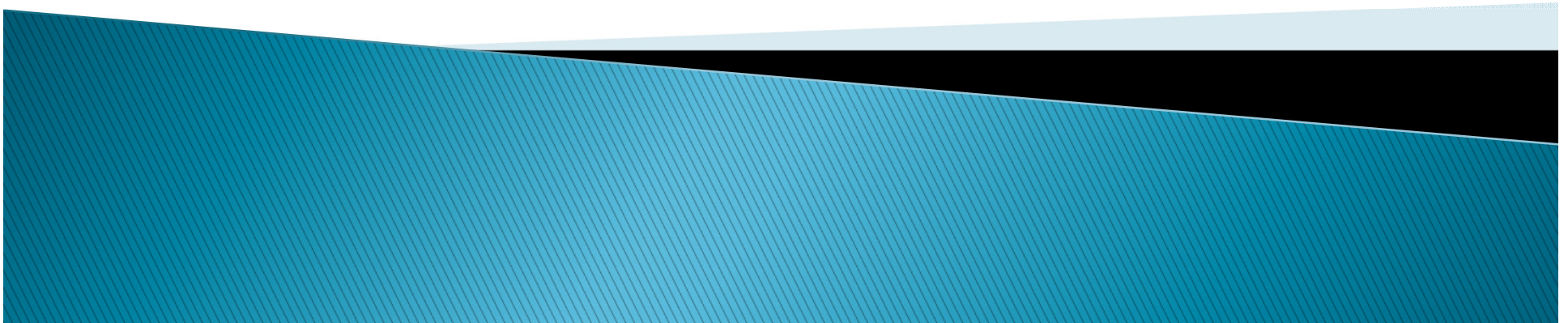


# Saltzman and Maasch 1991 Climate Model

An Overview  
Andrew Gallatin  
21 August 2014



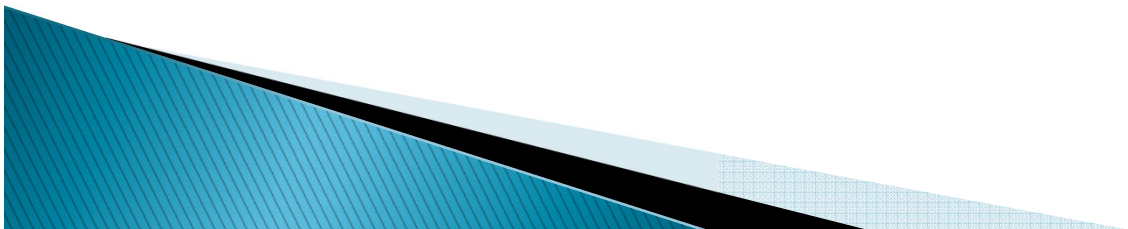
# The Model

$$\dot{X} = -X - Y - vZ - uf(t) + W_x(t)$$

$$\dot{Y} = -pZ + rY - sY^2 - Y^3 + W_y(t)$$

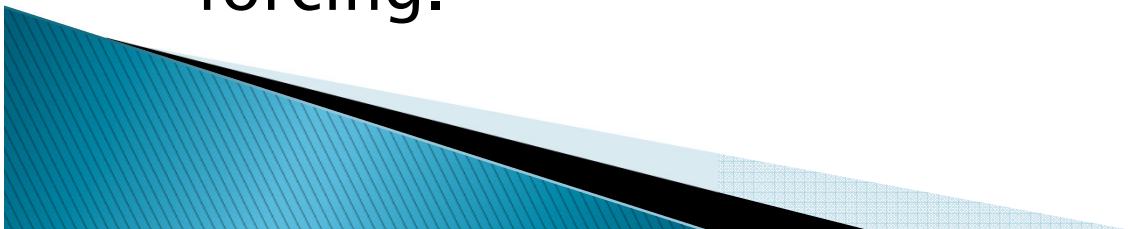
$$\dot{Z} = -q(X + Z) + W_z(t)$$

- X corresponds to the continental ice mass
- Y corresponds to the atmospheric CO<sub>2</sub>
- Z corresponds to the deep ocean temperature
- f(t) is the normalized insolation forcing at 65° N in July
- W<sub>i</sub> is the stochastic forcing with amplitude less than the orbital forcing



