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Credit Risk Measurement and Management

FRM二级知识框架图

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Learning Structure

Introduction of Credit Risk

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2. The Credit Analyst
3. Introduction of Credit Risk

Credit Risk Measurement ★★★

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Credit Risk Measurement

Credit VaR

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Key Parameter

PD

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10. Default Intensity Models

LGD

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Portfolio

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Section 1~2

The Credit Decision
The Credit Analyst

Credit Risk & Credit Risk Evaluation

了解

Credit Risk definition	Credit risk is the probability that a borrower will ***not pay back*** a loan in accordance with the terms of the credit agreement.
Credit Risk Evaluation	• ***The capacity and willingness*** of the obligor (borrower, counterparty, issuer, etc.) • The ***external environment*** (operating conditions, country risk, business climate, etc.) • The ***characteristics*** of the relevant credit instrument (product, facility, issue, debt security, loan, etc.) • The quality and sufficiency of any ***credit risk mitigants*** (collateral, guarantees, credit enhancements, etc.)

Credit Analysis Comparison

了解

	Consumers	Corporations	Financial Institutions	Sovereigns
Capacity	Wealth, salary, or incoming cash, expenses, assets, net cash	Liquidity, cash flow combined with earnings capacity and profitability	Similar to nonfinancial firms but bank specific.	Financial factors including the country's external debt load; tax receipts.
Willingness	Reputation of individual, payment history.	Quality of management, historical debt service.	Quality of management; ***qualitative analysis is even more important for financial films***.	Credit analysis for sovereigns is often ***more subjective*** than for financial and nonfinancial firms.
Methods of evaluation	***Credit scoring models***	Detailed manual analysis including financial statement analysis, interviews with management.	Similar to nonfinancial firms.	Similar to financial and nonfinancial firms but with increased subjective analysis of the political environment.
Loan size/type	Large exposures: secured (e.g., mortgage). Smaller exposures: unsecured (e.g., credit card loans).	Typically larger exposures	Similar to nonfinancial firms (i.e., large).	Similar to financial and nonfinancial firms (i.e., large)

The Credit Analyst

了解

Job Description

- Consumer Credit Analyst
- Credit Modeling Analyst
- Corporate Credit Analyst
- Counterparty Credit Analyst
- Credit Analysts at Rating Agencies
- Sell-Side and Buy-Side Fixed Income Analysts
- Bank Examiners and Supervisors

Functional Objective

- Risk Management
- Investment Selection
- Rating Agency

Type of Entity Analyzed

- Corporate Credit Analyst
- Bank and Financial Institution Credit Analyst
- Sovereign/Municipal Credit Analyst

Classification by Employer

- Banks, Nonbank Financial Institutions, and Institutional Investors
- Rating Agencies
- Government Agencies
- Rating Advisor

Section 3

Introduction of Credit Risk

Credit Risky Securities

Types of Credit Risk

Corporate debt
Sovereign debt
Credit derivatives
Structured credit products

Debt Seniorities

- Senior debt
- characteristics of both equity and debt securities
  - Preferred stock
  - Convertible bonds
  - Payment in kind (PIK) bonds

Credit Contract Frictions 名词掌握

- ***Asymmetric information*** refers to different parties to a transaction having different information sets.
- ***Principal-agent problems*** arise: the agent has better information than the principal.
- ***Risk shifting*** occurs when risks and rewards are transferred from one group of market participants to another group holding different positions in the firm's capital structure.
- ***Moral hazard*** arises when buying insurance or some protection that reduces the incentive of the insured party to avoid the insured event.
- ***Adverse selection*** occurs when parties to a transaction have asymmetric information.
- ***Externalities*** are costs or benefits that occur when one party's actions cause others to absorb the cost or benefit.
- ***Collective action problems*** (i.e., coordination failures) occur when a group of individuals were to benefit collectively if they all took a course of action, but would not benefit if an individual alone took the same course of action. Example: Prisoner's Dilemma

Default and Recovery ★

Expected loss (EL)

EL = PD x LGD x exposure

$$E\left[loss|default\right] = LGD = \frac{EL}{P\left[default\right]} = \frac{EL}{PD}$$

Compare Risk-free bonds with Risky bond

The investor may ***prefer the risky bond:***

- (1 - PD)(l + r + z) + (PD)(RR) >1 + r

expected loss = PD(1 - RR)

unexpected loss at default = (1 - PD)(l - RR)

Credit Vs. Market Risk Events ★

对比

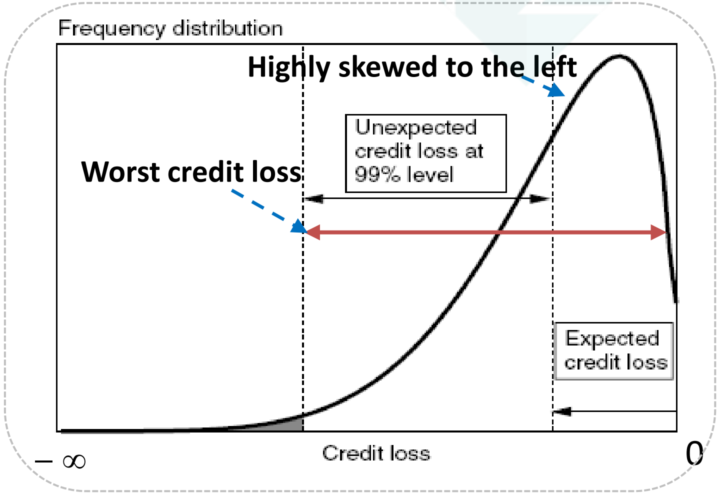
Market risk	the risk of economic losses from ***movements in market prices***
Credit risk	the risk of borrower ***default*** on contractual obligations, but also includes other risks including ***credit downgrades***.
Mark-to-market risk	A change in the perception of credit quality, even if it does not result in credit migration, may cause spreads to increase and give rise to credit risk “the ***ambiguity*** in the distinction between credit and market risk.”

Section 4

Credit VaR

Credit VaR ★★★ 名词掌握

Type of credit loss	Description	Formula
Expected credit loss (credit provision	Represent the average credit loss. Pricing of the portfolio covers expected loss.	$ECL_i = PD_i \times (1 - f_i) \times CE_i$ $ECL = \sum_{i=1}^{N} PD_i \times (1 - f_i) \times CE_i$
Worst credit loss	Represent the loss that will not be exceeded at some level of confidence.	$1 - c = \int_{WCL}^{\infty} f(x)dx$
Unexpected credit loss	Deviation from the expected loss like VAR Economic capital covers the unexpected loss	$UCL = CVAR = WCL - ECL$



	Market Risk	Credit Risk
Definition	compares a ***future value*** with a ***current value***	compares ***two future values***
Distributions	Symmetric	***Skewed to the left***
Time Horizon	Short Term (Days)	***Long*** Term (Years)

Risk Contribution ★★★ 计算

Step 1: Expected loss and unexpected loss of individual asset

$$EL_i = AE_i \times LGD_i \times EDF_i$$

$$UL_i = AE_i \times \sqrt{EDF_i \times \sigma_{LGD_i}^2 + LGD_i^2 \times \sigma_{EDF_i}^2}$$

where, $\sigma_{EDF_i}^2 = EDF_i \times (1 - EDF_i)$

Step 2: Expected loss and unexpected loss of N-asset portfolio

$$EL_P = \sum_i EL_i = \sum_i AE_i \times LGD_i \times EDF_i;\ UL_P = \sqrt{\sum_i \sum_j \rho_{ij} UL_i UL_j} \le \sum_i UL_i$$

For a two-asset portfolio, $UL_P = \sqrt{UL_1^2 + UL_2^2 + 2\rho_{12} UL_1 UL_2}$

Step 3: The risk contribution (RC) of each asset

$$RC_i = UL_i \frac{\partial UL_p}{\partial UL_i} = \frac{UL_i \sum_j UL_j \rho_{ij}}{UL_p}$$ and $$\sum_i RC_i = UL_P$$

$$RC_1 = UL_1 \frac{\partial UL_p}{\partial UL_1} = \frac{UL_1^2 + \rho_{12} UL_1 UL_2}{UL_p}$$ and $$RC_2 = UL_2 \frac{\partial UL_p}{\partial UL_2} = \frac{UL_2^2 + \rho_{12} UL_1 UL_2}{UL_p}$$

Section 5

Binomial Trees of PD

Binomial Trees of PD ★★★

Marginal default probability	the probability that a borrower will default in ***any given year***.
Survival Rate	a borrower will not default ***over a specified multi-year period***. • Survival Rate at end of two years: $S_2=(1-d_1)(1-d_2)$ • Survival Rate at end of three years: $S_3=(1-d_1)(1-d_2)(1-d_3)$
Cumulative default probability	a borrower will default ***over a specified multi-year period***. • Cumulative default probability at end of two years: $C_2 = 1 - S_2 = d_1 + (1 - d_1)d_2$ • Cumulative default probability at end of three years: 计算 $C_3 = 1 - S_3 = d_1 + (1 - d_1)d_2 + (1 - d_1)(1 - d_2)d_3$
Average survival rate and average default rate	$S_3 = (S^a)^3 \Rightarrow S^a$ $\quad S^a = 1 - d^a \Rightarrow d^a$

Change on probability of default ★★★ 对比

- For ***investment-grade credits***, the increase of cumulative default probability is ***more*** than proportional with the horizon.
- For ***speculative-grade credits***, the increase of cumulative default probability is ***less*** than proportional with the horizon.
- The marginal probability of default ***increase***s with maturity for initial ***high*** credit ratings, but ***decreases*** for initial ***low*** credit ratings. (Reasons: ***mean reversion effect***)

Section 6

Credit Ratings

Credit Ratings and Agencies

	Standard & Poor's/Fitch	Moody's
Highest grade	AAA	Aaa
High grade	AA	Aa
Upper medium grade	A	A
Medium grade	BBB	Baa
Lower medium grade	BB	Ba
Speculative	B	B
Poor standing	CCC	Caa
Highly speculative	CC	Ca
Lowest quality, no interest	C	C
In default	D	
Modifiers: A+, A, A-, and A1, A2, A3	$A+ > A > A- > BBB+ > BBB > BBB- > BB+$	
	$A1 > A2 > A3 > Baa1 > Baa2 > Baa3 > Ba1$	

Investment grade: Highest grade, High grade, Upper medium grade, Medium grade

Speculative grade: Lower medium grade, Speculative, Poor standing, Highly speculative, Lowest quality, no interest, In default

Internal Credit Rating ★ 名词掌握

At-the-point approach (short horizon) & Through-the-cycle approach (long horizon)

- Through-the-cycle approaches: ***high-rated firms may be underrated during growth*** periods and overrated during the decline of a cycle.
- The use of ***at-the-point*** approaches may be ***procyclical***. The changes in ratings and lending policies can *lag* the economic cycle.
- Through-the-cycle approach and at-the-point approach should not be mixed.

