- Transformadas Laplace
  - → Tabla (EDO)
  - → definición $\int_0^{\infty} e^{-st} f(t)\, dt$
    - ↓ demostrar

$f(t) \longrightarrow F(s)$

E.D.O — polinomio

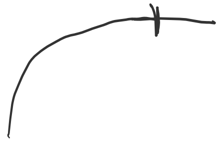
Dominio tiempo

- EDO
  - → Lineales $y(t) = e^{-} + \int$ (Forma Generica)
  - → homogenea ------ $= 0$
  - → particular $= F$
  - → euler → operador diferencial
  - → Bernoulli → Formular

→ Propiedades (Demostrar) $\int_0^\infty e^{-st} f(t)\, dt$

EDO $t=0$ Dominio tiempo $\longrightarrow \mathcal{L}$ $s \to \infty$ T.V.I

EDO $t=\infty$ Dominio tiempo $\longrightarrow \mathcal{L}$ $s \to 0$ T.V.F



→ ¿conceptual?

Ec. Bessel

$$x^2 y'' + x y' + (x^2 - r^2) y = 0 \quad , \quad r \geq 0$$

$$y^2 x'' + y x' + (y^2 - \alpha^2) x = 0 \quad , \quad \alpha \geq 0$$

$$\cancel{z}^2 \frac{dy^2}{dt} + z \frac{dy}{dt} + (z^2 - \beta) y = 0 \quad , \quad \beta \geq 0$$

Solución Ec. Bessel
$$J_r(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \cdot \Gamma(r+n+1)} \left(\frac{t}{2}\right)^{r+2n}$$

Función Gamma

$J_0(0) = 1$ es la condición inicial c.i

# Transformada $\mathcal{L}$ inversa

$\mathcal{L} \rightarrow$ tiempo

2. a) $f(t) = te^{-2t} \cdot \operatorname{sen} \omega t$

p. 6     p. 3

$\rightarrow \mathcal{L}\{\operatorname{sen} \omega t\} = \dfrac{b}{s^2+b^2} = \dfrac{\omega}{s^2+\omega^2}$    $b = \omega$

$\mathcal{L}\{e^{-2t} \cdot \underbrace{\boxed{\operatorname{sen} \omega t}}_{f(t)}\} = \mathcal{L}\{e^{at} \cdot f(t)\} = \boxed{F(s-a)}$

$= \dfrac{\omega}{(s--2)^2+\omega^2}$    $a = -2$    $\boxed{(s-(-2))}$

2 a) $f(t) = t \cdot e^{-2t} \cdot \operatorname{sen}(wt)$

$$\mathcal{L}\{t \cdot e^{-2t} \cdot \operatorname{sen}(wt)\} = \mathcal{L}\{t\} \cdot \mathcal{L}\{e^{-2t} \cdot \operatorname{sen}(wt)\}$$

$$= \mathcal{L}\{e^{-2t} \cdot \underbrace{t \cdot \operatorname{sen}(wt)}_{p.11}\}$$

$$\rightarrow \mathcal{L}\{t \cdot \operatorname{sen} wt\} = \mathcal{L}\{t^1 \cdot f(t)\}$$

$$= (-1)^n \cdot \frac{d^n}{ds^n} L(f) \qquad n = 1$$

$$\boxed{= (-1) \cdot \frac{d}{ds} \frac{w}{s^2+w^2}}$$

$$\boxed{\mathcal{L}\{\operatorname{sen} wt\} = \frac{b}{s^2+b^2} = \frac{w}{s^2+w^2}} = L(f)$$

p.3

$f \circ g = f(g(x))$

$$\rightarrow \mathcal{L}\{e^{-2t} \cdot f(t)\} = \mathcal{L}\{e^{-2t} \cdot T \cdot \operatorname{sen} wt\}$$

$$= -\frac{d}{ds}\left(\frac{w}{s^2+w^2}\right)\Big|_{s = s-a} \qquad L(f)\Big/_{s-a} = F(s-a)$$

p.10

$$= \frac{-d}{ds} \frac{w}{(s-a)^2+w^2} = \frac{2w\,(s+2)}{(s+2)^2+w^2}, \quad s>0$$

