# Independent subsets of a finite set

(web version)

## 1 Independent nontrivial subsets

```
pairs_independent_nontrivial_subsets(n) :=
block([a, b, d, s : 0 ],
  for a:1 thru n-1 do
    for d:1 thru a do
       ( b : n*d / a,
         if integerp(b) and b<n
            then
              s : s +
                   binomial(n,a) *
                     binomial(a,d) *
                       binomial(n-a,b-d) ) ,
    s)  $
```

```
pairs_independent_nontrivial_subsets(4);
```

24

```
L30 : makelist(i, i, 0, 30);
```

[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
19,20,21,22,23,24,25,26,27,28,29,30]

```
L_nontrivial_30  :  map(pairs_independent_nontrivial_subsets, L30);
```

[0,0,0,0,24,0,720,0,7000,15120,126000,0,1777776,0
,23543520,55855800,274565720,0,5337775872,0,
63026049424,117920013120,995265791520,0,15265486117744
,14283091977000,216344919117600,240142901941800,
2854493961432480,0,55689696384165720]

```
a(n) :=
sum(
   sum(
        (b : n*d / a,
        if integerp(b) and b<n then
           binomial(n,a) *
             binomial(a,d) *
               binomial(n-a,b-d) else 0),
   d,1,a),
a,1,n-1) $
```

```
a(6);
```

720

```
L_a_30 : map(a, L30);
```

[0,0,0,0,24,0,720,0,7000,15120,126000,0,1777776,0
,23543520,55855800,274565720,0,5337775872,0,
63026049424,117920013120,995265791520,0,15265486117744
,14283091977000,216344919117600,240142901941800,
2854493961432480,0,55689696384165720]

```
is(L_a_30 = L_nontrivial_30);
```

*true*

```
is(map(pairs_independent_nontrivial_subsets, L30) = map(a,L30) );
```

*true*

# 2 Independent proper subsets

```
pairs_independent_proper_subsets(n) :=
  if is(n=0) then 0 else a(n) + 2*(2^n -1)  - 1 $
```

```
L_proper_30  :  map(pairs_independent_proper_subsets, L30);
```

[0,1,5,13,53,61,845,253,7509,16141,128045,4093,
1785965,16381,23576285,55921333,274696789,262141,
5338300157,1048573,63028146573,117924207421,
995274180125,16777213,15265519672173,14283159085861,
216345053335325,240143170377253,2854494498303389,
1073741821,55689698531649365]

```
OEIS_A158345  :  [ 1, 5, 13, 53, 61, 845, 253, 7509, 16141, 128045,
   4093, 1785965, 16381, 23576285, 55921333, 274696789,
   262141, 5338300157, 1048573, 63028146573, 117924207421,
   995274180125, 16777213, 15265519672173, 14283159085861 ];
```

[1,5,13,53,61,845,253,7509,16141,128045,4093,
1785965,16381,23576285,55921333,274696789,262141,
5338300157,1048573,63028146573,117924207421,
995274180125,16777213,15265519672173,14283159085861]

```
rest(rest(L_proper_30), -5) ;
```

[1,5,13,53,61,845,253,7509,16141,128045,4093,
1785965,16381,23576285,55921333,274696789,262141,
5338300157,1048573,63028146573,117924207421,
995274180125,16777213,15265519672173,14283159085861]

```
is( rest(rest(L_proper_30), -5)  =  OEIS_A158345);
```

*true*

# 3 All independent subsets

```
pairs_independent_subsets(n) :=
  if is(n=0)  then 1 else a(n) + 4*(2^n -1) $
```

```
pairs_independent_subsets(6);
```

972

```
L_independent_30  :  map(pairs_independent_subsets, L30);
```

[1,4,12,28,84,124,972,508,8020,17164,130092,8188,
1794156,32764,23609052,55986868,274827860,524284,
5338824444,2097148,63030243724,117928401724,
995282568732,33554428,15265553226604,14283226194724,
216345187553052,240143438812708,2854495035174300,
2147483644,55689700679133012]

```
OEIS_A121312  :  [ 1, 4, 12, 28, 84, 124, 972, 508, 8020, 17164,
    130092, 8188, 1794156, 32764, 23609052, 55986868,
    274827860, 524284, 5338824444, 2097148, 63030243724,
    117928401724, 995282568732, 33554428, 15265553226604,
    14283226194724, 216345187553052 ];
```

[1,4,12,28,84,124,972,508,8020,17164,130092,8188,
1794156,32764,23609052,55986868,274827860,524284,
5338824444,2097148,63030243724,117928401724,
995282568732,33554428,15265553226604,14283226194724,
216345187553052]

```
is(rest(L_independent_30, -4) = OEIS_A121312);
```

*true*

```
transpose(matrix(L30,L_independent_30, L_proper_30, L_nontrivial_30
```

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 0 |
| 1 | 4 | 1 | 0 |
| 2 | 12 | 5 | 0 |
| 3 | 28 | 13 | 0 |
| 4 | 84 | 53 | 24 |
| 5 | 124 | 61 | 0 |
| 6 | 972 | 845 | 720 |
| 7 | 508 | 253 | 0 |
| 8 | 8020 | 7509 | 7000 |
| 9 | 17164 | 16141 | 15120 |
| 10 | 130092 | 128045 | 126000 |
| 11 | 8188 | 4093 | 0 |
| 12 | 1794156 | 1785965 | 1777776 |
| 13 | 32764 | 16381 | 0 |
| 14 | 23609052 | 23576285 | 23543520 |
| 15 | 55986868 | 55921333 | 55855800 |
| 16 | 274827860 | 274696789 | 274565720 |
| 17 | 524284 | 262141 | 0 |
| 18 | 5338824444 | 5338300157 | 5337775872 |
| 19 | 2097148 | 1048573 | 0 |
| 20 | 63030243724 | 63028146573 | 63026049424 |
| 21 | 117928401724 | 117924207421 | 117920013120 |
| 22 | 995282568732 | 995274180125 | 995265791520 |
| 23 | 33554428 | 16777213 | 0 |
| 24 | 15265553226604 | 15265519672173 | 15265486117744 |
| 25 | 14283226194724 | 14283159085861 | 14283091977000 |
| 26 | 216345187553052 | 216345053335325 | 216344919117600 |
| 27 | 240143438812708 | 240143170377253 | 240142901941800 |
| 28 | 2854495035174300 | 2854494498303389 | 2854493961432480 |
| 29 | 2147483644 | 1073741821 | 0 |
| 30 | 55689700679133012 | 55689698531649365 | 55689696384165720 |