Curve Construction

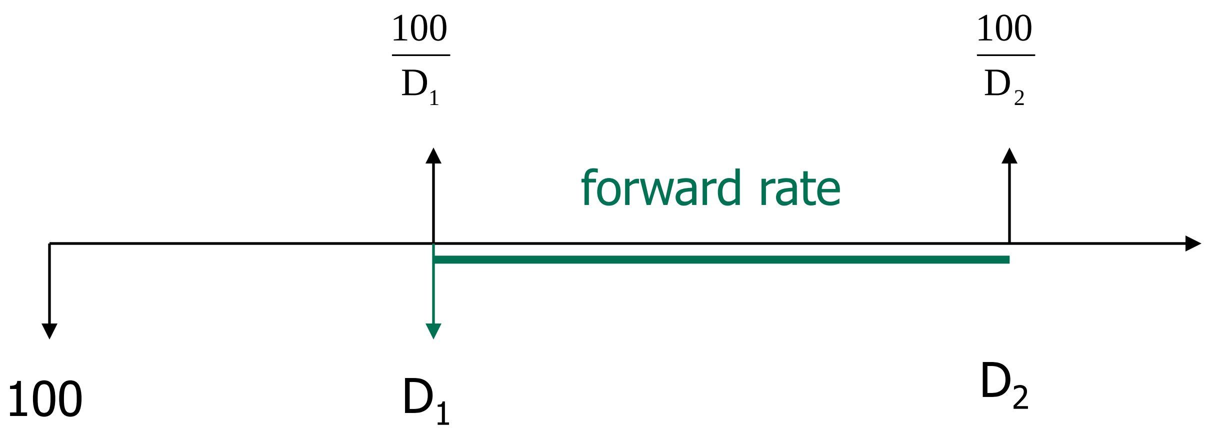
Yield Curve

- Market rate of interest for
  - Theoretical zero coupon instruments that
  - Matures at any future date
- Derived from
  - Prices of real financial instruments that
  - Trade in a liquid market

Input points

- Liquid market instruments do not exist for every possible date in the future
- Benchmark set or key points
  - Cash Rates
    - ON, TN, 3M, 6M, 1Y
  - Swaps Rates
    - 2Y, 3Y, 4Y, 5Y, 7Y, 10Y, 15Y, 20Y, 30Y
  - Liquid number of futures contract

Discount Factor & Forward Rate



$$\text{forward rate} = \left(\frac{\frac{100}{D_2}}{\frac{100}{D_1}} - 1 \right) \times \frac{\text{year}}{\text{period}} = \left(\frac{D_1}{D_2} - 1 \right) \times \frac{\text{year}}{\text{period}}$$

Discount Factor & Forward Rate

$$\text{forward rate} = \left(\frac{\frac{100}{D_2}}{\frac{100}{D_1}} - 1 \right) \times \frac{\text{year}}{\text{period}} = \left(\frac{D_1}{D_2} - 1 \right) \times \frac{\text{year}}{\text{period}}$$

$$D_2 = \frac{D_1}{1 + \text{forward rate} \times \frac{\text{period}}{\text{year}}}$$

Fixed Date Conventions

Short Dates

Overnight (O/N)	Starting today and maturing tomorrow
"Tom-next" (T/N)	Starting tomorrow and maturing the next day
Spot-next (S/N)	Starting on the spot date and maturing the day after spot
Spot-one week (S/W)	Starting spot and maturing seven days later

Fixed Date Conventions (Modified Following)

End/End Rule

If the spot date is a month-end, then all forward fixed dates will be month end

Month-End Roll Back

If the forward date lands on a month-end and that happens to be a weekend or a holiday, then it cannot be rolled forward to the next month. Settlement will be rolled back to the last working day of the same month

Example:

A two-month Eurodeposit booked in London on 26 February will be for value 28 February, the spot date. Since this a month-end, the deposit will mature on 30 April. If 30 April is a Sunday, the deposit will mature on 28 April.

Cash

Cash rates on 8 March 2007 (Thursday, Base date)

ON	0.57%
TN	0.57%
3M	0.70625%

Basis is ACT/360, Number of days to spot is 2.

