# 1 Performance Parameters

A real-world system such as social networks are being set up for sharing information. A piece of information flows from one node to another - known as information spread - has occurred. It is considered in the same way as an epidemic spreads. The *spread rate* is a measurable quantity that is to be computed based on how many susceptible neighbors get infected in the epidemic model.

The basic epidemic model is the Susceptible-Infected (SI) model [**?**], where the population, $N_E$ divide into two groups: S(susceptible) and I(infected). Let $i(t)$ be the rate of infection at any time $t$, **spread rate** of infection. and $s(t)$ be the susceptible ratio, so $s(t) + i(t) = 1$. Hence $N_E(s(t)) + N_E(i(t)) = N_E$ [**?**]. Let transition probability from suscepitible to infected or contact rate be $\beta$, then there are $N_E\beta s(t)i(t)$ susceptible user move to infected and $\beta siN_E$ is the increments in infected volume.

$$N_E\frac{di}{dt} = \beta siN_E$$
$$\frac{di}{dt} = \beta si$$

$$i = \frac{Increment\ in\ infected\ volume}{Total\ suscepitible\ volume}$$

## 1.1 The Kendall's rank correlation coefficient ($\tau$)

It is a measure of relationships between the columns of ranked data [**?**], [**?**]. Let $(p_i, q_i)$ and $(p_j, q_j)$ be the result of the random pairs of an operation from the list $P$ and $Q$, respectively. There are two conditions, if $p_i > p_j$ and $q_i > q_j$ or $p_i < p_j$ or $q_i < q_j$, the tuple are said to be concordant and if $p_i > p_j$ and $q_i < q_j$ or $p_i < p_j$ or $q_i > q_j$, they are said to be dis-concordant.

$$\tau = \frac{2(P - Q)}{(p + Q)((P + Q) - 1)} \tag{1}$$

where $P + Q$ is the result of total random pairs of operations, the pairs of $P$ and $Q$ are the number of concordant pair and dis-concordant pair, respectively.
The Eq. 1 is simplified as:

$$\tau = \frac{N_c - N_d}{0.5N_{total}(N_{total} - 1)} \tag{2}$$

where $N_{total} = P + Q$, $N_c$ and $N_d$ are $P$ and $Q$ respectively.

## 1.2 Average Similarity Between Layers

The similarity between layers determines the spreading behavior. A layer is considered to be important when it consists of nodes with higher ranks. There exist are two metrics for finding the similarity between layers: degree-degree correlation and average similarity of neighbors. Degree-Degree correlation is an analogy for comparing layers in multilayer networks, which defines the number of neighbors of a node $i$ in adjacent layers. The average similarity of neighboring layers is defined as:

$$AVG_{siml} = \sum_i \frac{k^\theta(i)}{k^\alpha(i) + k^\beta(i) - k^\theta(i)} \tag{3}$$

where $k^\alpha(i)$ is the number of neighbors of node $i$ in layer $\alpha$, $k^\beta(i)$ is the number of neighbors of node $i$ in layer $\beta$, $k^\theta(i)$ is the number of common neighbors of node $i$ in layers $\alpha$ and $\beta$.

## 1.3 Intersection Similarity Between Metrics

Given two vectors $X$ and $Y$ and assume that $x_k$ and $y_k$ are the top-ranking elements in each vector $X$ and $Y$, respectively. The intersection similarity is

$$sim_{\|i\|}(X, Y) = \frac{1}{|s|}\sum_{k=1}^{|s|}\frac{|x_k\Delta y_k|}{2k} \tag{4}$$

where $|x_k\Delta y_k|$ is the symmetric difference between $x_k$ and $y_k$, $|s|$ is the cardinality of the set. The $sim_{\|i\|}$ value for any two metrics lies in the range $0 \leq sim_{\|i\|}(x, y) \leq 1$. If the vectors are similar to each other, $sim_{\|i\|}(x, y) = 0$, otherwise $sim_{\|i\|}(x, y) = 1$. The smaller the value of $sim_{\|i\|}(x, y)$, the more similar nodes are present in both sets.