New Product Varieties and the Measurement of International Prices

By Robert C. Feenstra*

The high income elasticity of demand often estimated for U.S. imports may be a spurious result of omitting new product varieties from the import price indexes. The purpose of this paper is to demonstrate how to incorporate new product varieties into a constant-elasticity-of-substitution aggregate of import prices. This method is applied to U.S. imports of six disaggregate manufactured goods. It is shown that the corrected indexes are able to account for part—but not all—of the high income elasticities. (*JEL* C43, F14)

While models of product differentiation and international trade are now common, empirical application lags far behind. This deficiency is most serious when dealing with situations where it is believed that *new* product varieties are important. The entry of firms producing new varieties of goods is central to the theoretical models, but it is unclear how to incorporate new varieties into indexes of international prices or quantities.

The difficulties that arise from ignoring new product varieties can be seen from the estimation of U.S. import demand. Since the work of Henrik S. Houthakker and Stephen P. Magee (1969), it has been known that the estimated income elasticity of demand for U.S. imports exceeds unity and also exceeds the foreign income elasticity of demand for U.S. exports. If these estimates are correct, they imply that equal growth in the United States and abroad will lead to a worsening in the U.S. trade balance. One explanation for the high income elasticity, however, is that it is a spurious result of omitting new product varieties from indexes of U.S. import prices (see William L. Helkie and Peter Hooper, 1988; Hooper, 1989; Paul R. Krugman, 1989). According to this argument, over the past several decades the United States has experienced an expansion in the range of new imports from rapidly growing, developing countries, but no corresponding decrease in import prices. Then the rising share of imports is attributed to a high income elasticity in the import-demand equations.

Helkie and Hooper (1988) attempt to correct the estimation of aggregate U.S. import demand by including a measure of foreign country's capital stock, as a proxy for the supply of new products. It would be preferable to incorporate the new product varieties directly into the import price index. The purpose of this paper is to show how this can be done for the case in which the import varieties enter a constant-elasticity-of-substitution (CES) aggregator function. This method will be applied to data on U.S. imports of six disaggregate manufactured goods.

In Section I, the exact price index corresponding to a CES unit-cost function is derived, allowing for both new product varieties and taste or quality change in existing varieties. The bias that would have resulted from ignoring new varieties is shown to depend on their share in total expenditure, along with the elasticity of substitution be-

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tween all varieties. Thus, measuring the price index requires that this elasticity be estimated.

The import data are presented in Section II. For each product, imports are treated as differentiated across the countries of supply, as in Paul S. Armington (1969). This means that all varieties of a particular good are aggregated *within* each country. It is shown that, because of this aggregation, an increase in the number of varieties within a country acts in the same manner as an increase in the taste or quality parameter for that country's imports. In other words, when dealing with imports distinguished by country of origin, it is crucial to allow the taste parameters to vary over time.

A procedure for estimating the elasticity of substitution between the varieties from each country, using the generalized method of moments, is discussed in Sections III and IV. This estimator allows for random changes in the taste parameters for imports by country and corrects for simultaneous-equation bias and measurement error in the prices. The resulting elasticities of substitution and import price indexes are presented in Sections V and VI.

In Section VII, the import-demand equations for the manufactured goods are estimated. Using indexes that correct for new product varieties accounts for part—but not all—of the high income elasticities. The demand equations also show other empirical support for the corrected price indexes. Conclusions are given in Section VIII. The Appendix contains proofs, while details of the estimation procedure are described in a separate appendix, available from the author upon request.

I. CES Model

Suppose that the minimum cost of obtaining one unit of services from the varieties i of some product are given by the CES function:

$$(1)\quad c(\mathbf{p}_t, \mathrm{I}_t, \mathbf{b}_t) = \left(\sum_{i \in \mathrm{I}_t} b_{i,t} p_{i,t}^{1-\sigma} \right)^{1/(1-\sigma)} \qquad \sigma > 1$$

where σ denotes the elasticity of substitution, which is assumed to exceed unity; $\mathrm{I}_t \subset \{1, \dots, N\}$ is the set of varieties available in period t with prices $p_{i,t} > 0$, $i \in \mathrm{I}_t$; $b_{i,t} > 0$ denotes a taste parameter for variety i, which is allowed to vary over time; and $\mathbf{p}_t$ and $\mathbf{b}_t$ denote the corresponding vectors of prices and taste parameters in period t. Note that an increase in $b_{i,t}$ will lower unit costs and also raise demand for variety i (as shown in the next section), and it can be thought of as either a taste or quality parameter.

To briefly review known results, suppose that the same set of product varieties I are available in periods $t-1$ and t and that the taste parameters are constant, so $b_{i,t} = b_{i,t-1} = b$ for $i \in \mathrm{I}$. Further suppose that the quantity vectors, denoted by $\mathbf{x}_{t-1}$ and $\mathbf{x}_t$, are cost-minimizing for the prices $\mathbf{p}_{t-1}$ and $\mathbf{p}_t$, respectively. W. Erwin Diewert (1976) defines an *exact* price index as a function $P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$ depending on observed prices and quantities, such that

$$(2)\quad c(\mathbf{p}_t, \mathrm{I}, \mathbf{b}) / c(\mathbf{p}_{t-1}, \mathrm{I}, \mathbf{b}) = P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I}).$$

The remarkable feature of (2) is that the price index does not depend on the unknown parameters b_i, $i \in \mathrm{I}$. From Kazuo Sato (1976) and Yrjo O. Vartia (1976), the exact price index for the CES unit-cost function can be written as

$$(3a)\quad P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I}) \equiv \prod_{i \in \mathrm{I}} (p_{i,t} / p_{i,t})^{w_{i,t}(\mathrm{I})}.$$

This is a geometric mean of the individual price changes, where the weights $w_{i,t}(\mathrm{I})$ are computed using the cost shares $s_{i,t}(\mathrm{I})$ in the two periods, as follows:

$$(3b)\quad s_{i,t}(\mathrm{I}) \equiv p_{i,t} x_{i,t} \Big/ \sum_{i \in \mathrm{I}} p_{i,t} x_{i,t}$$

$$(3c)\quad w_{i,t}(\mathrm{I}) \equiv \frac{\left(\dfrac{s_{i,t}(\mathrm{I}) - s_{i,t-1}(\mathrm{I})}{\ln s_{i,t}(\mathrm{I}) - \ln s_{i,t-1}(\mathrm{I})} \right)}{\sum_{i \in \mathrm{I}} \left(\dfrac{s_{i,t}(\mathrm{I}) - s_{i,t-1}(\mathrm{I})}{\ln s_{i,t}(\mathrm{I}) - \ln s_{i,t-1}(\mathrm{I})} \right)}.$$

The numerator on the right of (3c) is the logarithmic mean of $s_{i,t}$ and $s_{i,t-1}$ and lies between these cost shares. Then the weights

$w_{i,t}(\mathrm{I})$ are normalized versions of the logarithmic means and add up to unity.[1]

The price index in (3) requires that the same varieties are available in the two periods. It will now be shown how the price index can be generalized to allow for different, but overlapping, sets of goods in the two periods and for changes in some of the taste parameters. To this end, suppose that there is a set of varieties $\mathrm{I} \neq \varnothing$ that are available in *both* periods, and for which the taste parameters are constant. Let $P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$ denote the price index in (3) that is computed using data on only this set of goods: this is referred to as a "conventional" price index, in the sense that it ignores new and disappearing product varieties. The exact price index should equal the ratio $c(\mathbf{p}_t, \mathrm{I}_t, \mathbf{b}_t)/c(\mathbf{p}_{t-1}, \mathrm{I}_{t-1}, \mathbf{b}_{t-1})$. The next result, proved in the Appendix, shows how the exact index can be measured with observed prices and quantities.

PROPOSITION 1: *If* $b_{i,t-1} = b_{i,t}$ *for* $i \in \mathrm{I} \subseteq (\mathrm{I}_t \cap \mathrm{I}_{t-1})$, $\mathrm{I} \neq \varnothing$, *then*

$$c(\mathbf{p}_t, \mathrm{I}_t, \mathbf{b}_t)/c(\mathbf{p}_{t-1}, \mathrm{I}_{t-1}, \mathbf{b}_{t-1})$$
$$= \pi(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$$
$$\equiv P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})(\lambda_t/\lambda_{t-1})^{1/(\sigma-1)}$$

where

$$(4) \qquad \lambda_r \equiv \sum_{i \in \mathrm{I}} p_{i,r} x_{i,r} \Big/ \sum_{i \in \mathrm{I}_r} p_{i,r} x_{i,r} \qquad \text{for } r = t-1, t.$$

This result states that the ratio of unit costs is measured by the exact price index $\pi(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$, which is defined to equal the conventional price index $P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$, times the additional term, $(\lambda_t/\lambda_{t-1})^{1/(\sigma-1)}$. To interpret this term, note that λ_t equals the fraction of expenditure in period t on the goods $i \in \mathrm{I}$ relative to the entire set $i \in \mathrm{I}_t$. Alternatively, λ_t measures 1 minus the share of expenditure in period t on the product varieties that are new or have a change in their taste parameters. If these new and upgraded varieties have a substantial share of expenditure, then λ_t will be small, and this will tend to make the exact index $\pi(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$ significantly lower than the index $P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$. In other words, the introduction of new or upgraded product varieties will *lower* the exact price index. The term λ_{t-1} equals 1 minus the share of expenditure in period $t-1$ on the product varieties that are not available in t or have a change in their taste parameters. Thus, if there are many disappearing varieties between the two periods, this will tend to make λ_{t-1} small, and *raise* the exact price index.