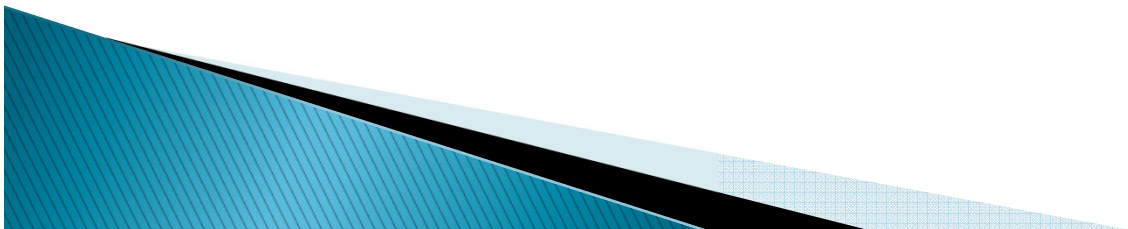
# Physical Backing to Model

- ▶ Reduced from a more general Paleoclimate dynamics system.
  - Based on the feedback loops generated by atmospheric CO<sub>2</sub>, deep ocean temperature, ice sheet volume, and Earth-orbital forcing through a general circulation model (surface climate).
- ▶ Interesting dynamics are found in the carbon equation (e.g. Y).
- ▶ A reduction in the CO<sub>2</sub> to a low enough value (bifurcation point) allows Northern Hemisphere ice sheets to form and excites the internal main free oscillator which phase-locks with the orbital forcing.



# Full Simulation

- ▶ Normalized insolation at  $65^{\circ}$  N in July.
- ▶ Stochastic forcing.
- ▶ Combined solution of the tectonic equilibriums and departure equations.
  - Dynamical system is the departure – seen by the beginning of the 100ky oscillations
  - Equilibrium solution is the general trend – seen pre-100ky oscillations



# Parameter Choices

- ▶ Changing parameters to simulate tectonic decrease in CO<sub>2</sub>.

- Vary  $r$  and  $s$  slowly over 5 My.
- Equations taken from Saltzman and Maasch's
- 1991 paper.

$$\dot{X} = -X - Y - vZ - uf(t) + W_x(t)$$

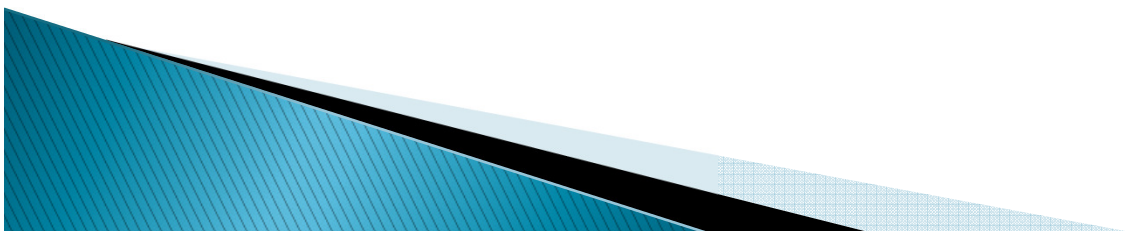
$$\dot{Y} = -pZ + rY - sY^2 - Y^3 + W_y(t)$$

$$\dot{Z} = -q(X + Z) + W_z(t)$$

- ▶ Fixed parameter values taken from Saltzman and Maasch's 1991 paper.

- With exception to  $u$  which was scaled to fit the data record more closely using variance ratios of early and late Pleistocene.

The Model



# Parameters

► Fixed parameter values:

- $p = 1.0$
- $q = 2.5$
- $u = 1.55$
- $v = 0.2$

$$\dot{X} = -X - Y - vZ - uf(t) + W_x(t)$$

$$\dot{Y} = -pZ + rY - sY^2 - Y^3 + W_y(t)$$

$$\dot{Z} = -q(X + Z) + W_z(t)$$

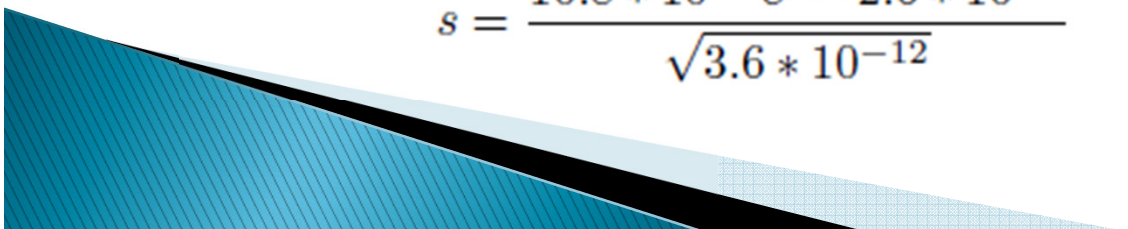
► Varying parameter values:

