Sociological Methodology
2017, Vol. 47(1) 1–40

DOI: 10.1177/0081175017709295
http://sm.sagepub.com
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DYNAMIC NETWORK ACTOR MODELS: INVESTIGATING COORDINATION TIES THROUGH TIME

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Abstract

Important questions in the social sciences are concerned with the circumstances under which individuals, organizations, or states mutually agree to form social network ties. Examples of these coordination ties are found in such diverse domains as scientific collaboration, international treaties, and romantic relationships and marriage. This article introduces dynamic network actor models (DyNAM) for the statistical analysis of coordination networks through time. The strength of the models is that they explicitly address five aspects about coordination networks that empirical researchers will typically want to take into account: (1) that observations are dependent, (2) that ties reflect the opportunities and preferences of both actors involved, (3) that the creation of coordination ties is a two-sided process, (4) that data might be available in a time-stamped format, and (5) that processes typically differ between tie creation and dissolution (signed processes), shorter and

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longer time windows (windowed processes), and initial and repeated creation of ties (weighted processes). Two empirical case studies demonstrate the potential impact of DyNAM models: The first is concerned with the formation of romantic relationships in a high school over 18 months, and the second investigates the formation of international fisheries treaties from 1947 to 2010.

Keywords

social networks, coordination ties, time-stamped data, stochastic actor-oriented models, `goldfish`, *longitudinal network models, relational event models, international cooperation, romantic ties, DyNAM*

1. INTRODUCTION

The formation and dissolution of relations in social networks are often not a one-sided decision but the result of a coordination process between two actors. Examples include research cooperation between individuals or organizations, political coordination between states in specific issue areas, and romantic relationships and marriage (Jackson and Watts 2002; Jackson and Wolinsky 1996; Kalmijn 1998; Kinne 2013; Newman 2001; Roth and Sotomayor 1992). In all those cases, a *coordination tie* is established only when both actors agree on its creation. Coordination ties are common in social and political sciences but have been difficult to study.

Questions of relevance about change in coordination networks are concerned with the preferences of actors about their coordination partners, given their opportunities (Hedström and Bearman 2009). They address which individual characteristics relate to the attractiveness of actors as a partner or consider *network mechanisms* that explain, for example, whether homophily, local clustering, or preferential attachment facilitate the formation of ties (Kadushin 2012). Empirical studies could also be concerned with the circumstances under which actors tend to form multiple ties with one another (*weighted* ties), how recent tie changes affect future changes (*windowed* ties), and whether the creation and dissolution of ties follow distinct social mechanisms (*sign* of ties). These properties are currently difficult to investigate with available statistical models.

This article introduces a dynamic network actor model (DyNAM) for the study of coordination networks through time. It conceives of the two-sided nature of coordination ties as the outcome of an agreement

Table 1. Exemplary Data in Coordination Networks

	Time	Actors	Sign
1	2016-06-01	1↔2	Create
2	2016-06-02	1↔3	Create
3	2016-06-04	2↔3	Create
4	2016-06-07	1↔2	Dissolve

process in which both actors express their individual preferences given the available opportunities. In particular, it allows researchers to study the effect of actors' network positions and covariates (monadic, dyadic, and global) for generic, weighted, windowed, and signed processes. Such precise analyses are possible as the model makes use of the fine-grained data typically available in studies of coordination ties.

Coordination data are often public, leaving a historical data record for analysis (e.g., political agreements), or they can be targeted for data collection (e.g., research cooperation or marriage). Table 1 shows typical data related to coordination ties.

Each data point in Table 1's ordered list represents the creation or dissolution of a coordination tie between two actors at a specific point in time. That precise time observations are available is a chief advantage of such data sets; that we see only the outcome of a potentially complex agreement process constitutes a difficulty.

We develop a statistical network model for these types of data because simple assumptions about observational independence between data points are typically not possible. The first and second observations in Table 1, for example, might be dependent as they both involve actor 1 (network dependence) and further occur in close temporal proximity (temporal dependence). The third observation, for example, may be related to the first two observations through an attempt by actors 2 and 3 to close a triangular structure that involves actors 1, 2, and 3.

The model that we propose is rooted in the tradition of social network analysis that has developed a variety of approaches to deal with such interconnected, dynamic systems.

None of the statistical tools to date, however, permit the accurate study of coordination networks as the ones sketched previously. The DyNAM model that we propose simultaneously builds on stochastic actor-oriented models (Snijders 1996; Snijders, van de Bunt, and Steglich 2010) and statistical network models for time-stamped data

(Butts 2008; Stadtfeld and Geyer-Schulz 2011). The model is tailored to the specific characteristics of the dynamic coordination networks mentioned previously. In particular, the model has the following features:

1. It is a *network model* and allows researchers to control for and explicitly test hypotheses about observational dependence.
2. It is *actor-oriented* and conceives of network ties as the expression of actors' preferences and available opportunities.
3. It is *two-sided* and models coordination between two actors involved in the creation of a tie.
4. It is *more precise* than existing panel-based models as it enables researchers to draw inference from more granular time-stamped data.
5. It has *additional properties* compared with existing network models, and in particular, it permits simultaneous tests of hypotheses about weighted, windowed, and signed processes.

The remainder of this article is structured as follows. Section 2 reviews a variety of statistical models and their applicability to dynamic coordination networks. Section 3 introduces the dynamic network actor model (DyNAM). Section 4 discusses the estimation strategy implemented in a new software package called goldfish. Section 5 sketches a variety of potential model specifications that enable researchers to test detailed hypotheses about change in coordination networks. Section 6 applies DyNAM models to a case study about the emergence of romantic relationships in a U.S. high school over a period of 18 months. In particular, the model tests a hypothesis developed by Bearman, Moody, and Stovel (2004) about the avoidance of short cycles and illustrates the usage of a novel class of windowed effects. Section 7 applies DyNAM models in the context of the emergence of international fishery treaties between 1947 and 2010. The study revisits and extends some earlier findings by Kinne (2013) that were based on panel data analyses and illustrates the usage of a novel class of weighted effects. Section 8 offers concluding remarks and proposes future directions.

2. EXISTING MODELS FOR COOPERATION NETWORKS

We consider the formation (or deletion) of cooperative ties within a system as a series of events that transpire over a given period of time. Thus, the first class of statistical models for empirical analysis that comes to mind are event history models (Box-Steffensmeier and Jones 2004). Event history models in general and the Cox proportional hazard model in particular have proved to be useful tools in the social sciences over the past decade. However, regardless of whether the events under analysis happen on the same timeline, such as the probability of dropping out of a particular presidential race, or with different starting times, such as transition to first marriage for a specified group, in the classical applications, potential endogeneity of the system is generally regarded as a nuisance. In other words, the time until a particular event is realized is modeled primarily depending on a set of exogenous factors. In contrast, when modeling the emergence and structure of a coordination network, there is a strong interest in how the realization of past events impacts the realization of future events. Hence, compared to event history modeling with independent events, inference on endogenous mechanisms—how past agreements influence future agreements—is of explicit interest to the researcher.

Two important classes of statistical models were designed to deal with and model endogenous mechanisms of network formation: the exponential random graph model (ERGM; Lusher, Koskinen, and Robins 2013) and the stochastic actor-oriented model (SAOM; Snijders 1996, 2001; Snijders et al. 2010). Even though both apply to binary networks in which ties indicate states of a relationship (e.g., alliances or friendships), it is instructive to review them here to outline how endogenous network mechanisms can be modeled. Both models have been widely used to analyze networks in the social sciences and in particular to study coordination networks (e.g., Ferligoj et al. 2015; Hollway and Koskinen 2016; Manger, Pickup, and Snijders 2012; Milewicz et al. forthcoming).

In both models, the likelihood of a tie to exist (in cross-sectional analysis) or come about (in longitudinal analysis) depends on its embedding in configurations or substructures of other ties within the network. These configurations include, for example, transitive triads—that is, when both partners of a tie are connected to a mutual third party. By associating a statistical parameter with the likelihood of observing the existence or formation of a tie in this specific substructure, we can draw inference on whether agents in the network prefer their ties to be embedded in such configurations or try to avoid these configurations.

While ERGMs and SAOMs are similar in many ways, they differ in subtle but important details (Block, Stadtfeld, and Snijders forthcoming). ERGMs model in which configurations ties are more likely to exist, while SAOMs model in which configurations ties are more attractive to a sending actor. For the latter, this means that (1) ties are weighed against one another by a sending actor and (2) ties' positions in substructures depend on which actor sends the tie. The relative weighting of potential ties against one another by actors creates a dependence between ties beyond the configurations that are modeled by themselves. Intuitively, this is because if one option to form a tie becomes more likely (because it is transitively embedded or reciprocated), all other options become relatively less likely. Due to these differences, the ERGM is also referred to as a "tie-based" model, while the SAOM is described as an "actor-oriented" model. We build the current model as an actor-oriented model.