Credit Transition Matrices ★★★ 计算

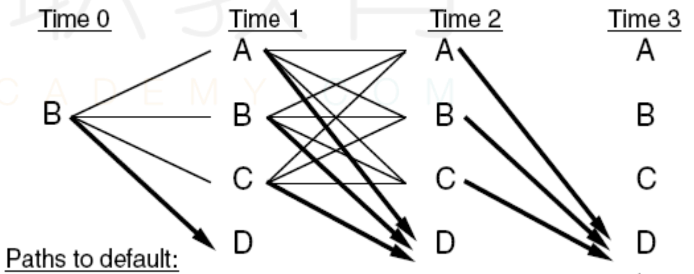
State Starting	Ending A	B	C	D	Total Prob.
A	0.97	0.03	0.00	0.00	1.00
B	0.02	0.93	0.02	0.03	1.00
C	0.01	0.12	0.64	0.23	1.00
D	0	0	0	1.00	1.00

- In the first year, the probability of default

$$P(D_1 \mid B_0) = 3\%$$

- In the second year, the probability of default

$$P(D_2 \mid A_1)P(A_1) + P(D_2 \mid B_1)P(B_1) + P(D_2 \mid C_1)P(C_1)$$
$$= 0.00 \times 0.02 + 0.03 \times 0.93 + 0.23 \times 0.02 = 3.25\%.$$



Time 1	Time 2	Time 3
B→D 0.03=0.0300	B→A→D 0.02*0.00=0.0000	B→A→A→D 0.02*0.97*0.00=0.0000
	B→B→D 0.93*0.03=0.0279	B→A→B→D 0.02*0.03*0.03=0.0000
	B→C→D 0.02*0.23=0.0046	B→A→C→D 0.02*0.00*0.23=0.0000
		B→B→A→D 0.93*0.02*0.00=0.0000
		B→B→B→D 0.93*0.93*0.03=0.0259
		B→B→C→D 0.93*0.02*0.23=0.0043
		B→C→A→D 0.02*0.00*0.00=0.0000
		B→C→B→D 0.02*0.12*0.03=0.0001
		B→C→C→D 0.02*0.64*0.23=0.0029
Default prob: 0.0300	0.0325	0.0333
Cumulative: 0.0300	0.0625	0.0958

Section 7

Credit Scoring Models

Credit Scoring Models

Quantitative methodologies for credit analysis and scoring ★★★ 概念对比

Type of credit scoring models	Feature	Example
Fisher Linear discriminant analysis	• ***Parametric*** technique • A process that segregates ***a larger group into homogeneous subgroups***.	z-score model
Parametric discrimination	• ***Parametric*** technique • Use a ***score function*** to determine the members of the subgroups. Whether the value of the score falls above or below a certain threshold determines which subgroup the observation is placed in	logit and probit model
K-nearest neighbor approach	• ***Non-parametric*** technique • Use the properties of firms that have already fallen into the categories of interest (e.g., default or not default) and ***categorizes a new entrant*** by how closely it resembles the members already in each of the groups.	
Support vector machines	• ***Parametric*** technique • Use the characteristics of observations (firms) to create an equation that does the best job of ***dividing the larger group*** into two subgroups.	Linear and nonlinear

Decision Rules in Credit Analysis ★★

概念对比

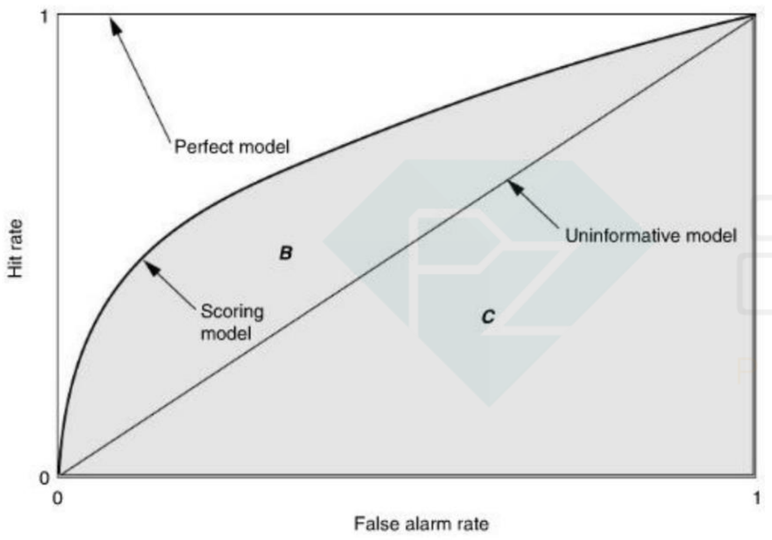
Type	Feature
Minimum error	• uses Bayes' theorem to determine a conditional probability • p(C given default) x p(default) > or < p(C given not default) x p(not default)
Minimum risk	• a class of rules that try to either ***minimize the probability of misclassification*** (incorrectly lending to risky firm) or minimize the loss associated with that error.
Neyman-Pearson	use the statistical concept of Type I and Type II errors. • A Type I error is lending to a risky firm because it was incorrectly accepted as a non-risky firm. • A Type II error is not lending to a non-risky firm because it incorrectly rejected as being risky. $\frac{p(\text{"conditions" given default})}{p(\text{"conditions" not given default})} > \text{threshold value}$
Minimax	• minimizing the maximum error or risk.

The ***minimum error rule*** makes a decision based on calculated ***probabilities***.
The ***other three*** methods use ***optimization*** techniques

Measures of Performance ★★★

名词掌握

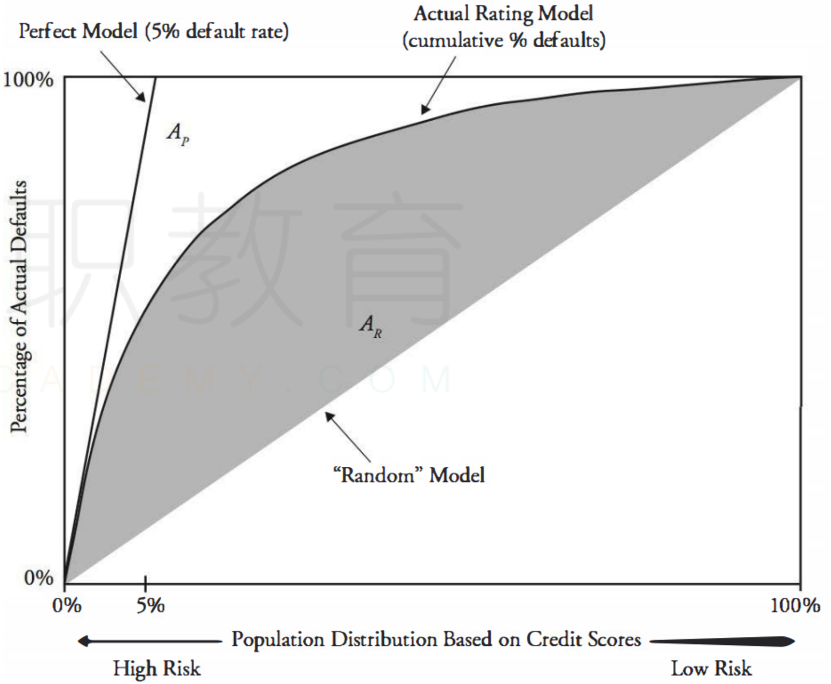
The receiver operating characteristic (ROC)



$$1.\ Y = \frac{\text{number of defaults correctly predicted}}{\text{number of defaults}}$$

$$2.\ X = \frac{\text{number of firms predicted to default and did not}}{\text{number of firms that did not default}}$$

The cumulative accuracy profile (CAP, GINI curve)



The ***accuracy ratio (AR) is defined as $A_R/A_p + A_R$***, with a ratio ***close to 1*** implying a ***more accurate*** model.

Retail Credit Risk Management

了解

Retail Banking ➡ The retail banking industry revolves around receiving deposits from and lending money to ***consumers and small businesses***.

Retail Credit Risk vs. Corporate Credit Risk

- Retail credit ***exposures are relatively small***. A ***commercial*** credit portfolio often consists of ***large*** exposures.
- Due to the ***diversification*** of a retail credit portfolio, estimating the default percentage allows a bank to treat this loss as a cost of "doing business". A commercial credit portfolio is subjected to the risk that its ***losses may exceed the expected threshold***.

Credit Risk Scoring Models

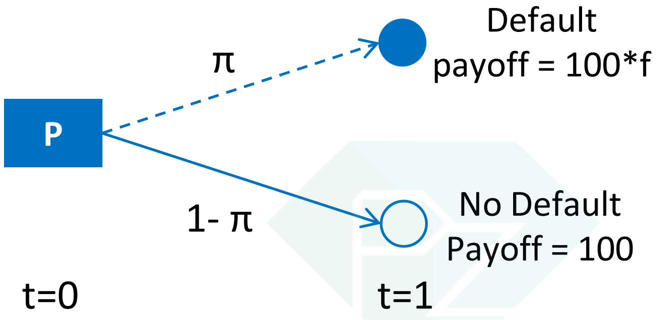
Credit bureau scores	an applicant's ***FICO score***
Pooled model	built by ***outside parties***, is more costly than implementing a credit bureau score model; however, it offers the advantage of flexibility to tailor it to a specific industry.
Custom model	***created by the lender itself*** using data specifically pulled from the lender's own credit application pool.

Section 8

Infer Credit Risk from Corporate Bond Prices

Infer Credit Risk from Corporate Bond Prices ★★★ 计算

Risk-Neutral Probability of Default



$$P=\frac{\$100}{(1+YTM)}=\left[\frac{\$100}{(1+R_f)}\right]\times(1-\pi)+\left[\frac{f\times\$100}{(1+R_f)}\right]\times\pi$$

$$\Rightarrow \pi=\frac{1}{(1-f)}\left[1-\frac{(1+R_f)}{(1+YTM)}\right]$$

$$\Rightarrow YTM\approx R_f+\pi(1-f)\Rightarrow YTM-R_f\approx\pi(1-f)$$

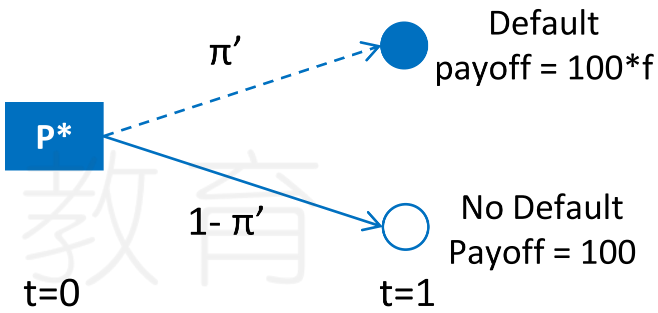
YTM: Market-determined yield

R_f: Risk-free yield

π: Risk-neutral PD

f: Recovery rate

Objective probability of default



$$P^*=\frac{\$100}{(1+YTM)}=\frac{\$100\times(1-\pi')+f\times\$100\times\pi'}{(1+R_f+\text{Risk Premium})}$$

$$\Rightarrow YTM-R_f-RP\approx\pi(1-f)$$

Where π' is the real-world PD and RP is the risk premium (liquidity premium and tax effects).

Spread Risk

Spread Measure	Definition
Yield spread	YTM risky bond - YTM benchmark government bond
I-spead	YTM risky bond - linearly interpolated YTM on benchmark government bond or swap rate
Z-spread	Basis points added to each spot rate on a benchmark curve
Asset-swap spread	Spread on floating leg of asset swap on a bond
CDS spread	Market premium of CDS of issuer bond
Option adjusted spread(OAS)	Z-spread adjusted for optionality of embedded options
Discount margin	Fixed spread above current LIBOR needed to price bond correctly

了解

Influencing factors of Credit spread ★ 结论

- Credit spreads are ***reduced*** during times of ***economic recovery***. This phenomenon is called flight to quality.
  Economy $\uparrow \Rightarrow$ Buy risky bonds $\uparrow \Rightarrow$ $V_{\text{Risky bond}} \uparrow \Rightarrow$ $\text{Yield}_{\text{Risky bond}} \downarrow \Rightarrow$ Credit spreads $\downarrow$
- The credit spread of ***callable*** bonds (redeemable bond) ***widens*** (narrows) as ***volatility of interest rate increases*** (decreases).
  $\sigma_{\text{interest}} \uparrow \Rightarrow V_{call} \uparrow \Rightarrow V_{\text{Callable bond}} \downarrow \Rightarrow$ Credit spread $\uparrow$
- The credit spread of puttable bonds narrows (widens) as volatility of interest rate increases (decreases).