ON (Key Point 1)
Discount factor for Friday, 9 March. How? (Ref: Curve_ppt.xls)

TN (Key Point 2)
Discount factor for Monday, 12 March. How? (Ref: Curve_ppt.xls)

3M cash rate ended on ? Discount factor is ? (Ref: Curve_ppt.xls)

Exponential Interpolation

Given 2 points x_1 and x_2 and a point x between them.

We assume that (x_1, y_1) and (x_2, y_2) lie on the curve

$$y = ke^{mx} \text{ where } k \text{ and } m \text{ are constants}$$

Solving for k and m,

$$m = \frac{\ln(y_2 / y_1)}{x_2 - x_1}$$

$$k = y_1 e^{-mx_1}$$

$$y = y_1 \left(\frac{y_2}{y_1}\right)^{\left(\frac{x - x_1}{x_2 - x_1}\right)}$$

What's the implication for ON rates?

Linear Interpolation

Given 2 points x_1 and x_2 and a point x between them.

We interpolate y such that

$$y = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Log Linear Interpolation

Given 2 points x_1 and x_2 and a point x between them.

We interpolate y such that

$$\ln y = \ln y_1 + \left(\frac{\ln y_2 - \ln y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y = y_1 \left(\frac{y_2}{y_1}\right)^{\left(\frac{x - x_1}{x_2 - x_1}\right)}$$

Futures Quotation

Price of contract is quoted not as a rate of interest but as 100 minus the rate of interest

Futures

IMM futures price*:

22-Mar-07	99.307	20-Jun-07	99.3082
19-Sep-07	99.2235	19-Dec-07	99.1369
19-Mar-08	99.0504	18-Jun-08	98.968
17-Sep-08	98.8927	17-Dec-08	98.8175

*Assume convexity adjusted.

Mar-07 futures contract start on Thursday, 22-Mar-07 and runs to 20-Jun-07.

If discount factor for 22-Mar-07 is known

Implied interest rate for the period? $\frac{100 - 99.307}{100} = 0.693\%$

Discount factor for 20-Jun-07 ? (Ref: Curve_ppt.xls)

Futures

First futures contract March 2007, started on Thursday, 22 March, 2007 (Note: 21 March 2007 is a TOK holiday).

DF_1st Futures should satisfy:

$$DF_{2nd\ Futures} = \frac{DF_{1st\ Futures}}{1 + \left(\frac{100 - Price_{1st\ Futures}}{100} \times \frac{Date_{2nd\ Futures} - Date_{1st\ Futures}}{360}\right)}$$

$$DF_{3M} = DF_{1st\ Futures} \times \left(\frac{DF_{2nd\ Futures}}{DF_{1st\ Futures}}\right)^{\left(\frac{Date_{3M} - Date_{1st\ Futures}}{Date_{2nd\ Futures} - Date_{1st\ Futures}}\right)}$$

Futures

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*Assume convexity adjusted.

Jun-07 futures contract start on Wednesday, 20-Jun-07 and runs to 19-Sep-07.