Combining an event framework with network ideas is not a completely new innovation. Two existing relational event models (REMs) incorporate ideas from structural network analysis and event history modeling. The slightly older of the two lies in the tradition of ERGMs as it models which event of all possible events in a network is likely to transpire next (Butts 2008). Butts's (2008) model is very flexible and can model both directed and undirected relational events. This flexibility is key for bilateral cooperation, and coordination represents undirected relational events. However, as a tie-based model, actors are left little room to compare the attractiveness of different tie choices. Instead, each potential tie is evaluated independently of other available choices of actors. Kitts et al. (2013) demonstrate how REMs can be applied for the empirical study of dynamic interorganizational networks. Other tie-oriented network event models have been proposed—for example, by de Nooy (2011) and Lerner et al. (2013).

In contrast, the other model is in the tradition of SAOMs as it models which actor is likely to send a relational event to which alter in the network (Stadtfeld 2012; Stadtfeld and Geyer-Schulz 2011). This actor orientation fits well with institutional theories that explicitly make space for agency and the notion that actors weigh the expected gain from different possible coordination agreements and choose the one that is most attractive. However, this model conceives of actors as primarily issuing ties, and it cannot deal with undirected ties that are the outcome of a mutual agreement between two involved parties. Related actor-oriented models have been proposed, for example, by Vu et al. (2011) and Perry and Wolfe (2013). The conceptual differences between SAOMs and ERGMs as presented previously apply equally to actor-oriented and tie-oriented relational event models.

We build off of these developments and present a model that allows the modeling of undirected ties from an actor-oriented perspective—a challenge that is also now being explored for panel data in SAOMs (Ripley et al. 2016) and further elaborated in Snijders and Pickup (forthcoming). We propose an approach that is related to the "one-sided opportunity mutual" model of Snijders and Pickup (forthcoming) but specify a symmetric change micro model rather than a combination of a multinomial and binary logit model.

3. DYNAMIC NETWORK ACTOR MODELS FOR COOPERATION NETWORKS

The behavioral model that we introduce is based on the intuition that for a coordination tie $i \leftrightarrow j$ to come about, both actors i and j *choose* each other from a set of alternative partners.[1] The generic micro-model for coordination ties that we propose is inspired by SAOMs and is illustrated in Figure 1.

Conceptually, the model assumes that a coordination tie $i \leftrightarrow j$ comes about as a three-step process. First, actor i who is part of system x considers proposing a new coordination tie. This consideration may occur at any point in time. Second, the actor chooses to *propose* the creation of a new tie to actor j rather than to any other actor and rather than deciding not to propose any tie at all. Third, after the proposal by i, actor j evaluates all potential partners from its point of view and chooses i as its preferred partner. By choosing i as well, rather than any other actor k, actor j *accepts* i's proposal. Only if proposal and acceptance match does a new tie come about and the process state is updated. In any other case, the process state remains unchanged. Each realized change relates to one specific event (one observed data point) in a data set like the one shown in Table 1.

Formally, the micro-process of change is described as a continuous-time Markov process (Snijders 2001; Waldmann and Stocker 2004) in which choices of actors about their preferred partners in the proposal and acceptance step are expressed as mutual multinomial choices (McFadden 1974). Nonrealized changes (e.g., when a proposal was not

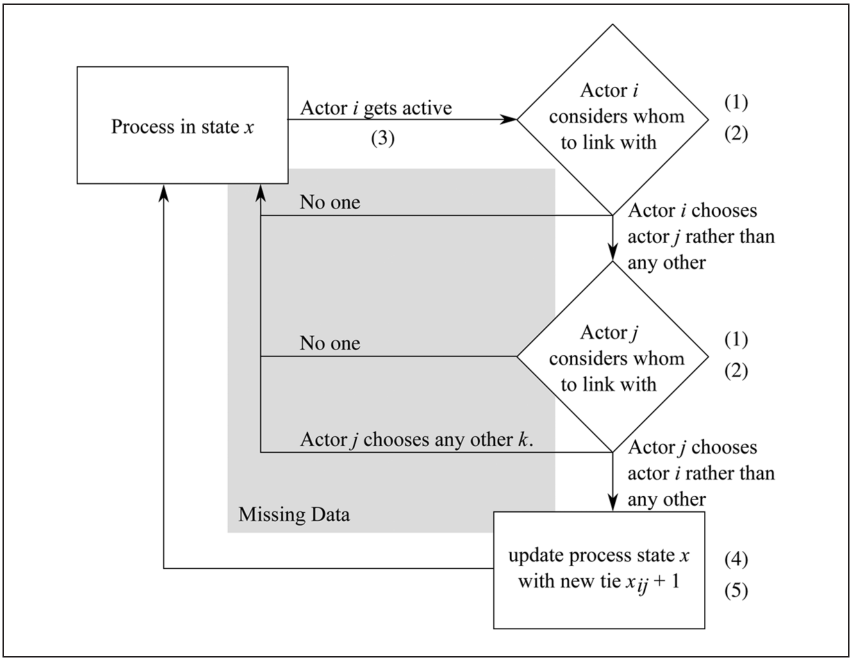


Figure 1. Conceptual micro-model describing changes in a coordination network. The numbers in parentheses refer to equations in the mathematical description.

accepted) are considered missing data since nonaccepted proposals are generally harder to trace in empirical studies.

The mathematical model is introduced in five steps. The numbers of the following items refer to the numbers of the equations in this section. The order of these items does not accord with the time order of the modeled process.

1. A linear objective function is introduced by which actors evaluate changes in the process states.
2. A multinomial choice probability models how actors propose and accept new ties.
   →Items 1 and 2 relate to "Actor *i*/*j* considers whom to link with" in Figure 1.
3. Individual Poisson rates determine the waiting times of actors between two subsequent tie proposals (that are not necessarily realized).
   →Item 3 relates to the first arrow "Actor *i* gets active" in Figure 1.

4. Tie-level Poisson rates determine the waiting time between two subsequent (realized) changes of a specific tie. These Poisson rates combine items 2 and 3.
5. A multinomial probability indicates the chances that a specific tie changes next within the network.
   →Items 4 and 5 relate to realized changes ("update process state x") in Figure 1.

We start explaining the mathematical framework by providing some notation. The process state (the upper left box in Figure 1) at time t is denoted by variable $x(t)$. In case the actual time t does not matter, we use the simplified notation x:

$$\begin{aligned}&x=(x^{(1)},x^{(2)},\ldots,z^{(1)},z^{(2)},\ldots,\mathcal{A}),\\&x\in X\end{aligned}\;.$$

The process state may consist of the state of multiple networks $x^{(\cdot)}$ and the value of multiple individual, tie-level, and system-level covariates $z^{(\cdot)}$. In particular, the modeled coordination network is an element of the process state (w.l.o.g: $x^{(1)}$). x denotes the space of all possible process states. The set of all actors is also part of the process state and is denoted by

$$\begin{aligned}&\mathcal{A}=\{1,\ldots,N\},\\&i,j\in\mathcal{A}.\end{aligned}$$

The process state after the creation of a new tie $i\leftrightarrow j$ is denoted by $x^{i\leftrightarrow j}$. Except for an updated tie $i\leftrightarrow j$ in the coordination network, $x^{i\leftrightarrow j}$ equals x; typically, this is an increase of $x_{ij}^{(1)}$ and $x_{ji}^{(1)}$ by 1. An actor may decide not to change the coordination network at all (this relates to the "No one" arrows in Figure 1). For that reason, we introduce the notation

$$x^{i\leftrightarrow i}=x.$$

The first important element of the actor-oriented process that we introduce is a linear *objective function* (Snijders 2005) by which an actor i evaluates possible changes in the process state x that are due to the creation of a specific network tie $i\leftrightarrow j$.

$$f_i(x^{i\leftrightarrow j},\beta)=\sum_{m=1}^{M}\beta_m s_m(i,x^{i\leftrightarrow j}). \tag{1}$$

Actors evaluate *structures* in a changed process state from their individual perspective. Structures are, for example, the number of ties that an actor has, the number of triads that an actor is embedded in, or the values of node covariates of all others that the actor has a coordination tie with. These structures relate to actors' preferences and are expressed through M statistic functions s_m on the process state x:

$$s_m : X \to \mathbb{R}.$$

The statistic functions can be of arbitrary complexity (and count, e.g., the number of ties, triads, and connected alters with a specific attribute) and are discussed in detail in a Section 5. They determine the dependence structures of the dynamic system (Frank and Strauss 1986). An actor's evaluation of a potential tie change further depends on a parameter vector β of length M that indicates whether the increase of a statistic function after a tie update (e.g., having one more tie within a triad) would increase ($\beta_m>0$) or decrease ($\beta_m<0$) actor i's evaluation of the process state.

The second element that we introduce is the probability function by which actors compare the attractiveness of specific tie changes. The probability that actor i will propose or accept a tie to actor $j\in\mathcal{A}$ is modeled as a multinomial choice model (McFadden 1974). The probability is proportional to the change in the objective function in Equation 1 and is defined as follows:

$$p_{i\to j}(x,\beta)=\frac{\exp(f_i(x^{i\leftrightarrow j},\beta))}{\sum_{k\in\mathcal{A}}\exp(f_i(x^{i\leftrightarrow k},\beta)).} \tag{2}$$

An actor will compare the objective functions that relate to the creation of ties with any of the other actors. Typically, in coordination networks, multiple ties between actors are possible. Ties can thus be created to any actor. The notation uses a directed arrow ($\to$) to indicate that the probability relates to actor i's tendency to propose or accept a coordination tie $x_{i\leftrightarrow j}$.