Spread '01 (DVCS) ★★ 名词掌握、结论

- The spread ’01 is analogous to the DV01. It measures ***the price change implied by a one basis point change in the credit spread***.
- The ***smaller*** the z-spread, the ***larger the effect*** on the bond price (i.e., the greater the credit spread sensitivity). The DVCS exhibits ***convexity***.

The Credit Spread Curve ★

- The first step to creating the curve is to plot the ***most liquid credit spreads*** observable in the market, generally ***from CDS premiums or bond spreads***.
- Plotting the curve is further complicated by the ***choice of reference***.
- An alternative method uses the credit spread around a ***single, liquid observation*** (e.g., credit spread with five years to maturity) to map the entire curve.

Section 9

Infer Credit Risk from Equity Prices

Merton Model ★★★

- Value of ***equity*** = Call option on firm = value of firm's asset (V) – value of risky debt
- Value of risky ***debt*** = Risk-free debt **–** ***put*** option on firm

Equity: Call Option

- Equity can be viewed as a call option on the firm value with strike price equal to the face value of debt.

$$\text{Stock}_t = \text{Call} = \underbrace{V}_{\text{Firm value}}N(d_1) - \underbrace{F}_{\text{Face value of debt}}e^{-r(T-t)}N(d_2)$$

$$\frac{\ln(V/Fe^{-r(T-t)})}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2} = d_1$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

- $N(-d_2)$ is the risk-neutral probability of default.

Corporate debt

- Long corporate bond = Long free-risk bond + Short put option on the firm value (or Short CDS)

$$Bond = Fe^{-r(T-t)} - Put = VN(-d_1) + Fe^{-r(T-t)}N(d_2)$$

$$ECL_T = Put \times e^{r(T-t)} = -Ve^{r(T-t)}N(-d_1) + FN(-d_2)$$

- $A_t = E_t + D_t$ = (European call with strike at F) + Fe^{-rT} - (European put with strike at F)

Determining Firm Value and Volatility

- ***$\sigma_S * S = \Delta * \sigma_V * V$***
- The distribution of equity values is not constant (***volatility smirk***). The non-constant volatility of equity is a ***violation of the Black-Scholes-Merton model.***

Credit spread

$$\text{credit spread} = -\left[\frac{1}{T-t}\right] * \ln\left(\frac{D}{F}\right) - R_F$$

$$y_t - r = \frac{1}{T-t}\log\left[\left(1-e^{-r(T-t)}\right)F + \text{European put option with strike at F}\right] - r$$

- As time increases there is greater probability that the value received will be less than par.
  - Studies have shown that ***as time to maturity increases, credit spreads tend to widen*** (i.e., increase).
  - For ***very risky debt***, it may be the case that ***credit spreads narrow***.
- As the ***risk-free rate increases***, the expected value of the firm at maturity increases, which in turn ***decreases the risk of default***.

Subordinate Debt in Merton Model

- Subordinate debt can be valued in a portfolio as a ***long position in a call option on the firm with an exercise price equal to the face value of senior debt, F, and a short position on a call option on the firm with an exercise price equal to the total principal due on all debt, U + F.***
- Subordinate debt values behave like ***equity*** when the ***firm has low values***, as during periods of financial distress, and they behave like ***senior debt*** when the ***firm is not in financial distress.***

Calculate PD and LGD

$$PD = N\left[\frac{\ln(F)-\ln(V)-(\mu)(T-t)+0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}}\right]$$

$$EL = F*PD - Ve^{\mu(T-t)}*N\left[\frac{\ln(F)-\ln(V)-(\mu)(T-t)+0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}}\right]$$

$$\text{expected } LGD = \frac{[e^{r(T-t)}\left(European\ put\ option\ with\ strike\ at\ D\right)]}{PD}$$

The Merton model rests on a number of assumptions:

- The market value of assets, A_t, and expected return, μ, are related.
- ***The value of the firm is observable*** and follows a ***lognormal*** diffusion process (geometric Brownian motion).
- The ***risk-free interest rate is constant*** through time.
- Debt consists of a ***single zero-coupon bond*** with a nominal payment of maturing at time T. The model assumes that the firm ***can default only on the maturity date*** of the bond.
- Equity consists of ***common shares only.***
- Debt-holders have ***limited liability and have no recourse*** to any other assets once equity is eliminated.
- ***Trading in markets*** occurs not only for the firm's equity and debt securities, but also for its ***assets***.
- There are ***no cash flows prior to the maturity*** of the debt (including dividends).

The drawbacks of Merton Model

- The Merton model makes some ***unrealistic assumptions***.
- It could result in ***low default probability values and high recovery rates*** for firms with high leverage. ***Firms with high leverage in reality would typically have higher default probabilities*** and lower recovery rates.

The KMV Approach ★★★

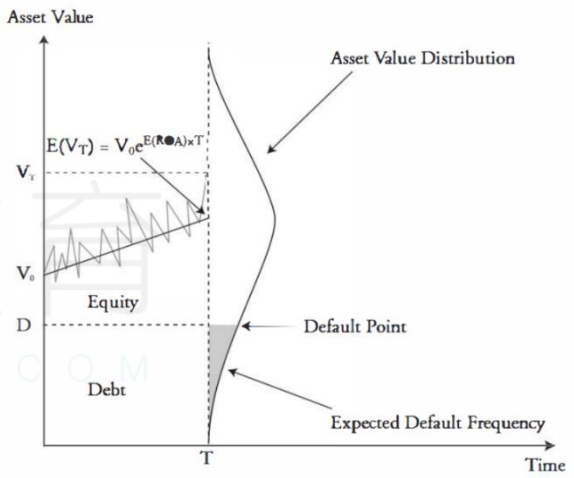
First step: calculate the distance to default (DD)

$$DD=\frac{\text{expected asset return}-\text{default threshold}}{\sigma_{\text{expected asset returns}}}=\frac{\text{A-K}}{\sigma_A}$$

Determining the default point (i.e., default threshold) is:

- If the ratio of long-term-liabilities-to-short-term-liabilities is less than 1.5:
  - short-term liabilities + 0.5 × long-term liabilities
- If this ratio is greater than 1.5:

$$\text{short-term liabilities}+\left(0.7-\frac{0.3\times\text{short-term liabilities}}{\text{long-term liabilities}}\right)\times\text{long-term liabilities}$$



$$DD=\frac{\log(V)-\log(\text{default threshold})+\left[E(ROA)-\frac{\sigma_V^2}{2}\right]\times\text{maturity}}{\sigma_V\times\sqrt{\text{maturity}}}$$

Final step: Report estimated default frequency (EDF)

- Once the distance to default is computed, the expected default frequency can be found.
- The EDF will be associated with a particular credit rating (e.g., BBB).

Summary: Estimation Approaches

Type	Characteristics
Historical Data Approach	• A ***transition matrix*** is helpful in calculating default probabilities because it identifies the ***historical probabilities*** of credit rating migration between periods. • This methodology assumes the transition matrix is ***constant*** over time and hence ***unaffected by the business cycle***, an observation not supported by empirical evidence. In general, credits are more likely to be downgraded than upgraded.
Risk-Neutral Approach	• The risk-neutral probability of default is derived from the ***observed credit spread and the market price of a traded credit security.*** • Empirical evidence indicates that ***risk-neutral default probabilities*** are significantly ***larger than real-world default probabilities***.
Equity-Based Approaches	• The ***Merton model*** uses ***equity market data*** to estimate default probabilities. • The ***KMV approach*** uses a ***proprietary approach*** built on the Merton model, but it ***relaxes*** several of its ***assumptions***.

Rating Assignment Methodologies

了解

Experts-based Approaches	Statistical-based Models	Heuristic and Numerical Approaches
Structured Experts-based Systems	Structural approaches: ***based on economic and financial theoretical assumptions***.	'Heuristic methods': opposed to algorithms-based approaches, as 'expert systems' based on artificial intelligence techniques.
Agencies' ratings	Reduced form approaches: using the ***most statistically suitable set of variables and disregarding the theoretical and conceptual causal relations*** • Statistical method • Unsupervised techniques: ***'Cluster analysis' & 'Principal component analysis'***	'Numerical methods': reach optimal solutions adopting ***'trained' algorithms to take decisions in highly complex environments***
Experts-based Internal Ratings Used by Banks	Cash Flow Simulations	

Section 10

Default Intensity Models

Poisson Distribution & Exponential Distribution ★ 结论

Bernoulli trial	• ***Default risk for a single company*** can be represented as a ***Bernoulli trial.*** • An important property of the Bernoulli distribution is that ***each trial is conditionally independent. (memoryless property)***
Binomial Distribution	This series of ***independent and identically*** (i.e., same probability of default) distributed Bernoulli trials is characterized by a ***binomialdistribution.***
Poisson random variable	• $p(X=x)=\frac{\lambda^x e^{-\lambda}}{X!}$ • The mean and variance of a **Poisson distributed random variable** is equal toλ and, as it turns out, the mean of the **exponential distribution** is equal to $1/\lambda$ and the variance is equal to $1/\lambda^2$

Exponential Distribution	• The exponential distribution is often used to model ***waiting times*** such as how long it takes an employee to serve a customer or the time it takes a company to default.
	• $f(x)=\frac{1}{\beta}\times e^{-x/\beta}, x>0$
	  • The ***scale parameter,β,*** is greater than zero and is the reciprocal of the ***"rate" parameter***(i.e., $\lambda=1/\beta$)
	  • The rate parameter measures ***the rate at which it will take an event to occur***. In the context of waiting for a company to default, the rate parameter is known as ***the hazard rate and indicates the rate at which default will arrive.***

Hazard Rates ★★★

名词掌握、计算

The ***hazard rate*** (i.e., default intensity) is represented by the (constant) parameter λ and ***the probability of default over the next, small time interval, dt, is λdt.***

Survival rate & cumulative PD	• The survival distribution: $P(t^*\geq t)=1-F(t)=e^{(-\lambda t)}$. • The cumulative default time distribution F(t) represents the probability of default over (0,t): $P(t^* < t) = F(t) = 1 - e^{-\lambda t}$ • As ***t increases, the cumulative default probability approaches 1*** and the survival probability approaches 0.
Marginal default probability	$\lambda e^{-\lambda t}$

Risk-neutral Hazard Rates

- $e^{-(r_\tau+z_\tau)\tau} = e^{-r_\tau \tau}\left[e^{-\lambda_\tau^* \tau} \times 1 + (1 - e^{-\lambda_\tau^* \tau}) \times \mathrm{RR}\right]$
- $\lambda_\tau^* \approx \frac{z_\tau}{1-RR}$

Credit Risk Measurement

Advantages of Using CDS Market

- ***CDS spreads are observable.***
- Our previous analysis on estimating hazard rates did not fully capture the ***complexities of the bond market.***
- CDS can overcome these difficulties because ***liquid contracts exist for several maturities.***

Hazard Rate Curves ★ 结论

Time-varying hazard rate: λ(t) ➡

$$\pi_t = 1 - e_0^{-\int_0^t \lambda(s)ds}$$

$$\lambda(t) = \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{Bmatrix} \text{ for } \begin{Bmatrix} 0 < t \le 1 \\ 1 < t \le 3 \\ 3 < t \le 5 \\ 5 < t \le 7 \\ 7 < t \end{Bmatrix}$$