A useful way to interpret the effect of the new and disappearing varieties on the exact price index is by treating the price of a variety when it is *not* available as equal to its reservation price, where demand equals zero (see John R. Hicks, 1940; Franklin M. Fisher and Karl Shell, 1972). For the CES function with $\sigma > 1$, the demand for any good approaches zero only as its price approaches infinity, so this is the reservation price. Then new varieties can be thought of as having a price *fall* from infinity to the actual level, and this acts to lower the overall price index. Disappearing varieties can be thought of as having a price *increase* to infinity, which raises the index.

The exact price index $\pi(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$ also depends on the elasticity of substitution σ. If σ is high so that $1/(\sigma-1)$ approaches zero, then $(\lambda_t/\lambda_{t-1})^{1/(\sigma-1)}$ will be close to unity, in which case the conventional price index $P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$ is quite adequate. It is intuitive that ignoring new and disappearing varieties should not matter if they are close substitutes for existing varieties. At the other extreme, as σ approaches unity and $1/(\sigma-1)$ approaches infinity, then $(\lambda_t/\lambda_{t-1})^{1/(\sigma-1)}$ approaches zero (infinity) as $\lambda_t < (>)\ \lambda_{t-1}$, indicating that the new and disappearing varieties have a very significant effect on unit costs.[2]

[1]Using L'Hôpital's rule, it is readily shown that as $s_{i,t-1} \to s_{i,t}$ for all i, then the weights $w_{i,t}$ approach $s_{i,t}$.

[2]For $\sigma < 1$ all goods are essential to achieve positive production or utility, which is why a new or disappearing variety has an infinite effect on unit costs. For this reason, $\sigma < 1$ is excluded from the analysis.

TABLE 1—DATA ON U.S. IMPORTS

Import	Developing countries 1967	Developing countries 1977	Developing countries 1987	Industrial countries 1967	Industrial countries 1977	Industrial countries 1987
Men's leather athletic shoes						
Number of suppliers	5	11	17	11	9	10
Value ($million)	0.07	191	966	3.2	49	79
Men's and boys' cotton knit shirts						
Number of suppliers	11	21	40	13	9	11
Value ($million)	2.8	59	462	3.8	4.3	25
Stainless steel bars (cold rolled)						
Number of suppliers	1	4	6	9	8	9
Value ($million)	0.0014	1.3	5.3	3.5	24	35
Carbon steel sheets (cold rolled)						
Number of suppliers	2	6	12	9	12	14
Value ($million)	0.13	101	247	139	816	640
Color television receivers (over 17″)						
Number of suppliers	2[a]	4	9	4	3	5
Value ($million)	0.46	46	513	63	258	148
Portable typewriters						
Number of suppliers	5	9	10	12	10	4
Value ($million)	0.75	17	64	41	94	33
Gold bullion						
Number of suppliers	n.a.	11[b]	21	n.a.	8	11
Value ($million)	n.a.	317	308	n.a.	542	752
Silver bullion						
Number of suppliers	7[c]	9	10	4	2	5
Value ($million)	8.8	148	330	61	165	132

Note: The data for all products begin in 1964, except as noted below.
[a]For television receivers, the 1967 entry is for 1970, the first year of data.
[b]For gold bullion, the 1977 entry is for 1978, the first year of data.
[c]For silver bullion, the 1967 entry is for 1969, the first year of data.

One difficulty in the construction of the exact index $\pi(\mathbf{p}_{t-1},\mathbf{p}_t,\mathbf{x}_{t-1},\mathbf{x}_t,\mathrm{I})$ is that the researcher may not know which varieties (if any) have no change in the taste parameters $b_{i,t}$ and so should be included in the set I. If varieties that have actually experienced an increase in their qualities are included in the set I, with a corresponding increase in market share, this will tend to bias the measured index upward (since the increase in the market share should properly be measured by a low value of λ_t). On the other hand, there is no harm in excluding varieties from I for which the taste parameters are actually constant, since the index $\pi(\mathbf{p}_{t-1}, \mathbf{p}_t,\mathbf{x}_{t-1},\mathbf{x}_t,\mathrm{I})$ is still exact provided that $\mathrm{I} \neq \varnothing$. Generally, it is useful to experiment with several choices for I, and results of this type will be reported in Section VI.

II. Import Data and Aggregation

In Table 1, the data used to measure the import price indexes are summarized. Annual imports of six manufactured products are considered: men's leather athletic shoes, men's and boys' cotton knit shirts, stainless steel bars (cold rolled), carbon steel sheets (cold rolled), color television receivers (over 17-inch), and portable typewriters.[3] The sample period for each product begins in 1964 or later, depending on data availability, and continues until 1987. All of these products are at the most disaggregate level available in import statistics, which is generally finer than the eight-digit SIC level. In addition, annual imports of gold and silver bullion are included, to check how the estimation methods perform on perfectly homogeneous goods.

The manufactured products in Table 1 were chosen as commonly recognized goods which, on initial inspection, showed an increase in the number of countries supplying the United States over time. Included in Table 1 are the c.i.f. import value from the developing and industrial countries (using the IMF classification) and also the number of supplying countries, for selected years. All the manufactured products have considerable growth in the number of developing countries supplying the United States, while the number of industrial countries shows much less fluctuation and no trend in most cases.

In addition to the import values, quantities in physical units were obtained, so that the unit value from each supply country could be computed. The unit value will be used as a measure of the import price. This means that all varieties of a particular good are aggregated *within* each country, which leads to several sources of error. First, there is the well-known measurement error since the unit values are not proper price indexes; this error will be addressed at the end of Section IV. Second, there is an error through ignoring changes in the number of varieties supplied from *each* exporting country, and the implications of this aggregation are derived next.

Suppose that there are any number of (unobserved) varieties produced in each country and that all the varieties enter a CES function in the United States. The exact price index for the imports from each country can be constructed as in Proposition 1. Thus, letting i denote an index of supplying countries, $\pi_{i,t}$ equals the (unobserved) exact price index of the varieties from country i. Letting $b_{i,t}$ denote the taste parameter for the imports of country i, the cost of obtaining one unit of import services can be written as in (1):

$$(1')\quad c(\boldsymbol{\pi}_t,\mathbf{I}_t,\mathbf{b}_t)=\left(\sum_{i\in\mathbf{I}_t} b_{i,t}\pi_{i,t}^{1-\sigma}\right)^{1/(1-\sigma)}\qquad \sigma>1.$$

From Proposition 1, the exact index can be written $\pi_{i,t}=P_{i,t}(\lambda_{i,t}/\lambda_{i,t-1})^{1/(\sigma-1)}$, where $P_{i,t}$ is a conventional price index that ignores new product varieties and $\lambda_{i,t}$ is defined as in (4) but applied to the product varieties of *each* country. This can be substituted into (1′) to obtain

$$(5)\quad c(\boldsymbol{\pi}_t,\mathbf{I}_t,\mathbf{b}_t)=\left(\sum_{i\in\mathbf{I}_t}(b_{i,t}\lambda_{i,t-1}/\lambda_{i,t})P_{i,t}^{1-\sigma}\right)^{1/(1-\sigma)}.$$

The significant feature of (5) is that the taste parameter for imports from each country now takes the form $(b_{i,t}\lambda_{i,t-1}/\lambda_{i,t})$, where $\lambda_{i,t}$ $(\lambda_{i,t-1})$ equals 1 minus the share of expenditure on new (disappearing) varieties within country i. *Thus, a change in the number of varieties within a country acts in the same manner as a change in the taste or quality parameter for that country's imports.* This point can also be seen by calculating the expenditure share on the imports from

[3]The data were collected from U.S. Bureau of the Census, *U.S. General Imports for Consumption*. The TSUSA numbers corresponding to these products, together with the data and programs used in estimation, are available from the author upon request. Note that in a number of cases the product data would split during the sample period, such as when portable typewriters split into electric and nonelectric beginning in 1971. In these cases, I simply formed a unit value of the two (or more) series and used this as the price variable. This procedure is consistent with the price variable that was used before the product was split, which was a unit value by necessity.

country i:

$$(6)\quad s_{i,t} = c(\boldsymbol{\pi}_t, \mathrm{I}_t, \mathbf{b}_t)^{\sigma-1} \times P_{i,t}^{-(\sigma-1)}(b_{i,t}\lambda_{i,t-1}/\lambda_{i,t}).$$

Thus, an increase in the number of varieties available from country i will lower the value of $\lambda_{i,t}$ and thereby raise the share of expenditure on imports from that country, in the same manner as an increase in the taste parameter $b_{i,t}$.

This interpretation of the taste parameters will be used to motivate the error term in the demand equation specified below. In addition, it may be useful in explaining several results in the literature. For example, Alan C. Stockman and Linda L. Tesar (1991) find that the variations in consumption and prices across industrial countries are consistent with an intertemporal optimizing model only if the model includes fluctuations in taste parameters: these changes in tastes can be explained by the appearance of new domestic and imported goods. Richard H. Clarida (1994) also relies on fluctuations in taste parameters in his study of aggregate U.S. import demand, which can be interpreted in the same way. In another context, computable-general-equilibrium (CGE) models that distinguish imports by country of origin and hold the CES taste parameters *fixed* during simulations would give misleading results if the range of product varieties should actually change. Of course, this limitation of the Armington approach has already been recognized, and recent CGE models allow for product differentiation by firms rather than by countries.

