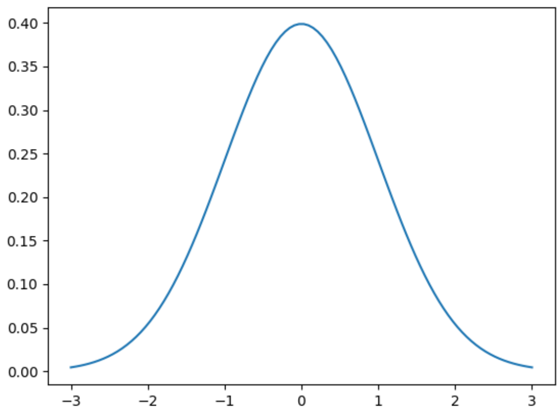
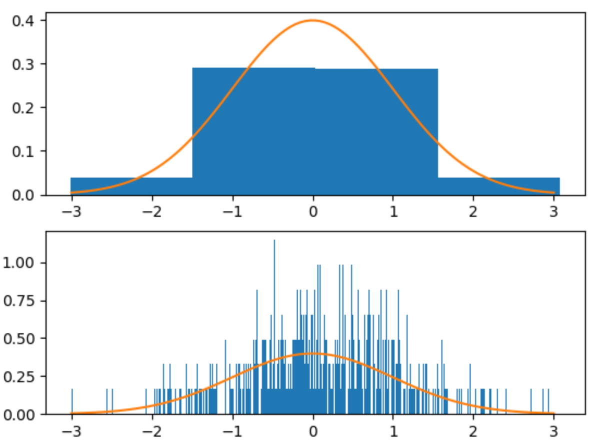
Exercise 1

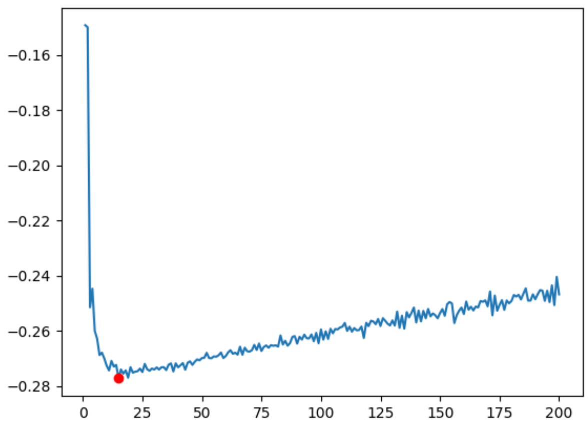
1a)

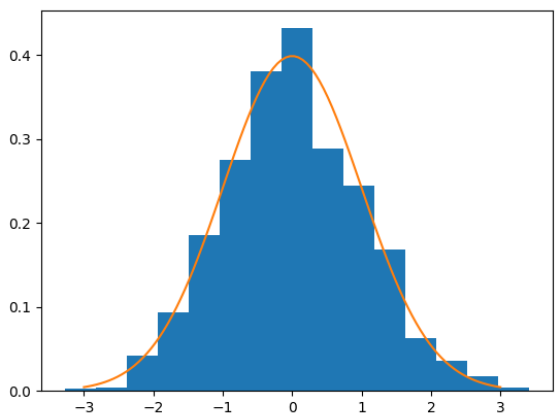


1b)



1c)


−0.16
−0.18
−0.20
−0.22
−0.24
−0.26
−0.28
0
25
50
75
100
125
150
175
200


0.4
0.3
0.2
0.1
0.0
−3
−2
−1
0
1
2
3

Exercise 2

2a-i)

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 2 \\ 6 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu) = -\frac{1}{2}\begin{bmatrix} x_1-2 & x_2-6 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} x_1-2 \\ x_2-6 \end{bmatrix}$$

$$\det\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = 4-1 = 3 \qquad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

$$-\frac{1}{2}\begin{bmatrix} x_1-2 & x_2-6 \end{bmatrix}\begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}\begin{bmatrix} x_1-2 \\ x_2-6 \end{bmatrix}$$

$$= -\frac{1}{2}\left[\frac{2x_1 \cancel{-4} - x_2 \overset{2}{\cancel{+6}}}{3} \quad \frac{-x_1 \cancel{+2} + 2x_2 \overset{-10}{\cancel{-12}}}{3}\right]\begin{bmatrix} x_1-2 \\ x_2-6 \end{bmatrix}$$