The hazard rates used in default models are not constant but will not vary each instant in time

Constructing the hazard rate curve uses a bootstrapping methodology

$$\int_0^t \lambda(s)ds = \begin{Bmatrix} \lambda_1 t \\ \lambda_1 + (t-1)\lambda_2 \\ \lambda_1 + 2\lambda_2 + (t-3)\lambda_3 \\ \lambda_1 + 2\lambda_2 + 2\lambda_3 + (t-5)\lambda_4 \\ \lambda_1 + 2\lambda_2 + 2\lambda_3 + 2\lambda_4 + (t-7)\lambda_5 \end{Bmatrix} for \begin{Bmatrix} 0 < t \le 1 \\ 1 < t \le 3 \\ 3 < t \le 5 \\ 5 < t \le 7 \\ 7 < t \end{Bmatrix}$$

- The ***hazard rate was extracted from the default probability***.
- The fact that the ***CDS swap spread is observable allows for the inference of default probability for the 1-year maturity*** by equating (PV of expected payments in default) and (PV of expected premiums paid). The ***hazard rate can be inferred for the first period.***
- The bootstrapping procedure is then employed so that the ***hazard rate for the first period is used to infer the hazard rate for the second period from the piecewise function.***

Section 11

Loss Given Default and Recovery Rate

Factors Effecting Recovery Rates 了解

Factor	Impact
Seniority of the debt	Seniority $\uparrow \Rightarrow$ Recovery rates $\uparrow$
Collateral	Market value $\uparrow \Rightarrow$ Recovery rates $\uparrow$ Liquidity $\uparrow \Rightarrow$ Recovery rates $\uparrow$ Uniqueness $\uparrow \Rightarrow$ Recovery rates $\downarrow$
State of economy	Expansion $\Rightarrow$ Recovery rates $\uparrow$ Recession $\Rightarrow$ Recovery rates $\downarrow$
Obligor's characteristics	Tangible assets $\uparrow \Rightarrow$ Recovery rates $\uparrow$ Previous rating $\uparrow \Rightarrow$ Recovery rates $\uparrow$
Jurisdiction	Bankruptcy law $\uparrow \Rightarrow$ Recovery rates $\uparrow$ Facilitate liquidation $\uparrow \Rightarrow$ Recovery rates $\uparrow$ Encourage reorganization $\uparrow \Rightarrow$ Recovery rates $\uparrow$

Section 12

Credit Exposure

Credit Exposure Metrics ★★★

概念对比

Key terms	Definition
Expected mark to market (MtM)	The ***expected value*** of a transaction at a ***given point*** in the future.
Expected exposure (EE)	The amount that is ***expected to be lost*** if there is positive MtM and the counterparty defaults. It is ***larger than expected MtM***
Potential future exposure (PFE)	The ***worst exposure*** that could occur at a ***given time*** in the future at a given confidence level.
Maximum PFE	The ***highest PFE*** value over a stated time frame
Expected positive exposure (EPE)	The ***average EE*** through time
Negative exposure	The exposure from the ***counterparty's point of view***, is represented by negative future values. The ***expected negative exposure (ENE)*** and the negative expected exposure (NEE) are the exact opposite of EPE and EE.
Effective EE & Effective EPE	***Effective EE*** is equal to ***nondecreasing EE***. Effective EPE is the average of the effective EE.

Figure 1: Potential Future Exposure

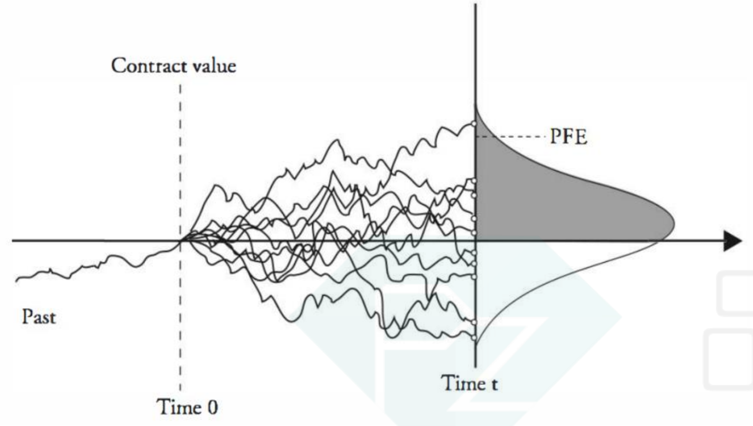


Figure 2: Credit Exposures

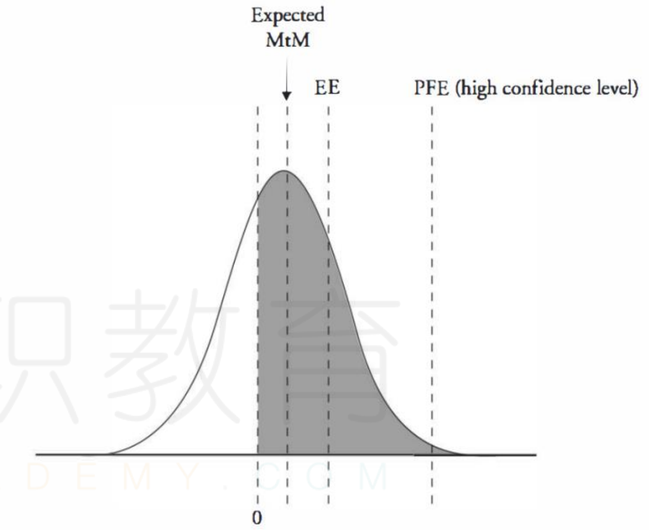


Figure 3: Expected Positive Exposure

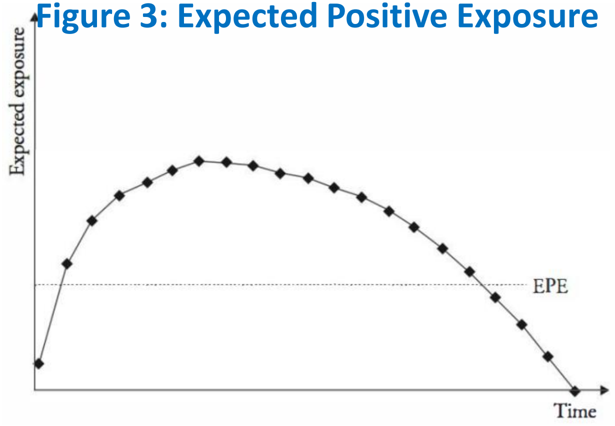
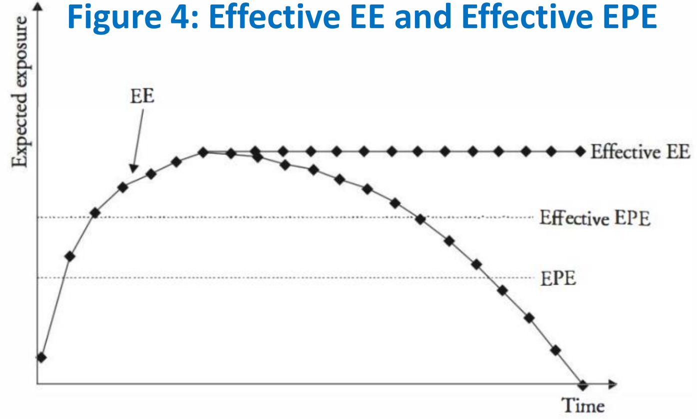


Figure 4: Effective EE and Effective EPE



Credit Exposure Factors

了解

- ***Future uncertainty***. In situations where there is a ***single payout*** at the end of the life of a contract, ***uncertainty regarding the value of the final exchange*** increases over time. Example: Foreign exchange forwards and FRAs
- ***Periodic cash flows***: Unlike the situation where there is a single payout, when ***cash flows occur regularly***, ***the negative impact of the future uncertainty factor is reduced***. Example: interest rate swap
- ***Combination of profiles***: when the credit exposure of a product results from the ***combination of multiple underlying risk factors***. Example: cross-currency swap.
- ***Optionality\ Exercise decisions*** (e.g., a swap-settled interest rate swaption) will have an impact on credit exposure.
- ***Collateral*** will also have a ***significant impact on credit exposure***, as it typically reduces the level of credit exposure.

Exposure Profiles of Different Security Types

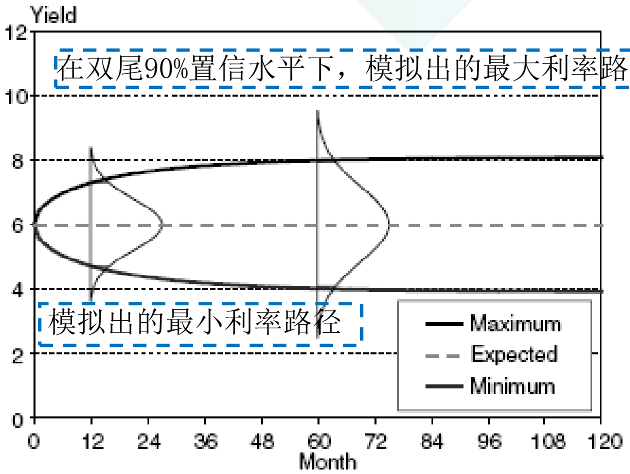
★★★ 结论、计算

Interest Rate Swaps

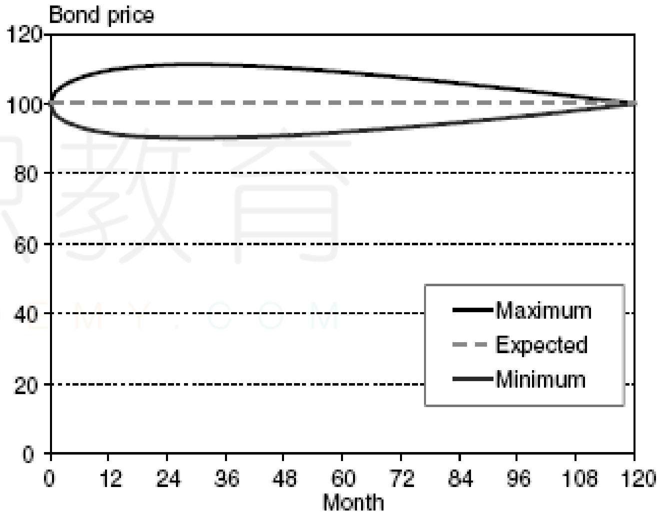
Stochastic process for the interest rate

均值复归的速度　长期平均利率　利率的波动率

$$dr_t = \underbrace{\kappa(\theta - r_t)dt}_{\text{Drift term}} + \underbrace{\sigma r_t^{\gamma} dz_t}_{\text{Diffusion term}}$$



For par bond with the fixed rate



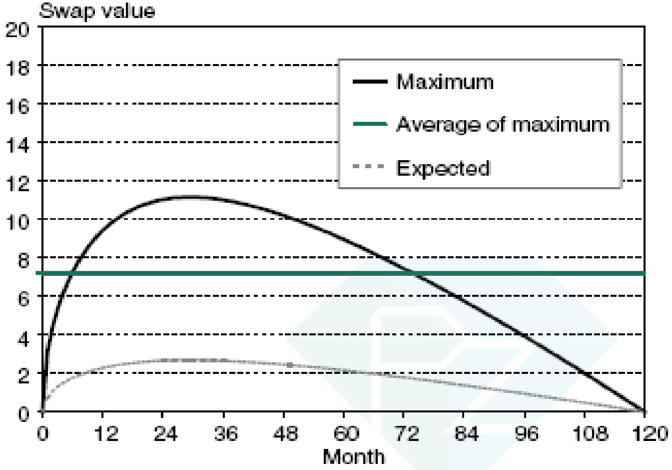
Diffusion effect (DE, 扩散效应):

