Independent subsets of a finite set

(web version)

1 Independent nontrivial subsets

```
pairs_independent_nontrivial_subsets(n) :=
block([a, b, d, s : 0 ],
  for a:1 thru n-1 do
    for d:1 thru a do
        ( b : n*d / a,
          if integerp(b) and b<n
              then
                s : s +
                    binomial(n,a) *
                      binomial(a,d) *
                        binomial(n-a,b-d) ) ,
    s)  $
```

```
pairs_independent_nontrivial_subsets(4);
```

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```
L30 : makelist(i, i, 0, 30);
```

[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
19,20,21,22,23,24,25,26,27,28,29,30]

```
L_nontrivial_30  :  map(pairs_independent_nontrivial_subsets, L30);
```

[0,0,0,0,24,0,720,0,7000,15120,126000,0,1777776,0
,23543520,55855800,274565720,0,5337775872,0,
63026049424,117920013120,995265791520,0,15265486117744
,14283091977000,216344919117600,240142901941800,
2854493961432480,0,55689696384165720]

```
a(n) :=
sum(
   sum(
        (b : n*d / a,
        if integerp(b) and b<n then
           binomial(n,a) *
             binomial(a,d) *
               binomial(n-a,b-d) else 0),
   d,1,a),
a,1,n-1) $
```

```
a(6);
```

720

```
L_a_30 : map(a, L30);
```

[0,0,0,0,24,0,720,0,7000,15120,126000,0,1777776,0
,23543520,55855800,274565720,0,5337775872,0,
63026049424,117920013120,995265791520,0,15265486117744
,14283091977000,216344919117600,240142901941800,
2854493961432480,0,55689696384165720]

```
is(L_a_30 = L_nontrivial_30);
```

true

```
is(map(pairs_independent_nontrivial_subsets, L30) = map(a,L30) );
```

true

2 Independent proper subsets

```
pairs_independent_proper_subsets(n) :=
  if is(n=0) then 0 else a(n) + 2*(2^n -1)  - 1 $
```

```
L_proper_30  :  map(pairs_independent_proper_subsets, L30);
```

[0,1,5,13,53,61,845,253,7509,16141,128045,4093,
1785965,16381,23576285,55921333,274696789,262141,
5338300157,1048573,63028146573,117924207421,
995274180125,16777213,15265519672173,14283159085861,
216345053335325,240143170377253,2854494498303389,
1073741821,55689698531649365]

```
OEIS_A158345  :  [ 1, 5, 13, 53, 61, 845, 253, 7509, 16141, 128045,
   4093, 1785965, 16381, 23576285, 55921333, 274696789,
   262141, 5338300157, 1048573, 63028146573, 117924207421,
   995274180125, 16777213, 15265519672173, 14283159085861 ];
```

[1,5,13,53,61,845,253,7509,16141,128045,4093,
1785965,16381,23576285,55921333,274696789,262141,
5338300157,1048573,63028146573,117924207421,
995274180125,16777213,15265519672173,14283159085861]

```
rest(rest(L_proper_30), -5) ;
```

[1,5,13,53,61,845,253,7509,16141,128045,4093,
1785965,16381,23576285,55921333,274696789,262141,
5338300157,1048573,63028146573,117924207421,
995274180125,16777213,15265519672173,14283159085861]

```
is( rest(rest(L_proper_30), -5)  =  OEIS_A158345);
```

true

3 All independent subsets

```
pairs_independent_subsets(n) :=
  if is(n=0)  then 1 else a(n) + 4*(2^n -1) $
pairs_independent_subsets(6);
```

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```
L_independent_30  :  map(pairs_independent_subsets, L30);
```

[1,4,12,28,84,124,972,508,8020,17164,130092,8188,
1794156,32764,23609052,55986868,274827860,524284,
5338824444,2097148,63030243724,117928401724,
995282568732,33554428,15265553226604,14283226194724,
216345187553052,240143438812708,2854495035174300,
2147483644,55689700679133012]

```
OEIS_A121312  :  [ 1, 4, 12, 28, 84, 124, 972, 508, 8020, 17164,
    130092, 8188, 1794156, 32764, 23609052, 55986868,
    274827860, 524284, 5338824444, 2097148, 63030243724,
    117928401724, 995282568732, 33554428, 15265553226604,
    14283226194724, 216345187553052 ];
```

[1,4,12,28,84,124,972,508,8020,17164,130092,8188,
1794156,32764,23609052,55986868,274827860,524284,
5338824444,2097148,63030243724,117928401724,
995282568732,33554428,15265553226604,14283226194724,
216345187553052]

```
is(rest(L_independent_30, -4) = OEIS_A121312);
```

true

```
transpose(matrix(L30,L_independent_30, L_proper_30, L_nontrivial_30
```

0	1	0	0
1	4	1	0
2	12	5	0
3	28	13	0
4	84	53	24
5	124	61	0
6	972	845	720
7	508	253	0
8	8020	7509	7000
9	17164	16141	15120
10	130092	128045	126000
11	8188	4093	0
12	1794156	1785965	1777776
13	32764	16381	0
14	23609052	23576285	23543520
15	55986868	55921333	55855800
16	274827860	274696789	274565720
17	524284	262141	0
18	5338824444	5338300157	5337775872
19	2097148	1048573	0
20	63030243724	63028146573	63026049424
21	117928401724	117924207421	117920013120
22	995282568732	995274180125	995265791520
23	33554428	16777213	0
24	15265553226604	15265519672173	15265486117744
25	14283226194724	14283159085861	14283091977000
26	216345187553052	216345053335325	216344919117600
27	240143438812708	240143170377253	240142901941800
28	2854495035174300	2854494498303389	2854493961432480
29	2147483644	1073741821	0
30	55689700679133012	55689698531649365	55689696384165720