The Model

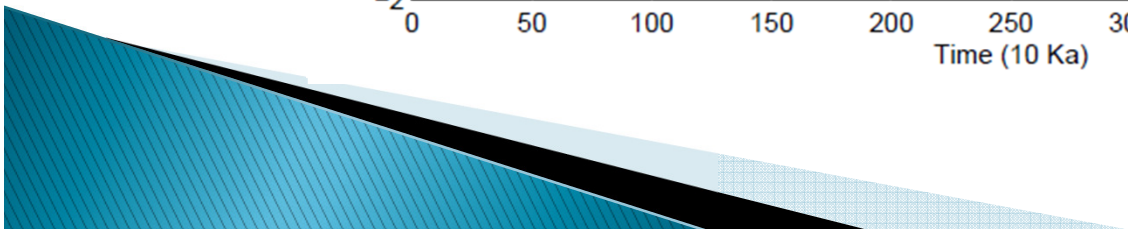
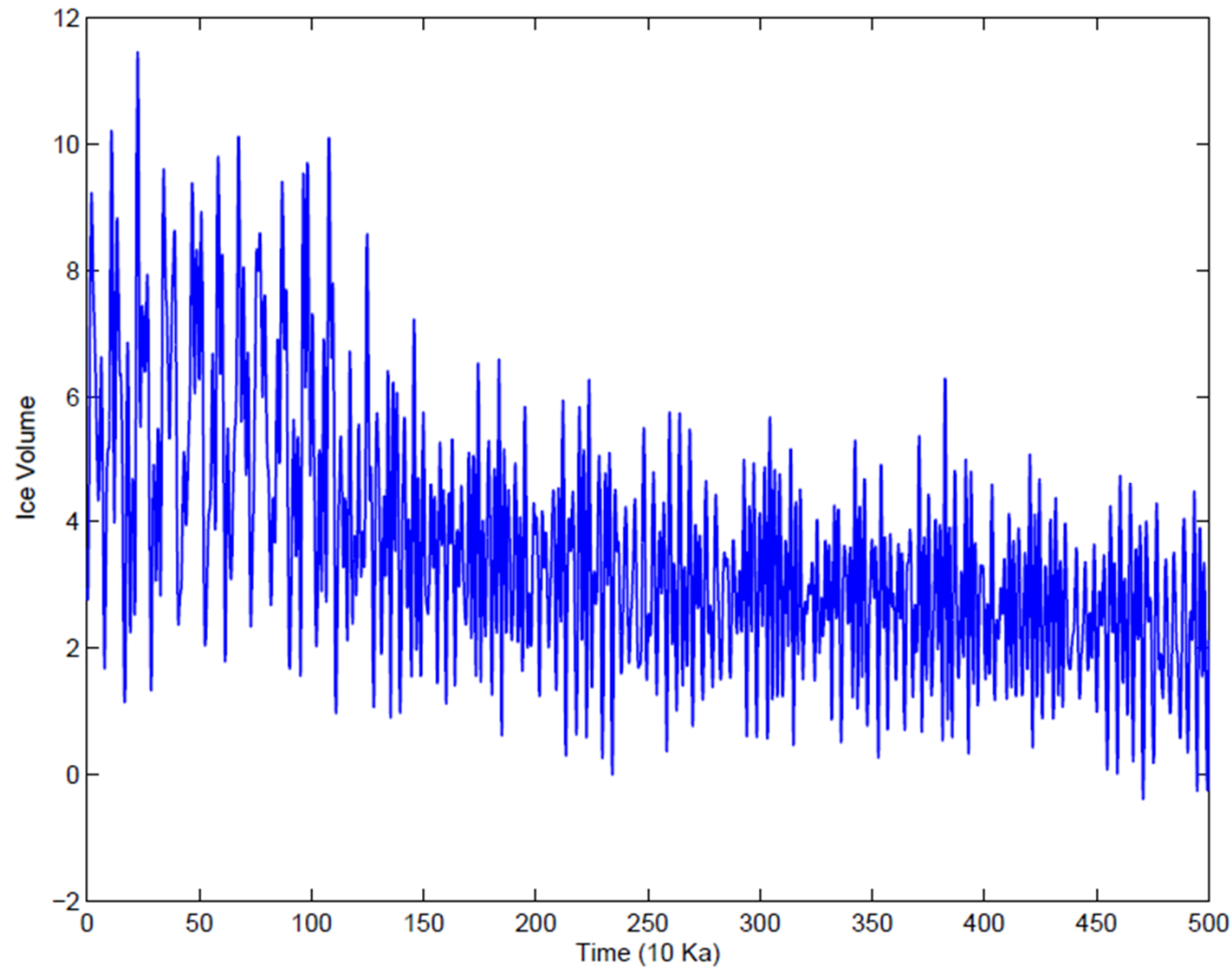
$$U = -35t + 425$$

$$r = \frac{5.2 * 10^{-5}U - 10.8 * 10^{-8}U^2 - 6.3 * 10^{-3}}{1 * 10^{-4}} + 2$$