The third element of the mathematical model is the individual activity rates of actors. This activity is governed by a Poisson process that determines the time intervals between two subsequent active moments of an actor (which does not need to result in actual changes, as shown in Figure 1). A parameter of the Poisson process is defined as follows:

$$\tau_i(x,\theta) = \exp(\theta_0 + \sum_{l=1}^{L} \theta_l s_l^{\tau}(i,x)). \tag{3}$$

The general propensity of actors to create a network tie may also depend on an individual evaluation of the process state x at a certain point in time. This evaluation is operationalized by statistic functions s^{τ}. Those can, for example, show that an actor's covariate or network position may have an effect on its activity. The direction and magnitude of these effects are expressed by parameters θ_l. Parameter θ_0 works as an intercept and expresses the baseline activity of actors.

The fourth element of the mathematical model is concerned with the waiting times between two subsequent *tie realizations*. These waiting times can be understood as a Poisson process even though some of the information about the process in Figure 1 (indicated by the gray box) is empirically missing. Under the assumption that the individual rates $\tau_i(x,\theta)$ and the actor's choice probabilities $p_{i\to j}(x,\beta)$ are conditionally independent given x, the product of individual Poisson rates (Equation 3) and choice probabilities (Equation 2) constitutes a Poisson process λ (Waldmann and Stocker 2004). This combined Poisson process refers to the propensity of dyads $i \leftrightarrow j$ to be updated in the network:

$$\begin{aligned}\lambda_{i\leftrightarrow j}(x,\beta,\theta) &= \tau_i(x,\theta)p_{i\to j}(x,\beta)p_{j\to i}(x,\beta)\\ &\quad + \tau_j(x,\theta)p_{j\to i}(x,\beta)p_{i\to j}(x,\beta)\\ &= (\tau_i(x,\theta)+\tau_j(x,\theta))p_{i\to j}(x,\beta)p_{j\to i}(x,\beta).\end{aligned} \tag{4}$$

The Poisson rates $\lambda_{i\leftrightarrow j}$ consist of two summands, as a tie $i \leftrightarrow j$ can come about by i or j proposing its formation and the respective other being the one that accepts. Under the assumption that those two rates are also conditionally independent given x, the dyad rate can be constructed by summing up the two Poisson rates. The proposal and agreement probabilities are symmetric, and so the two terms can be combined. We will use the abbreviated notations in the following equations:

$$\begin{aligned}\lambda_{ij}(x) &= \lambda_{i\leftrightarrow j}(x,\beta,\theta)\\ \tau_i(x) &= \tau_i(x,\theta)\\ p_{ij}(x) &= p_{i\to j}(x,\beta).\end{aligned}$$

The first four elements introduced so far are not sufficient to construct a likelihood function that can be fitted to the data: Neither the individual rates nor the multinomial choice probabilities are directly observed. It is possible that a large number of proposals are not accepted (missing data in Figure 1) before a new cooperation tie is realized.

The fifth element of the mathematical model thus defines the probability that a specific tie change $i \leftrightarrow j$ is the *next* to be observed rather than a tie change on any other dyad $k \leftrightarrow l$. Under the assumption that the dyadic rates λ_{kl} are also conditionally independent given the process state x, we may define the probability of a change on the dyad $i \leftrightarrow j$ as follows.

$$P_{i\leftrightarrow j}(x) = \frac{\lambda_{ij}(x)}{\sum_{k,\, l\in\mathcal{A};k\leq l}\lambda_{kl}(x)}. \tag{5}$$

The missing data that were identified in Figure 1 then do not matter anymore as the model expresses the waiting time between actually realized changes. In a simplified model, we may assume that the general activity of actors in the process is constant (and thus depends only on the parameter θ_0 in Equation 3). In that case, the individual rates τ cancel out from Equation 5, and we are left with a simplified model in which the probability of a specific tie change depends only on all actors' simultaneous, multinomial evaluation of their current change opportunities:

$$P_{i\leftrightarrow j}(x) = = \frac{p_{ij}(x)\; p_{ji}(x)}{\sum_{k,\, l\in\mathcal{A};k\leq l} p_{kl}\;(x)\; p_{lk}\;(x)}. \tag{6}$$

This "constant rate" model will be applied in the empirical case studies presented later in this article. Its estimation method will be elaborated in the following section.

4. ESTIMATION METHOD

An estimation method for the aforementioned model is implemented in a new software package called `goldfish`.[2] It allows testing hypotheses about change in coordination networks based on time-stamped data (see Table 1 for example).

The estimation routine uses a maximum likelihood (ML) optimization. The probabilities in Equations 5 and 6 may be interpreted as likelihood functions, and we can estimate optimal parameter vectors $\hat{\beta}$ and $\hat{\theta}$ that maximize the likelihood. In this section, we focus on the ML estimation of the constant rate model in Equation 6 and its parameter vector β. Equation 6 is thus reformulated as a likelihood:

$$L_\omega(\beta; i_\omega \leftrightarrow j_\omega, x_\omega) = \frac{p_{i_\omega \to j_\omega}(x_\omega, \beta)\ p_{j_\omega \to i_\omega}(x_\omega, \beta)}{\sum_{k,l \in \mathcal{A}; k \le l} p_{k \to l}(x_\omega, \beta)\ p_{l \to k}(x_\omega, \beta)}. \tag{7}$$

The variable ω refers to a specific observation (an *event*) in an event sequence Ω (i.e., all coordination ties created within a specific period) in which a network tie $i_\omega \leftrightarrow j_\omega$ at process state x_ω is updated. An exemplary event sequence was shown in Table 1. The likelihood of a sequence Ω is defined as the product of $|\Omega|$ such expressions. In the following equation, we focus—without loss of generality—on the likelihood of a single product term, and we use a simplified notation similar to Equation 6:

$$L_\omega(\beta) = \frac{p_{ij}(x, \beta)\ p_{ji}(x, \beta)}{\sum_{k \le l} p_{kl}(x, \beta)\ p_{lk}(x, \beta)}.$$

The event log likelihood is thus

$$\log L_\omega(\beta) = \log\ p_{ij}(x, \beta) + \log p_{ji}(x, \beta) - \log\left(\sum_{k \le l} p_{kl}(x, \beta)\ p_{lk}(x, \beta))\right). \tag{8}$$

We propose a Newton-based estimation strategy. The default procedure iteratively updates a vector of parameter estimates based on the vector of first derivatives and the Hessian matrix H of second derivatives, both with respect to the parameters β_k. Rather than calculating the Hessian matrix H explicitly, we replace it with the negative Fisher information matrix F in the Newton-Raphson updating steps. The latter matrix is easier to calculate and a reasonable proxy for the Hessian given that it is the negative of its expectation:

$$F = -E(H).$$

This estimation strategy is called the Gauss/Fisher scoring method, and it is further explicated in various textbooks—for example, by Cramer (2003). The first derivative of a likelihood term with respect to parameter β_m (the mth element of the score vector) is

$$\begin{aligned}\frac{\partial logL_\omega(\beta)}{\partial\beta_m} &= \frac{p'_{ij}}{p_{ij}} + \frac{p'_{ji}}{p_{ji}} - \frac{\sum_{k\leq l} p'_{kl}\, p_{lk} + p_{kl}\, p'_{lk}}{\sum_{k\leq l} p_{kl}\, p_{lk}} \\ &= \frac{p'_{ij}}{p_{ij}} + \frac{p'_{ji}}{p_{ji}} - \sum_{k\leq l}\left(P_{kl}\left(\frac{p'_{kl}}{P_{kl}} + \frac{p'_{lk}}{P_{lk}}\right)\right).\end{aligned} \tag{9}$$

The term p'_{kl} is an abbreviation for the partial derivative of p_{kl} with respect to β_m:

$$p'_{kl} = \frac{\partial p_{kl}}{\partial\beta_m}.$$

Following the calculation in Cramer (2003, Equation 8.13), the terms $\frac{p'_{kl}}{p_{kl}}$ can be replaced with

$$\frac{p'_{kl}}{p_{kl}} := s_m(k, x^{k\to l}) - \sum_h p_{kh} s_m(k, x^{k\to h}).$$

This equation can be interpreted as the difference between the realized and expected value of a statistic function s_m when actor k chooses a tie to l from all choice options h (that include l). The information matrix of one event F_ω is constructed by first deriving a generic Hessian matrix H with the element (m, n) defined as

$$H_{\omega;mn} = \sum_{k\leq l}\left(\frac{y_{kl}}{P_{k\leftrightarrow l}(x_\omega)}\frac{\partial^2 P_{k\leftrightarrow l}(x_\omega)}{\partial\beta_m\partial\beta_n} - \frac{y_{kl}}{P_{k\leftrightarrow l}(x_\omega)^2}\frac{\partial P_{k\leftrightarrow l}(x_\omega)}{\partial\beta_m}\frac{\partial P_{k\leftrightarrow l}(x_\omega)}{\partial\beta_n}\right). \tag{10}$$

