The Sieve of Eratosthenes for finding primes

Reference: Ziegenbalg: Algorithmen von Hammurapi bis Gödel, 4. Auflage,
   Springer-Spektrum, Wiesbaden 2016, section 3.2.3

1 A version essentially using arrays

```
(%i1) Eratosthenes_Array(UpperLimit) :=
        /* as similar as possible to the BYTE version - with correct result */
        block([A, i, k],
          A : make_array(fixnum, UpperLimit+1),
          for i : 0 thru UpperLimit do (A[i] : i),
          A[1] : 0,
          i : 2,
          while i*i <= UpperLimit do
           (k : i+i,
            while k <= UpperLimit do
              (A[k] : 0,
               k : k+i),
            i : i+1),
          delete(0, listarray(A))  ) $ ;          /*  the result is a list with the zeroes removed */
```

```
(%i2) Eratosthenes_Array(1000);
```

(%o2) *[*2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,
83,89,97,101,103,107,109,113,127,131,137,139,149,151,157,163,167,173,
179,181,191,193,197,199,211,223,227,229,233,239,241,251,257,263,269,
271,277,281,283,293,307,311,313,317,331,337,347,349,353,359,367,373,
379,383,389,397,401,409,419,421,431,433,439,443,449,457,461,463,467,
479,487,491,499,503,509,521,523,541,547,557,563,569,571,577,587,593,
599,601,607,613,617,619,631,641,643,647,653,659,661,673,677,683,691,
701,709,719,727,733,739,743,751,757,761,769,773,787,797,809,811,821,
823,827,829,839,853,857,859,863,877,881,883,887,907,911,919,929,937,
941,947,953,967,971,977,983,991,997*]*

2 A version using lists and delete

(%i3) Eratosthenes_List(UpperLimit) :=
```
block([E, i, k],
  E : makelist(j, j, 1, UpperLimit),
  E[1] : 0,
  i : 2,
  while i*i <= UpperLimit do
   (k : i+i,
    while k <= UpperLimit do
      (E[k] : 0,
       k : k+i),
    i : i+1),
  E : delete(0, E),
  E) $ ;
```

(%i4) Eratosthenes_List(1000);

(%o4) *[*2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,
83,89,97,101,103,107,109,113,127,131,137,139,149,151,157,163,167,173,
179,181,191,193,197,199,211,223,227,229,233,239,241,251,257,263,269,
271,277,281,283,293,307,311,313,317,331,337,347,349,353,359,367,373,
379,383,389,397,401,409,419,421,431,433,439,443,449,457,461,463,467,
479,487,491,499,503,509,521,523,541,547,557,563,569,571,577,587,593,
599,601,607,613,617,619,631,641,643,647,653,659,661,673,677,683,691,
701,709,719,727,733,739,743,751,757,761,769,773,787,797,809,811,821,
823,827,829,839,853,857,859,863,877,881,883,887,907,911,919,929,937,
941,947,953,967,971,977,983,991,997*]*

3 A version using sublist with predicate instead of delete

(%i5) Eratosthenes_SubList(UpperLimit) :=
```
        /* for avoiding the destructive delete-command */
block([E, i, k],
  E : makelist(j, j, 1, UpperLimit),
  E[1] : 0,
  i : 2,
  while i*i <= UpperLimit do
   (k : i+i,
    while k <= UpperLimit do
      (E[k] : 0,
       k : k+i),
    i : i+1),
  sublist(E, lambda([x], not(is(x=0)))) ) $ ;
```

(%i6) Eratosthenes_SubList(1000);

(%o6) *[*2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,
       83,89,97,101,103,107,109,113,127,131,137,139,149,151,157,163,167,173,
       179,181,191,193,197,199,211,223,227,229,233,239,241,251,257,263,269,
       271,277,281,283,293,307,311,313,317,331,337,347,349,353,359,367,373,
       379,383,389,397,401,409,419,421,431,433,439,443,449,457,461,463,467,
       479,487,491,499,503,509,521,523,541,547,557,563,569,571,577,587,593,
       599,601,607,613,617,619,631,641,643,647,653,659,661,673,677,683,691,
       701,709,719,727,733,739,743,751,757,761,769,773,787,797,809,811,821,
       823,827,829,839,853,857,859,863,877,881,883,887,907,911,919,929,937,
       941,947,953,967,971,977,983,991,997*]*

       Some tests regarding equality

(%i7) 2+3=5;
(%o7) $5 = 5$

(%i8) is(2+3=5);
(%o8) ***true***

(%i9) is(lambda([x], not(x=0))(2))   /* the "non-zero"-predicate */;
(%o9) ***true***