$$s = \frac{10.8 * 10^{-8}U - 2.6 * 10^{-5}}{\sqrt{3.6 * 10^{-12}}}$$

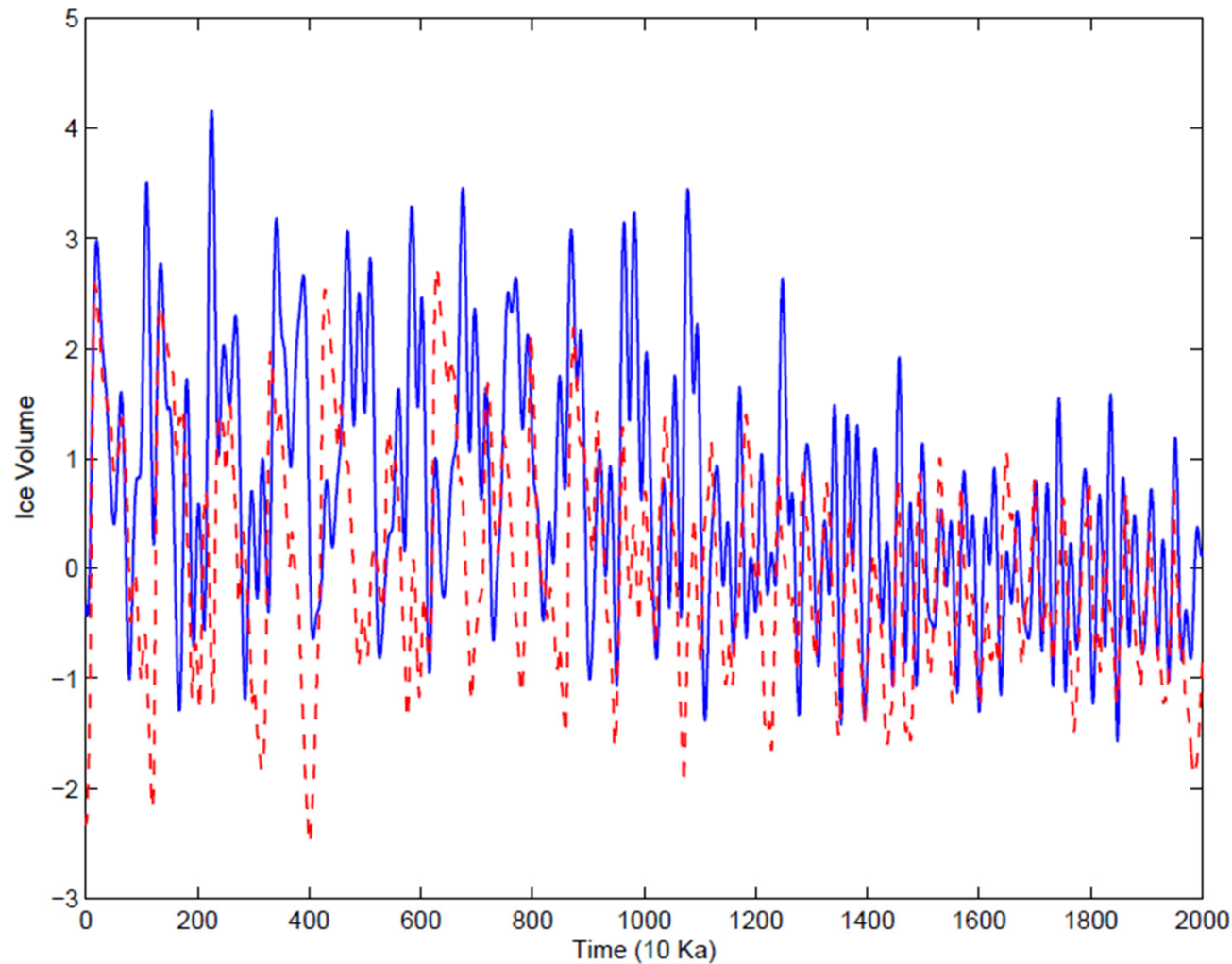


# Full Model Simulation





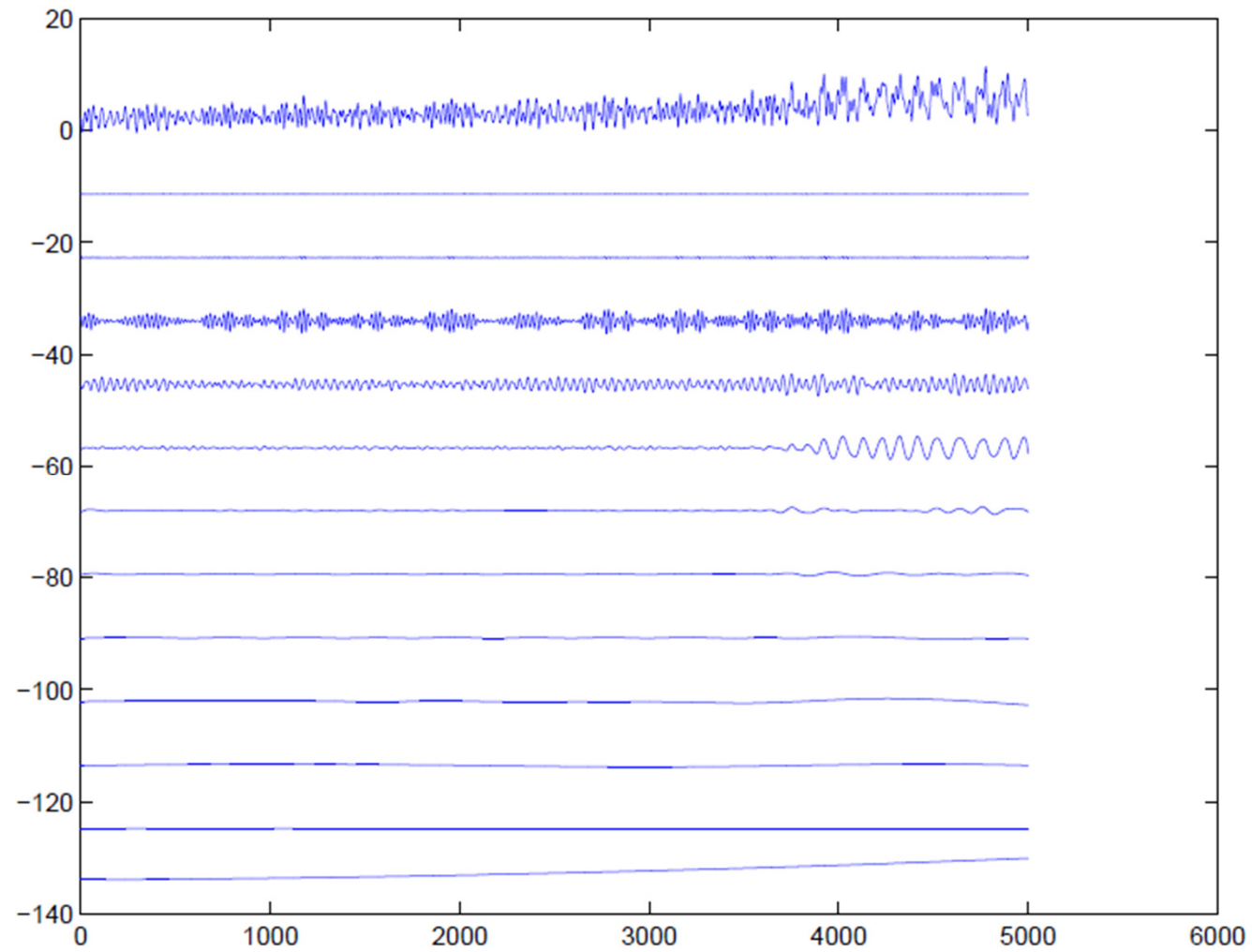
# Past 2Ma Model and DO18



Blue line is model output and red dashed line is DO18 ice record.



# Full Model EEMD



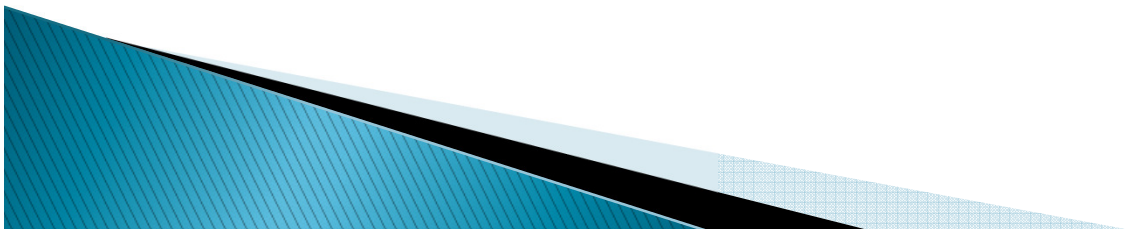
# Bifurcation Discussion

- ▶ This model undergoes a sub-critical Hopf bifurcation at around 1.25 Ma.
  - This is consistent with the late Pleistocene transition that appears in the DO18 record.
- ▶ The bifurcation is driven by the changing parameters tied to the tectonic decline in CO<sub>2</sub>.