The probabilities $P_{k\leftrightarrow l}(x_\omega)$ have been introduced in Equations 5 and 6. The terms y_{kl} are the observed outcomes of tie $k \leftrightarrow l$ in event ω, and they are zero for all cases but $y_{i_\omega j_\omega} = 1$. These observed outcomes are replaced with their expectations to obtain the negative Fisher information matrix:

$$E(y_{kl}) = P_{k\leftrightarrow l}(x_\omega).$$

Performing this substitution in Equation 10 yields the Fisher information matrix F. The first term in the sum of Equation 10 then sums up to zero, and the second term is simplified to the following equation:

$$F_{\omega;mn}(\beta) = \sum_{k\leq l}\left(P_{k\leftrightarrow l}(x_\omega)\frac{\partial P_{k\leftrightarrow l}(x_\omega)}{\partial\beta_m}\frac{\partial P_{k\leftrightarrow l}(x_\omega)}{\partial\beta_n}\right). \tag{11}$$

An updating step of a parameter vector β^t to β^{t+1} in iteration $t+1$ can then be calculated by inserting the score vector and the information F over all events for parameter vector β^t into the general Newton-Raphson update equation. The calculation in each updating step is rather straightforward as only the change probabilities $P_{k \leftrightarrow l}(x_\omega)$ and the score vector (Equation 9) have to be calculated.

$$\begin{aligned} \beta^{t+1} &= \beta^t + F(\beta^t)^{-1} \frac{\partial logL(\beta^t)}{\partial \beta^t}, \\ F(\beta^t) &= \sum_{\omega \in \Omega} F_\omega(\beta^t) \\ \log L(\beta^t) &= \sum_{\omega \in \Omega} L_\omega(\beta^t). \end{aligned} \tag{12}$$

The mathematical formulations in this section closely follow Cramer (2003) about the application of the Gauss/Fisher scoring method for the estimation of standard multinomial logit models. Repeated iterations will eventually approximate the vector $\hat{\beta}$ of maximum likelihood estimates. ML estimates of the activity Poisson process parameter vector $\hat{\theta}$ can in principle be calculated in a similar fashion.

5. MODEL SPECIFICATIONS

At the core of the actor-oriented model that we propose are the model specifications by which actors evaluate potential tie creations. These evaluations are expressed as effect functions s_m in the objective function in Equation 1. In the case of the actor activity rates, those were introduced as functions s_l^τ in Equation 3. We categorize the types of effect functions into four classes: (1) generic, (2) signed, (3) weighted, and (4) windowed. A brief summary of model specifications that are implemented in the `goldfish` software package is provided in Appendix B.

5.1. *Generic Effects*

As a network model, our example draws on exogenously given covariates to explain network evolution and network effects that act endogenously with the structure of the network. This section is concerned with structures in a binary representation of coordination networks. We will later discuss the effects that take into account multiple ties (weighted effects), the sign of ties, and process changes within certain time windows. Such effects can be formulated similarly to the five generic effects that we present in this section:

1. Effects can be defined on the level of the individual. These effects may represent differential attractiveness, ability, or interest to be involved in such a cooperative network.
2. Effects can be defined on the level of the dyad. Such effects could capture how particular combinations of qualities, such as homophily (or heterophily), make coordination more or less likely or how ties between pairs of actors in one dimension lead to a tie in another dimension, such as how communication or exchange can lead to agreement, or vice versa.
3. Effects can be defined on the level of the triad. Such effects capture how groups of more than two come together in their smallest configuration. A simple endogenous example is of course transitivity, but transitivity can be specified in such a way that it invokes attributes.
4. Higher order effects and effects that involve more than three nodes can be specified. These extend relational algebra to implicate further ties in dependencies. Examples include geometrically weighted edgewise shared partners' effects (for a detailed discussion, see Snijders et al. 2006) or attractiveness mechanisms such as preferential attachment.
5. Global effects on exogenously given or endogenously created data can be specified to account for time heterogeneity in the evolution of the network. For example, global structural features such as the bipolarity of the cold war can be exogenously defined to apply for a certain period or specified endogenously to relate to, say, community structures in the network.

Tables 2 and 3 give an overview of nomenclature, graphical representations, and mathematical definition of such basic effect specifications. Table 2 lists a number of endogenous network effects in which structures in the dependent network explain future changes. Table 3 presents effects that are related to endogenous or exogenous covariates on an actor-, dyad-, or global-process level. In the mathematical specification of effects, the following abbreviations are used:

$$\dot{x}_{ik}^{(1)} = I(x_{ik}^{(1)} > 0)$$
$$\dot{x}_{i+}^{(1)} = \sum\nolimits_k \dot{x}_{ik}^{(1)},$$

Table 2. Distinguishing Dimensions of Generic Effects: Structural Network Effects

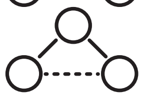
Outdegree		$s_1(i,x) = \dot{x}^{(1)}_{i+}$
Degree popularity		$s_2(i,x) = \sum_k \dot{x}^{(1)}_{ik} \dot{x}^{(1)}_{k+}$
Assortativity		$s_3(i,x) = \sum_k \dot{x}^{(1)}_{ik} \sqrt{\dot{x}^{(1)}_{i+}} \sqrt{\dot{x}^{(1)}_{k+}},$
Transitivity		$s_4(i,x) = \sum_{k,l} \dot{x}^{(1)}_{ik} \dot{x}^{(1)}_{il} \dot{x}^{(1)}_{kl}$

Table 3. Distinguishing Dimensions of Generic Effects: Effects Related to a Monadic Actor-level Covariate ($v^{(1)}$), a Dyadic Covariate ($v^{(2)}$), and a Global Covariate ($v^{(3)}$).

Covariate activity		$s_5(i,x) = v^{(1)}_i \dot{x}^{(1)}_{i+}$
Covariate attractiveness		$s_6(i,x) = \sum_k v^{(1)}_k \dot{x}^{(1)}_{ik}$
Covariate heterophily		$s_7(i,x) = \sum_k \lvert v^{(1)}_i - v^{(1)}_k \rvert \dot{x}^{(1)}_{ik}$
Dyadic covariate		$s_8(i,x) = \sum_k v^{(2)}_{ik} \dot{x}^{(1)}_{ik}$
Global covariate		$s_9(i,x) = v^{(3)} \dot{x}^{(1)}_{i+}$

with I being an indicator function that is one (zero) when the condition argument is true (false). $\dot{x}^{(1)}$ is thus a binary projection of the weighted dependent network $x^{(1)}$; $\dot{x}^{(1)}_{i+}$ is the (binary) outdegree of a node i in a network and counts the number of unique partners rather than the total number of ties that can be larger in networks where multiple ties between two actors are possible.

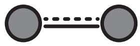
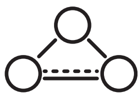
5.2. *Signed Effects*

Signed effects are concerned with the distinction between the creation and deletion of ties. In SAOMs, this choice is called creation and endowment, though typically these two processes—making new ties or keeping existing ones—are modeled jointly. Although our model can treat them jointly, too, we believe that users will find more utility

Table 4. Distinguishing Dimensions of Signed Effects

Creation of ties	
Deletion of ties	

Table 5. Distinguishing Dimensions of Weighted Effects

Multiple ties		$s_{10}(i,x) = \sum_k \left(x_{ik}^{(1)}\right)^2$
Multiple dyadic effects		$s_{11}(i,x) = \sum_k v_{ik}^{(2)} x_{ik}^{(1)}$
Multiple heterophily		$s_{12}(i,x) = \sum_k \lvert v_i^{(1)} - v_k^{(1)} \rvert x_{ik}^{(1)}$
Multiple transitivity		$s_{13}(i,x) = \sum_{k,l} x_{ik}^{(1)} \dot{x}_{il}^{(1)} \dot{x}_{kl}^{(1)}$

Note: Any of the generic effects may be tested. The effects combine weighted ($x^{(1)}$) and binary ($\dot{x}^{(1)}$) network ties.

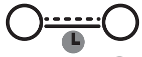
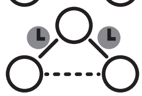
in separating them out into dependent subprocesses, as illustrated in Table 4.

In the case studies in this article, we are concerned only with the creation of coordination ties. The dissolution of ties can be modeled similarly but could make use of a different model—for example, a model that allows one-sided dissolution of ties.

5.3. *Weighted Effects*

Weighted effects are concerned with not only whether these elements matter for new relationships but also for the deepening of these relationships through multiple concurrent ties. Table 5 shows graphical representations and the mathematical formulations of four exemplary weighted effects. The multiple ties effect, for example, relates to a tendency toward *deepening* relations with multiple ties of coordination. This first effect is squared in tie creation or tie deletion models as otherwise the effect would be collinear with the binary outdegree effect in Table 2. All effects evaluate the actual *weighted* network of coordination ties $x^{(1)}$ rather than its binary projection $\dot{x}^{(1)}$.