Interest rate uncertainty ↑ ⇒ DE ↑

Amortization effect (AE, 摊销效应):

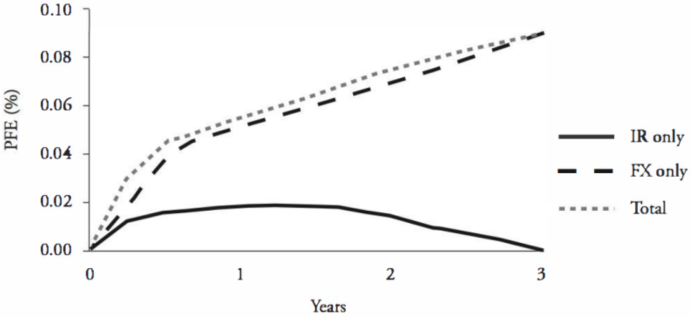
Bond's duration ↓ ⇒ AE ↑

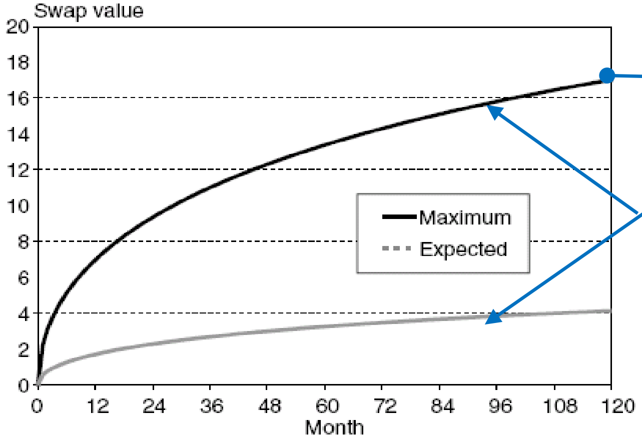
Exposure of interest rate swap



- Assume that the bond's (modified) duration is proportional to the remaining life, D = k(T − t) at any date t.
- The volatility from 0 to time t: $\sigma_{r_t - r_0} = \sigma\sqrt{t}$
- Hence the swap volatility: $\sigma(V) = [k(T - t)] \times \sigma\sqrt{t}$
- Thus peak exposure occurs : $t_{MAX} = \frac{1}{3}T \Leftarrow \frac{d\sigma(V)}{dt} = 0$

Currency Swaps





Peak exposure at the end (no amortization effect)

For credit exposure:
Currency swap > Interest rate swap

Other Security Types

Types	Description
Loans, Bonds and Repos	• The ***exposures*** of bonds, loans and repos can usually be considered ***almost deterministic and approximately equal to the notional value***. • Bonds typically pay a ***fixed rate*** and therefore will have some ***additional uncertainty*** since, if interest rates decline, the exposure may increase.
Options	The general ***exposure*** profile of a ***long option*** position tends to ***increase*** until exercise due to the ***increased possibility that the option can be highly in the money***
Credit Derivatives	The ***increase in exposures in early years*** is the result of the ***CDS premium*** (or credit spread) ***widening***. The maximum exposure for the CDS occurs at a credit event where the notional value is paid less the recovery value.

Risk Mitigation Techniques ★★★ 结论

Modeling Netting Agreements

The benefit of correlation

- ***Positive correlations have lower netting benefits*** than negative correlations, with perfect positive correlation providing the least netting benefit.
- ***Negative correlations provide stronger netting benefits***, with perfect negative correlation leading to the greatest netting benefit.

Netting factor

$$\text{netting factor} = \frac{\sqrt{n+n(n-1)\bar{\rho}}}{n}$$

- Netting benefit improves (i.e., netting factor declines) with a larger number of exposures and a lower correlation.
- It is important to note that the ***netting benefit also depends on the initial MtM of transactions***.

Impact of Collateral on Credit Exposure

Certain parameters impact the effectiveness of collateral

- ***Remargin period***: the time between the call for collateral and its receipt.
- ***Threshold***: exposure level below which collateral is not called. It represents an amount of uncollateralized exposure.
- ***Minimum transfer amount***: the minimum quantity or block in which collateral may be transferred. Quantities below this amount represent uncollateralized exposure as well.
- ***Independent amount***: an amount posted independently of any subsequent collateralization. This is also referred to as the ***initial margin.***
- ***Rounding***: the process by which a collateral call amount will be adjusted (rounded) to a certain increment.

Impact of Collateral on Credit Exposure	
Remargin Period	The remargin period, also known as the margin call frequency, is ***the period from which a collateral call takes place to when collateral is actually delivered.*** Expected exposure (EE): $EE = \frac{1}{\sqrt{2\pi}} \times \sigma_E \times \sqrt{T_M} \approx 0.4 \times \sigma_E \times \sqrt{T_M}$ Potential future exposure (PFE): $PFE = k \times \sigma_E \times \sqrt{T_M}$
Imperfect collateralization	• Collateralization may be deficient due to terms in the collateral agreement, such as ***threshold, minimum transfer amount, and rounding***. $\text{exposure}_{t-\Delta} > \text{collateral}_{t-\Delta}$ • Exposure could increase ***between margin calls***. $\text{exposure}_{t} > \text{collateral}_{t-\Delta}$

Section 13

Counterparty Risk

Introduction of Counterparty Risk

★★★ 概念对比

A type of credit risk that one of the parties to a transaction will ***not fulfill its obligations***. Counterparty risk is typically a ***two-way transaction***.

Counterparty Risk & Market Risk

- ***Market risk*** is the risk that the value of an underlying position will move against the trader due to adverse market factors, which may result in a ***negative NPV*** of the investment.
- Counterparty risk is the conditional risk that the ***NPV is positive***, however, the ***counterparty fails to perform its obligations***.

Counterparty Risk & Lending Risk

Lending risk has two notable characteristics

- The ***principal*** amount at risk is usually ***known.***
- ***Only one party*** takes on risk.

Counterparty risk

- the value of the underlying instrument is ***uncertain***
- Counterparty risk is ***bilateral*** in that each party takes on the risk that the counterparty will default

Counterparty Risk Terminology	
Credit exposure	***Credit exposure*** is the loss that is "conditional" on the counterparty defaulting.
Credit migration	There is ***mean reversion in credit quality***, so the implication is that counterparties with strong credit ratings tend to deteriorate and those with weak credit ratings tend to improve.
Recovery	LGD = 1 - recovery rate
Mark-to-market	MtM = present value of all expected inflows less the present value of expected payments

Managing and Mitigating Counterparty Risk ★★★ 概念对比

Approaches	Description
Close-out	The ***immediate closing of all contracts*** with the defaulted counterparty.
Collateralization (i.e., margining)	Occur in the form of a collateral agreement between two counterparties that reduces exposure by requiring sufficient collateral to be posted.
Walkaway feature	A ***walkaway feature*** allows a party to ***cancel the transaction if the counterparty defaults.*** It is advantageous if a party has a ***negative MtM*** and the counterparty defaults.
Mark-to-market	MtM = present value of all expected inflows less the present value of expected payments
Diversification	Limit credit exposure to any given counterparty
Exchange and centralized clearinghouses	***Exchange and centralized clearinghouses take on the role of the counterparty*** and guarantee all trades by removing all counterparty risk from trades.
Netting	commonly used to mitigate counterparty risk
Hedging	Using credit derivatives allows an organization to reduce counterparty exposure to its own clients

Credit Value Adjustment (CVA)

★★★ 名词掌握、结论

- The credit value adjustment (CVA) is defined as ***the expected value or price of counterparty credit risk***. A positive value represents a ***cost*** to the counterparty that bears a greater propensity to default.
- ***Risky value = risk-free value - CVA***

CVA as a spread

- This would be a charge to the weaker counterparty.

$$\text{CVA as a spread} = X^{\text{CDS}} \times \text{EPE}$$

Impact of Changes in Credit Spread assumptions on CVA

Credit Spread:

- The CVA will most often ***increase*** given an ***increase in the credit spread.***
- However, the impact will not be linear because default probabilities are limited to 100%. If a counterparty is very close to default, the CVA will actually decrease slightly, and in default the CVA will fall to zero.

Credit spread curve

- The CVA will be ***lower*** for an ***upward-sloping curve*** compared to a flat and a downward-sloping curve, and the CVA will be ***higher*** for a ***downward-sloping*** curve compared to a flat and an upward-sloping curve.

Incremental CVA	Change (or increment) in CVA that a ***new trade will create***, ***taking netting into account.***
Marginal CVA	• By using a marginal CVA measure, it will be possible to ***break down a CVA for any number of netted trades into trade level contributions*** that sum to the total CVA. • This metric allows for more rigorous analysis as it is ***useful for better understanding which trades have the greatest impact*** on a counterparty's CVA.

Wrong-Way Risk Vs. Right-Way Risk

★★★ 名词掌握、结论

Wrong-Way Risk

- An outcome of any association, dependence, linkage, or interrelationship between ***exposure and counterparty creditworthiness*** that generates an overall ***increase in counterparty risk.***
- An increase in the amount of the credit value adjustment (CVA).

Right-Way Risk

- Any dependence, linkage, or interrelationship between the ***exposure and default probability of a counterparty*** producing an overall ***decrease in counterparty risk*** is described as right-way risk.
- RWR decreases the CVA and increases the DVA

Examples of Wrong-Way Risk and Right-Way Risk	
Over-the-Counter Put Option	***WWR*** (Assume the ***counterparty and the underlying issuer are the same***.)
Over-the-Counter Call Option	***RWR***
Credit Default Swaps (CDSs)	***WWR*** (The 2007-2009 credit crisis offers a classic example)
Foreign Currency Transactions	***WWR*** (for the financial institution in the developed economy)
Interest rate swaps	***WWR*** (***The decline in the euro swap rate also increased the counterparty risk exposure.*** Deteriorating economic conditions also ***increased the default probability*** of Italian financial institutions)
Commodities	***WWR***

Section 14

Portfolio Credit Risk

Credit Risk Portfolio Models ★★ 概念对比

Models	Description
CreditRisk+ (Credit Suisse)	• Each obligor has its own sensitivity to each of the ***common risk factors.*** • The model allows for ***only two outcomes (default or non-default)*** for a loss of a fixed size. • The model assumes that defaults are ***uncorrelated across obligors.***
CreditMetrics (J.P. Morgan)	Steps in calculating credit VaR using CreditMetrics: 1. Determine rating class for debt claim. 2. Use historical rating transition matrix to determine the probability that claim will migrate. 3. Estimate the distribution of value for claim by computing the expected values for one year. 4. Use the 1-year forward zero curves rates to get current price of zero-coupon bond. 5. Assume annual coupons to compute value of bond for each possible rating for next year. 6. Compute the expected bond value $E(BV_p) = \sum p_i BV_i$. Then compute the credit value at risk (VaR) for a given confidence level.
Moody's KMV	• Use of ***current equity values*** in the model. This allows for the impact of a current event to immediately affect the probability of default. Ratings changes occur with a considerable lag. • The use of equity values allows for ***probabilities of default to change continuously*** as equity values change. In the ***CreditMetrics*** approach, ***the value of the firm change without any impact*** on the probability of default.
CreditPortfolioView	• Model the transition matrices ***using macroeconomic or economic cycle data.*** • CreditPortfolioView estimates an ***econometric model for an index that drives the default rates of an industrial sector.***

Comparison of Credit Risk Models

	CreditMetrics	CreditRisk+	Moody's KMV	Credit Portfolio View
Originator	JPMorgan	Credit Suisse	KMV	McKinsey
Model type	**Bottom-up**	**Bottom-up**	Bottom-up	**Top-down**
Risk definition	Market value (MtM)	**Default losses(DM)**	Default losses (MTM/DM)	Market value (MtM)
Risk drivers	Asset values	Default rates	Asset values	Macro factors
Credit events	Rating change/ default	**Default**	**Continuous** default probability	Rating change/default
Probability	**Unconditional**	**Unconditional**	**Conditional**	**Conditional**
Volatility	Constant	Variable	Variable	Variable
Correlation	From equities (structural)	Default process (reduced-form)	From equities (structural)	From macro factors
Recovery rates	Random	**Constant within band**	Random	Random
Solution	Simulation/ analytic	Analytic	Analytic	Simulation

Default Correlation for Credit Portfolios ★★

Default correlation: $\rho_{12} = \dfrac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$

计算

结论

Drawbacks in Credit Portfolio Framework

- ***The number of required calculations:*** There are 2^n event outcomes. The number of correlations is equal to n(n-1).
- ***Certain characteristics of credit positions do not fit well in the default correlation*** credit portfolio model.
- ***The limited data for estimating defaults:*** Firm defaults are relatively rare events.

