Difference Equations

- Part 2 -

Cobweb diagrams

("Spinnweben"-Diagramme)

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References

Dürr R. / J. Ziegenbalg: Mathematik für Computeranwendungen: Dynamische Prozesse und ihre Mathematisierung durch Differenzengleichungen, Schöningh Verlag, Paderborn 1989
Second edition: Mathematik für Computeranwendungen; Ferdinand Schöningh Verlag, Paderborn 1989

Goldberg S.: Introduction to Difference Equations; John Wiley, New York 1958

Rommelfanger H.: Differenzen- und Differentialgleichungen; B.I., Zürich 1977

J. Ziegenbalg: Figurierte Zahlen; Springer-Spektrum, Wiesbaden 2018

Generating the basic data list

The following program generates a list subsequently to be processed for graphical representation.

```
AnnuityList[y0_, A_, B_, k_] :=
 Module[{i = 0, y = y0, AL = {}},
  AL = Append[AL, {i, y}];
  While[i < k,
   i = i + 1; y = A * y + B; AL = Append[AL, {i, y}]];
  Return[AL] ]
```

```
AnnuityList[100000, 1.05, -10000, 15]
```

```
{{0, 100000}, {1, 95000.}, {2, 89750.}, {3, 84237.5}, {4, 78449.4}, {5, 72371.8},
 {6, 65990.4}, {7, 59290.}, {8, 52254.5}, {9, 44867.2}, {10, 37110.5},
 {11, 28966.1}, {12, 20414.4}, {13, 11435.1}, {14, 2006.84}, {15, -7892.82}}
```

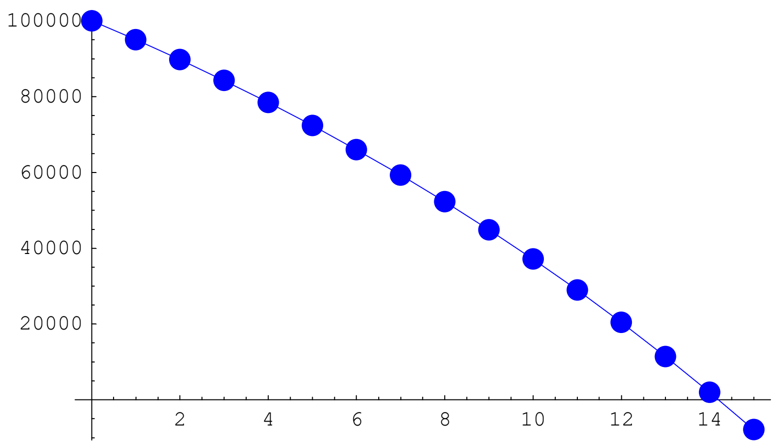
The standard plot ("timeline plot")

▪ Implementation of the timeline plot

```
TimelinePlot[y0_, A_, B_, k_] :=
 (TP1 = ListPlot[AnnuityList[y0, A, B, k],
    PlotStyle → {RGBColor[0, 0, 1], PointSize[0.03]}, DisplayFunction → Identity];
  TP2 = ListPlot[AnnuityList[y0, A, B, k] , PlotStyle → {RGBColor[0, 0, 1]},
    PlotJoined → True, DisplayFunction → Identity];
  Show[{TP1, TP2}, PlotRange → All, ImageSize → {360, 360} ,
   DisplayFunction → $DisplayFunction])
```

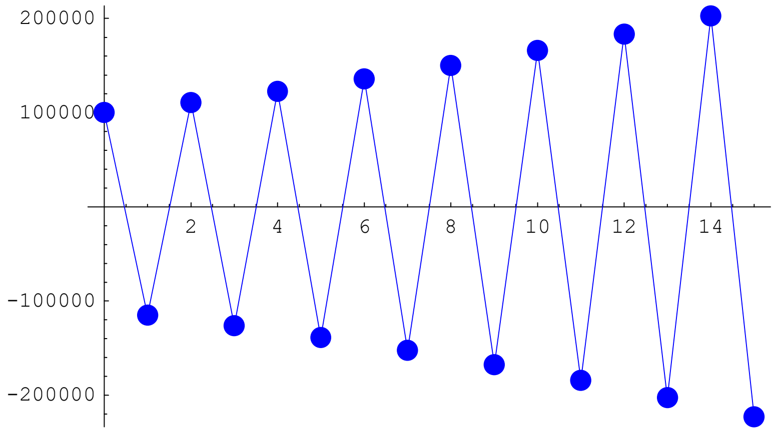
▪ Experiments and Results

```
TimelinePlot[100000, 1.05, -10000, 15]
```




```
- Graphics -
```

```
TimelinePlot[100000, -1.05, -10000, 15]
```




```
- Graphics -
```

The "cobweb" plot

The program AnnuityList generates a list of pairs, the annuity list. The first component of each pair can be interpetreted as "time". The complete liste gives the development of the magnitude y in time (see previous graphic).