Table 6. Distinguishing Dimensions of Windowed Effects

Windowed multiple ties		$s_{14}(i,x) = \sum_{k} (x_{ik}^{(2)})^2$
Windowed degree popularity		$s_{15}(i,x) = \sum_{k} \dot{x}_{ik}^{(2)} \dot{x}_{k+}^{(2)}$
Windowed transitivity		$s_{16}(i,x) = \sum_{k,l} \dot{x}_{ik}^{(2)} \dot{x}_{il}^{(2)} \dot{x}_{kl}^{(2)}$

Note: Any of the generic effects may be tested. Network $\dot{x}^{(2)}$is the binary network that includes all the ties that have been created within a specific time window prior to the current event time.

5.4. *Windowed Effects*

Windowed effects are concerned with whether ties continue to be salient for their entire duration (uniform) or whether their contribution to an actor's objective is of limited temporal validity. The introduction of these windowed effects relaxes the relatively tight assumption of SAOMs that existing ties have a uniform effect as part of the network structure, regardless of how long ago they were first created.

Without loss in generality, let $x^{(2)}$ denote a network that includes all ties that have been created within a specific time window prior to the current event (say, three years). Three exemplary windowed effects that evaluate structures in this network are listed in Table 6. The first refers to the tendency of actors to create multiple ties with the same partner within a specific period of time, the second and third to preferential attachment and transitive mechanisms within a specific time window.

It is important to note that by including time window effects in a model, the model is no longer first-order Markovian. Although we could reinstate this feature by amending the process state with information about lagged events (that relate to process state updates at the end of a window), this possibility is not further explored in this article. The lagged changes at the end of a window determine the end of a right-censored positive interval. These are not taken into account in the default model definition in this article.

6. CASE STUDY: CHAINS OF AFFECTION EVENTS

In this section, we illustrate the utility of our model by reanalyzing a subset of the "chains of affection" data of Bearman et al. (2004). The

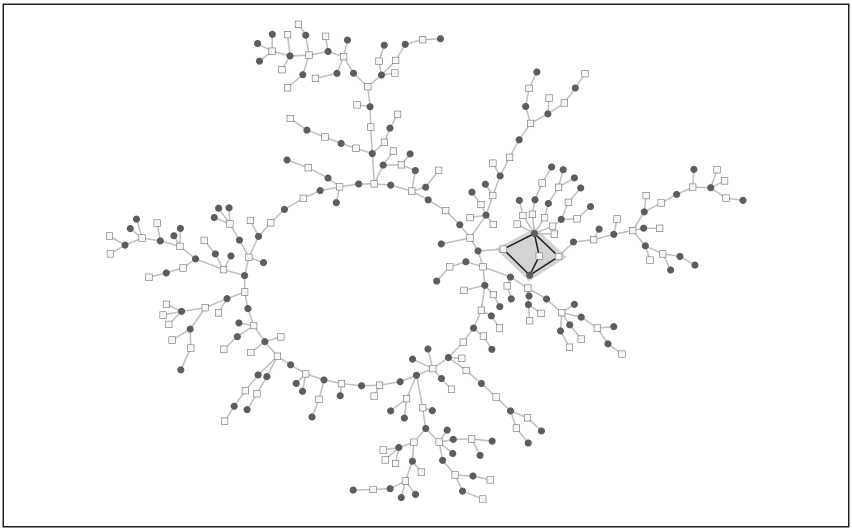


Figure 2. The main component of the "chains of affection" network—the sample used in this case study. The component includes only three cycles of length four (highlighted in gray).

data that we use are self-reported romantic relationships of 288 students of "Jefferson High School" that were created or dissolved within 18 months prior to data collection. The data stem from the Add Health study that was designed by J. Richard Udry and Peter Bearman. The students in the sample at hand report 291 incidences of formation and dissolution of romantic relationships with one another. This number is above the high school average as the sampling of those students was based on the fact that they together form the largest interconnected component of a network in which all romantic ties were aggregated over time. Figure 2 shows the network component that constitutes the sample in this case study.[3] For a visualization of the complete network, we refer to Bearman et al. (2004) (Figure 2). The color and shape of the nodes indicate gender (females are light rectangles), which is the only exogenous variable that we use in this case study. Even though individuals in the sample are typically only in one romantic relationship at a time, individuals with many subsequent relationships appear as nodes with a high degree in the aggregated network.

The data include information about the time of the creation and breakup of romantic relationships. The data are time-stamped but

Table 7. Instances of Tie Creation and Tie Dissolution over Time

	Week		Actors		Sign
1	1105	18	↔	172	Create
2	1105	23	↔	205	Create
3	1105	163	↔	280	Create
4	1106	113	↔	215	Create
	⋮				
8	1111	84	↔	241	Create
	⋮				
20	1120	84	↔	241	Dissolve
	⋮				

categorized by week. Within a time window, we analyze the data in a randomly determined order. The robustness of decisions of that type can in principle be tested by analyzing alternative permutations of events within time windows.[4] Generally, DyNAM models can be applied to such time window data if dependence between observations within a window can be assumed to be small. The first few lines of the data set are shown in Table 7.

Applying a DyNAM model for coordination networks allows us to test hypotheses that relate to the circumstances under which romantic relationships are newly formed. In particular, we are interested in demonstrating the test of hypotheses that relate to our argument about the higher precision and windowed mechanisms. The following three hypotheses are tested without detailing their theoretical foundation:

Hypothesis 1: Individuals will be more likely to form new relationships in the immediate period after a breakup (a windowed version of the covariate attractiveness effect in Table 3 where $v_k^{(1)}$ is a binary variable indicating whether an individual had a breakup in the prior 10 weeks).

Hypothesis 2: Individuals who have been in a romantic relationship at least once will be more likely to form new relationships (a time-dependent version of the covariate attractiveness effect in Table 3 where $v_k^{(1)}$ is a binary variable indicating prior romantic experience).

Hypothesis 3: Individuals will tend to avoid forming cycles of length four in the romantic partnership network (generic structural effect). It

Table 8. Case Study 2 Results

Effects	Coefficient	SE	t
Popularity	–.01	.07	–.11
Different gender	2.29***	.29	7.84
Different experience	.29***	.08	3.68
Four-cycle	.46	.26	1.78
No more than one tie	.23	.12	1.87
Recent breakup	.52**	.17	3.12
Experienced partner	–.80***	.15	–5.17
Log likelihood	–2,855.63		

**$p < .01$. ***$p < .001$.

is thus unlikely that individuals date their former partner's new partner's ex-partner.

Hypothesis 3 is of specific interest as Bearman et al. (2004) argue that only mechanisms that preclude the formation of short cycles may lead to networks such as the one shown in Figure 2 that appear like "spanning trees" (networks with few short cycles and long chains of connected nodes): "[F]rom the perspective of males or females . . ., a relationship that completes a cycle of length 4 can be thought of as a 'seconds partnership,' and therefore involves a public loss of status" (Bearman et al. 2004:75).

At the same time, we control for the fact that individuals will typically not maintain more than one romantic relationship at a time (generic structural mechanism) and are more likely to form mixed-gender romantic relationships (monadic generic mechanism). We further test two generic mechanisms that relate to the degree of nodes in a network in which all prior romantic relationships are aggregated. We test whether individuals with a high degree (many prior relations) are more likely to be involved in romantic relationships (degree popularity in Table 2).

The results of the model are presented in Table 8. In our sample, we find strong evidence for the first-time window hypothesis. Individuals are more likely to form ties in the period after a breakup (line 6: .52, $p < .01$). However, we find an effect that points in the reverse direction regarding the second time–related hypothesis. In fact, individuals are preferred when they have no prior romantic experience (line 7: .80,

$p < .001$). At the same time, mixed experience partnerships are preferred over same partnerships where both have the same level of experience (line 3: .29, $p < .001$). The fact that inexperienced actors are preferred over experienced, however, is a side effect of the data sampling in which only nodes with at least one relationship occur. Appendix A discusses this issue in more detail. In stochastic actor-oriented models for panel data (see Section 2), such precise tests of time window hypotheses are not available.

We find no evidence of an avoidance of four cycles (avoiding the formation of a romantic tie with the former partner of an ex-partner's new partner, line 4: .46, *ns*). The insignificant effect is positive (not negative as hypothesized), which indicates that four cycles are in fact closed more often than would be expected by the opportunities to do so. An alternative explanation why spanning tree–like structures emerge are the opportunities available to "single" nodes at the time of partnership formation. We could argue that the chance that two individuals are simultaneously singles after their ex-partners start dating is rather low. But once such an opportunity exists, it is not taken with less probability than expected by chance. An apparent avoidance of four cycles is thus potentially created by vacancy chain mechanisms that create few paths of length three where the two individuals at the start and the end are simultaneously singles. Opportunity restrictions of that kind are explicitly taken into account in our model. A recent cross-sectional analysis of the same data by Rolls et al. (2015) also did not find support for the avoidance of four cycles. Our preliminary, dynamic nonfindings are not conclusive, though. More research on a larger data set with the DyNAM model is needed to fully disentangle opportunity effects from actor preferences.

We find no evidence that individuals with many prior relationships are more attractive partners (line 1: .01, *ns*). This finding corresponds with the cross-sectional visualization in Figure 2 in which degree centralization does not seem to be a dominant pattern. There are weak patterns of not having more than one relationship at a time (line 5: .23, $p =$ .06) and strong tendencies for forming mixed-gender relations (line 2: 2.29, $p < .001$.). The fact that multiple relationships do not seem to be a strict taboo is in line with the empirical fact that a high proportion of relationship overlap is reported in the data: 27 percent of all individuals who start a relationship are found to have an ongoing relationship at the same time.

