Volatility Models in Option

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Volatility models

- **Standard deviation(SD)**

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})}$$

- **Exponentially Weighted Moving Average(EWMA)**

$$\sigma_n = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

- **Generalized AutoRegressive Conditional Heteroskedasticity(GARCH)**

$$\sigma_n = (1-\alpha-\beta)V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

- **Stochastic Volatility(SA)**

$$d\nu_t = \alpha_{\nu,t}dt + \beta_{\nu,t}dB_t$$

Option pricing models

- **Simple Black-Scholes**

$$Call = Se^{-qt}N(d_1) - Ke^{-rt}N(d_2)$$

$$Put = Ke^{-rt}N(-d_2) - Se^{-qt}N(-d_1)$$

$$d_{1,2} = \frac{log(\frac{S}{K}) + (r - q \pm \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

- **The Heston Model**

$$dS_t = \mu S_t d_t + \sqrt{v_t}S_t dW_1$$

$$dv_t = \alpha(S_t, v_t, t)d_t + \sigma\beta(S_t, v_t, t)dW2$$

$$dW_1 \cdot dW_2 = \rho dt$$

$$parameter = (v_0, \theta, \rho, \kappa, \xi)$$

Results

3month-call option on Apple Inc. with following parameter.

- K = 120, S = 148.11, R = 0.0421, T = 0.25, Q = 0.0061, Actual call price = 31.35

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##                      BSM_SD   BSM_EWMA   BSM_GARCH      BSM_SV Actual
## 1 In-the-Money 26.74217757  27.1084363  26.91855601 30.34989122  31.35
## 2 Out-of-Money           0 1.129831294 0.6713966474 2.089795823   2.19
```

Conclude:

- The results of standard deviation, EWMA, GARCH seems to be inaccurate. The most possible reason is that the calculation formula is based on European style, while the option provided by APPLE Inc.(AAPL) is American type. The actual trading option price must be higher than or equal to the results computed by Black-Scholes equation.

- Since the stochastic volatility models are calibrated to satisfy the implied volatility from the real market, the model capture the forward looking factors in the market. As a consequence, The Heston Model offers more precise prediction in option pricing.