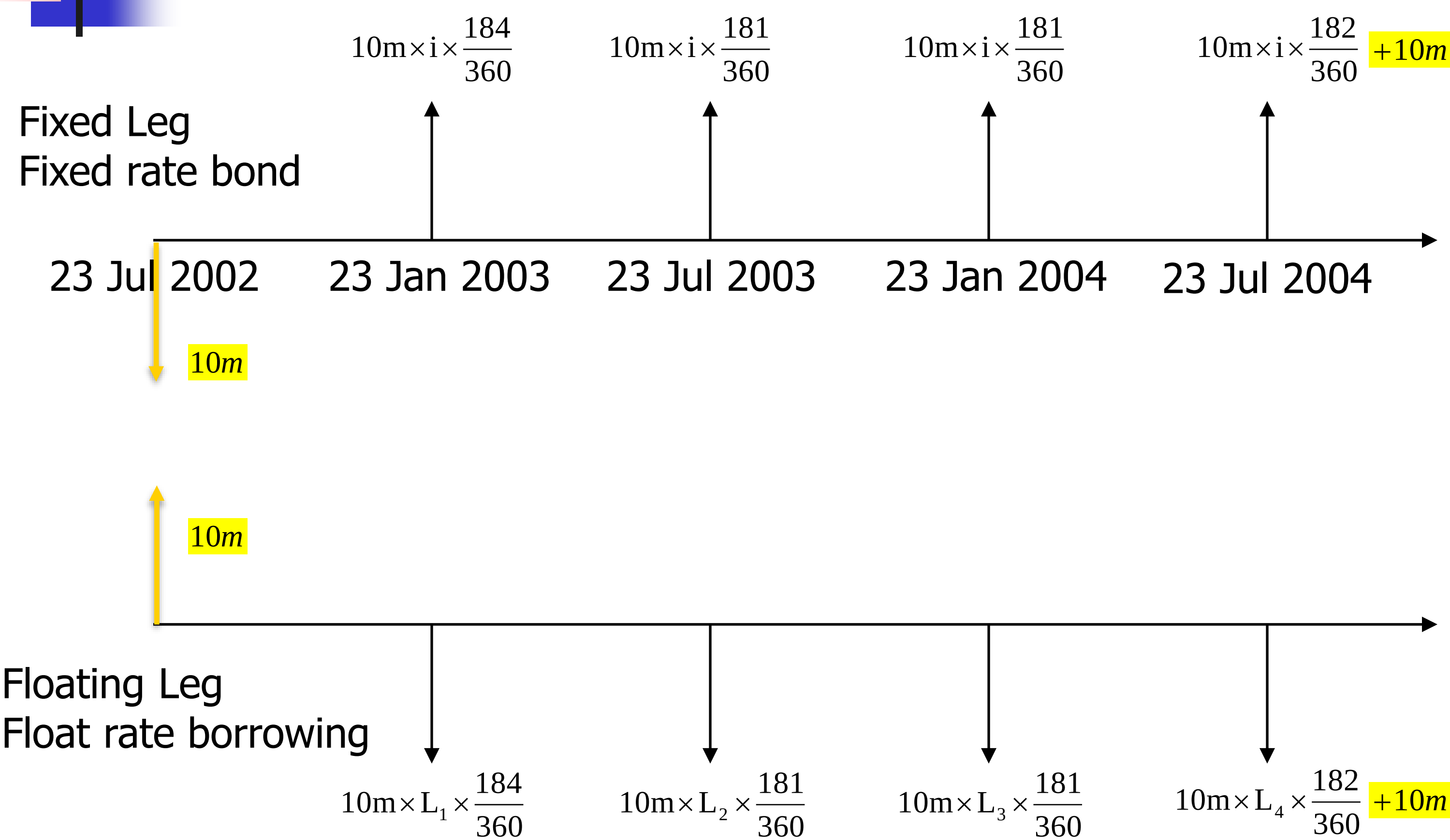
Discount factor for 19-Sep-07 ? (Ref: Curve_ppt.xls)