III. Stochastic Model

The next task is to specify the import demand and supply equations, used to obtain estimates of the elasticity of substitution between the varieties of each product. The term $\varepsilon_{i,t} \equiv \Delta\ln(b_{i,t}\lambda_{i,t-1}/\lambda_{i,t})$ from (6) is treated as an unobserved random variable, reflecting changes in the number and quality of product varieties available from each country. Then, writing the expenditure shares from (6) in first differences yields

$$(7)\quad \Delta\ln s_{i,t} = \phi_t - (\sigma-1)\Delta\ln P_{i,t} + \varepsilon_{i,t}$$

where

$$\phi_t \equiv (\sigma-1)\ln[c(\boldsymbol{\pi}_t, \mathrm{I}_t, \mathbf{b}_t)/c(\boldsymbol{\pi}_{t-1}, \mathrm{I}_{t-1}, \mathbf{b}_{t-1})]$$

is a random effect, since $\mathbf{b}_t$ is random. Thus, $\varepsilon_{i,t}$ appears as an error term in this demand equation.

In order to estimate the elasticity σ, the possible correlation of $\varepsilon_{i,t}$ with the price and market-share variables must be specified. It is very plausible that the data on U.S. import prices suffer from two problems: correlation of $\varepsilon_{i,t}$ with both $\Delta\ln s_{i,t}$ and $\Delta\ln P_{i,t}$, due to the simultaneous determination of import prices and quantities; and measurement error in the prices, which are unit values of the product from each supplying country. The simultaneity issue will be addressed first; the measurement-error problem will be addressed at the end of this section and the next.

Letting E_t denote total expenditure on an import product, the quantity of imports from country is $x_{i,t} \equiv s_{i,t}E_t/P_{i,t}$. The supply curve for these imports from country i is specified in first differences as:

$$(8)\qquad \Delta\ln P_{i,t} = \omega\,\Delta\ln x_{i,t} + \xi_{i,t}$$

where $\omega \geq 0$ is the inverse supply elasticity, and $\xi_{i,t}$ is a random error that is assumed to be independent of $\varepsilon_{i,t}$. For convenience, the supply elasticity is treated as equal across all supplying countries.

Because the demand equation is expressed in terms of expenditure shares, while the supply equation uses the import quantity $x_{i,t}$, it will be useful to eliminate $x_{i,t}$ from the latter. Using the definition of $x_{i,t}$, the equilibrium price is solved from (7) and (8) as

$$(9) \quad \Delta \ln P_{i,t} = \psi_t + \rho\varepsilon_{i,t}/(\sigma-1) + \delta_{i,t} \qquad 0 \le \rho < 1$$

where $\psi_t = \omega(\phi_t + \Delta \ln E_t)/(1+\omega\sigma)$ is a random effect, $\delta_{i,t} \equiv \xi_{i,t}/(1+\omega\sigma)$ is the error, and $\rho \equiv \omega(\sigma-1)/(1+\omega\sigma)$. The parameter ρ satisfies $0 \le \rho < (\sigma-1)/\sigma < 1$, and equals zero if and only if $\omega = 0$, meaning that the supply curve is horizontal. To interpret (9), the value $\varepsilon_{i,t}/(\sigma-1)$ equals the vertical shift in the demand curve due to changes in the taste parameters for country i. Then the corresponding change in the equilibrium price equals $\rho\varepsilon_{i,t}/(\sigma-1)$. Thus, ρ is the correlation between the vertical shifts in the demand curve and the change in the equilibrium price. Equation (9) can be interpreted as a "reduced-form" supply curve, and its error $\delta_{i,t}$ is independent of $\varepsilon_{i,t}$.

The unusual feature of the demand and supply equations (7) and (9) is that no exogenous variables are included, as would normally be used to identify and estimate the elasticities. Instead, the demand and supply elasticities will be consistently estimated by exploiting the panel nature of the data set: for each import, the data extend over multiple years and countries. The estimator that will be used can be motivated from Edward E. Leamer (1981), who considers the time-series estimation of a demand and supply system in the absence of instruments. Assuming that the errors in the demand and supply equations are uncorrelated, he shows that the maximum-likelihood estimates of the demand and supply elasticities lie on a hyperbola defined by the second moments of the data; the nonuniqueness of the estimates illustrates the identification problem.

Now consider a panel data set, such as used here, where the elasticity of demand for a given product is equal across the countries (due to the CES demand structure), and the elasticities of supply for *each* country are also equal (by assumption). Then by using the time-series data for each country of supply, a hyperbola of elasticity estimates is obtained, as in Leamer. Combining these over all countries, multiple hyperbolas are obtained, whose intersection defines estimates for the demand and supply elasticities, provided that an identification condition discussed below is satisfied.[4]

While the estimate of the elasticity of substitution will be obtained in essentially this manner, the method used here differs in one important respect from that used by Leamer: because of the measurement error in the import prices (unit values), I avoid using physical quantities, which would have measurement error correlated with the unit values (Murray C. Kemp, 1962). Instead, the expenditure shares will be used, which should not be influenced by the measurement error. This means that the geometric interpretation used by Leamer will not carry over directly, though my method is closely related to his.[5]

IV. Estimation Method

To write (7) and (9) in a form more suitable for estimation, eliminate the (random) terms ϕ_t and ψ_t in each of them by subtracting the same equation for source k, to obtain

$$\begin{aligned}\tilde{\varepsilon}_{i,t} &= (\Delta \ln s_{i,t} - \Delta \ln s_{k,t}) \\ &\quad + (\sigma-1)(\Delta \ln P_{i,t} - \Delta \ln P_{k,t}) \\ \tilde{\delta}_{i,t} &= (\Delta \ln P_{i,t} - \Delta \ln P_{k,t}) \\ &\quad - \rho\tilde{\varepsilon}_{i,t}/(\sigma-1) \\ &= (1-\rho)(\Delta \ln P_{i,t} - \Delta \ln P_{k,t}) \\ &\quad - [\rho/(\sigma-1)](\Delta \ln s_{i,t} - \Delta \ln s_{k,t})\end{aligned}$$

[4] In small samples, the hyperbolas will not generally intersect at a point, so the estimates are chosen to minimize the sum of squared distances to the hyperbola. See also footnote 6.

[5] The estimator used is also related in spirit to Jerry A. Hausman and Zvi Griliches (1986), who argue that measurement error can be overcome in the context of panel data sets. They rely on different autocorrelations of the true variables and the measurement errors, whereas I focus on independence of contemporaneous errors across the demand and supply equations. See also footnote 7.

where $\tilde{\varepsilon}_{i,t} \equiv \varepsilon_{i,t} - \varepsilon_{k,t}$ and $\tilde{\delta}_{i,t} \equiv \delta_{i,t} - \delta_{k,t}$. In order to take advantage of the independence of $\tilde{\varepsilon}_{i,t}$ and $\tilde{\delta}_{i,t}$, these two equations are multiplied together and then divided by $(1-\rho)(\sigma-1)>0$, to obtain

$$(10) \qquad Y_{i,t} = \theta_1 X_{1i,t} + \theta_2 X_{2i,t} + u_{i,t}$$

where

$$(11a) \qquad Y_{i,t} \equiv (\Delta \ln P_{i,t} - \Delta \ln P_{k,t})^2$$

$$(11b) \qquad X_{1i,t} \equiv (\Delta \ln s_{i,t} - \Delta \ln s_{k,t})^2$$

$$(11c) \qquad X_{2i,t} \equiv (\Delta \ln s_{i,t} - \Delta \ln s_{k,t}) \times (\Delta \ln P_{i,t} - \Delta \ln P_{k,t})$$

$$(11d) \qquad u_{i,t} \equiv \tilde{\varepsilon}_{i,t}\tilde{\delta}_{i,t}/(1-\rho)(\sigma-1)$$

$$(11e) \qquad \theta_1 \equiv \frac{\rho}{(\sigma-1)^2(1-\rho)}$$

$$\theta_2 \equiv \frac{(2\rho-1)}{(\sigma-1)(1-\rho)}.$$

Since the prices and expenditure shares are correlated with the errors $\varepsilon_{i,t}$ and $\delta_{i,t}$, then $u_{i,t}$ is correlated with $X_{1i,t}$ and $X_{2i,t}$ in equation (10). A consistent estimator can be obtained, however, by averaging (10) over all t. Let $\bar{Y}_i$, $\bar{X}_{1i}$, $\bar{X}_{2i}$, and $\bar{u}_i$ denote the sample means of the variables in (11). Then (10) can be rewritten as

$$(10') \qquad \bar{Y}_i = \theta_1 \bar{X}_{1i} + \theta_2 \bar{X}_{2i} + \bar{u}_i$$

where this equation has observations equal to the number of supplying countries less one (since differences have been taken with one country k). Note that the data in (10′) are the second moments of the changes in price and expenditure shares, while $\bar{u}_i$ is the cross-moment of the errors in the demand and supply equations. From the assumption that these errors are independent, $E(\bar{u}_i)=0$. Under weak additional conditions, as the number of time periods T approaches infinity then $\text{plim}(\bar{u}_i)=0$, which means that the error in (10′) vanishes.