■ Implementation of the cobweb plot

The following module CobList takes the annuity list as its input and transforms it into a list suitable for display in the form of a *cobweb diagram*.

```
CobList[AnnuityList_] :=
 Module[{L = Map[Last, AnnuityList] , CobL = {}},
  (If[Length[L] > 0, CobL = {{First[L], 0}} ];
   While[
    Length[L] > 1,
    CobL = Append[CobL, {L[[1]], L[[2]]} ] ;
    CobL = Append[CobL, {L[[2]], L[[2]]} ] ;
    L = Delete[L, 1] ]);
  Return[CobL] ]
```

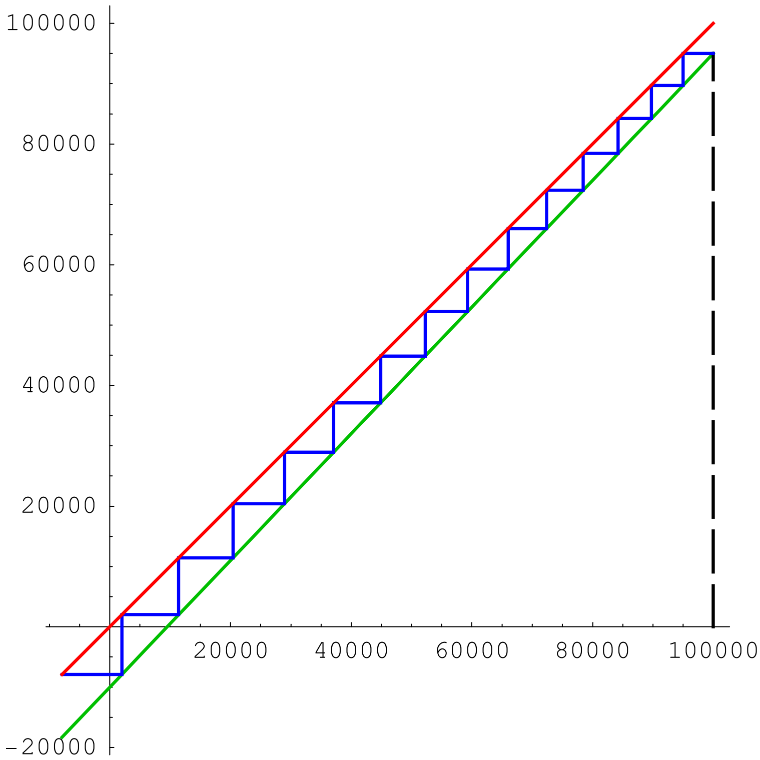
```
CobList[AnnuityList[100000, 1.05, -10000, 15]]
```

```
{{100000, 0}, {100000, 95000.}, {95000., 95000.}, {95000., 89750.},
 {89750., 89750.}, {89750., 84237.5}, {84237.5, 84237.5}, {84237.5, 78449.4},
 {78449.4, 78449.4}, {78449.4, 72371.8}, {72371.8, 72371.8}, {72371.8, 65990.4},
 {65990.4, 65990.4}, {65990.4, 59290.}, {59290., 59290.}, {59290., 52254.5},
 {52254.5, 52254.5}, {52254.5, 44867.2}, {44867.2, 44867.2}, {44867.2, 37110.5},
 {37110.5, 37110.5}, {37110.5, 28966.1}, {28966.1, 28966.1}, {28966.1, 20414.4},
 {20414.4, 20414.4}, {20414.4, 11435.1}, {11435.1, 11435.1}, {11435.1, 2006.84},
 {2006.84, 2006.84}, {2006.84, -7892.82}, {-7892.82, -7892.82}}
```

```
CobWebPlot[y0_, A_, B_, k_] :=
 Module[
  {CL = CobList[AnnuityList[y0, A, B, k]],
   xmin, xmax, g0, g1, g2, g3, g4, g5},
  xmin = Min[CL];
  xmax = Max[CL]; g0 = {Dashing[{0.06, 0.02}],
    {Thickness[0.005], {RGBColor[0, 0, 0], Line[{CL[[1]], CL[[2]]}]}}};
  g1 = {Thickness[0.005], {RGBColor[0, 0, 1], Line[Drop[CL, 1]]}};
  g2 = {Thickness[0.005], {RGBColor[1, 0, 0], Line[{{xmin, xmin}, {xmax, xmax}}]}};
  g3 = Plot[Function[x, A * x + B][t], {t, xmin, xmax}, AspectRatio → Automatic,
    PlotRange → All, TextStyle → {FontSize → 12}, PlotStyle →
     {Thickness[0.005], RGBColor[0, 0.75, 0]}, DisplayFunction → Identity];
  Show[g3, Graphics[g0], Graphics[g1], Graphics[g2], AspectRatio → Automatic,
   PlotRange → All, ImageSize → {360, 360} , DisplayFunction → $DisplayFunction]]
```