(%i10) E1: [0,1,0,2,0,0,3,0,0,0,4,0,0,0];
(E1)   *[*0,1,0,2,0,0,3,0,0,0,4,0,0,0*]*

(%i11) sublist(E1, lambda([x], not(is(x=0))));
(%o11) *[*1,2,3,4*]*

(%i12) functions;
(%o12) *[*Eratosthenes_Array*(*UpperLimit*)*,Eratosthenes_List*(*UpperLimit*)*,
       Eratosthenes_SubList*(*UpperLimit*)]*

4   *Timing*

(%i13) timer(all);
(%o13) *[*Eratosthenes_Array,Eratosthenes_List,Eratosthenes_SubList*]*

(%i14) timer;
(%o14) *[*Eratosthenes_SubList,Eratosthenes_List,Eratosthenes_Array*]*

(%i15) Eratosthenes_Array(10000);

(%o15) [2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,
83,89,97,101,103,107,109,113,127,131,137,139,149,151,157,163,167,173,
179,181,191,193,197,199,211,223,227,229,233,239,241,251,257,263,269,
271,277,281,283,293,307,311,313,317,331,337,347,349,353,359,367,373,
379,383,389,397,401,409,419,421,431,433,439,443,449,457,461,463,467,
479,487,491,499,503,509,521,523,541,547,557,563,569,571,577,587,593,
599,601,607,613,617,619,631,641,643,647,653,659,661,673,677,683,691,
701,709,719,727,733,739,743,751,757,761,769,773,787,797,809,811,821,
823,827,829,839,853,857,859,863,877,881,883,887,907,911,919,929,937,
941,947,953,967,971,977,983,991,997,1009,1013,1019,1021,1031,1033,
1039,1049,1051,1061,1063,1069,1087,1091,1093,1097,1103,1109,1117,
1123,1129,1151,1153,1163,1171,1181,1187,1193,1201,1213,1217,1223,
1229,1231,1237,1249,1259,1277,1279,1283,1289,1291,1297,1301,1303,
1307,1319,1321,1327,1361,1367,1373,1381,1399,1409,1423,1427,1429,
1433,1439,1447,1451,1453,1459,1471,1481,1483,1487,1489,1493,1499,
1511,1523,1531,1543,1549,1553,1559,1567,1571,1579,1583,1597,1601,
1607,1609,1613,1619,1621,1627,1637,1657,1663,1667,1669,1693,1697,
1699,1709,1721,1723,1733,1741,1747,1753,1759,1777,1783,1787,1789,
1801,1811,1823,1831,1847,1861,1867,1871,1873,1877,1879,1889,1901,
1907,1913,1931,1933,1949,1951,1973,1979,1987,1993,1997,1999,2003,
2011,2017,2027,2029,2039,2053,2063,2069,2081,2083,2087,2089,2099,
2111,2113,2129,2131,2137,2141,2143,2153,2161,2179,2203,2207,2213,
2221,2237,2239,2243,2251,2267,2269,2273,2281,2287,2293,2297,2309,
2311,2333,2339,2341,2347,2351,2357,2371,2377,2381,2383,2389,2393,
2399,2411,2417,2423,2437,2441,2447,2459,2467,2473,2477,2503,2521,
2531,2539,2543,2549,2551,2557,2579,2591,2593,2609,2617,2621,2633,
2647,2657,2659,2663,2671,2677,2683,2687,2689,2693,2699,2707,2711,
2713,2719,2729,2731,2741,2749,2753,2767,2777,2789,2791,2797,2801,
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2917,2927,2939,2953,2957,2963,2969,2971,2999,3001,3011,3019,3023,
3037,3041,3049,3061,3067,3079,3083,3089,3109,3119,3121,3137,3163,
3167,3169,3181,3187,3191,3203,3209,3217,3221,3229,3251,3253,3257,
3259,3271,3299,3301,3307,3313,3319,3323,3329,3331,3343,3347,3359,
3361,3371,3373,3389,3391,3407,3413,3433,3449,3457,3461,3463,3467,
3469,3491,3499,3511,3517,3527,3529,3533,3539,3541,3547,3557,3559,
3571,3581,3583,3593,3607,3613,3617,3623,3631,3637,3643,3659,3671,
3673,3677,3691,3697,3701,3709,3719,3727,3733,3739,3761,3767,3769,
3779,3793,3797,3803,3821,3823,3833,3847,3851,3853,3863,3877,3881,
3889,3907,3911,3917,3919,3923,3929,3931,3943,3947,3967,3989,4001,
4003,4007,4013,4019,4021,4027,4049,4051,4057,4073,4079,4091,4093,
4099,4111,4127,4129,4133,4139,4153,4157,4159,4177,4201,4211,4217,
4219,4229,4231,4241,4243,4253,4259,4261,4271,4273,4283,4289,4297,
4327,4337,4339,4349,4357,4363,4373,4391,4397,4409,4421,4423,4441,
4447,4451,4457,4463,4481,4483,4493,4507,4513,4517,4519,4523,4547,
4549,4561,4567,4583,4591,4597,4603,4621,4637,4639,4643,4649,4651,
4657,4663,4673,4679,4691,4703,4721,4723,4729,4733,4751,4759,4783,
4787,4789,4793,4799,4801,4813,4817,4831,4861,4871,4877,4889,4903,
4909,4919,4931,4933,4937,4943,4951,4957,4967,4969,4973,4987,4993,