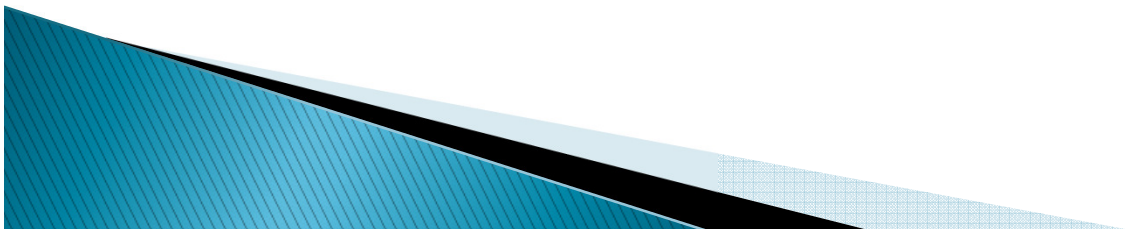
# Comparing to DO18 Data

- ▶ Auto-correlation to Earth-orbital forcing
  - Correlation to obliquity (IMF 4)
    - Early Pleistocene (pre-transition) – 1.4 to 2 Ma
    - Late Pleistocene (post-transition) – 0 to 1.2 Ma
    - Full Pleistocene – 0 to 2 Ma
  - Correlation to eccentricity (IMF 5 and 5/6)
    - Late Pleistocene (post-transition) – 0 to 1.2 Ma
- ▶ Auto-correlation directly to data
  - IMF 4, 5, 5/6, 4/5/6
  - Early, late, and full Pleistocene

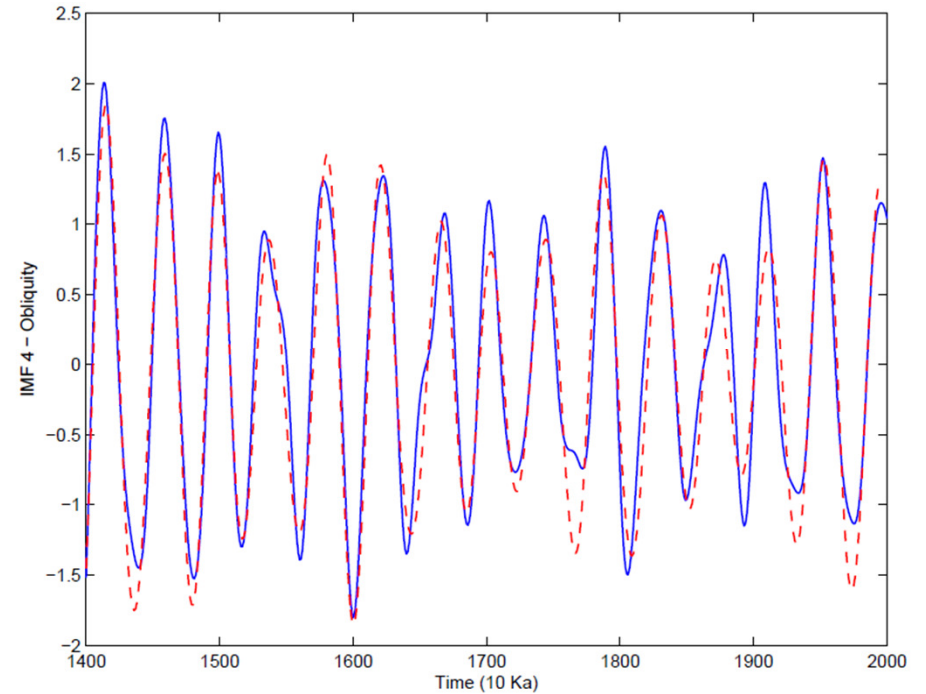
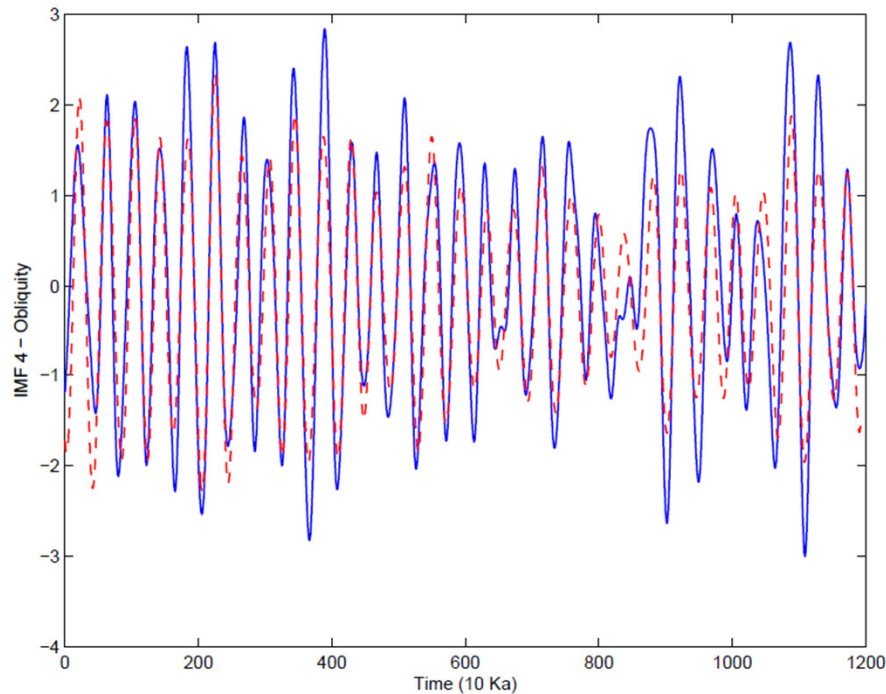