Let $\hat{\theta}_1$ and $\hat{\theta}_2$ denote estimates of θ_1 and θ_2 obtained by running weighted least squares (WLS) on (10′). This estimator corresponds to Lars P. Hansen's (1982) generalized method of moments: the moment condition $E(u_{i,t})=0$ is approximated by choosing θ_1 and θ_2 to minimize the (weighted) sum of squared sample moments $\bar{u}_i$. Then $\hat{\theta}_1$ and $\hat{\theta}_2$ can be treated as consistent estimates provided that an identification condition is satisfied: the vectors $\bar{\mathbf{X}}_1 = (\bar{\mathbf{X}}_{1i})$ and $\bar{\mathbf{X}}_2 = (\bar{\mathbf{X}}_{2i})$, which are the regressors in (10′), cannot be proportional as $T \to \infty$. If these vectors were proportional asymptotically, so that the regressors become colinear, then it is immediate that θ_1 and θ_2 could not be separately estimated from (10′). It turns out that the probability limits of $\mathbf{X}_1$ and $\mathbf{X}_2$ are *not* proportional provided that there exist countries $i \neq k$ and $j \neq k$ such that

$$(12) \qquad \left(\frac{\sigma_{\varepsilon i}^2 + \sigma_{\varepsilon k}^2}{\sigma_{\varepsilon j}^2 + \sigma_{\varepsilon k}^2}\right) \neq \left(\frac{\sigma_{\delta i}^2 + \sigma_{\delta k}^2}{\sigma_{\delta j}^2 + \sigma_{\delta k}^2}\right).$$

Condition (12) requires that there must be some differences in the relative variances of the demand and supply curves across the countries.[6] Provided that this condition is satisfied, the estimates of θ_1 and θ_2 obtained from running WLS on (10′) are consistent. This claim is demonstrated formally in Feenstra (1991), where the identification condition (12) is derived.

Equivalent to running WLS on (10′), an instrumental-variable (IV) estimator can be used on equation (10), where the instruments are dummy variables across the countries $i \neq k$. Regressing the variables on the right of (10) on the instruments results in vectors with elements $\bar{X}_{1i}$ and $\bar{X}_{2i}$ repeated each time country i appears, along with $\bar{u}_i$ in the error term. This is identical to (10′),

[6]To relate this condition back to the discussion of Leamer (1981), if expression (12) did not hold, then the hyperbolas for the various countries would all be identical asymptotically. Then it is immediate that the demand and supply elasticities would not be identified. As pointed out by Leamer (1981 p. 321), expression (12) cannot be expected to hold by simply splitting a time-series sample in half and comparing the variances in the two halves: creating an "artificial" panel of this type will not lead to identification.

except that observation i in (10′) is repeated each year that country i supplies. This IV estimator yields consistent estimates $\hat{\theta}_1$ and $\hat{\theta}_2$, along with their standard errors. In addition, *efficient* estimates can be obtained by correcting the IV estimator for heteroscedasticity: the identification condition (12) implies that the variance of $\bar{u}_i$ changes over i, and correcting for this yields the efficient estimates reported in the next section.[7]

Having obtained the consistent (and efficient) estimates $\hat{\theta}_1$ and $\hat{\theta}_2$, the values of $\hat{\sigma}$ and $\hat{\rho}$ can be solved from the quadratic equations (11e). So long as $\hat{\theta}_1 > 0$ these equations yield two solutions for $\hat{\sigma}$, one greater and the other less than unity. Attention is restricted to the solution that exceeds unity.

PROPOSITION 2: *So long as $\hat{\theta}_1 > 0$, then the estimates of σ and ρ are as follows:*

(a) if $\hat{\theta}_2 > 0$ then

$$\hat{\rho} = \frac{1}{2} + \left(\frac{1}{4} - \frac{1}{4 + \left(\hat{\theta}_2^2 / \hat{\theta}_1 \right)} \right)^{1/2}$$

(b) if $\hat{\theta}_2 < 0$ then

$$\hat{\rho} = \frac{1}{2} - \left(\frac{1}{4} - \frac{1}{4 + \left(\hat{\theta}_2^2 / \hat{\theta}_1 \right)} \right)^{1/2}$$

and in either case,

$$\hat{\sigma} = 1 + \left(\frac{2\hat{\rho} - 1}{1 - \hat{\rho}} \right) \frac{1}{\hat{\theta}_2} > 1.$$

As $\hat{\theta}_2 \to 0$, then $\hat{\rho} \to \frac{1}{2}$ and $\hat{\sigma}_1 \to 1 + \hat{\theta}_1^{-1/2}$.

In the event that $\hat{\theta}_1$ is negative, then the formulas in Proposition 2 fail to provide estimates for σ and ρ that are both in the ranges $\hat{\sigma} > 1$ and $0 \leq \hat{\rho} < 1$. For example, if $\hat{\theta}_1$ is only slightly less than zero ($-\hat{\theta}_2^2/4 < \hat{\theta}_1 < 0$), then $\hat{\rho} \notin [0,1]$, and $(\hat{\sigma} - 1)$ has the same sign as $-\hat{\theta}_2$. In this case it is still possible that a value of $\hat{\sigma}$ exceeding unity is obtained, as will be found for one of the import products. However, if $\hat{\theta}_1$ is too low ($\hat{\theta}_1 < -\hat{\theta}_2^2/4$) then imaginary values for $\hat{\rho}$ and $\hat{\sigma}$ are obtained. In general, a value of $\hat{\sigma}$ greater than unity must be obtained in order to apply Proposition 1.

Finally, the implications of measurement error in the prices (which are actually unit values) should be addressed. This measurement error is most serious for the imports of gold and silver, because the price from each country is calculated as an annual average (i.e. the annual value of shipments divided by the number of ounces). Since at any instant in time the prices of these products are obviously identical across supplying countries, *all* of the cross-country variation in the annual prices can be attributed to measurement error. What are the implication of this error for the parameter estimates?

It can be demonstrated that the estimates are robust to the simplest form of measurement error, with equal variance across supplying countries provided that a constant term is added to equations (10) and (10′). To see this, note that the left-hand-side variable in equation (10) or (10′) is the second moment of the unit values, which in large samples will equal the variance of the true prices plus the variance of the measurement error. Thus, the constant term in the equation will reflect the variance of the measurement error. On the right-hand side, the expenditure shares are assumed to be uncorrelated with the measurement error in unit values. This means that second moment of the shares, and the cross-moment of the unit values and shares, will not be affected asymptotically by the measurement error. It follows that the estimates of θ_1 and θ_2 are still consistent, provided that a constant term, denoted by θ_0, is included.

V. Estimation Results

To estimate (10), a country k must be selected to form the differences ($\Delta \ln P_{i,t} -$

[7]The details of this IV estimation procedure are described in an appendix, which is available from the author upon request, along with the SAS programs used to obtain the efficient estimates and the data. Joshua D. Angrist (1991) also discusses the use of dummy variables over cross-sectional units to correct for measurement error in a panel data set.

TABLE 2—PARAMETER ESTIMATES

Product	L	θ_0	θ_1	θ_2	σ	ρ	$1/\omega$
Athletic shoes	420	0.013	0.050	0.068	6.23	0.58	2.85
		(0.007)	(0.009)	(0.064)	(0.72)	(0.07)	(0.71)
					[4.4, 10.6]		
Knit shirts	651	0.044	0.068	0.123	5.83	0.61	2.01
		(0.015)	(0.016)	(0.050)	(0.506)	(0.04)	(0.50)
					[4.2, 11.0]		
Steel bars	220	−0.0003	0.051	−0.254	3.59	0.25	6.58
		(0.0037)	(0.009)	(0.075)	(0.35)	(0.06)	(1.75)
					[2.8, 5.3]		
Steel sheets	353	0.0079	−0.0015	−0.317	4.21	−0.015	−213
		(0.0057)	(0.0048)	(0.084)	(0.94)	(0.055)	(718)
					[3.0, 10.0]		
TV receivers	133	0.058	0.145	0.931	8.38	0.89	−0.063
		(0.052)	(0.034)	(0.219)	(2.70)	(0.05)	(0.153)
					[6.4, 12.3]		
Typewriters	312	0.041	0.174	−0.171	2.96	0.40	1.94
		(0.015)	(0.023)	(0.065)	(0.15)	(0.04)	(0.37)
					[2.5, 3.6]		
Gold bullion	236	0.0092	0.0013	−0.0034	27.2	0.48	27.8
		(0.0015)	(0.0006)	(0.015)	(9.09)	(0.10)	(6.82)
					[17.7, 124]		
Silver bullion	223	0.0119	0.0018	0.051	42.9	0.76	12.4
		(0.0025)	(0.0008)	(0.018)	(15.2)	(0.08)	(3.11)
					[21.3, 4,876]		

Notes: Standard errors are in parentheses. For σ, 95-percent confidence intervals are in brackets. L is the number of observations, over years and supplying countries. Confidence intervals are computed by repeatedly applying Proposition 2 (a more detailed explanation is contained in an unpublished appendix, which is available from the author upon request). For gold and silver bullion, the confidence interval also includes negative or imaginary values for σ.