(%i16) Eratosthenes_List(10000);

(%o16) [2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,
83,89,97,101,103,107,109,113,127,131,137,139,149,151,157,163,167,173,
179,181,191,193,197,199,211,223,227,229,233,239,241,251,257,263,269,
271,277,281,283,293,307,311,313,317,331,337,347,349,353,359,367,373,
379,383,389,397,401,409,419,421,431,433,439,443,449,457,461,463,467,
479,487,491,499,503,509,521,523,541,547,557,563,569,571,577,587,593,
599,601,607,613,617,619,631,641,643,647,653,659,661,673,677,683,691,
701,709,719,727,733,739,743,751,757,761,769,773,787,797,809,811,821,
823,827,829,839,853,857,859,863,877,881,883,887,907,911,919,929,937,
941,947,953,967,971,977,983,991,997,1009,1013,1019,1021,1031,1033,
1039,1049,1051,1061,1063,1069,1087,1091,1093,1097,1103,1109,1117,
1123,1129,1151,1153,1163,1171,1181,1187,1193,1201,1213,1217,1223,
1229,1231,1237,1249,1259,1277,1279,1283,1289,1291,1297,1301,1303,
1307,1319,1321,1327,1361,1367,1373,1381,1399,1409,1423,1427,1429,
1433,1439,1447,1451,1453,1459,1471,1481,1483,1487,1489,1493,1499,
1511,1523,1531,1543,1549,1553,1559,1567,1571,1579,1583,1597,1601,
1607,1609,1613,1619,1621,1627,1637,1657,1663,1667,1669,1693,1697,
1699,1709,1721,1723,1733,1741,1747,1753,1759,1777,1783,1787,1789,
1801,1811,1823,1831,1847,1861,1867,1871,1873,1877,1879,1889,1901,
1907,1913,1931,1933,1949,1951,1973,1979,1987,1993,1997,1999,2003,
2011,2017,2027,2029,2039,2053,2063,2069,2081,2083,2087,2089,2099,
2111,2113,2129,2131,2137,2141,2143,2153,2161,2179,2203,2207,2213,
2221,2237,2239,2243,2251,2267,2269,2273,2281,2287,2293,2297,2309,
2311,2333,2339,2341,2347,2351,2357,2371,2377,2381,2383,2389,2393,
2399,2411,2417,2423,2437,2441,2447,2459,2467,2473,2477,2503,2521,
2531,2539,2543,2549,2551,2557,2579,2591,2593,2609,2617,2621,2633,
2647,2657,2659,2663,2671,2677,2683,2687,2689,2693,2699,2707,2711,
2713,2719,2729,2731,2741,2749,2753,2767,2777,2789,2791,2797,2801,
2803,2819,2833,2837,2843,2851,2857,2861,2879,2887,2897,2903,2909,
2917,2927,2939,2953,2957,2963,2969,2971,2999,3001,3011,3019,3023,
3037,3041,3049,3061,3067,3079,3083,3089,3109,3119,3121,3137,3163,
3167,3169,3181,3187,3191,3203,3209,3217,3221,3229,3251,3253,3257,
3259,3271,3299,3301,3307,3313,3319,3323,3329,3331,3343,3347,3359,
3361,3371,3373,3389,3391,3407,3413,3433,3449,3457,3461,3463,3467,
3469,3491,3499,3511,3517,3527,3529,3533,3539,3541,3547,3557,3559,
3571,3581,3583,3593,3607,3613,3617,3623,3631,3637,3643,3659,3671,
3673,3677,3691,3697,3701,3709,3719,3727,3733,3739,3761,3767,3769,
3779,3793,3797,3803,3821,3823,3833,3847,3851,3853,3863,3877,3881,
3889,3907,3911,3917,3919,3923,3929,3931,3943,3947,3967,3989,4001,
4003,4007,4013,4019,4021,4027,4049,4051,4057,4073,4079,4091,4093,
4099,4111,4127,4129,4133,4139,4153,4157,4159,4177,4201,4211,4217,
4219,4229,4231,4241,4243,4253,4259,4261,4271,4273,4283,4289,4297,
4327,4337,4339,4349,4357,4363,4373,4391,4397,4409,4421,4423,4441,
4447,4451,4457,4463,4481,4483,4493,4507,4513,4517,4519,4523,4547,
4549,4561,4567,4583,4591,4597,4603,4621,4637,4639,4643,4649,4651,
4657,4663,4673,4679,4691,4703,4721,4723,4729,4733,4751,4759,4783,
4787,4789,4793,4799,4801,4813,4817,4831,4861,4871,4877,4889,4903,
4909,4919,4931,4933,4937,4943,4951,4957,4967,4969,4973,4987,4993,