This case study illustrates how DyNAM models allow us to zoom in on the individual decision level in dynamic networks of romantic relationships. In particular, we demonstrated the use of a small number of generic and windowed effects in a straightforward dynamic network model. The interpretation of effects and nonfindings is limited because the sample available to us is very small, is based on a nonstandard sampling procedure, and lacks relevant covariates and network structures that could be important determinants of change. However, it invites future research retesting the "avoidance of four cycles" hypothesis of Bearman et al. (2004) on a complete data set and contrasting it to opportunity effects that might generate certain macro-level network structures. Future research can use the full tool box that we developed to test the theoretical propositions by Bearman et al. and extend them with novel mechanisms that take into account new generic effects (e.g., the role of multivariate homophily and friendship ties within the high school) and windowed effects (e.g., the role of the recent changes in the romantic relationship or friendship network).

7. CASE STUDY: THE EMERGENCE OF INTERNATIONAL FISHERIES TREATIES

This section illustrates the utility of our model by applying it to the case of states' bilateral fisheries treaties with one another. The creation of a bilateral fisheries treaty between two states represents the process of entering into a cooperative relationship to solve one of two problems. First, in the case of adjacent states, they may make a treaty to allocate a shared fish stock that straddles their maritime boundaries. Second, states that are further apart may enter into such arrangements so that one state can gain access to the underexploited fish stocks residing in the Exclusive Economic Zone (EEZ) of the other, often in exchange for aid, investment, or some other support or concession. From a conceptual perspective, this study goes beyond the simplified replication of the study by Bearman et al. (2004) as states can over time form multiple treaties with one another. Therefore, the testing of generic and weighted mechanisms is at the core of this case study.

We use the data on bilateral fisheries agreements from Hollway and Koskinen (2016), which was constructed by merging and further developing two prominent data sets in the field of international environmental politics (ECOLEX 2011; Mitchell 2013). The data have the general

Table 9. Instances of Tie Creation and Tie Dissolution over Time

	Time	States Involved			Sign
1	1947-03-14	PHL	↔	USA	Create
2	1947-12-19	CHE	↔	ITA	Create
3	1948-04-29	NOR	↔	SWE	Create
4	1948-12-30	BEL	↔	DNK	Create
5	1949-01-25	MEX	↔	USA	Create
6	1949-01-28	NOR	↔	SWE	Create
7	1949-04-13	ITA	↔	SRB	Create
8	1949-06-13	FIN	↔	NOR	Create
9	1950-03-24	CAN	↔	USA	Create
10	1950-06-30	PHL	↔	USA	Dissolve
	⋮				

Note: PHL = Philippines; USA = United States of America; CHE = Switzerland; ITA = Italy; NOR = Norway; SWE = Sweden; BEL = Belgium; DNK = Denmark; MEX = Mexico; SRB = Yugoslavia; FIN = Finland; CAN = Canada.

structure that we identified in the first section. A subset of the data is shown in Table 9.

We model the creation of bilateral fisheries agreements with two different specifications to illustrate opportunities introduced by the model. The first reproduces a SAOM-like model specification using basic effects (Kinne 2013), with outdegree effects (including interactions with the two node covariates), two structural effects (popularity and transitivity), one dyadic covariate (contiguity), one annually aggregated changing covariate (capacity, as measured by GDP), and one date-stamped changing covariate (regime, as coded in the Polity IV data set). The dyadic covariate, contiguity, was chosen for its linkage to the role of geographic adjacency in driving some bilateral fisheries agreements. The monadic covariates, regime and capacity, were chosen for their connection to the heterophily expected to drive other bilateral fisheries agreements. All effects chosen also appear in Kinne (2013).

The results shown in Table 10 are relatively straightforward to interpret. States do not cooperate with many other states on fisheries topics (negative outdegree). When they consider proposing a tie, this is thus likely to be a new tie with an existing partner (which does not increase the binary statistic function in Table 2). Large democratic states tend to create ties to fewer partners, controlling for popularity effects (negative

Table 10. Case Study 2 Results

		Generic		Weighted	
	Effect	Coefficient	SE	Coefficient	SE
1	Outdegree	–3.156***	(.064)	–2.547***	(.080)
2	Regime outdegree	–.009*	(.004)	–.017	(.009)
3	Capacity outdegree	–.240***	(.049)	.087	(.050)
4	Weighted outdegree			.078***	(.005)
5	Popularity	.168***	(.004)	.106***	(.007)
6	Weighted popularity			.059***	(.005)
7	Transitivity	–.123***	(.027)	–.048	(.029)
8	Weighted transitivity			–.043*	(.020)
9	Contiguity	1.426***	(.064)	1.405***	(.083)
10	Weighted contiguity			–.028	(.056)
11	Regime alter	.013**	(.004)	.032***	(.009)
12	Weighted regime alter			–.020**	(.008)
13	Regime diff	–.001	(.003)	.002	(.004)
14	Weighted regime diff			–.004	(.003)
15	Capacity alter	.364***	(.070)	.245***	(.059)
16	Weighted capacity alter			.663***	(.056)
17	Capacity diff	–.094	(.049)	–.148*	(.062)
18	Weighted capacity diff			–.211***	(.046)
	Log likelihood	–5,614.4		–5,302.9	
	Akaike Information Criterion	11,249.1		10,642.7	

*$p < .05$. **$p < .01$. ***$p < .001$.

regime outdegree and capacity outdegree). Those two effects are operationalized as the covariate activity effects introduced in Table 3. However, when countries do choose to cooperate with new partners, they do so with certain preferences. The contiguity effect suggests that spatial contiguity is a major factor in bilateral fisheries cooperation. States that do cooperate choose partners that are large and democratic (positive regime alter and capacity alter). Those effects are the covariate attractiveness shown in Table 3. There is no evidence that states have a homophilic or heterophilic preference to tie to those that differ from themselves on these attributes (covariate heterophily).

In addition to these effects defined with reference to exogenous covariates, both structural effects, popularity and transitivity (Table 2), are significant. That popularity is positive is unsurprising and suggests that there is centralization in the network—that ties have accumulated around a few actors, such as the European Union or United States. That transitivity is negative is more surprising though since in both Kinne

(2013) and Hollway and Koskinen (2016), analogous effects were positive. Hollway and Koskinen (2016) used an entirely different class of models (cross-sectional multilevel ERGMs), they had a different set of controls, and transitivity was calculated in a different way (alternating), so the comparability should not be overstated. However, Kinne (2013) used a more comparable longitudinal, actor-oriented SAOM; a similar set of effects; and the same formula for calculating the related statistics. To test hypotheses within the SAOM framework, Kinne (2013) had to aggregate the time-stamped data presented previously into somewhat artificial binary panel data. The SAOM estimation method would then simulate possible chains of events between those panels rather than drawing statistical inference on the actually observed chains of events. The model is nevertheless comparable as SAOMs are also network models (feature 1), actor-oriented (feature 2), and can take into account the two-sided nature of tie formation (feature 3). Due to the analysis of panel data in SAOMs, some of the additional properties cannot be tested, though. We would argue that since DyNAM gets much closer to dependencies in the data, it should be considered the more precise. Indeed, the negative result of transitivity accords with our understanding of these treaties as centralized. Controlling for contiguity, popularity, and country attributes (mechanisms that all potentially induce transitive structures), a country is not likely to cooperate with its partners' partners.

The second specification introduces weighted effects for each of the base effects included, except for the outdegree-related effects. These weighted effects get at mechanisms driving the creation of additional bilateral fisheries agreements but with existing partners. Note that the introduction of these effects means that we are now explaining more of the events in the event sequence since those agreements that were created atop existing relationships were only captured by the outdegree-related effects in the previous model. A SAOM, as applied in the study by Kinne (2013), cannot model weighted effects as the data are typically analyzed in a binarized form where only the initial creation of ties is observed.

First, we find that although the outdegree intercept is still negative, weighted outdegree is positive. The latter effect was introduced as the multiple ties effect in Table 5. As it is squared, the positive parameter suggests that states generally prefer to reinforce relationships with many ties rather those with few ties should they consider not to initiate a new tie. Contiguity continues to be important for the selection of partners, but simply being contiguous does not guarantee countries will deepen their cooperative relationship. This may be because only one comprehensive agreement is necessary to resolve shared fisheries allocation issues.

Again, democratic regimes are preferred partners, but it is autocratic partners that attract multiple agreements (negative weighted regime alter). This may be because partnerships with autocratic regimes require more reinforcement, more frequently changed agreements, or more specific agreements. Countries with large capacity are also still preferred partners, but they also attract multiple agreements (positive weighted capacity alter), possibly because this capacity enables them to pursue a more frequent treaty agenda. Both broad and weighted capacity difference are negatively significant. This suggests that countries with similar GDPs are both more likely to partner and more likely to reinforce this partnership with additional treaties.