2.b) $f(t) = (t^2 - 3t + 2)\ \text{sen}\ 3t$

$$f(t) = t^2\ \text{sen}\ 3t - 3t\ \text{sen}\ 3t + 2\ \text{sen}\ 3t$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2\ \text{sen}\ 3t - 3t\ \text{sen}\ 3t + 2\ \text{sen}\ 3t\} =$$

$$= \mathcal{L}\{t^2\ \text{sen}\ 3t\} - \mathcal{L}\{3t\ \text{sen}\ 3t\} - \mathcal{L}\{2\ \text{sen}\ 3t\}$$

$$= \frac{d^2}{ds^2}\left(\frac{3}{s^2+9}\right) - \frac{d}{ds}\left(\frac{3}{s^2+9}\right) - \frac{6}{s^2+9}$$

$$\rightarrow \mathcal{L}\{t^2\ \text{sen}\ 3t\} \overset{P.11}{=} (-1)^2 \frac{d^2}{ds^2} \frac{3}{s^2+9} = \frac{d^2}{ds}\left(\frac{3}{s^2+9}\right)$$

$n = 2$

$$\underbrace{\mathcal{L}\{\text{sen}\ 3t\}}_{P.3} \rightarrow L(f) = \frac{3}{s^2+9}$$

$\hookrightarrow b = 3$

$$\rightarrow 3\mathcal{L}\{t\ \text{sen}\ 3t\} = (-1)\ \frac{d}{ds}\ \frac{3}{s^2+9}$$

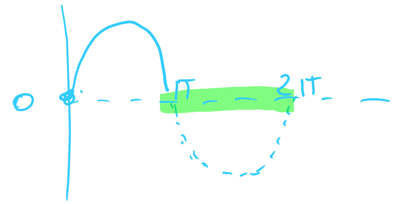
$n = 1$

$$\rightarrow 2\ \mathcal{L}\{\text{sen}\ 3t\} = 2 \cdot \frac{3}{s^2+9}$$

Determinar → tabla ? consulta profe

Calcula → integral , definición $\mathcal{L}$ consulta profe

$$f(t) = \begin{cases} \text{sen } t & 0 < t < \pi \\ 0 & t > \pi \end{cases}$$




conceptual %. $\int$ → area bajo curva

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t) e^{-st} dt = \int_0^\pi f(t)\Big|_0^\pi e^{-st} dt + \int_\pi^\infty f(t)\Big|_\pi^\infty e^{-st} dt$$

$$= \int_0^\pi \text{sen}(t)\, e^{-st} dt + \int_\pi^\infty 0 \cdot e^{-st} dt$$

integrales ciclica

$$= \int_0^\pi e^{-st}\, \text{sen}(t)\, dt$$

$u = e^{-st}$ $\quad du = -se^{-st}$

$dv = \text{sen } t$ $\quad v = -\cos t$

$$\int_0^\pi e^{-st} \cdot \text{sen}(t) = u \cdot v - \int v\, du$$

$$= -e^{-st} \cdot \cos t - \int -\cos t - se^{-st}\, dt$$

$$= -e^{-st} \cos t + \int \cos t \cdot se^{-st}\, dt$$

$$s\left(\int \cancel{s} \cdot e^{-st} \cos t \, dt\right) =$$

$u = e^{-st}$ $\quad du = -se^{-st}$

$dv = \cos t$ $\quad v = \operatorname{sen} x$

$$s\left(e^{-st} \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot -se^{-st} \, dt\right)$$

$$\boxed{se^{-st} \cdot \operatorname{sen} x + s^2 \int \operatorname{sen} x \, e^{-st} \, dt}$$

$$\int e^{-st} \operatorname{sen} t \, dt = -e^{-st} \cos t + se^{-st} \operatorname{sen} x + s^2 \int \operatorname{sen} x \, e^{-st} dt$$

$$(1 - s^2) \int_0^{\pi} e^{-st} \operatorname{sen} t \, dt = -e^{-st} \cos t + se^{-st} \operatorname{sen} t$$

$$\int_0^{\pi} e^{-st} \operatorname{sen} t \, dt = \frac{1}{(1-s^2)} -e^{-st} \cos t + \frac{1}{(1-s^2)} se^{-st} \operatorname{sen} t$$

$$= \frac{e^{-st}}{(1-s^2)} \left( \cos(t) + s \operatorname{sen}(t) \right) \Big|_0^{\pi}$$

$\mathcal{L}$ inversa

$$\mathcal{L}\{t\,f(t)\} = \frac{1}{s(s^2+1)}$$

* descomponen en F. Parciales (2)

descomprir polinomio (1)

$$\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$$= A(s^2+1) + s(Bs+C)$$

$$= As^2 + A + s^2B + sC$$

$$= (A+B)s^2 + (C)s + A$$

$$A = 1$$

$$C = 0$$

$$A+B = 0 \Rightarrow B = -1$$

$$= \frac{1}{s} + \frac{-s+0}{s^2+1} = \frac{1}{s} - \frac{s}{s^2+1} \quad t \cdot f(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s}{s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$$

$$= 1 - \cos(t)$$

$$t \cdot f(t) = 1 - \cos(t)$$

---

$$t^n \cdot f(t) = (-1)^n \cdot \frac{d^n}{ds^n} L(f) = \frac{1}{s} - \frac{s}{s^2+1}$$