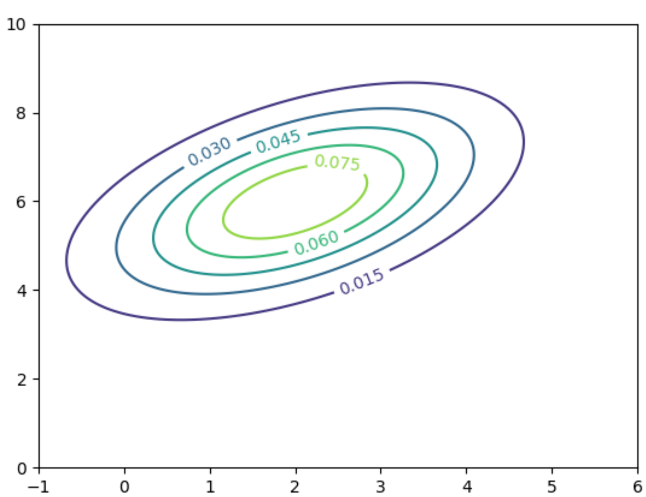
$$-\frac{1}{2}\left[\frac{2x_1 - x_2 + 2}{3} \quad \frac{-x_1 + 2x_2 - 10}{3}\right]\begin{bmatrix} x_1-2 \\ x_2-6 \end{bmatrix}$$

$$g(x_1, x_2) = -\frac{1}{6}(x_1-2)(2x_1-x_2+2) + -\frac{1}{6}(x_2-6)(-x_1+2x_2-10)$$

$$\frac{1}{\sqrt{(2\pi)^2 \left|\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right|}} = \frac{1}{\sqrt{(2\pi^2)}\sqrt{10}}$$

$$f_X(x_1, x_2) = \frac{1}{\sqrt{(2\pi)^2}\sqrt{10}} \cdot \exp(g(x_1, x_2))$$

2a-ii)



2b)

2b) i) $Y = AX + \vec{b} \rightarrow \vec{\mu}_y = A\vec{\mu}_x + \vec{b}$ $\quad \mu_x = \vec{0}$

$$\vec{\mu}_y = A\mu_x + \vec{b} = A\vec{0} + \vec{b} = \boxed{\vec{b} = \vec{\mu}_y}$$

$$\Sigma_Y = E[(Y-\mu_y)(Y-\mu_y)^T]$$

$$Y - \mu_y = (AX + \vec{b}) - \vec{b} = AX \cancel{+\vec{b}} \cancel{-\vec{b}}$$

$$Y - \mu_y = AX$$

$$\Sigma_Y = E[(AX)(AX)^T]$$

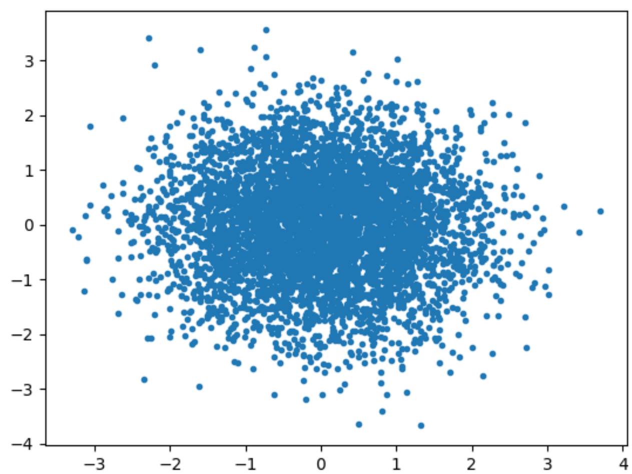
$$= E[A(XX^T)A^T]$$

$$= A\,E[XX^T]A^T$$

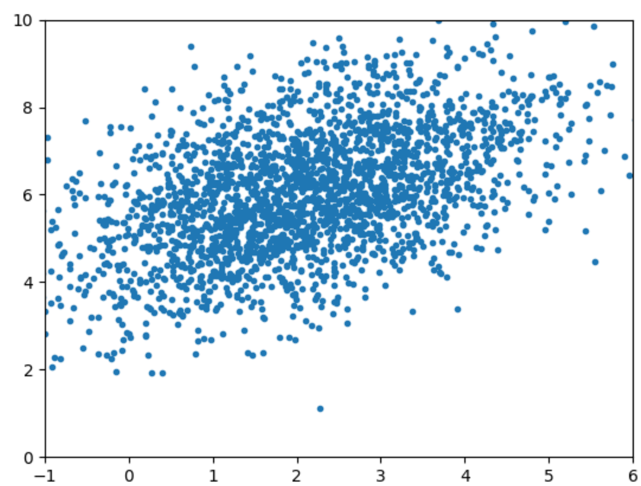
$$= A\Sigma_x A^T \qquad \Sigma_x = I$$

$$= AIA^T = AA^T$$

2c-i)