$\Delta \ln P_{k,t}$) and ($\Delta \ln s_{i,t} - \Delta \ln s_{k,t}$) in (11). This country should be a supplier of the product in every year. For each of the six manufactured products in the present study, there were a number of countries (primarily industrial countries) that could play this role, but the only country that supplied all six of the products to the United States in every year was Japan. For this reason, Japan was chosen as country k, and the data were differenced to construct the variables in (11). For gold and silver bullion, Canada was chosen as country k, since it was the dominant supplier. The parameters of (10) were then estimated while pooling the data over the industrial and developing countries.

The estimation results are reported in Table 2. In addition to the estimated coefficients $\hat{\theta}_i$, $i = 0,1,2$, the parameters $\hat{\sigma}$ and $\hat{\rho}$ are shown as calculated in Proposition 2, along with the supply elasticity ($1/\hat{\omega}$). The standard errors of $\hat{\sigma}$ and $\hat{\rho}$ (shown in parentheses) are obtained by linearizing the quadratic equations (11e), and using the variance and covariance of $\hat{\theta}_1$ and $\hat{\theta}_2$ to compute the same for $\hat{\sigma}$ and $\hat{\rho}$. In addition to this procedure, a 95-percent confidence interval for σ is computed by using all values of $\boldsymbol{\theta} = (\theta_1, \theta_2)$ in their 95-percent confidence ellipse, defined using the F distribution as $(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})'V(\hat{\boldsymbol{\theta}})^{-1}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \leq 2F_{[0.95]}(2, L-3)$. For each value of $\boldsymbol{\theta}$ in this region, the formulas in Proposition 2 were applied, and the minimum and maximum values obtained for σ define its 95-percent confidence interval.

Looking first at the results for athletic shoes and knit shirts, the constant term $\hat{\theta}_0$ is significant (marginally so for athletic shoes), indicating some measurement error in the unit values. The values of the elasticity of substitution are both around 6, with standard errors less than unity. However, the 95-percent confidence intervals for $\hat{\sigma}$, obtained by repeatedly applying Proposition 2, are substantially larger than the standard errors in the second rows would suggest. In particular, the confidence intervals include a greater range of values above the point estimate $\hat{\sigma}$ than below, reflecting the nonlinear nature of the formulas in Proposition 2. It appears that these confidence intervals provide a better indication of the range of values for $\hat{\sigma}$ than do the standard errors obtained by linearizing (11e).

For the two steel products, the constant $\hat{\theta}_0$ is not significant, so in these cases measurement error in the unit values is not important. Surprisingly, the estimates of $\hat{\sigma}$ of about 4 are less than those obtained for athletic shoes and knit shirts, though the confidence intervals still overlap substantially. Because the value of $\hat{\theta}_1$ for steel sheets is negative, a value of $\hat{\rho}$ lying outside the interval [0,1] is obtained. The highest estimate of the elasticity of substitution among the manufactured goods is obtained for color television receivers ($\hat{\sigma} = 8.38$), and the lowest estimate is that for portable typewriters ($\hat{\sigma} = 2.96$). Generally, the pattern of point estimates does not seem to correspond well to prior beliefs about the homogeneity of the various products, though again, the confidence intervals for many of the products overlap significantly.

Turning to gold and silver bullion, much higher estimates of the elasticity of substitution are obtained, with confidence intervals including values in excess of 100 (these intervals also included negative or imaginary values for $\hat{\sigma}$). Thus, in these extreme cases of perfectly homogeneous products, the estimation technique gives the expected results. In addition, it is noteworthy that the constant term $\hat{\theta}_0$ is highly significant for both of these products, confirming the presence of measurement error in the unit values computed as annual averages. Generally, it is recommended that a constant be included whenever this estimation technique is applied to unit values.

VI. Price Indexes

The price indexes constructed according to Proposition 1 are shown in Figures 1–8, using the elasticity estimates from Table 2. The index labeled P1 is constructed using data for each country that supplied in the current *and* past year, using the Sato-Vartia formula in (3). Thus, this index ignores data on new supplying countries in their first year and on disappearing countries in their last year, as is conventionally done. The index set (I) used in constructing this conventional index, is formally defined as:

$$\text{(13a)} \quad \mathrm{I}(t)_1 \equiv \{\text{all countries supplying in years } t \text{ and } t-1\}$$

where this index set changes over time. Note that P1 shown in the figures is the *cumulative* price index, obtained by multiplying the annual price indexes defined in (3).

Several exact price indexes are also constructed, which take account of the new and disappearing countries and also allow for quality change in some subset of countries. For this purpose, the index set I within which the countries do *not* exhibit quality change in their products must be specified. As discussed after Proposition 1, it is useful to experiment with several different sets I, and compare the results obtained in each case. In addition to (13a), two other index sets are used:

$$\text{(13b)} \quad \mathrm{I}(t)_2 \equiv \{\text{all industrial countries supplying in years } t \text{ and } t-1\}$$

$$\text{(13c)} \quad \mathrm{I}(t)_3 \equiv \{\text{all industrial countries except Japan, supplying in } t \text{ and } t-1\}.$$

The motivation for using $\mathrm{I}(t)_2$ is that it may be the developing countries, in particular, that have changed the quality of their products over the period of estimation. Then it is appropriate to exclude these countries from the calculation of the conventional index $P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I}(t)_2)$ in Proposition 1, while incorporating the developing countries into the terms λ_t and λ_{t-1}. To the extent that the developing countries have a rising import share, without a corresponding fall in their unit values, excluding them from the set $\mathrm{I}(t)_2$ will lower the exact index. The third case also allows for quality change in the Japanese products and excludes Japan from the set $\mathrm{I}(t)_3$.

