t)]dt = \\
\int_1^2 [6t^{-10}(y_0(t)+G(t))^2\ y_n(t) - 2t^{-10}(y_0(t)+G(t))^3 - 10t^2] \\
[-\ddot{e}_j(t) + 6t^{-10}(y_0(t)+G(t))^2\ e_j(t)]dt
\end{aligned} \tag{16}
$$