(%i17) Eratosthenes_SubList(10000);

(%o17) [2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,
83,89,97,101,103,107,109,113,127,131,137,139,149,151,157,163,167,173,
179,181,191,193,197,199,211,223,227,229,233,239,241,251,257,263,269,
271,277,281,283,293,307,311,313,317,331,337,347,349,353,359,367,373,
379,383,389,397,401,409,419,421,431,433,439,443,449,457,461,463,467,
479,487,491,499,503,509,521,523,541,547,557,563,569,571,577,587,593,
599,601,607,613,617,619,631,641,643,647,653,659,661,673,677,683,691,
701,709,719,727,733,739,743,751,757,761,769,773,787,797,809,811,821,
823,827,829,839,853,857,859,863,877,881,883,887,907,911,919,929,937,
941,947,953,967,971,977,983,991,997,1009,1013,1019,1021,1031,1033,
1039,1049,1051,1061,1063,1069,1087,1091,1093,1097,1103,1109,1117,
1123,1129,1151,1153,1163,1171,1181,1187,1193,1201,1213,1217,1223,
1229,1231,1237,1249,1259,1277,1279,1283,1289,1291,1297,1301,1303,
1307,1319,1321,1327,1361,1367,1373,1381,1399,1409,1423,1427,1429,
1433,1439,1447,1451,1453,1459,1471,1481,1483,1487,1489,1493,1499,
1511,1523,1531,1543,1549,1553,1559,1567,1571,1579,1583,1597,1601,
1607,1609,1613,1619,1621,1627,1637,1657,1663,1667,1669,1693,1697,
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1907,1913,1931,1933,1949,1951,1973,1979,1987,1993,1997,1999,2003,
2011,2017,2027,2029,2039,2053,2063,2069,2081,2083,2087,2089,2099,
2111,2113,2129,2131,2137,2141,2143,2153,2161,2179,2203,2207,2213,
2221,2237,2239,2243,2251,2267,2269,2273,2281,2287,2293,2297,2309,
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2399,2411,2417,2423,2437,2441,2447,2459,2467,2473,2477,2503,2521,
2531,2539,2543,2549,2551,2557,2579,2591,2593,2609,2617,2621,2633,
2647,2657,2659,2663,2671,2677,2683,2687,2689,2693,2699,2707,2711,
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2803,2819,2833,2837,2843,2851,2857,2861,2879,2887,2897,2903,2909,
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3259,3271,3299,3301,3307,3313,3319,3323,3329,3331,3343,3347,3359,
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3571,3581,3583,3593,3607,3613,3617,3623,3631,3637,3643,3659,3671,
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3779,3793,3797,3803,3821,3823,3833,3847,3851,3853,3863,3877,3881,
3889,3907,3911,3917,3919,3923,3929,3931,3943,3947,3967,3989,4001,
4003,4007,4013,4019,4021,4027,4049,4051,4057,4073,4079,4091,4093,
4099,4111,4127,4129,4133,4139,4153,4157,4159,4177,4201,4211,4217,
4219,4229,4231,4241,4243,4253,4259,4261,4271,4273,4283,4289,4297,
4327,4337,4339,4349,4357,4363,4373,4391,4397,4409,4421,4423,4441,
4447,4451,4457,4463,4481,4483,4493,4507,4513,4517,4519,4523,4547,
4549,4561,4567,4583,4591,4597,4603,4621,4637,4639,4643,4649,4651,
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4787,4789,4793,4799,4801,4813,4817,4831,4861,4871,4877,4889,4903,
4909,4919,4931,4933,4937,4943,4951,4957,4967,4969,4973,4987,4993,

(%i18) timer_info();

(%o18)

function	time//call	calls	runtime	gctime
Eratosthenes_SubList	3.453 sec	1	3.453 sec	0
Eratosthenes_List	4.828 sec	1	4.828 sec	0
Eratosthenes_Array	0.172 sec	1	0.172 sec	0
total	2.817666666666666 sec	3	8.453 sec	0