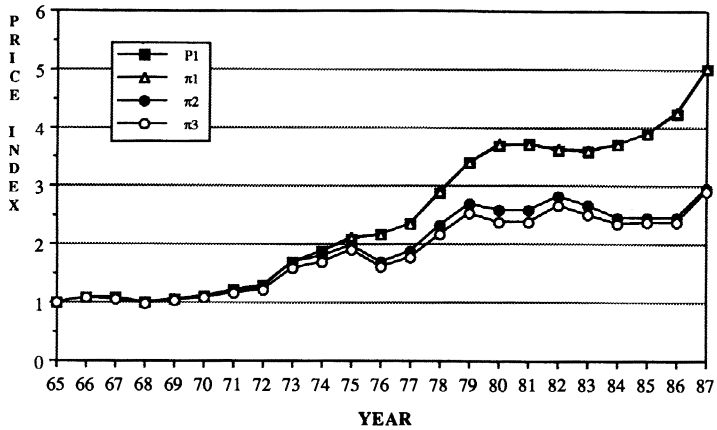

FIGURE 1. ATHLETIC SHOES

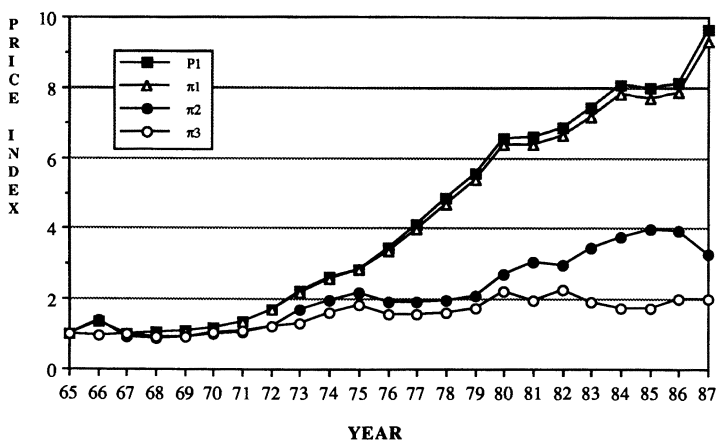

FIGURE 2. COTTON KNIT SHIRTS

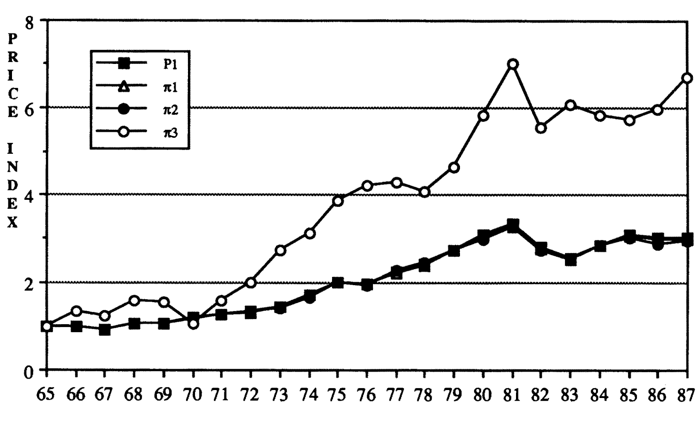

FIGURE 3. STAINLESS STEEL BARS

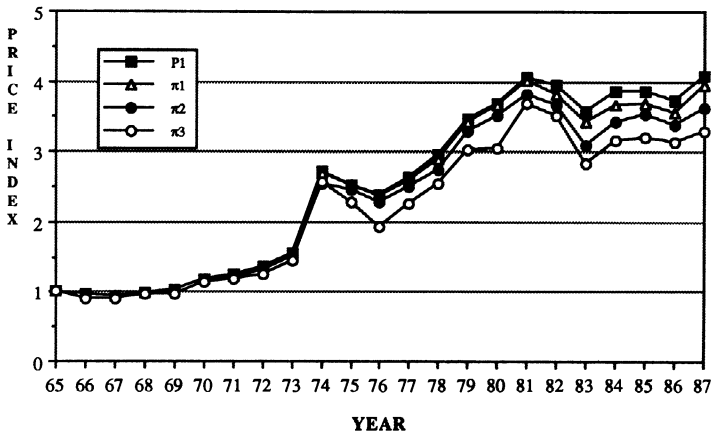

FIGURE 4. CARBON STEEL SHEETS

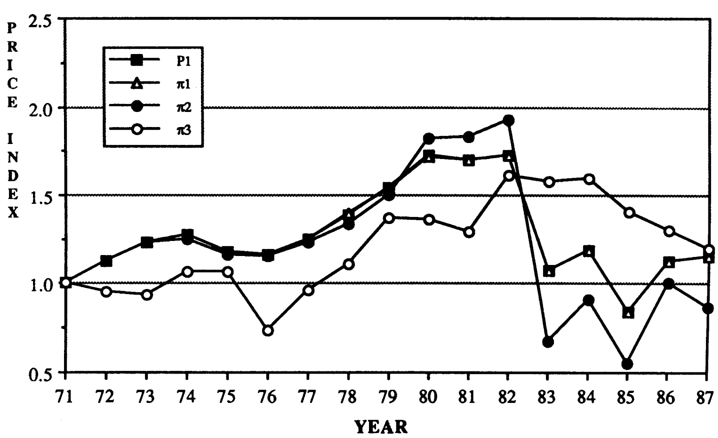

FIGURE 5. COLOR TV RECEIVERS

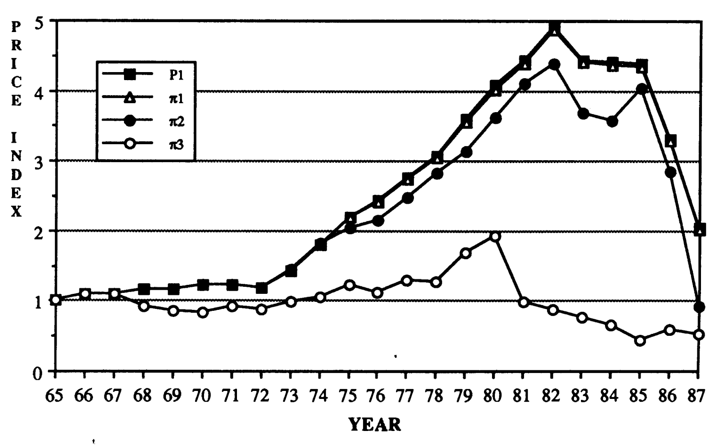

FIGURE 6. PORTABLE TYPEWRITERS

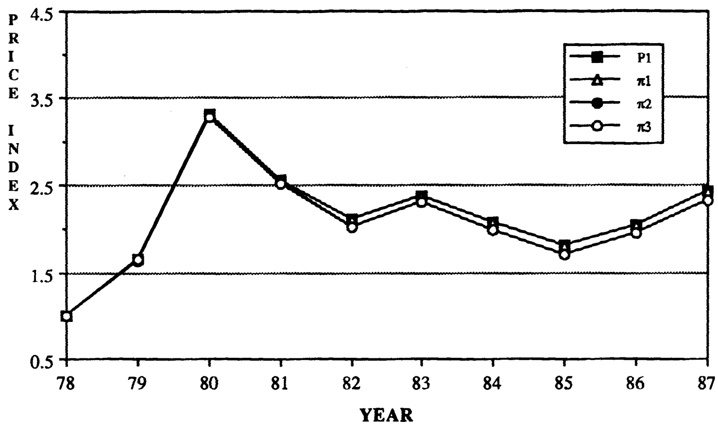

FIGURE 7. GOLD BULLION

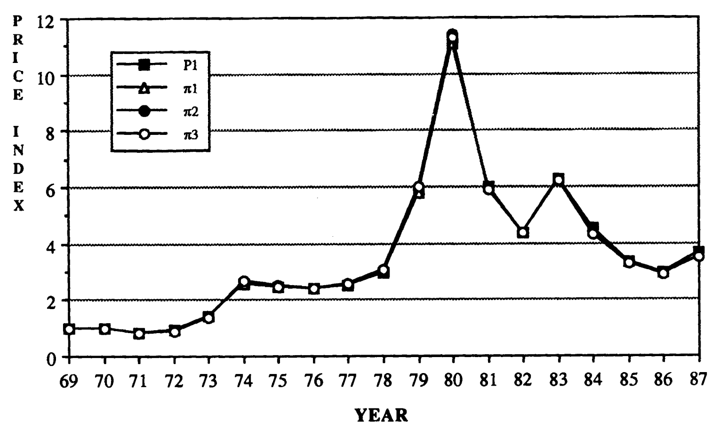

FIGURE 8. SILVER BULLION

The cumulative exact indexes obtained using the sets $I(t)_1$, $I(t)_2$, and $I(t)_3$ are denoted by $\pi 1$, $\pi 2$, and $\pi 3$ in Figures 1–8. The first of these indexes, $\pi 1$, differs from P1 by taking into account the market shares of new supplying countries in their first year and of disappearing countries in their last year. Since these market shares are small relative to total U.S. imports of each product, there is little difference between P1 and $\pi 1$ for most products.

The second index, $\pi 2$ generally lies below P1, reflecting the rising share of imports from the developing countries. For example, for athletic shoes the conventional index P1 increases fivefold over 1965–1987, while $\pi 2$ increases only threefold. Substantial differences between the conventional index P1 and corrected index $\pi 2$ can also be seen for cotton knit shirts, color TV receivers, and portable typewriters. On the other hand, for stainless steel bars the developing countries did *not* capture a significant market share relative to industrial countries over the sample period (see Table 1), and in this case the indexes P1 and $\pi 2$ are nearly identical. In addition, for gold and silver bullion there is little or no difference between the indexes, reflecting the very high elasticities of substitution obtained for these two products.

The final index, $\pi 3$, is calculated allowing for quality change in all the developing countries and Japan. For three of the manufactured goods (athletic shoes, cotton knit shirts, and carbon steel sheets), this index lies somewhat below $\pi 2$, reflecting a rising Japanese share, but it is otherwise similar to $\pi 2$. However, for the other three manufactured products, the index $\pi 3$ is very different, and each of these cases will be discussed in turn.

For stainless steel bars, $\pi 3$ lies everywhere *above* the other indexes in Figure 3. This is because the Japanese share of U.S. imports for this product fell from 95 percent in 1965 to 50 percent in 1987. There were a number of industrial countries that experienced a rising import share in this product, and the greatest increase was for Spain, whose share increased from zero in 1965 to 16 percent in 1987. The most dramatic change in imports can be traced to the years 1971–1973, when the Japanese share fell from 89 percent to 50 percent and the Spanish share rose from 2 percent to 25 percent, and $\pi 3$ diverges substantially from the other indexes during these years.

If the actual elasticity of substitution between stainless steel bars from these countries is higher than the estimate of 3.59 in Table 2, then the erratic behavior of $\pi 3$ for this product is a spurious result. This case illustrates a potential limitation of my analysis, as can occur when two countries with product varieties that are close substitutes have very large changes in their market shares. This limitation was not found for gold or silver, however, since very high elasticities were estimated in those cases.

Turning to color TV receivers, the index $\pi 3$ is substantially different from $\pi 2$. This difference is in part due to a surge in the Japanese import share just prior to the application of import quotas in 1977, which lowered $\pi 3$ in 1976. Furthermore, there was a dramatic fall in the unit value of Japanese color TV receivers, from \$477 to \$113 during 1982–1983, with little change in the market share. Since the Japanese established several plants in the United States during the period of import quotas (1977–1982), it is likely that this fall in the unit value reflects the import of *unfinished* receivers. The index $\pi 3$ shows much smoother behavior than $\pi 2$ over 1982–1983, which I presume is a more accurate measure of the import prices. Indeed, this index will perform much better in the import-demand equations that are estimated in the next section.

For portable typewriters, the index $\pi 3$ lies substantially below the other indexes shown in Figure 6, reflecting very large increases in the Japanese share of U.S. imports: from 18 percent in 1965 to a high of 73 percent in 1985. Unfortunately, data are not available to estimate the import-demand equation for typewriters and to compare the results from using these various indexes. It is likely that the level of imports relative to U.S. production has risen substantially for this product, and quite possible that the index $\pi 3$ could explain more of this increase than the other indexes.

VII. Import Demand

In this section, the various price indexes are used in regressions of import demand, to determine whether they influence the income elasticity. As a functional form, it is desirable to continue working with expenditure shares and also to allow for nonhomothetic preferences. These considerations suggest using the almost ideal demand system (AIDS), due to Angus Deaton and John Muellbauer (1980). For each manufactured product, a two-good AIDS over total imports and U.S. supply will be estimated.

The data for U.S. consumption of the domestically produced manufactured goods were obtained at the same level of disaggregation as for the data on import products.[8] This was not possible, however, for two of the products: portable typewriters, where U.S. data have been suppressed since the 1970's due to the small number of U.S. producers; and men's and boys' cotton knit shirts, where the U.S. data were available only at a significantly broader level of aggregation. For the four remaining manufac-

[8]These data were obtained from *Current Industrial Reports* (U.S. Bureau of the Census, various years).