2c-ii)



Exercise 3

3b)

$\vec{y}$ = Measured data points

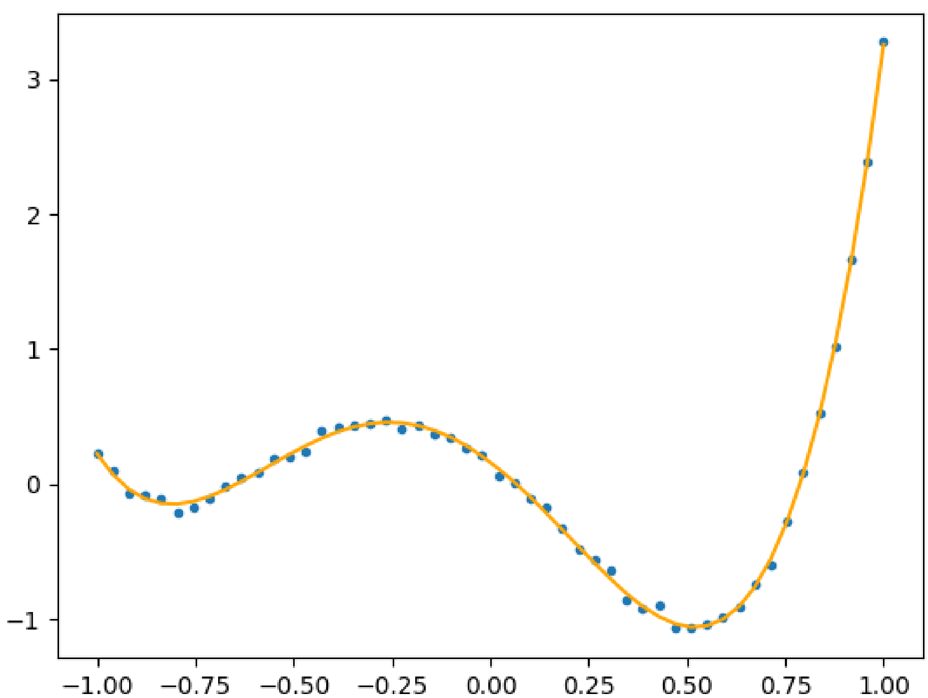
X = input data points plugged into Legendre polynomial like so

$$\begin{bmatrix} L_0(x_0) & L_1(x_0) & L_2(x_0) & L_3(x_0) & L_4(x_0) \\ L_0(x_1) & L_1(x_1) & L_2(x_1) & L_3(x_1) & L_4(x_1) \\ & & \vdots & & \\ L_0(x_N) & L_1(x_N) & L_2(x_N) & L_3(x_N) & L_4(x_N) \end{bmatrix}$$

B = weights

$$B = (X^T X)^{-1} X^T \vec{y}$$

3a and 3c)



3e)

3e) $c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \\ \end{matrix}\right\} 5 = d \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} 50 = N \end{matrix}$ $\qquad x = \begin{bmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \\ \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{bmatrix}$

$$A = \begin{bmatrix} L_0(\Phi_1) & L_1(\Phi_1) & L_2(\Phi_1) & L_3(\Phi_1) & L_4(\Phi_1) & -1 & 0 & 0 & 0 \\ \vdots & & & & & 0 & -1 & 0 \cdots & 0 \\ & & & & & 0 & 0 & -1 & 0 \\ \vdots & & & & & & \vdots & & \ddots \\ L_0(\Phi_N) & L_1(\Phi_N) & L_2(\Phi_N) & L_3(\Phi_N) & L_4(\Phi_N) & 0 & 0 & 0 \cdots & -1 \\ -L_0(\Phi_1) & -L_1(\Phi_1) & -L_2(\Phi_1) & -L_3(\Phi_1) & -L_4(\Phi_1) & -1 & 0 & 0 \cdot & 0 \\ \vdots & & & & & 0 & -1 & 0 \cdots & 0 \\ \vdots & & & & & 0 & 0 & -1 \ddots & 0 \\ & & & & & & \vdots & & \\ -L(\Phi_N) & -L(\Phi_N) & -L(\Phi_N) & -L(\Phi_N) & -L(\Phi_N) & 0 & 0 & 0 \cdots & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \\ -y_1 \\ -y_2 \\ \vdots \\ -y_n \end{bmatrix}$$

ground truths

3d and 3f)