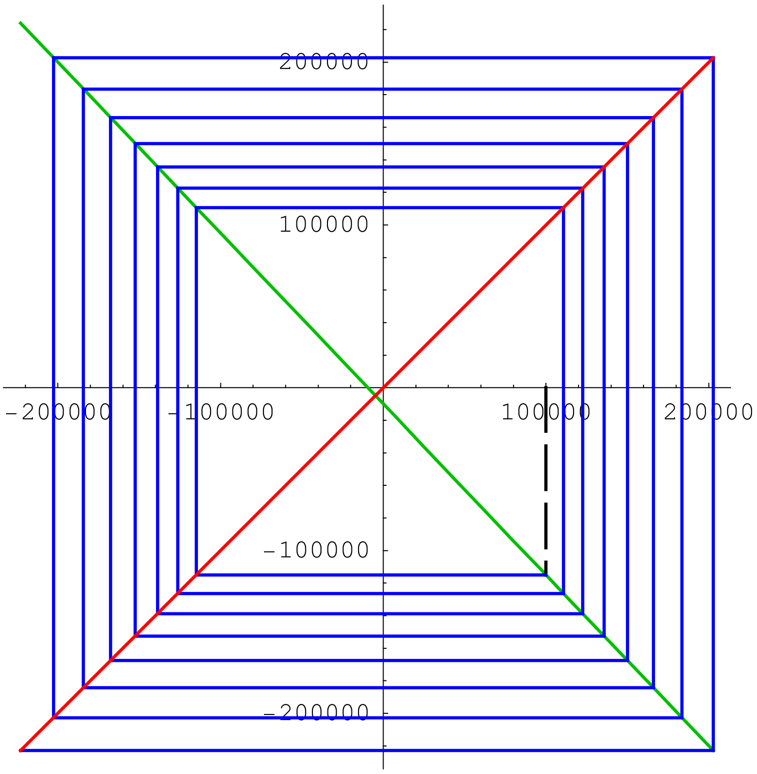
■ Experiments and Results

```
CobWebPlot[100000, 1.05, -10000, 15]
```