TABLE 3—SHARE OF IMPORTS IN U.S. CONSUMPTION

Product	Period	Mean share	Range
Men's leather athletic shoes[a]	1973–1987	0.64	0.35–0.89
Stainless steel bars (cold rolled)	1971–1987	0.10	0.04–0.19
Carbon steel sheets (cold rolled)[b]	1971–1982	0.12	0.10–0.18
Color TV receivers (over 17-inch)[c]	1979–1987	0.19	0.10–0.30

[a]The import shares are underestimated, since the U.S. data include women's and children's athletic shoes and also exports.
[b]Later years are omitted due to import quotas.
[c]Earlier years are omitted due to import quotas.

tured products the comparable U.S. data were obtained for 1973–1987 (1971–1987 for steel), though in some cases a shorter period will be used in the estimation.

The import shares of U.S. apparent consumption are shown in Table 3. For men's leather athletic shoes, the import share is underestimated somewhat, since the domestic data included women's and children's athletic shoes and also U.S. exports. For the remaining three products, an exact match between the import and domestic products was obtained, and exports were netted out from U.S. supply. While the import share for athletic shoes grew steadily over the sample period, the shares for the other three products are quite erratic. This is particularly the case for carbon steel sheets and color TV receivers. For these two products, there was a surge of imports just prior to the imposition of import quotas, which were applied in November 1982 for carbon steel sheets and in December 1978 for imports of color TV receivers from Taiwan and Korea.[9] These quotas should not in principle affect the estimation of import demand, provided that the price and quantity are still on the demand curve. However, the demand equations estimated below performed very poorly when the entire periods were used, so the years 1983–1987 were omitted for carbon steel sheets, and the years 1973–1978 were omitted for color TV receivers.

Turning to the AIDS demand system, let π_t denote the price index for aggregate imports of each product, P_t^* the corresponding home price of the U.S. variety, and E_t the total U.S. expenditure on the product. Then the share equation for imports can be written as

$$(14a)\quad s_t = \alpha_1 + \gamma_{11} \ln \pi_t + \gamma_{12} \ln P_t^* + \beta_1 \ln\left(E_t / \tilde{P}_t\right)$$

where $\tilde{P}_t$ is an index of the prices defined by

$$(14b)\quad \ln \tilde{P}_t = \alpha_0 + \alpha_1 \ln \pi_t + \alpha_2 \ln P_t^* + \frac{\gamma_{11}}{2}(\ln \pi_t)^2 + \frac{\gamma_{22}}{2}(\ln P_t^*)^2 + \gamma_{12} \ln \pi_t \ln P_t^*$$

and $\gamma_{12} = \gamma_{21}$ is used. The number of parameters in (14) is reduced by imposing the homogeneity restrictions $\Sigma_i \alpha_i = 1$ and $\Sigma_i \beta_i = \Sigma_i \gamma_{ij} = 0$. Substituting these restrictions into (14) and simplifying, a simple linear equation for the import share is obtained:

$$(15)\quad s_t = \eta_0 + \eta_1 \ln(\pi_t / P_t^*) + \eta_2 [\ln(\pi_t / P_t^*)]^2 + \beta_1 \ln(E_t / P_t^*)$$

where $\eta_0 = \alpha_1 - \beta_1 \alpha_0$, $\eta_1 = \gamma_{11} - \beta_1 \alpha_1$, and $\eta_2 = -\beta_1 \gamma_{11} / 2$.

Thus, the import share in (15) depends on the relative import price, its squared value, and real expenditure measured relative to the U.S. price P_t^*. It can be noted

[9]See cases M-14 and M-18 in Gary C. Hufbauer et al. (1986).

TABLE 4—ESTIMATES FOR IMPORT DEMAND

Product	η_1	η_2	β_1	σ	$\bar{R}^2$ [T] (DW)	Price elasticity	Income elasticity
Athletic shoes	−0.467	0.283	0.423	—	0.93 [15]	−1.73	1.66
	(0.098)	(0.110)	(0.047)		(2.67)	(0.15)	(0.074)
Using $\pi 2$	−0.336	0.367	0.279	—	0.93 [15]	−1.53	1.47
	(0.067)	(0.219)	(0.022)		(2.74)	(0.11)	(0.035)
Nonlinear	−0.211	0.601	0.240	4.90	0.85 [15]	−1.33	1.37
	(0.093)	(0.323)	(0.040)	(1.46)	(1.97)	(0.15)	(0.063)
Steel bars	−0.209	2.47	0.030	—	0.21 [17]	−3.02	1.29
	(0.165)	(1.03)	(0.094)		(0.88)	(1.60)	(0.91)
Using $\pi 2$	−0.212	2.60	0.019	—	0.34 [17]	−3.05	1.18
	(0.126)	(0.89)	(0.076)		(1.01)	(1.22)	(0.74)
Nonlinear	−0.211	2.49	0.011	2.47	0.33 [17]	−3.05	1.11
	(0.115)	(0.90)	(0.074)	(1.00)	(1.16)	(1.11)	(0.72)
Steel sheets	−0.131	1.14	−0.27	—	0.36 [12]	−2.07	0.78
	(0.057)	(0.41)	(0.034)		(2.04)	(0.47)	(0.28)
Using $\pi 2$	−0.163	1.54	−0.031	—	0.50 [12]	−2.34	0.75
	(0.056)	(0.49)	(0.030)		(2.45)	(0.46)	(0.25)
Nonlinear	−0.170	1.75	−0.028	2.55	0.50 [12]	−2.39	0.77
	(0.056)	(0.49)	(0.030)	(0.69)	(3.10)	(0.46)	(0.25)
TV receivers	0.210	0.393	0.381	—	0.85 [8]	0.13	3.05
	(0.107)	(0.354)	(0.081)		(1.88)	(0.58)	(0.43)
Using $\pi 3$	−0.194	−1.54	0.239	—	0.90 [8]	−2.04	2.29
	(0.097)	(0.895)	(0.041)		(2.83)	(0.52)	(0.22)
Nonlinear	−0.190	−1.59	0.238	8.03	0.86 [8]	−2.02	2.28
	(0.132)	(1.32)	(0.051)	(4.99)	(2.78)	(0.71)	(0.28)

Notes: For η_1, η_2, β_1, σ, and the elasticity, standard errors are in parentheses. T is the number of observations, and DW is the Durbin-Watson statistic. The price elasticity equals $-1+\hat{\eta}_1/\bar{s}_1$, and the income elasticity equals $1+\hat{\beta}_1/\bar{s}_1$, where $\bar{s}_1$ is the mean import share.

that this formulation of the share equation avoids the approximation to $\tilde{P}_t$ that is sometimes used and is an exact specification of the AIDS equation. While the values of β_1 and η_i, $i=0,1,2$, can be used to solve back to the underlying parameters in (14), there is nothing to be gained by doing so, since after imposing the homogeneity restrictions there are exactly four free parameters in both (14) and (15).

A conventional error term is added to (15), on the right. This error may reflect the appearance of new domestic and imported varieties that are not measured, as discussed in Section II. In addition, note that both imports and U.S. supply refer to *shipments*, rather than actual consumption (i.e., changes in inventories are not controlled for). One implication of this is that real expenditure E_t/P_t^* is measured with error and therefore correlated with the error term. For example, an unusually high value of U.S. shipments can be reflected in both a low import share and high "expenditure" —which is actually the value of shipments. This type of correlation is readily apparent in the data, and accordingly, instrumental variables are used to estimate (15).[10]

The results of estimating the import equation are shown in Table 4, where the

[10]The instruments used were the U.S. real personal consumption expenditures on shoes, radio and television receivers, and total U.S. apparent consumption (in tons) of carbon steel and stainless steel, along with the relative prices that are regressors in (15). See also footnote 13.

first row for each product is obtained using the import price index P1 (shown in Figures 1–5) to replace π_t in (15). In addition to reporting the estimated coefficients, the price and income elasticities of import demand are also computed, evaluated at the mean value of the import shares.[11] For athletic shoes, an income elasticity of 1.66 is obtained, with a low standard error. For the two steel products, on the other hand, the income elasticities are insignificantly different from unity. These two regressions explain a small portion of the variation in the import shares, which are very erratic. For television receivers, an income elasticity of 3.05 is obtained, which is significantly greater than unity. Thus, for this product and athletic shoes, the import demand equations estimated with a conventional price index show evidence of high income elasticities.

In the second regression for each product, the exact price indexes $\pi 2$ or $\pi 3$ from Figures 1–5 are used. These indexes correct for quality change and take account of new and disappearing supplying countries. In most cases $\pi 2$ gave the same or a superior fit to using $\pi 3$, but for TV receivers it turned out that $\pi 3$ (which allows for quality change in Japanese imports) had the greatest impact on the income elasticity, and so results from using $\pi 3$ are reported for this product in Table 4.[12] The results from using the exact indexes are of principal interest for athletic shoes and TV receivers, since these two products had high income elasticities initially. From the last column in Table 4, it can be seen that the income elasticities for both products are reduced by using the exact indexes but do not fall all the way to unity. Thus, the exact indexes are able to account for part—but not all—of the high income elasticities. Note that for TV receivers, a much improved price elasticity is also obtained using the exact index $\pi 3$.

One problem with the estimates reported in the second regression for each product is that the price indexes ($\pi 2$ or $\pi 3$) are constructed using the estimated parameters $\hat{\sigma}$ from Table 2. Thus, the price indexes should be treated as estimated values rather than data. This feature does not affect the consistency of the estimates, but it does affect the standard errors: the errors reported in the second regressions understate the true errors, since the price indexes are estimated. In order to obtain correct standard errors and further assess the importance of using the exact price indexes, the following method is used.