Lastly, on top of these effects, both popularity and weighted popularity are positive and significant. That is, countries with more partners are both more likely to attract further partners (preferential attachment) as well as further agreements from existing partners. Whereas transitivity is no longer significant, weighted transitivity is. This suggests that countries avoid reinforcing those transitive relations they are embedded in. In the initial creation of ties, however, the presence of transitive structure seems not to matter positively or negatively. In the case of transitivity effects, the parameter values and levels of significance are comparable in size, so this distinction should not be overstated.

By comparing the models with regard to the corrected version of Akaike's Information Criterion (AIC), we may conclude that the novel weighted specification clearly increased the fit of the model.

8. CONCLUSION

We have introduced a dynamic network actor model (DyNAM) that is tailored to the analysis of coordination networks through time. We argued that coordination ties in different social science disciplines share five specific features that should be taken into account when empirically studying their evolution. Those features were explicitly considered when introducing the new mathematical model and in two empirical applications of the model, to the case of romantic relationships in Jefferson

High School and the case of international fishery treaties between 1947 and 2010. We rooted the new model in the growing literature of statistical network methods. In particular, the model significantly builds on stochastic actor-oriented models (Snijders 1996; Snijders et al. 2010) and network models for the analysis for time-stamped network data (Butts 2008; Stadtfeld and Geyer-Schulz 2011), which are network extensions of classic event history models (Box-Steffensmeier and Jones 2004). In Appendix A, the new (actor-oriented) model was compared in some detail with the tie-oriented relational event model by Butts (2008) that can be specified to model undirected networks. We found that results of the two models in a simple case study were similar but discussed small differences that could be related to the actor- and tie-oriented nature of the two models. None of the models that we reviewed in the Section 2, however, satisfactorily took into account all of the five key features identified.

The first feature states that instances of coordination are dependent through time and through network structures and hence a dynamic network approach is required. In both case studies, we had to assume complex dependence structures such as transitivity (fishery treaties), four cycles (romantic ties), and degree popularity (both).

The second feature is the actor-oriented nature of coordination ties; actors choose their potential partners based on their preference structures as well as available opportunities. In high school "romantic markets" and in the complex institutional system of fishery treaties, it seemed reasonable to consider the observed tie formations as expressions of preferences (e.g., preferred connections with high GDP states) as well as opportunities (searching for a partner who is currently not in a romantic relationship).

The third feature is concerned with the two-sided nature of coordination ties. The mathematical model was closely linked to the conceptual idea that ties come about as a two-step process involving a proposal by one actor and acceptance by the other. This assumption is certainly reasonable in both empirical case studies—romantic ties and international treaties are the outcome of a mutual agreement process.

The fourth feature is concerned with the precision with which coordination network data are typically observed. The model we introduced builds on statistical network models for time-stamped data. This allowed us to draw inferences on almost 300 empirically observed creations of romantic relationships and 500 empirically observed instances of fishery treaties rather than artificially aggregating data into data panels or one static network observation. By getting closer to the actual dependencies represented in the fishery agreement data rather than simulating across panels, we were able to identify a negative transitivity effect, which accords better with the historical record. In the study of romantic ties, the higher precision allowed us to test a hypothesis about the avoidance of short cycles for which we found—other than previous studies with less temporal precision—no support.

The fifth feature relates to a variety of properties that empirical researchers might want to investigate in coordination networks specifically. Other statistical network models were developed for use in cases such as friendship networks, and therefore their properties to test coordination-specific hypotheses are somewhat scarce. Three classes of new effects were introduced and empirically applied. *Signed effects* relate to the distinction between processes of tie creation and tie deletion. We argued that in coordination networks it might often be unreasonable to assume that creation and tie deletion processes follow the same actor-preference structures. In this article, we studied only tie creation rather than deletion, but we used available information about tie deletion to update the process state. *Weighted effects* relate to the distinction of actor preferences for creating ties that create new partnerships from those that reinforce existing ones. For example, in the model about fishery treaties, we found evidence that contiguity matters for the formation of cooperative relationships with the first bilateral fisheries treaty. However, we found no evidence that contiguity drove countries to reinforce or update this relationship with additional treaties. *Windowed effects* relate to the recency of change in the coordination network. We argued that in the case of romantic ties in a high school, the time since the breakup of the previous relationship as well as the romantic experience of individuals should matter when new ties are formed. Indeed, we found significant effects for both time window processes, whereas in the case of romantic experience, the interpretation had to take into account the method by which the data were sampled. Although we illustrated such effects with relatively simple model specifications, these classes of effects can be combined productively to test more sophisticated hypotheses about network and temporal dependencies in richer data sets.

The approach that we introduced has certain constraints. In some empirical studies, data will not be available in a time-stamped format but will be aggregated into (e.g., annual) data panels. In such a case, the application of stochastic actor-oriented models or process-oriented, longitudinal extensions of exponential random graph models will be more appropriate. In the case of fine-grained panels such as the weekly data about romantic relationships, random permutations of the observed order within panels may be used to identify the robustness of estimates.

One key assumption of the model is its actor-oriented nature. Actors will be more likely to propose and accept ties that increase their individual objective function the most. In some empirical studies, this assumption might be too strict, and we might be instead interested in the relative probabilities of event realizations across the network. Researchers could then consider applying tie-oriented network methods such as the relational event model.

Another design decision relates to the rate parameters that model the general tendency of actors to propose ties. In the empirical part of this article, we assumed that these rates are constant and that varying degrees of actors are explained by their variation in attractiveness as partners.

Our models allow the inclusion of windowed effects. Those effects can express that within certain time windows the rates of some ties differ (e.g., individuals are more likely to be chosen as a partner in the period right after a breakup). At the end of the time window, the process state is updated, which means that the rates also change. In event history modeling, such intervals are called *right-censored* (Box-Steffensmeier and Jones 2004). By not modeling those intervals explicitly, a model with windowed effects is not purely Markovian anymore and assumes that the right-censored information is irrelevant for the parameter estimation. Future research should further investigate this topic and allow inclusion of right-censored intervals in the estimation.

The model assumes that proposal and acceptance of ties are symmetric processes. This might not be a realistic assumption in some contexts. However, if that is not the case, researchers should aim at collecting additional data about tie proposals and (rejected) tie acceptance. These proposal and acceptance data could then be explicitly modeled as two separate and nonsymmetric processes within an actor-oriented framework.

We perceive the DyNAM model as an important step towards a more precise and more accurate understanding of the dynamics of coordination networks. We illustrated its usability in two straightforward case studies about the creation of romantic ties in a high school over 18 months and bilateral fishery treaties between 1947 and 2010. Other potential use cases include the bilateral dynamics of financial treaties, research cooperation, marriage, trade agreements, military cooperation, rebel group coalitions, or knowledge exchange. In general, the model is very flexible and can be applied to study various types of dynamic coordination processes among individuals, organizations, or states. Hence, the DyNAM model has the potential to enable significant contributions to our understanding of how social actors form coordination ties through time.

APPENDIX A

Comparing the DyNAM Model with a Relational Event Model

Appendix A compares the dynamic network actor models (DyNAM) with the tie-based relational event model (REM; Butts 2008) based on the case study discussed in Section 6. With the REM, it is possible to fit models for undirected networks with specifications similar to the ones introduced in this article. Actor-oriented effects such as the covariate attractiveness effect in Table 2 can be replaced by an effect that evaluates the sum of the scores of both actors. To specify a model that is comparable to the one in Table 8, this is done for isolate scores, recent breakup scores, and experience scores (the effects in rows 5–7) that in the case of DyNAM relate to the relative attractiveness of a partner choice. A comparison of results is shown in Table A1.

On first glance, both models seem to be very similar when it comes to parameter signs and size, levels of significance, and likelihood-based model fit. The most significant differences are that the likelihood of the REM is slightly better (on average, event likelihoods are 1.3 percent higher[5]) and that the effect not more than one tie is above the .05 significance threshold in one model and slightly below in the other. The popularity effect is insignificant in both models, but the sign of the effect goes in different directions—a process that can be explained by the fact that in the DyNAM model, the average degree of actors chosen as partners is slightly below the expectations (of all potential partner choices of an actor), whereas in the REM, the average degrees of actors forming a tie is slightly above expectation (it is larger than the average sum of any two actors). The insignificant difference cannot be interpreted substantively but indicates how parameter interpretation is somewhat different: It relates to actor opportunities and choices in the DyNAM model and to the network position of ties in the REM. Overall, it seems that in the case of this simple data

Table A1. Comparison of Estimates of the DyNAM Model and a Similarly Specified Relational Event Model

Effects	Coefficient-DyNAM	SE	Coefficient-REM	SE
Popularity	–.01	.07	.06	.08
Different gender	2.29***	.29	4.56***	.58
Different experience	.29***	.08	.62***	.13
Four cycle	.46	.26	.78	.55
Not more than one tie	.23	.12	.27*	.12
Recent breakup	.52**	.17	.51**	.16
Experienced partner	–.80***	.15	–.88***	.14
Log likelihood	–2,855.63		–2,851.74	

Note: The attractiveness effects 5–7 have been replaced in the REM with similar effects evaluating the sum of the isolate scores, recent breakup scores, and experience scores of both actors involved in a tie. DyNAM = dynamic network actor models; REM = relational event model.
*$p < .05$. **$p < .01$. ***$p < .001$.

set and a straightforward model specification, no major differences can be found. However, some details show how conceptual differences between models can lead to practical differences.