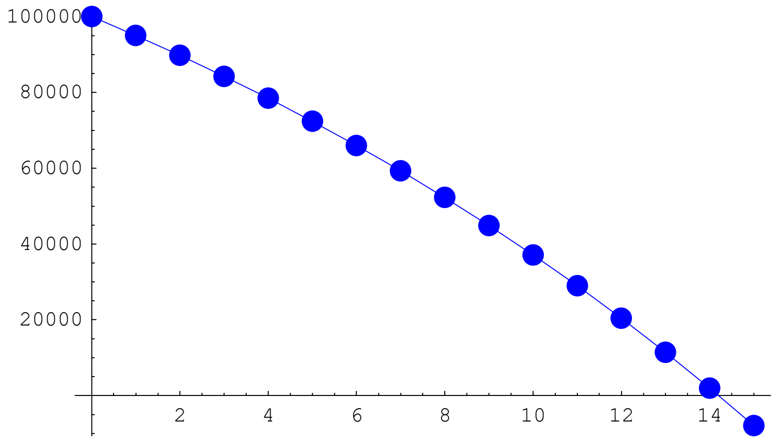

```
- Graphics -
```

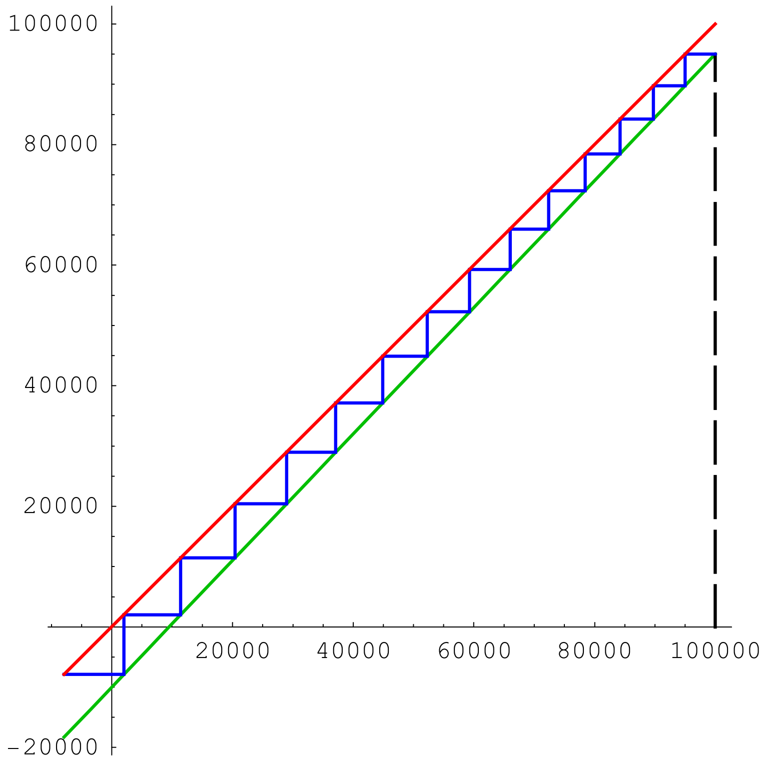
```
CobWebPlot[100000, -1.05, -10000, 15]
```