# Comparing to DO18 Data

	Model Max Correlation (Lag)	DO18 Data Max Correlation (Lag)
Obliquity (Early)	0.932821 (-6)	0.642243 (-9)
Obliquity (Late)	0.901264 (-7)	0.571306 (-11)
Obliquity (Full)	0.916191 (-7)	0.615503 (-11)
Eccentricity (IMF 5)	0.513051 (-46)	0.441474 (-9)
Eccentricity (IMF 5/6)	0.513665 (-46)	0.449656 (-9)



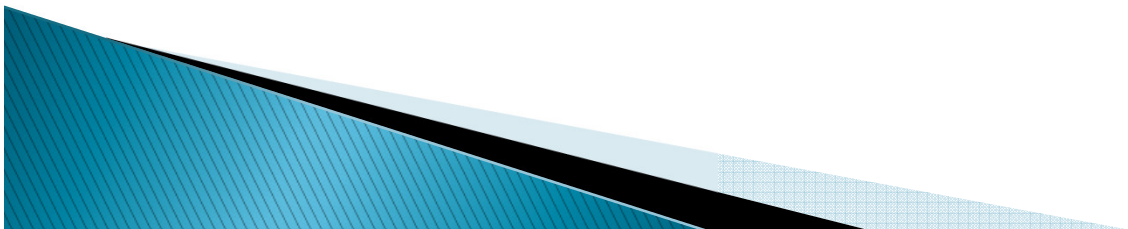
# Model Correlation to Obliquity



Blue line is model output and red dashed line is obliquity forcing taken from Laskar 2004.

# Obliquity Correlation Discussion

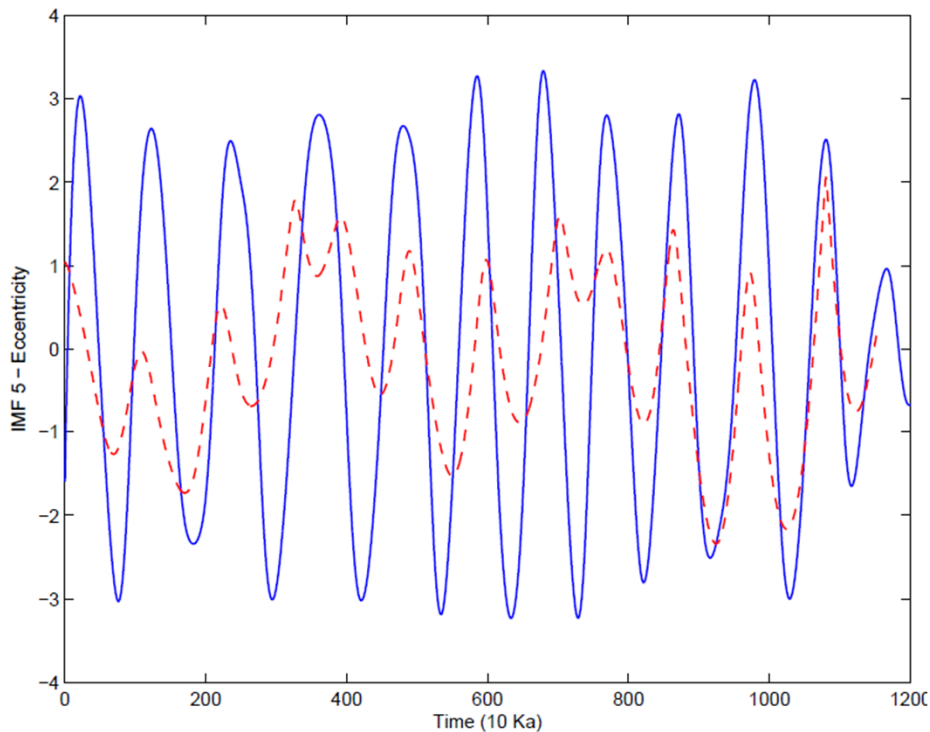
- ▶ Model is highly correlated to obliquity in the early and late Pleistocene
  - Suggests that the model has a persistent 41 ky cycle throughout the transition at about 1.25 Ma.
  - Because the model assumes direct influence of insolation forcing and is more correlated than the data, it is plausible to suggest that the physical cycles that drive the ice volume have some processes that delay or dampen the effects of obliquity in the system.
- ▶ Model has a shorter lag (7 ky) by about 3 ky compared to the data (10 ky).



