

# **Design and Analysis of Computer Algorithms**

## **Homework 1**

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August 31, 2016

## Homework Exercises: Problems 9.3-7, 9.3-8

1. (Exercise 9.3-7)

Algorithm

- (1) Find the median of set  $S$  with *SELECT* algorithm  $\Rightarrow O(n)$
- (2) Calculate the distances *Distance* (i.e.,  $\text{abs}(S[i] - \text{median})$ )  $\Rightarrow O(n)$
- (3) Find the  $K^{\text{th}}$  small element  $d_k$  in *Distance*  $\Rightarrow O(n)$
- (4) Traverse *Distance*, find elements that is smaller than  $d_k \Rightarrow O(n)$

Time Complexity:  $O(n)$

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**Algorithm 1** **SELECT-K**( $S, k$ )

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1: if ( $n == k$ ) then
2:   return  $S$ 
3: end if
4:  $\text{median} = \text{SELECT}(S, 1, n, n/2)$ 
5:  $\text{Distance} = [], \text{Result} = []$ 
6: for ( $i = 1$  to  $n$ ) do
7:    $\text{Distance}[i] = \text{abs}(S[i] - \text{median})$ 
8: end for
9:  $d_k = \text{SELECT}(\text{Distance}, 1, n, k)$ 
10: for ( $i = 1$  to  $n$ ) do
11:   if ( $\text{Distance}[i] \leq d_k$ ) then
12:      $\text{Result}[i] = S[i]$ 
13:   end if
14: end for
15: return  $\text{Result}$ 
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2. (Exercise 9.3-8) Suppose that  $X$  and  $Y$  are in increasing order. The algorithm used a various of binary search which can be generalized to find the  $k$ th element in two sorted arrays.

Algorithm

- (1) Get the two median  $mx$  and  $my$  from  $X$  and  $Y$ . Since  $X, Y$  are sorted, it costs  $O(1)$ .
- (2) If  $mx == my$ , then return  $mx$ .
- (3) If  $mx > my$ , then the median is present in one of the below two subarrays.
  - a. From first element of  $X$  to  $mx$  ( $X[0 \dots n/2]$ )
  - b. From  $my$  to the last of  $Y$  ( $Y[n/2 \dots n]$ )
- (4) If  $my > mx$ , then the median is present in one of the below two subarrays.
  - a. From  $mx$  to the last of  $X$  ( $X[n/2 \dots n]$ )

b. From first element to  $my$  ( $Y[0\dots n/2]$ )

Time Complexity:  $O(\lg(n))$

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**Algorithm 2**  $MEDIAN(X, x_1, x_2, Y, y_1, y_2)$

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1:  $mx = X[(x_1 + x_2)/2]$

2:  $my = Y[(y_1 + y_2)/2]$

3: **if** ( $mx == my$ ) **then**

4:   return  $mx$

5: **end if**

6: **if** ( $mx < my$ ) **then**

7:   return  $MEDIAN(X, (x_1 + x_2)/2 + 1, x_2, Y, y_1, (y_1 + y_2)/2 + 1)$

8: **else**

9:   return  $MEDIAN(X, x_1, (x_1 + x_2)/2, Y, (y_1 + y_2)/2 + 1, y_2)$

10: **end if**

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