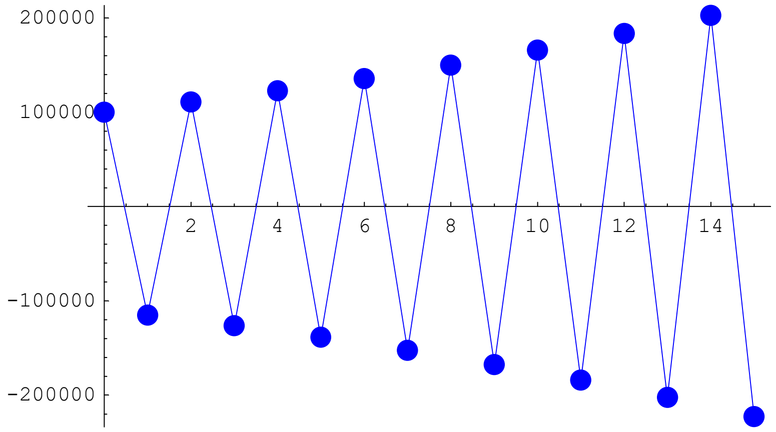


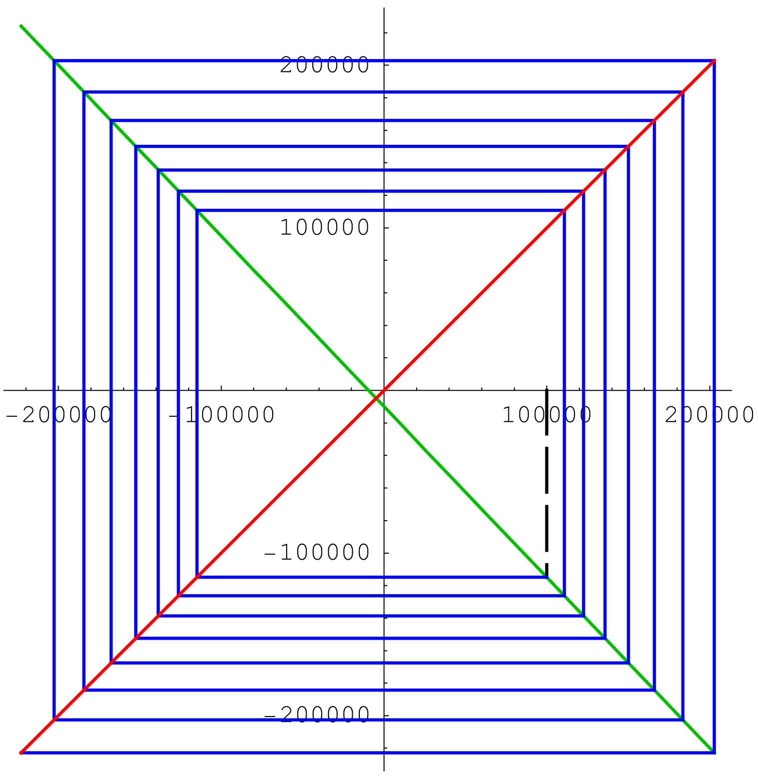
- Graphics -

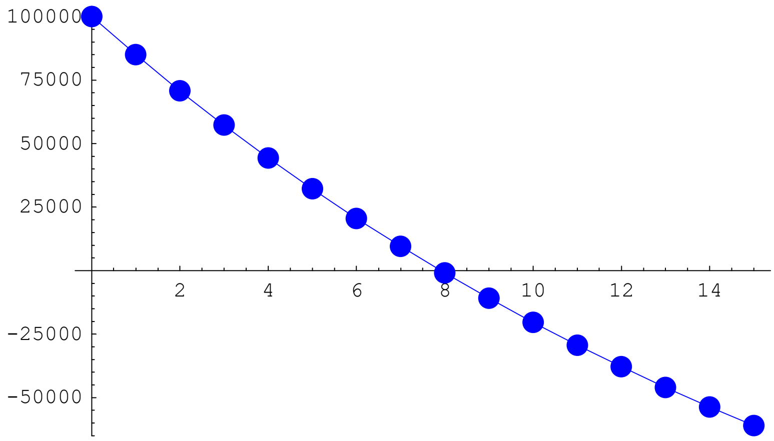
```
Demos =
 (TimelinePlot[100000, 1.05, -10000, 15];
  CobWebPlot[100000, 1.05, -10000, 15];
  TimelinePlot[100000, -1.05, -10000, 15];
  CobWebPlot[100000, -1.05, -10000, 15];
  TimelinePlot[100000, 0.95, -10000, 15];
  CobWebPlot[100000, 0.95, -10000, 15];
  TimelinePlot[100000, -0.95, -10000, 15];
  CobWebPlot[100000, -0.95, -10000, 15] )
```