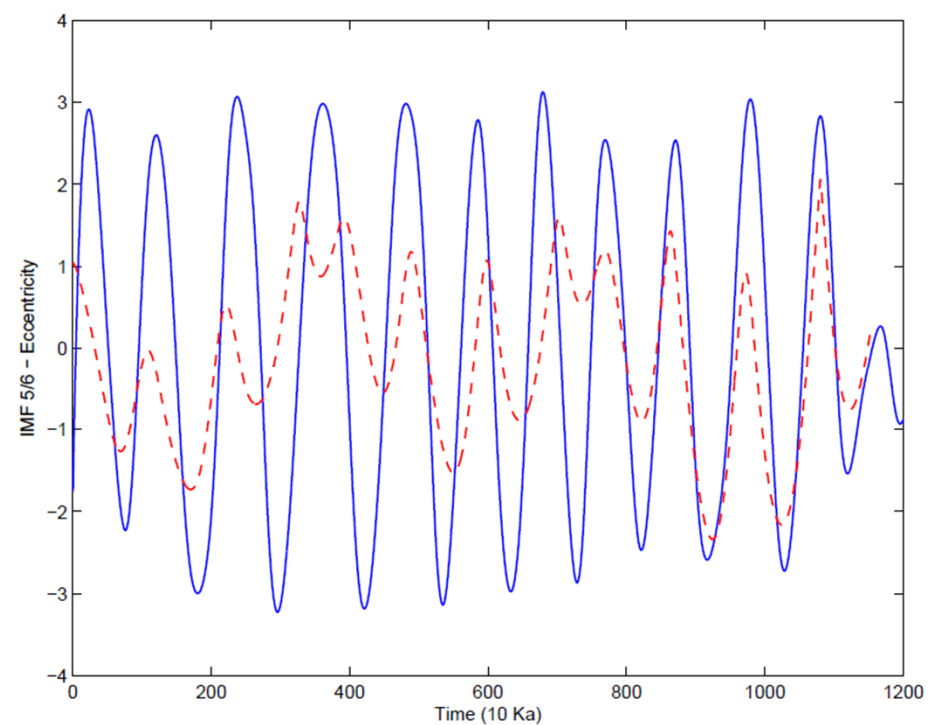


# Model Correlation to Eccentricity

IMF 5



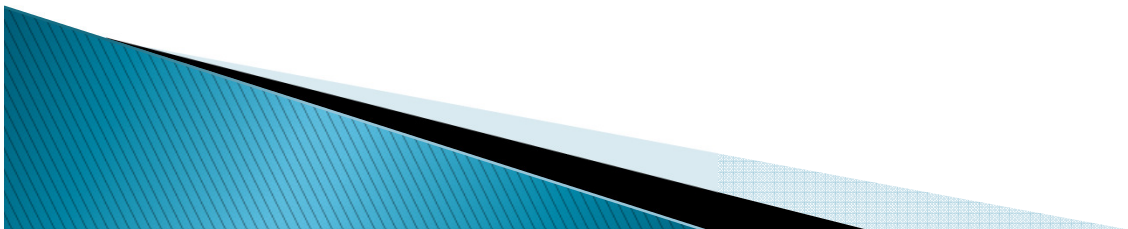
IMF 5/6



Blue line is model output and red dashed line is eccentricity forcing taken from Laskar 2004.

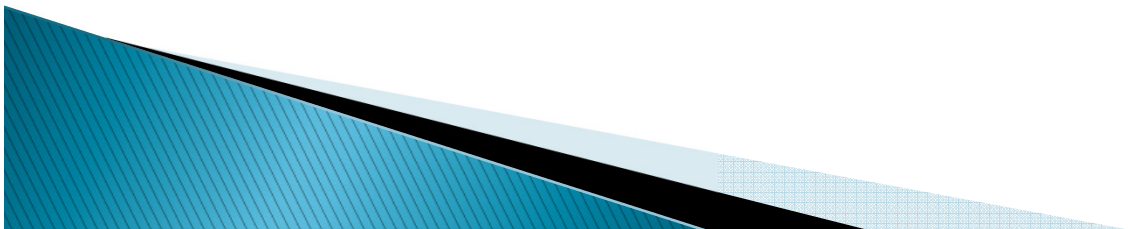
# Eccentricity Correlation Discussion

- ▶ Model max correlation to eccentricity occurs when the lag is close to 50 ky which is half a period out of phase with the eccentricity forcing.
  - Suggests possibility of multiple phases to phase-lock to depending on initial conditions.



# Key Ideas

- ▶ The interesting dynamics lie in the carbon equation (e.g. Y)
- ▶ Physically, a tectonic decrease in the  $\text{CO}_2$  permits the growth of northern hemisphere ice sheets and the internal excitation of the main free oscillator
- ▶ The cycle is possibly phase locked by the 100ky period of orbital forcing
- ▶ We would still have ice ages without Earth-orbital variations



# Where To Go

- ▶ Direct correlation to DO18 record
- ▶ Comparison of other models
- ▶ Phase-locking into multiple phases
- ▶ Physical origins of equations.