***Credit VaR & Default Correlation* (看基础班讲义例题)**

- ***Default correlation impacts the volatility and extreme quantiles of loss rather than the expected loss***.
  - ***If default correlation is 1***, then there are ***no credit diversification benefits,*** and the portfolio behaves as if there were just one credit position.
  - ***A default correlation equal to 0*** implies the portfolio is a ***binomial-distributed*** random variable because there is no correlation with other firms/ credits.
- "Granular": ***reducing the weight*** of each credit as a proportion of the total portfolio by ***increasing the number of credits.***
  - As a credit portfolio becomes ***more granular, the credit VaR decreases***. However, ***when the default probability is low, the credit VaR is not impacted as much*** when the portfolio becomes more granular

Single Factor Model ★ 结论

Firm i's individual asset return is defined as:

- (1) the return on a market factor m that denotes the correlation between default and the state of the economy, and (2) a shock ε_i that captures idiosyncratic risk.

$$\alpha_i = \beta_i m + \sqrt{1-\beta_i^2}\,\varepsilon_i$$

- Assuming that ***each ε_i is not correlated with other credits***, each return on asset, ***a_i, is a standard normal variate.*** The correlation between pairs of individual asset returns between two firm's i and j is $\beta_i\beta_j$

Conditional Independence

- The ***unconditional default distribution*** is a standard normal distribution.
- The ***conditional distribution*** is a normal distribution with a mean of $\beta_i \bar{m}$ and a standard deviation of $\sqrt{1-\beta_i^2}$.

Credit VaR with a single-factor model

The unconditional distribution used to calculate credit VaR

1. The default loss level is assumed to be a random variable *X* with realized values of x.
2. The market factor return, $\bar{m}$, for a given loss level, $\bar{x}$, is determined based on the following relationship

$$\Phi^{-1}(\bar{x}) = \left(\frac{k-\beta\bar{m}}{\sqrt{1-\beta^2}}\right)$$

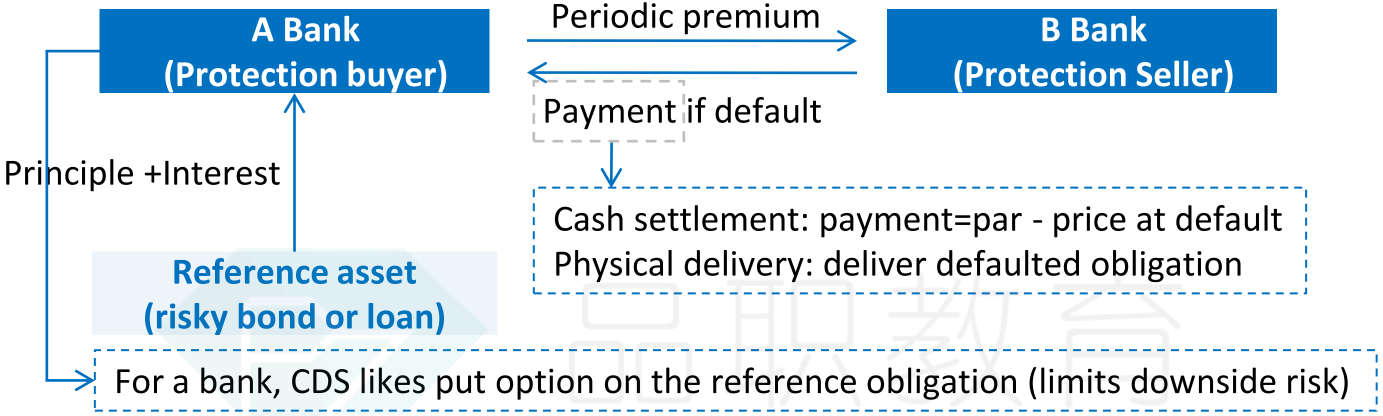
3. The market factor is assumed to be standard normal, and therefore, a loss level of 0.01 (99% confidence level) is equal to a value of -2.33 based on the standard normal distribution.
4. These steps are repeated for each individual credit to ***determine the loss probability distribution.***

Section 15

Credit Derivatives

- **Credit Event**
  - Credit Event (defined by ISDA)
    - Bankruptcy (破产)
      - Dissolution of the obligor (other than merger)
      - Insolvency or inability to pay its debt
      - Assignment of claims
      - Institution of bankruptcy proceeding
      - Appointment of receivership
      - Attachment of substantially all assets by third party
    - Failure to pay (支付失败)
    - Obligation/cross acceleration (交叉加速)
    - Obligation/cross default (交叉违约或连带违约)
    - Repudiation (拒付)
    - Moratorium (延期支付)
    - Restructuring (重组)
  - Other credit events: downgrading, currency inconvertibility, governmental action. (not defined by ISDA)

Credit Default Swaps ★★★ 结论



Long corporate bond = long risk-free bond + ***short credit default swap (CDS)***

Short put option on the firm value

Settlement methods	
Physical delivery	• The ***buyer*** of the CDS ***delivers the reference obligation*** to the seller of the swap and ***receives the par value.*** • Delivery squeeze • An ***advantage*** of physical delivery is that there is ***no need to determine the size of the loss***
Cash delivery	• (100 - Z)% * the notional principal (Z is midpoint) • There is ***no need to own or purchase the defaulted securities.*** • A problem arises because the market price is ***fluid***

Basket credit default swap 结论

First-to-default basket swap	Nth-to-default basket swap (N≥2)
$\rho_{ij} \downarrow \Rightarrow premuim \uparrow$	$\rho_{ij} \uparrow \Rightarrow premuim \uparrow$

Pricing Model

One-factor Gaussian copula model for time to default

Senior basket CDS & Subordinated basket CDS

Basket (A,B,C,D,E) $50 million

Payments for Senior basket CDS

Payments for Subordinated basket CDS

Final loss level ($30 million)

Pre-specified loss level ($20 million)

- **Senior basket CDS** (高级篮子信用违约互换) will not receive any compensatory payment until a pre-specified loss level is reached.
- **Subordinated basket CDS** (次级篮子信用违约互换) will receive compensatory payments for cumulative losses below the pre-specified loss level.

定价 ➡

$$V_{CDS} = PV_{payoff} - s\left(PV_{spread}\right) = \left[\sum_{t=1}^{T} \boxed{k_t}\left(1-\boxed{f}\right)PV_t\right] - \boxed{s}\left(\sum_{t=1}^{T} \boxed{S_{t-1}}PV_t\right) = 0$$

计算

Recover rate: f

Present value of a dollar: PV_t

spread: s

Survival rate: S_{t-1}

$$\boxed{k_t} = S_{t-1}\boxed{d_t}$$

Marginal default rate from now to year t: k_t

Marginal default rate in year t: d_t

Other Credit Derivatives

Total return swaps (TRS) ★★★ 结论

Interest on the loan + market value increase of the loan (or - market value decrease of the loan)

Pay total return on reference obligation

A Bank (Protection buyer, Total return payer, or risk seller)

Receive LIBOR + spread

B Bank (Protection seller, Total return receiver, or risk buyer)

Invest

Reference obligation (Loan or bond)

exposed to credit and interest risk

Vulnerable Option 了解

- An ***option with default risk***. The vulnerable option holder ***receives the promised payment only if the value of the counterparty firm, V, is greater than the required payment*** on the option. The payoff of the vulnerable option is: ***Max[Min(V, S - X), 0]***
- ρ(V, S): correlation between the value of the firm and the underlying asset value. If ***ρ(V, S) is strongly negative*** then vulnerable option has ***little value***. If ***ρ(V, S) is strongly positive*** then there is ***no credit risk***.