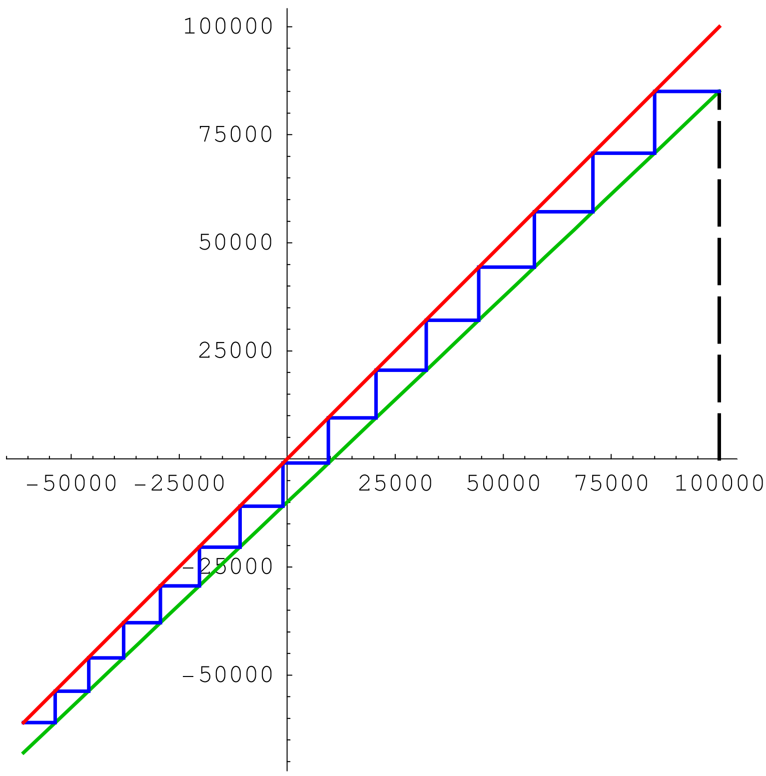

100000
80000
60000
40000
20000
2
4
6
8
10
12
14

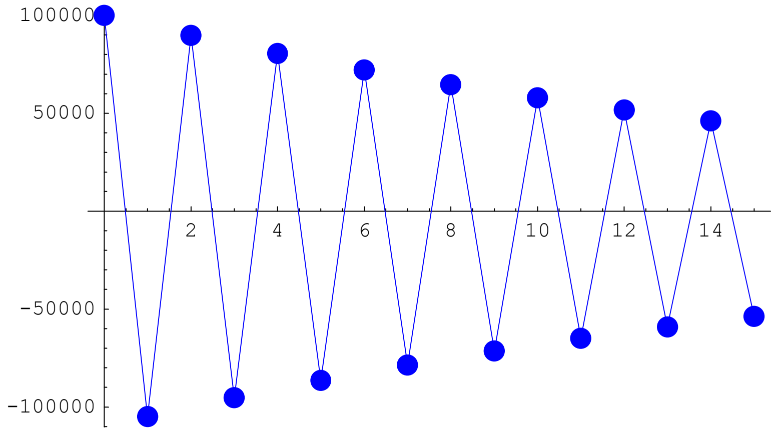

100000
80000
60000
40000
20000
-20000
20000
40000
60000
80000
100000

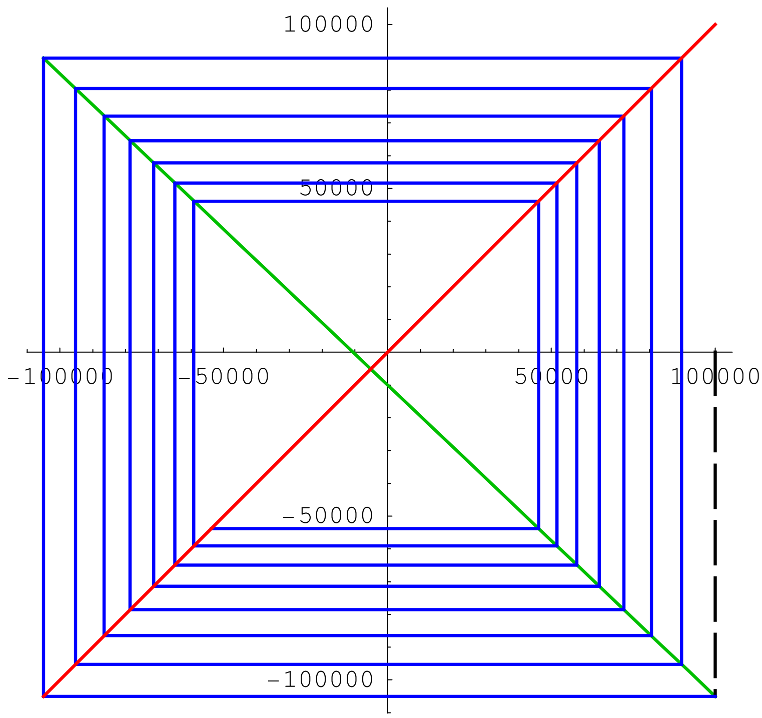

200000
100000
2
4
6
8
10
12
14
-100000
-200000


200000
100000
-200000
-100000
100000
200000
-100000
-200000


100000
75000
50000
25000
-25000
-50000
2
4
6
8
10
12
14


100000
75000
50000
25000
-50000
-25000
25000
50000
75000
100000
-25000
-50000


100000
50000
-50000
-100000
2
4
6
8
10
12
14


100000
50000
-50000
-100000
-100000
-50000
50000
100000

- Graphics -