Similar for other futures contracts

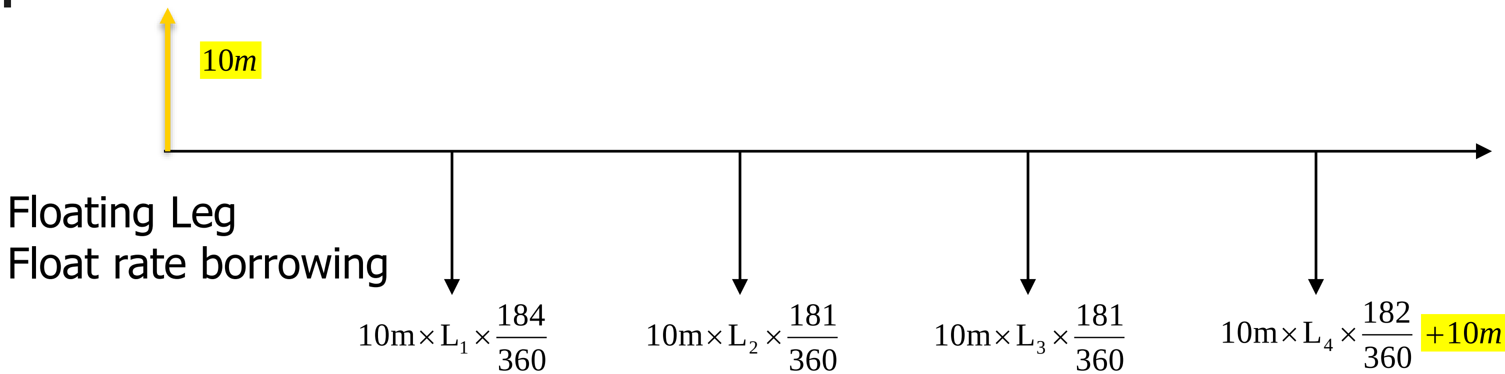
Valuation of Swap – Basics

- Long Swap = + Long a Fixed rate bond – Floating rate borrowing
- At inception, the NPV of the floating leg cashflow is zero. This is assumed that all future period floating rate are discounted at the then market floating borrowing rate, hence the NPV of floating leg = 0 at inception.
- Swap Value = + PV of Fixed leg cashflow – PV of Floating leg cashflow
- Hence, Swap Value = NPV of the Fixed leg cashflow
- At inception, NPV of all the cashflow = 0
- i.e. NPV of the Fixed leg cashflow = 0

+ Long a Fixed rate bond – Floating rate borrowing



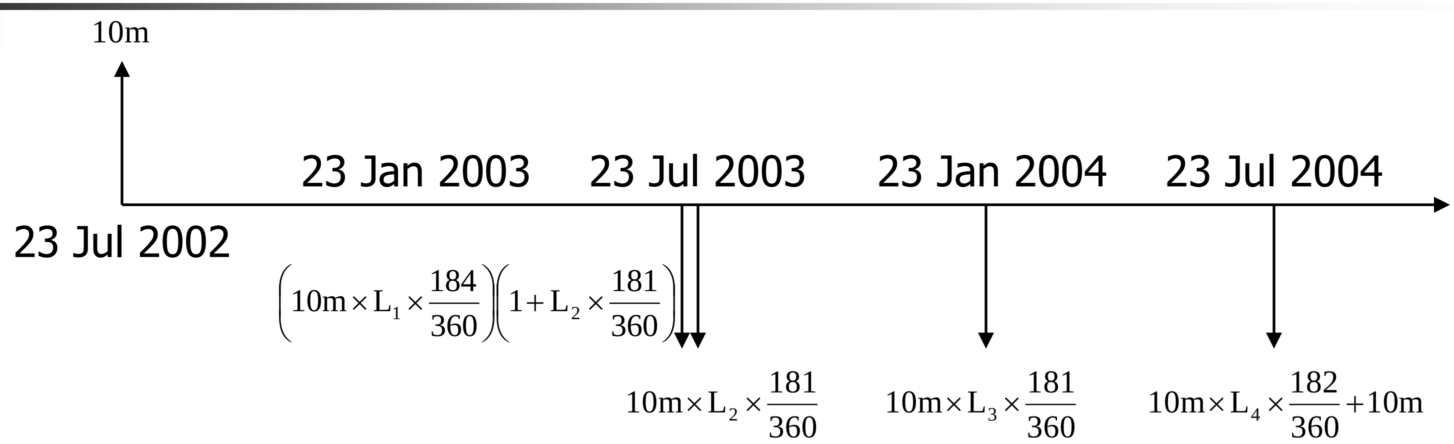
NPV of Floating Leg Cashflow



$$\mathrm{NPV} = 10\mathrm{m} - \frac{10\mathrm{m} \times L_1 \times \frac{184}{360}}{\left(1 + L_1 \times \frac{184}{360}\right)} - \frac{10\mathrm{m} \times L_2 \times \frac{181}{360}}{\left(1 + L_1 \times \frac{184}{360}\right)\left(1 + L_2 \times \frac{181}{360}\right)} - \frac{10\mathrm{m} \times L_3 \times \frac{181}{360}}{\left(1 + L_1 \times \frac{184}{360}\right)\left(1 + L_2 \times \frac{181}{360}\right)\left(1 + L_3 \times \frac{181}{360}\right)}$$