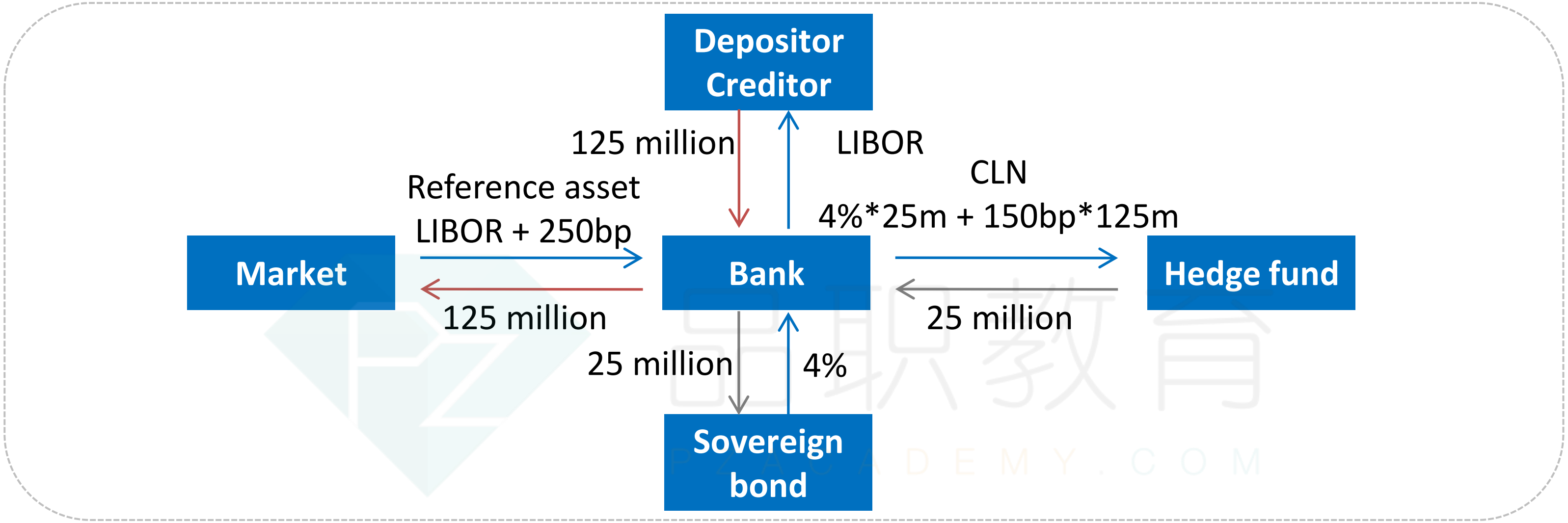
Derivatives With Credit Risks - Swap 了解

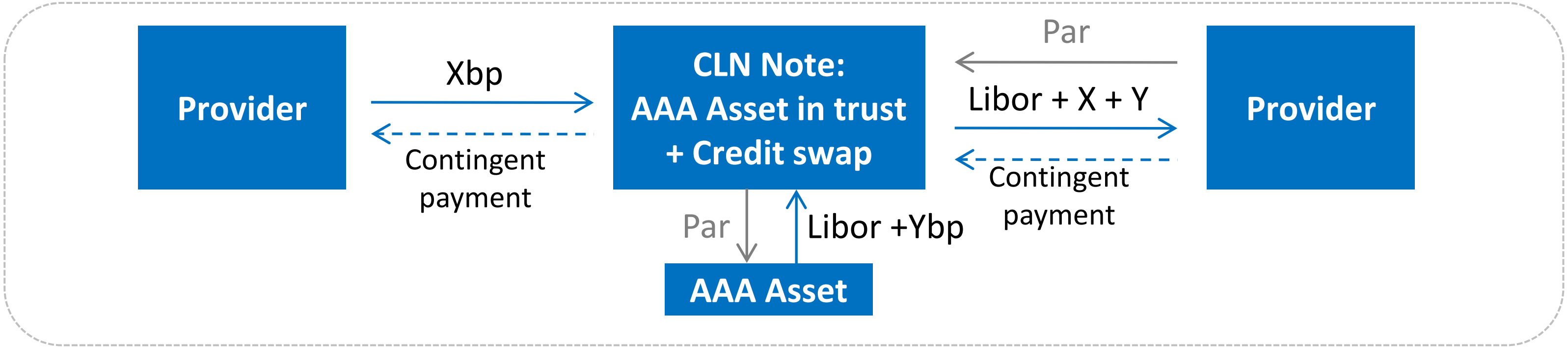
RC ← F — **MM**

RC — S → **MM**

(-)Max(F - S, 0) + Max[Min(S, V) - F, 0]

Credit-Linked Notes ★★ 结论





Portfolios of Credit Derivatives ★ 结论

CDS index: most popular

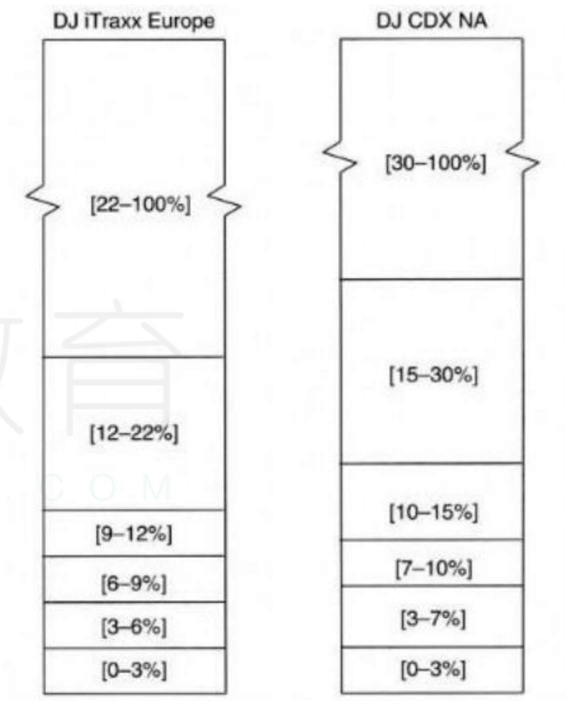
- The two most popular (and liquid) indices are the CDX NA IG and iTraxx Europe.
- CDS indices are created with a ***fixed maturity and static constituents***. If there is a significant credit event, the affected credit entity will be removed, but ***not replaced, from the index***.
- Each tranche is described by its ***attachment point (X%)*** and ***detachment point (Y%)***, denoted [X%, Y%], and the width of each tranche is Y% - X%.

Super senior risk

- Super-senior tranches represent the portion of the capital structure for credit indices that has the ***highest subordination level and lowest probability of incurring losses.*** Informally, these tranches are termed ***super triple-A and quadruple A*** tranches.

$$\text{number of defaults} = n\left(\frac{X\%}{1-\text{revovery}}\right)$$

- ***The primary risk of these tranches is counterparty risk (termed super-senior risk)*** as this risk is positively correlated to tranche seniority. That is, ***higher seniority tranches have higher levels of counterparty risk.***



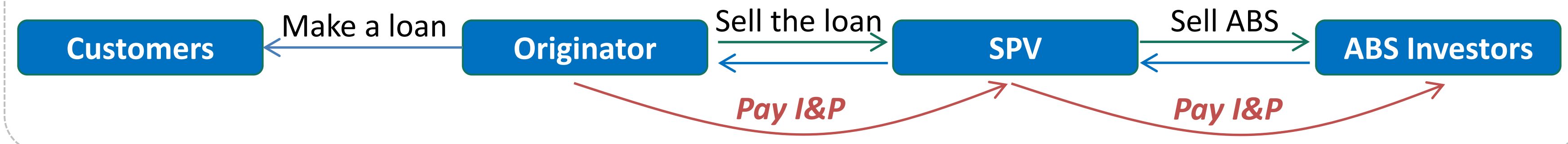
Section 16

Structured Credit Risk and Securitization

Types of Structured Products ★ 概念对比

Types of Structured Products	
Covered bonds	• ***Principal and interest is paid and guaranteed by the originator*** and is not based on the performance of the underlying assets themselves. Covered bonds are not true securitizations since the assets are ***not part of a bankruptcy-remote structure***
MPS	• MBS are ***true off-balance sheet*** securitizations. • Most pass-throughs are ***agency MBS*** that carry implicit or explicit government guarantee. The ***primary risk is due to prepayment*** of principal by the homeowner.
CMOs	• ***CMOs are MBSs that tranche*** cash flows into different securities. • The most basic structure is the ***waterfall or sequential pay*** structure. Tranche 1 will have a very low prepayment risk
Structured credit products	The difference is that structured credit products create ***tranches that have different amounts of credit risk.***
ABS	MBS is a special case of the more general ABS.

Securitization Process and Participants



Waterfall Structure

- The most ***senior*** tranches at the top of the capital structure will have the ***highest priority*** to receive principal and interest and earns a relatively low fixed coupon
- The ***equity*** tranche is the slice of the cash flow distribution with the ***lowest priority and will absorb the first losses up to a prespecified level.***
- Between the senior and equity tranches is the ***mezzanine*** tranche (i.e., the junior tranche). The mezzanine tranches will ***absorb losses only after the equity tranche*** is completely written down.

Three-Tiered Securitization Structure

- ***Inflows prior to maturity*** = interest on the collateral (L_t) + the recovery from the sale of any defaulted assets in the current period (R_t)
- The ***terminal cash flows in the final year*** = last interest payment + principal and recovery of defaulted assets.
- The ***outflows*** (B) = the coupon payments paid to senior + mezzanine note holder.

To determine the cash flow to equity, the following steps must be performed:

- Is the current period interest sufficient to cover the promised coupons: L_t-B ≥0?
  - ***If yes***, then the following overcollateralization test must be performed to see: L_t-B ≥ K?
    - If yes, then K is diverted to trust, and L_t-B - K flows to equity holders: OC_t = K.
    - If no, then L_t-B is diverted to trust, and nothing flows to equity holders: OC_t = L_t-B
  - ***If no***, then the interest is not sufficient to pay bondholders and all L_t flows to bondholders. Therefore, the shortfall is B- L_t.

Credit Enhancements	
External credit enhancement	insurance or wraps purchased from a ***third party***
Internal credit enhancement	Overcollateralization (hard credit enhancement)
	Excess spread (soft credit enhancement): the difference between the cash flows collected and the payments made to all bondholders
	Subordinating note classes (tranches)

SPV Structures	
Amortizing structure	• Principal and interest payments are made on an ***amortizing schedule*** to investors over the life of the product (residential mortgages, commercial mortgages, and consumer loans). • Because payments are made as coupons are received, this type of structure is referred to as a ***pass-through structure.***
Revolving structures	• ***Principal payments of the assets are paid in large lump sums*** rather than a pre-specified amortization schedule. (Credit card debt and auto loans) • Under a revolving structure, ***payments are not simply passed through.*** Rather, ***principal payments are often used to purchase new receivables***
Master trust structure	• A master trust structure ***allows an SPV to make frequent issues or multiple securitizations.*** • ***the master trust structure enables the SPV to issue multiple ABS through the single trust.*** • The ability of SPV master trust structures to sell multiple issues to investors that ***share excess spreads over these multiple series***

ABS and MBS Performance Tools

Performance Analysis Tools	Asset Type	Calculation
loss curves	auto loans	expected cumulative losses
absolute prepayment speed (APS)	auto loans	Prepayments/pool balance
delinquency ratio	credit cards	past due receivables/pool balance
default ratio	credit cards	Defaults/pool balance
monthly payment rate (MPR)	credit cards	receivables collected/pool balance
debt service coverage ratio (DSCR)	commercial mortgages	NOI / debt payments
weighted average coupon (WAC)	mortgages	weighted pool coupon payment
weighted average maturity (WAM)	mortgages	weighted pool maturity
weighted average life (WAL)	mortgages	$\sum(a/365)\ xPF(t)$
single monthly mortality (SMM)	mortgages, home-equity, student loans	prepayment / pool balance
constant prepayment rate (CPR)	mortgages, home-equity, student loans	$1-(1-SMM)^{12}$
Public Securities Association (PSA)	mortgages, home-equity, student loans	100PSA: [CPR/(0.2)(months)] x 100

Structured Credit Risk ★★★ 结论

Impact of Probability of Default and Default Correlation

PD

Increasing the probability of default will negatively impact the values of all tranches.

Convexity

- *Senior* tranches exhibit *negative convexity*. *Equity* tranches exhibit *positive convexity*. The *mezzanine*: negative convexity at low default rates, positive convexity at high default rates.

Correlation

Equity tranche

- A *low correlation implies a predictable, but positive, number of defaults.* In turn, the equity tranche will assuredly *suffer writedowns*.
- *the equity tranche increases in value from increasing correlation* as the possibility of zero (or few) credit losses increases from the high correlation.

Mezzanine tranche

- When *default rates are low*, increasing the correlation increases the likelihood of losses to the junior bonds (*similar to senior bonds*).
- When *default rates are relatively high*, increasing the correlation actually decreases the expected losses to mezzanine bonds (*similar to equity tranche*)

Senior tranches

- *increasing* correlation *decreases the value* of senior tranches as the pool is now more likely to suffer extreme losses.

Increasing Default Probability (Holding Correlation Constant)

	Mean Value	Credit VaR
Equity tranche	↓	↓
Mezzanine tranche	↓	↑then↓
Senior tranche	↓	↑

Increasing Correlation (Holding Default Probability Constant)

	Mean Value	Credit VaR
Equity tranche	↑	↑
Mezzanine tranche	↓(at low default rates) ↑(at high default rates)	↑
Senior tranche	↓	↑

Measuring Default Sensitivities: 1/20[(mean value/loss based on π+0.001) – (mean value/loss based on π – 0.001)]

Implied Correlation

- For securitized tranches, ***starting with observed market prices and a pricing function for the tranches, it is possible to back out the unique implied correlation*** to calibrate the model price with the market price.