Figure A1 shows the likelihood ratios for all 291 events (x-axis). Cases above the solid line are favored by the DyNAM model, cases below by the REM. The dashed line indicates the likelihood ratios at any time of the process state. Where the dashed line is above the solid line, the DyNAM event likelihood is on average higher up to that point and vice versa. It is notable that the single event likelihood ratios seem to drop toward the end of the observed period. Indeed, when observing the event likelihood ratio over time, we can see that the DyNAM model is slightly superior for most of the analyzed period but then drops below the solid line toward the end. This pattern is a side effect of the sampling strategy used to select the data set. The data set was selected based on the main component of a network, which means that all nodes must be connected in a single component by the end of the observation period. Indeed, in the last 30 events or so, each isolate must have a much stronger tendency to form a tie than usual to ensure that the end-state is reached. A tie-oriented network model does not assume that actors compare their alternative choices and that observed tie changes are thus nested within actors. It can therefore put a lot of probabilistic weight on all ties of specific nodes. In cases like the final events where isolate nodes have a strong tendency to form ties, it is thus advantaged in

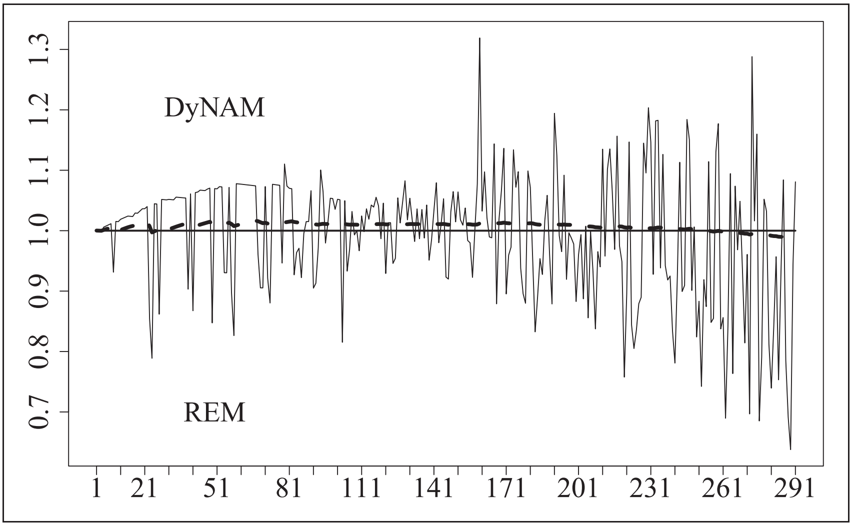


Figure A1. A comparison of relative event likelihoods of the two models over 291 events. The dashed line represents the mean relative event likelihoods; its end point is .987—the likelihood ratio of the two fitted models normalized by the number of events.

a likelihood-based model comparison, but this holds only where the data structure demands it.

Figure A2 shows the row sums of all event probability matrices for both models. The scatter plots thus indicate how much single row weights (all ties of one specific actor) differ between actors and between the two models. Figure A2(a) shows the distribution for all events, and panels A2(b) and A2(c) show the first 191 and the final 100 events. While row weights do not differ between models a lot in the first part of the process and few high values are observed (A2[b]), among the final 100 events, a large proportion of events can be found where there is significantly more weight on some actors (the upper right cluster of nodes in panel A2[c]), and these tendencies are stronger for the REM (indicated by the fact that the cluster is below the diagonal).

The REM seems to be better at dealing with actor inhomogeneities in the process. In contrast, the DyNAM model seems to be better at dealing with balanced structural effects such as the four-cycle effect where opportunities are more evenly distributed over the network. A DyNAM model without the highly centralized experience-related effects is in fact superior to the REM in terms of the likelihood ratios. Further, when diminishing the extreme centralization of

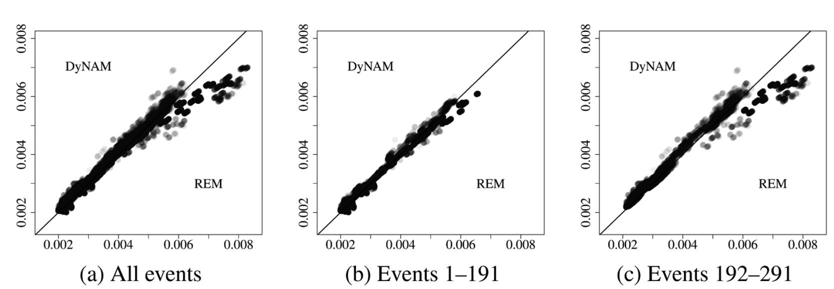


Figure A2. A comparison of the row sums of the empirical probability matrices of all events and both models.

probability weights around specific actors by artificially including 20 actors that are and remain isolates throughout the process, the estimated likelihoods become basically equal.

These findings are in line with a recently published argument by Block et al. (forthcoming) that suggests basing model choices on the theoretical assumptions underlying the process. We would argue that once explicit assumptions about underlying agreement processes are made, we should consider using the DyNAM model as parameters that can be interpreted in light of these. In Table A1, we can interpret the different experience and experienced partner effects in terms of actor choices: The probability of choosing an experienced partner if an actor is inexperienced (in the proposal or acceptance step) is about 60 percent as high as choosing a partner who is also inexperienced.[6] The interpretation of parameters in the REM is based on tie comparisons across the network: The probability that a tie between an experienced and an inexperienced node is observed next in the event sequence is 77 percent as high as observing a tie creation between two inexperienced actors.[7] One model is thus concerned with the choices of actors given their alternatives in the proposal or acceptance step (DyNAM) while the other is concerned with the propensity that certain ties rather than others are realized (REM).

A purely empirical/model fit–oriented reasoning might suggest choosing the DyNAM model for cases in which tie formation processes are rather homogeneous across actors, whereas REM might be preferable for cases of high actor heterogeneity. However, we recommend starting a model choice by reasoning about the fit of a model with theoretical assumptions and suggest additional comparisons of the two models in other theoretical and empirical settings. The goldfish estimation software that we developed allows us to fit both types of models facilitating such comparisons.

APPENDIX B

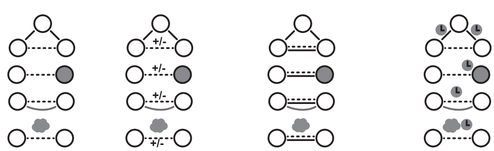
Overview of Effect Categories

The current implementation of dynamic network actor models (DyNAM) in the goldfish package allows the inclusion of four types of effect: (1) structural, (2) monadic, (3) dyadic, and (4) global. These effects can be tested for processes that relate to generic network mechanisms, signed processes, weighted ties, and windowed changes. Interactions across effects and processes are possible. Table B1 provides an overview of this classification with exemplary effects for transitivity (structural), attribute popularity (monadic), dyadic covariates, and global effects.

The structural signed example in the table is concerned with the differential tendencies of creating and dropping ties of partners that are embedded in transitive clusters. The monadic windowed effect in the table relates to the attractiveness of coordination partners that have recently changed an attribute (e.g., states that became democratic). The dyadic generic example relates to the general tendency to have ties with partners through which one is connected on a different network layer (e.g., forming romantic ties with friends or neighbors). The global weighted example describes the general tendency of actors to deepen relationships given a global variable (e.g., during times of financial crisis).

Table B1. Overview of Exemplary Model Specifications That Can Be Tested with the Current Implementation of the goldfish Software

		Process class			
		Generic	Signed	Weighted	Windowed
Effect type	Structural				
	Monadic				
	Dyadic				
	Global				



Acknowledgments

The authors thank Professor Peter Bearman for kindly sharing the data used in Section 6. Useful feedback and comments that considerably improved this work were graciously provided by members of the social network research groups in Groningen and Zürich; participants of the Swiss Networks Workshop in Zürich, the Sunbelt Conference in Brighton, and the International Studies Association Annual Convention in Atlanta; as well as three anonymous reviewers.

Funding

The authors disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The research has been funded by internal research grants of ETH Zurich and the Graduate Institute, Geneva.

Notes

1. Multinomial stochastic choice models were introduced by McFadden (1974) and first applied in the social networks literature by Snijders (1996) in the form of stochastic actor-oriented models (SAOMs).
2. The name refers to the (incorrect) assertion that `goldfish`—like Markov processes—are memoryless. It has been developed in R (R Core Team 2013).
3. Visualized with the visone software (Brandes and Wagner 2004).
4. In this case study, we observe a mean of 5.3 tie creations per time stamp. Mostly, those relate to distinct actors. In 61 cases, however, 2 individuals reported to have created and dissolved a romantic tie within one week. In 81 cases, individuals dissolved a romantic tie and created a new tie with a new partner within the same week.
5. This follows from a normalized likelihood ratio calculated as $\exp((\log L_1 - \log L_2)/291) = 1.0135$.
6. Calculated by interpreting the effects as log odds: exp(.29 – .8) = .6.
7. Calculated by interpreting the effects as log odds: exp(.62 – .88) = .77.

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