■ The cobweb plot - some special cases

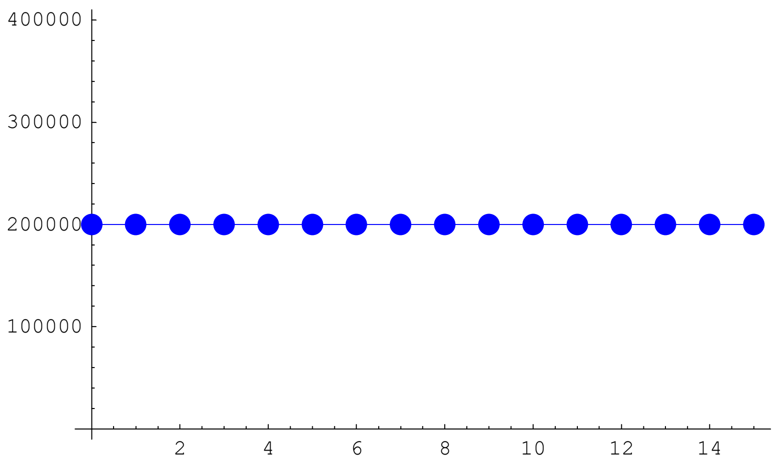
Case: $y_0 = \frac{B}{1-A}$

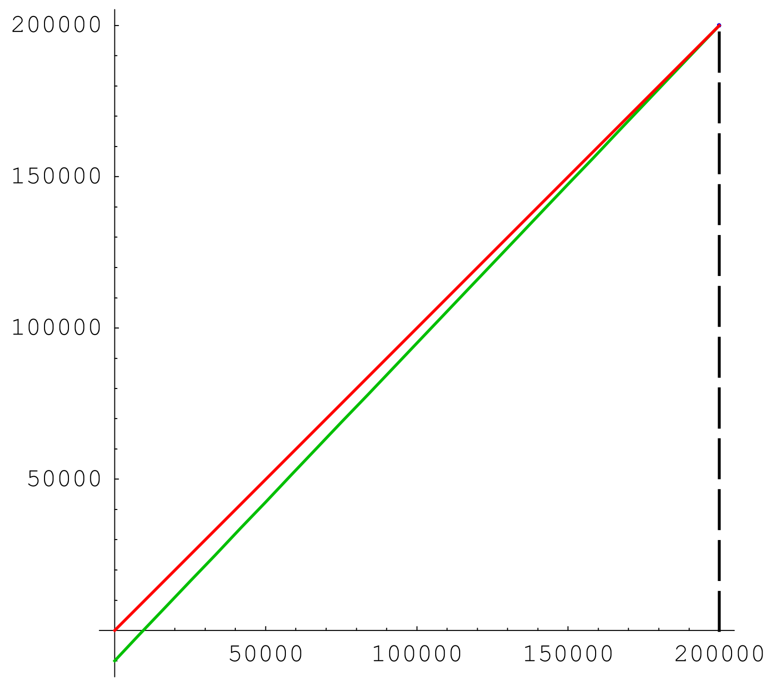
hence: $y_1 = A \cdot y_0 + B = A \cdot \frac{B}{1-A} + B = \frac{A \cdot B + B \cdot (1-A)}{1-A} = \frac{B}{1-A} = y_0$

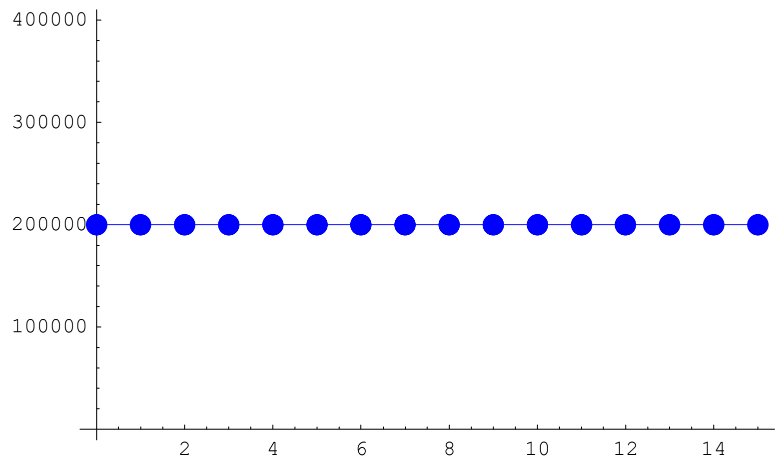
hence: $y_0 = y_1 = y_2 = y_3 = y_4 = \ldots$

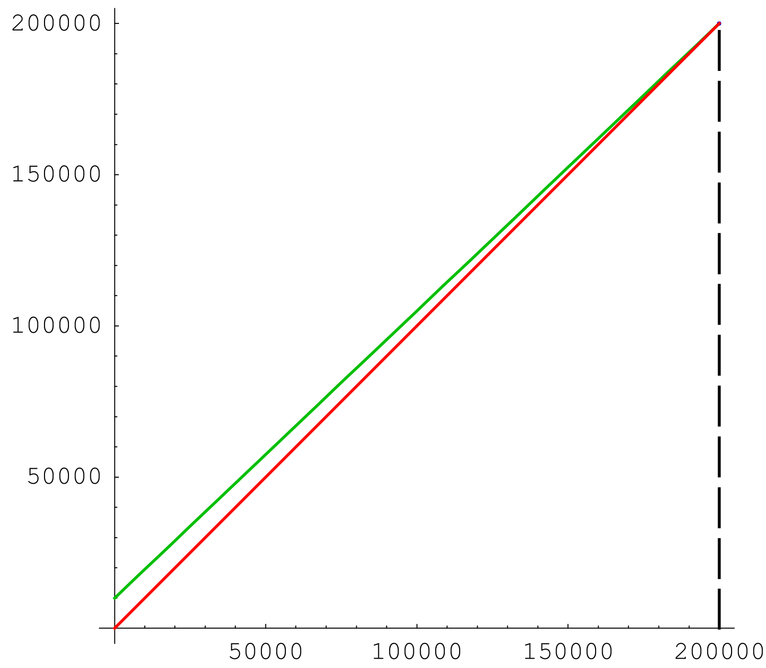
In the following diagram this fact is visualized with various slopes.

```
TimelinePlot[200000, 1.05, -10000, 15];
CobWebPlot[200000, 1.05, -10000, 15];
TimelinePlot[200000, 0.95, 10000, 15];
CobWebPlot[200000, 0.95, 10000, 15];
```