Other Risk Factors for Structured Products

- **Systematic risk:** high systematic risk expressed in high correlations can still severely damage a portfolio.
- **Tranche thinness:** The ***equity and mezzanine tranches*** are relatively ***thin***. The implication is that given that the tranche has been breached, the loss is likely very large.
- **Loan granularity:** ***Loan granularity*** references the ***loan level diversification***.

Section 17-18

The Securitization of Subprime Mortgage Credit
Collateralized Debt Obligations (CDOs)

了解

The Subprime Securitization Process

Prime loans that meet conforming standards are sold to government sponsored enterprises (GSEs).
The remaining loans are increasingly being sold and taken off the originators' balance sheet.

Frictions in Subprime Securitization	
Friction 1: mortgagor & originator	The lender may steer the borrower to products that are not suitable.
Friction 2: Originator and arranger	The arranger operates at an information disadvantage to the originator.
Friction 3: Arranger and third-parties	The arranger of the pool of mortgages will possess better information about the borrower than third parties (Resolution: Due diligence)
Friction 4: Servicer and mortgagor	The homeowner in financial difficulty does not have the incentive to upkeep tax payments, insurance, or maintenance on the property.
Friction 5: Servicer and third-parties	Moral hazard: Between servicer & asset manager; Between servicer & credit rating agencies
Friction 6: Asset manager and investor	It is difficult for the investor to comprehend the investment strategy (same ***moral hazard*** problem as shareholder-manager)
Friction 7: Investor and credit rating agencies	Rating agencies are compensated by the arranger. (***conflict of interest***)

Characteristic of The Subprime Mortgage Market

Protection to Investors

- **Subordination:** creating tranches of differing priority levels.
- **Excess spread:** the weighted average coupon (less servicing expenses) exceeds the weighted average payout.
- **Shifting interest:** senior investors receive all principal in the pool while mezzanine investors receive only interest.
- **Performance triggers:** release of overcollateralizion which is applied from the bottom of the capital structure up
- **Interest rate swaps:** Since the first few years of the pool are fixed, the pool faces interest rate risk. As protection, interest rate swaps are used where the pool will pay a fixed rate and receive a floating rate.

The Credit Ratings Process

Credit ratings for subprime securities differ from corporate ratings

- ***First, corporate bond ratings are based on the firm-specific characteristics*** of the issuer. ***Systematic risk*** and degree of correlation between assets is ***important*** in the ***latter*** but not the former.
- In addition, ***the forecasts for ABS incorporate future economic conditions*** since the cash flow stream is tied to the macro environment.
- Finally, while corporates and ABSs with the same rating may indicate similar default probabilities, the ***ABS will exhibit much wider variation in losses.***

Predatory Lending and Borrowing

- lying on the mortgage application allows the borrower to the buy the house with the expectation that continued appreciation will allow a favorable refinancing.

Flaws in The Securitization of Subprime Mortgages

- ***Originate-to-distribute model:*** OTD models have produced ***three primary benefits.***
  - The first benefit is that loan originators enjoy ***increased capital efficiency and decreased earnings volatility***
  - The second benefit is that ***investors have a wider array of diversification options for the fixed income portion*** of their portfolios
  - The third benefit is that ***borrowers have expanded access to credit and lowered borrowing costs.***
- Under the ***traditional originate-to-hold (OTH) lending model***, credit assets are retained at the business unit level.

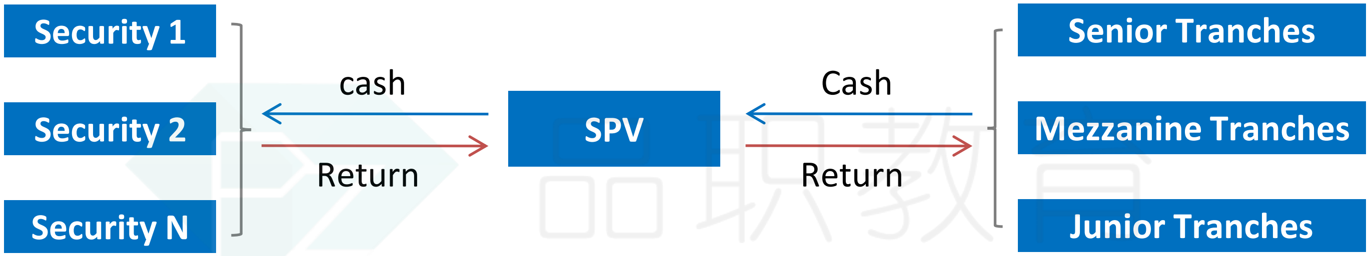
Credit Risk Mitigation Techniques

Type	Definition
Bond insurance	Purchase ***insurance***
Collateralization	The losses sustained by the lender will be offset by the value of the collateral
Termination	A certain trigger event, such as a downgrade, has occurred and the issuer was obligated to repay the loan early
Reassignment	Give the right to assign one's position as a counterparty to a third party
Netting	Net replacement value represents the true credit risk exposure.
Marking-to-market	periodically acknowledging the true market value of a transaction.
Loan syndication	multiple lenders all working together as a team to provide funding to a given borrower
Outright selling	Outright selling of a loan portfolio in the secondary market

Collateralized Debt Obligations (CDOs)

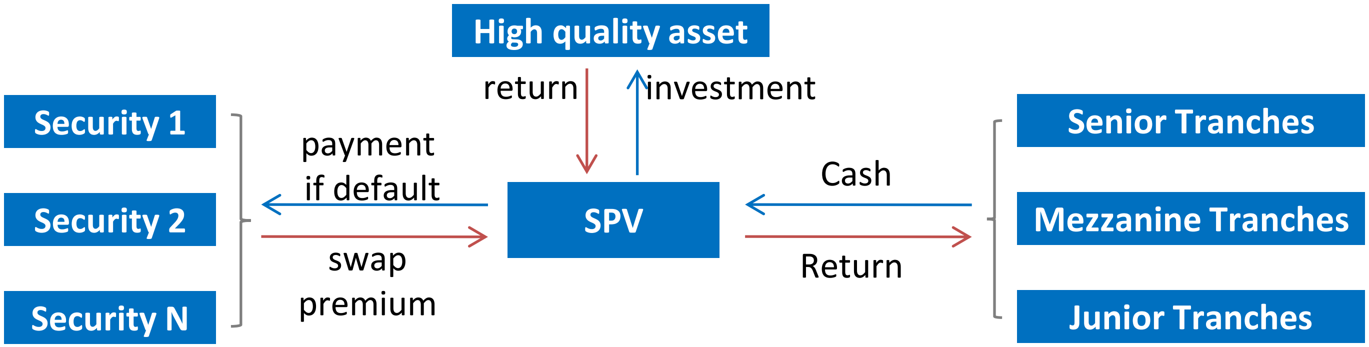
Cash-flow CDOs & Synthetic CDOs

Cash-flow CDO with N underlying securities



Synthetic CDO with N underlying securities

Characteristics:
- Sell CDS
- Buy high quality asset
- Off-balance sheet
- No operational risk



Section 19

Risk Mitigation Techniques

结论 ★

Netting and Close-Out

Netting and Close-Out Between Two Counterparties

- ***Payment netting:*** combining the cash flows from different contracts with a counterparty into a single net amount
- ***Close-out netting:*** the netting of contract values with a counterparty in the event of the counterparty's default

Netting and Close-Out Between Multiple Counterparties

- ***Netting arrangements would involve multiple counterparties to mitigate counterparty and operational risk.*** Typically, multilateral netting is achieved with a central entity, such as an exchange or clearinghouse.

Termination Features

Termination provisions

- Termination events allow institutions to ***terminate a trade before their counterparties become bankrupt.***

Walkaway clauses

- Under these clauses an entity can ***walk away from, or avoid, its net liabilities to a counterparty that is in default***, while still being able to claim in the event of a positive MtM exposure.

Trade compression

- An approach for utilizing multilateral netting without the need for a membership organization.
- Compression aims to ***reduce the gross notional amount and the number of trades*** (e.g., OTC derivatives transactions).

Collateral Management

- Collateral management has been highly standardized through the introduction of ISDA documentation. (CSA)
- There are three key parameters established with any CSA: ***Threshold, Minimum transfer amount, Independent amount***

Central Counterparties (CCP)

引入CCP

OTC市场容易违约 →

Strengths of a CCP

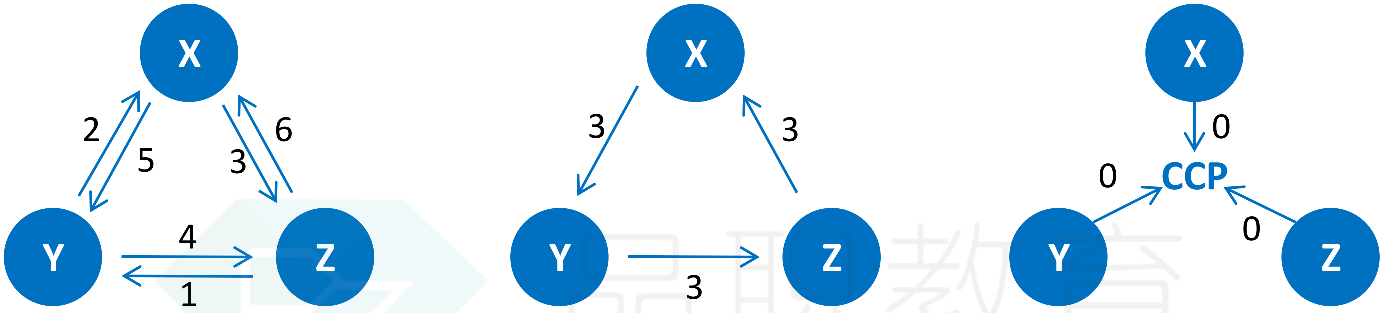
- ***Multilateral netting:*** Mitigate counterparty risk and reduces total credit exposures.
- ***Liquidity.*** values are set daily to adjust variation margins.
- ***Transparency.*** The process of daily valuation greatly enhances the transparency
- ***Legal and operational efficiency.*** A CCP can work directly with regulators
- ***Loss mutualization.*** This process reduces systemic risk as losses are distributed through a network of members.
- ***Default management through auction process.*** The defaulted positions are auctioned off to surviving members

Weaknesses of a CCP

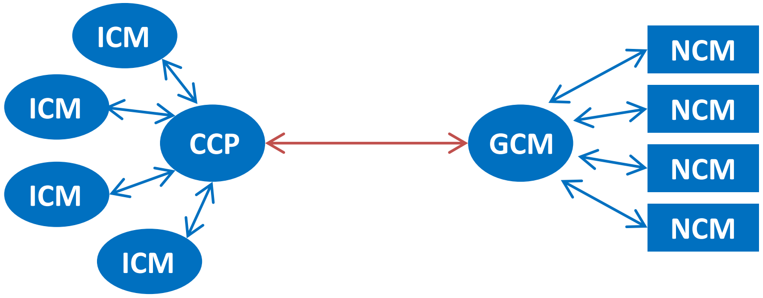
- ***Inability to eliminate counterparty risk.*** A CCP only reduces counterparty risk
- ***Undesirable consequences of CCP features and roles.*** moral hazard and adverse selection
- ***CCPs can potentially increase systemic risk.*** if a CCP defaults the result could be severe systemic problems.

CCP Netting Schemes

Reduction of Risk Exposure through Multilateral Netting



CCP Trade Participants



loss waterfall systems

Defaulted member's initial margin and default fund contribution

↓

Portion of CCP's equity

↓

Remaining CCP reserve fund

↓

Surviving member's capital contribution

↓

CCP remaining equity

↓

Insolvency of CCP or liquidity support

Thank You!