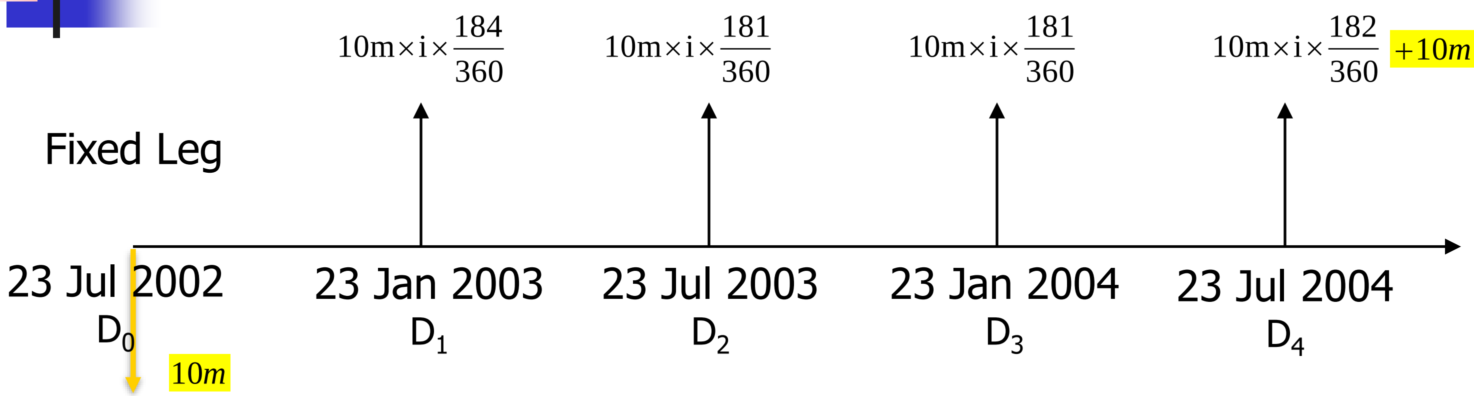
$$- \frac{10\mathrm{m} \times L_4 \times \frac{182}{360} + 10\mathrm{m}}{\left(1 + L_1 \times \frac{184}{360}\right)\left(1 + L_2 \times \frac{181}{360}\right)\left(1 + L_3 \times \frac{181}{360}\right)\left(1 + L_4 \times \frac{182}{360}\right)}$$

NPV of Floating Leg Cashflow



$$
\begin{aligned}
\text{NPV} = 10\text{m} &- \frac{\left(10\text{m}\times L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)+10\text{m}\times L_2\times\frac{181}{360}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)} - \frac{10\text{m}\times L_3\times\frac{181}{360}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)\left(1+L_3\times\frac{181}{360}\right)} \\
&- \frac{10\text{m}\times L_4\times\frac{182}{360}+10\text{m}}{\left(1+L_1\times\frac{184}{360}\right)\left(1+L_2\times\frac{181}{360}\right)\left(1+L_3\times\frac{181}{360}\right)\left(1+L_4\times\frac{182}{360}\right)}
\end{aligned}
$$

NVP of Fixed Leg



$$10m \times D_0 = \left(10m \times i \times \frac{184}{360}\right) \times D_1 + \left(10m \times i \times \frac{181}{360}\right) \times D_2$$

$$+\left(10m \times i \times \frac{181}{360}\right) \times D_3 + \left(10m \times i \times \frac{182}{360}\right) \times D_4$$

Swaps

3-year swap priced at 1.05625% against 6-month LIBOR. Spot, at 2 business days, Monday, 12 March 2007.

Date	Day Count		Discount factor
12 Mar 07 (Spot)	4	D_0	0.99993667
12 Sep 07	188	D_1	0.996373369
12 Mar 08	370	D_2	0.992289654
12 Sep 08	554	D_3	0.987313667
12 Mar 09	735	D_4	0.981676371
18 Mar 09 (futures)	741		0.981483207
14 Sep 09	921	D_5	?
12 Mar 10	1100	D_6	?

Swaps

The fixed leg cashflow must satisfy the following:

$$100D_0 = c_1D_1 + c_2D_2 + c_3D_3 + \ldots + c_5D_5 + (100 + c_6)D_6$$

D_5 and D_6 can be estimated by:

$$D_5 = 0.981483207 \times k^{180}$$
$$D_6 = 0.981483207 \times k^{(180+179)}$$
where k is the constant overnight discount factor

Solving iteratively, k = 0.999963597