200000
150000
100000
50000
50000
100000
150000
200000


400000
300000
200000
100000
2
4
6
8
10
12
14



Case: $A = -1$

hence: $y_1 = -y_0 + B$

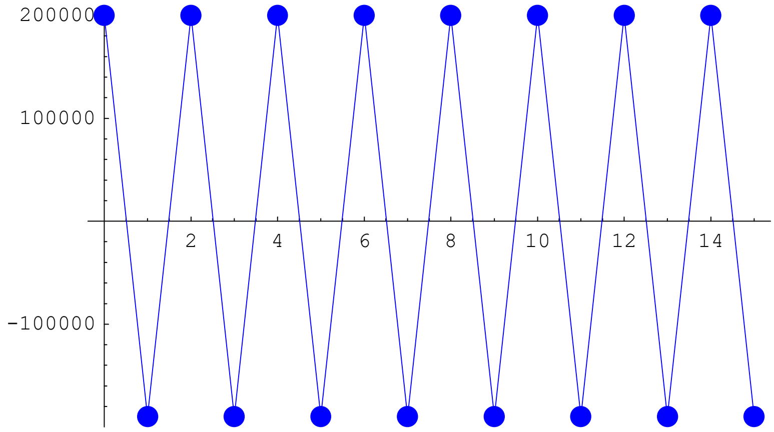
$$y_2 = -y_1 + B = (y_0 - B) + B = y_0$$

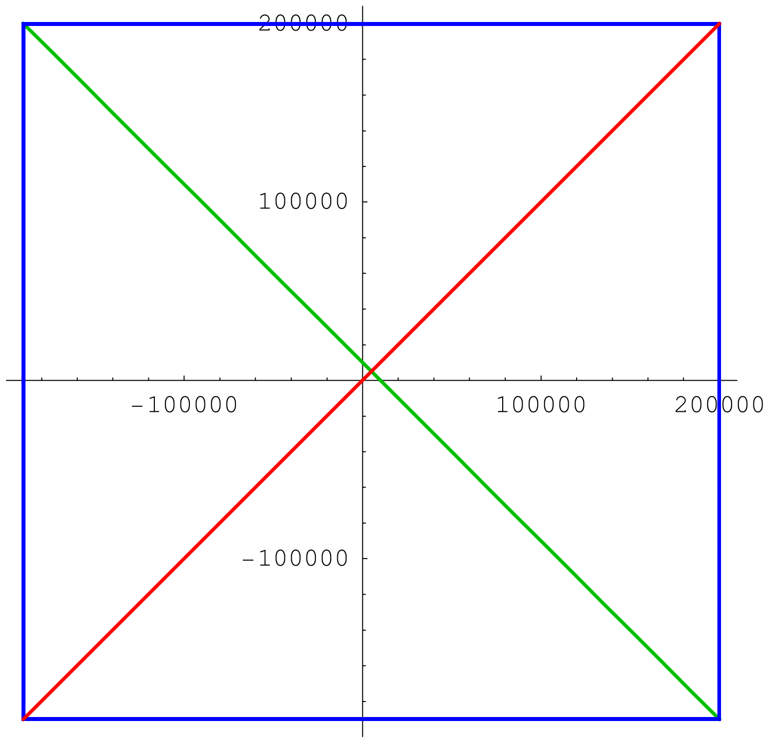
$$y_3 = -y_2 + B = -y_0 + B = y_1$$

$$y_4 = -y_3 + B = -y_1 + B = y_2 = y_0$$

Visualisation:

```
TimelinePlot[200000, -1, 10000, 15];
CobWebPlot[200000, -1, 10000, 15];
```





Auxiliary stuff