P. 11

$n = 1$

$$\Rightarrow -\frac{d}{ds}\left(L(f)\right) = \boxed{\frac{1}{s} - \frac{s}{s^2+1}} \quad / \int$$

$$-L(f) = \int \frac{1}{s} - \frac{s}{s^2+1} \, ds$$

$$-L(f) = \ln(s) - \frac{1}{2}(s^2+1)$$

$$L(f) = \frac{1}{2}(s^2+1) - \ln(s)$$

---

$$\frac{d}{ds}\left(-L(f)\right) = \frac{1}{s(s^2+1)} \quad / \int$$

$$+L(f) = -\ln \frac{s}{\sqrt{s^2+1}} = \ln\left(s \cdot (s^2+1)^{-1/2}\right)$$

$$= - - \frac{1}{2} \ln s \cdot (s^2+1)$$

$$f(t) = \boxed{L(t) = \frac{1}{2}\left(\ln(s) + \ln(s^2+1)\right)}$$

$$\mathcal{L}\{e^{-t} f(2t)\} = F(2(s - -1)) = F(2(s+2))$$

$f(t) \to f(2t)$

$L(t) =$

escalamiento en el tiempo

$$\mathcal{L}\{f(\underset{\downarrow}{at})\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

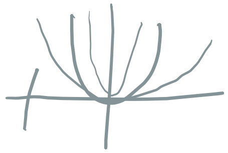
argumento

Transf. sobre la variable independiente

$f(t) \to f(at)$

$\boxed{2f(t) \text{ o } \quad x^2 \cdot f(t)}$

T. dependiente



$$\mathcal{L}\{f(at)\} = \int_0^\infty f(at) e^{-s\tau} dt$$

$t = \frac{\tau}{a}$ $\quad \boxed{\tau = at \to d\tau = a\,dt} \rightsquigarrow dt = \frac{d\tau}{a}$

$$= \int_0^\infty f(\tau)\cdot e^{-s\tau/a} \cdot \frac{d\tau}{a}$$

$$= \frac{1}{a}\int_0^\infty f(\tau)\cdot e^{-(s/a)\tau} d\tau$$

$$= \frac{1}{a}\mathcal{L}^{-1}\left\{\overbrace{\int_0^\infty f(\tau) e^{-(s/a)\tau} dt}^{\mathcal{L}}\right\}$$

$$= \frac{1}{a} F\left(\frac{s}{a}\right)$$

P. 10

$$\mathcal{L}\{t\cdot f(t)\} = \frac{1}{s(s^2+1)} \rightarrow \text{tiempo}$$

$$\mathcal{L}\{e^{-t} f(2t)\} =$$

$$\boxed{\mathcal{L}\{f(2t)\} = \frac{1}{2} F\left(\frac{s}{2}\right)} = \boxed{\frac{1}{2} \cdot -\frac{1}{2} \ln \frac{s}{2}\left(\frac{s^2}{2^2}+1\right) = -\frac{1}{4} \ln \frac{s}{2} \cdot s^2}$$

$$F(s) = L(t) = -\frac{1}{2} \ln s \cdot (s^2+1)$$

$$\boxed{F\left(\frac{s}{a}\right) = -\frac{1}{2} \ln \frac{s}{a} \cdot \left(\frac{s^2}{a^2}+1\right)}$$

$$F(s) = -\ln \frac{s}{\sqrt{s^2+1}} \rightarrow \boxed{F\left(\frac{s}{a}\right) = -\ln \frac{\left(\frac{s}{2}\right)}{\sqrt{\frac{s^2}{4}+1}}}$$

$$\mathcal{L}\{e^{-t} \cdot f(2t)\} = F(s) \rightsquigarrow F(s-(-1))$$

$$= F\left(\frac{s}{a}\right) = F\left(\frac{s+1}{a}\right) = -\ln\left(\frac{\frac{s+1}{2}}{\sqrt{\frac{(s+1)^2}{4}+1}}\right)$$

$$= -\ln \frac{(s+1)\;4}{2\sqrt{(s+1)^2+4}} = -\ln 2\,(s+1) \cdot$$