From Proposition 1, the exact price index can be written in logs as

$$\text{(16)} \quad \ln \pi(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I}(t)) = \ln P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I}(t)) + \frac{\ln(\lambda_t / \lambda_{t-1})}{\sigma - 1}.$$

Substituting this for $\ln \pi_t$ in (15), an import-share equation is obtained where the cumulative values of $\ln P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I}(t))$ and $\ln(\lambda_t / \lambda_{t-1})$ are used as data, and the parameter σ is estimated in a nonlinear fashion. Thus, this method does not use the estimates $\hat{\sigma}$ from Table 2 but, rather, obtains new estimates directly from the demand equation. Ignoring the earlier estimates means that this method is not fully efficient, but it has the advantage that the standard errors obtained are correct.

The results from substituting (16) into (15) and estimating the parameters using nonlinear instrumental variables, are shown in the third regression for each product. In most cases the nonlinear regression converged quickly, except for athletic shoes, where the estimates obtained were sensitive

[11] The price elasticity of import demand is $-1+[\eta_1 + 2\eta_2 \ln(\pi_t / P_t^*)]/s_{1t}$ from (15), and the income elasticity is $1+\beta_1/s_{1t}$. For convenience, the relative import price was normalized to have a mean value of zero over the sample period. Then the price elasticity reported in Table 4 is computed as $-1+\hat{\eta}_1/\bar{s}_1$, and the income elasticity is $1+\hat{\beta}_1/\bar{s}_1$, where $\bar{s}_1$ is the mean import share.

[12] When $\pi 2$ is used for TV receivers, the results are nearly the same as when P1 is used.

to the instrumental variables used.[13] For both athletic shoes and TV receivers, the standard errors of the income elasticity are increased somewhat, reflecting the fact that the errors in the second regression are understated. In addition, it is noteworthy that the estimates of σ across products display the same pattern as in Table 2 (i.e., highest for TV receivers, followed by athletic shoes and then the steel products). The higher standard errors for $\hat{\sigma}$ in Table 4 reflect the very small number of observations in each regression, as compared to the earlier panel estimation. The estimates of $\hat{\sigma}$ for each product in Tables 2 and 4 are generally within one standard error of each other. The similarity of these estimates can be regarded as additional evidence supporting the use of the exact price indexes, which correct for quality change and new supplying countries.

VIII. Conclusions

The contribution of this paper is threefold. First, an expression for the exact price index for a CES unit-cost function is obtained, allowing for different (but overlapping) sets of product varieties over time and quality change in some of the varieties. Second, a technique for estimating the elasticity of substitution is proposed that permits correlation between the (unobserved) taste parameters, prices, and quantities in a demand and supply equilibrium. Third, these methods are applied to six manufactured products imported into the United States over 1964–1987. Allowing for quality change in the imports from developing countries generally resulted in a price index that rose more slowly than conventional measures. Two of these products had high income elasticities of import demand when estimated with conventional indexes, and these elasticities were reduced by using the corrected index.

Further research is needed to determine whether the index proposed here can account for a substantial portion of the high income elasticity of imports for other goods. At an aggregate level, it is surprising that the efforts by the U.S. Department of Labor to correct for quality change (reported in William Alterman [1991]), result in an import price index that rises *faster* than conventional measures. While Alterman relies on survey information to control for quality change, the methods used here rely on the market share of suppliers to infer quality change. It is quite possible that these techniques could be combined. In particular, the surveys are designed to compare imported goods whose quality characteristics are unchanged over time, and such goods could then be included in the set I. Total expenditure on other varieties of that good could be incorporated into the terms $(\lambda_t/\lambda_{t-1})$, and if the expenditure share on these other varieties is rising, it would lower the overall index.

Finally, note that the results reported here for the CES price index readily carry over to the CES quantity index (Feenstra and James Markusen, 1992). In addition, it may be possible to extend some of the results beyond the CES case. Diewert (1976) derives the exact indexes for the "quadratic mean of order r" aggregator function, when the set of goods is constant over time. Since this function nests the CES, it is a useful starting point to examine changing sets of goods. My preliminary results show that it is possible for new goods to have either a larger or smaller impact on the exact indexes than in the CES case, though it may be necessary to rely on upper and lower bounds rather than the precise index formula obtained here.

[13] The instrumental variables used in the third regressions were identical to those used for the second regressions [i.e., the real-expenditure variable for each product described in footnote 10, along with the price variables $\ln(\pi_t/P_t^*)$ and $\ln(\pi_t/P_t^*)^2$ which are regressors in (15)]. However, when this set of instruments was used in the nonlinear regression for athletic shoes, it converged to a value for $\hat{\sigma}$ of about 1.3, and near-zero estimates for all other parameters. Instead, only the real-expenditure variable was used as an instrument for $\ln(E_t/P_t^*)$, and the results for athletic shoes shown in the third regression in Table 4 were then obtained. The sensitivity of these results to including the price variables as instruments is surprising, since they are not significant when used in the first-stage regression to explain $\ln(E_t/P_t^*)$.

Appendix

PROOF OF PROPOSITION 1:

For the unit-cost function in (1), express the expenditure share on each variety as $s_{i,r}(\mathrm{I}_r)^{1/(\sigma-1)} = c(\mathbf{p}_r, \mathrm{I}_r, \mathbf{b}_r) b_{i,r}^{1/(\sigma-1)} / p_{i,r}$ for $r = t-1, t$. Then for varieties with constant taste parameters,

$$\text{(A1)}\quad \frac{c(\mathbf{p}_t, \mathrm{I}_t, \mathbf{b}_t)}{c(\mathbf{p}_{t-1}, \mathrm{I}_{t-1}, \mathbf{b}_{t-1})} = \frac{p_{i,t} s_{i,t}(\mathrm{I}_t)^{1/(\sigma-1)}}{p_{i,t-1} s_{i,t-1}(\mathrm{I}_{t-1})^{1/(\sigma-1)}} \quad \text{for } i \in \mathrm{I}.$$

Note that $s_{i,r}(\mathrm{I}_r) = s_{i,r}(\mathrm{I})\lambda_r$, $r = t-1, t$, from the definitions in (3b) and (4). Then substitute this into (A1), and take a geometric mean across the varieties $i \in \mathrm{I}$ using the weights $w_{i,t}(\mathrm{I})$, to obtain

$$\text{(A2)}\quad \frac{c(\mathbf{p}_t, \mathrm{I}_t, \mathbf{b}_t)}{c(\mathbf{p}_{t-1}, \mathrm{I}_{t-1}, \mathbf{b}_{t-1})} = P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I}) \left(\frac{\lambda_t}{\lambda_{t-1}} \right)^{1/(\sigma-1)} \times \prod_{i \in \mathrm{I}} \left(\frac{s_{i,t}(\mathrm{I})}{s_{i,t-1}(\mathrm{I})} \right)^{w_{i,t}(\mathrm{I})/(\sigma-1)}$$

where $P(\mathbf{p}_{t-1}, \mathbf{p}_t, \mathbf{x}_{t-1}, \mathbf{x}_t, \mathrm{I})$ is defined by (3a). To show that the product on the right of (A2) equals unity, take its natural log to obtain

$$\sum_{i \in \mathrm{I}} \frac{w_{i,t}(\mathrm{I})}{(\sigma-1)} [\ln s_{i,t}(\mathrm{I}) - \ln s_{i,t-1}(\mathrm{I})] = \frac{\sum_{i \in \mathrm{I}} [s_{i,t}(\mathrm{I}) - s_{i,t-1}(\mathrm{I})] \big/ (\sigma-1)}{\sum_{i \in \mathrm{I}} \left(\dfrac{s_{i,t}(\mathrm{I}) - s_{i,t-1}(\mathrm{I})}{\ln s_{i,t}(\mathrm{I}) - \ln s_{i,t-1}(\mathrm{I})} \right)} = 0$$

where the first equality follows from the definition of $w_{i,t}(\mathrm{I})$ in (3c), and the second equality holds since the cost shares $s_{i,r}(\mathrm{I})$ sum to unity over $i \in \mathrm{I}$, $r = t-1, t$. Then Proposition 1 follows from (A2).

PROOF OF PROPOSITION 2:

Written in terms of the parameter estimates, (11e) becomes

$$\text{(A3)}\quad \hat{\theta}_1 = \hat{\rho} \big/ \left[(\hat{\sigma} - 1)^2 (1 - \hat{\rho}) \right]$$

$$\hat{\theta}_2 = (2\hat{\rho} - 1) \big/ \left[(\hat{\sigma} - 1)(1 - \hat{\rho}) \right].$$

Eliminate $(\hat{\sigma} - 1)$ in (A3) by writing

$$\hat{\theta}_2^2 / \hat{\theta}_1 = (2\hat{\rho} - 1)^2 \big/ \left[\hat{\rho}(1 - \hat{\rho}) \right].$$

It follows that

$$\hat{\rho}^2 \left[4 + \left(\hat{\theta}_2^2 / \hat{\theta} \right) \right] - \rho \left[4 + \left(\hat{\theta}_2^2 / \hat{\theta}_1 \right) \right] + 1 = 0.$$

The solution to this quadratic equation is

$$\text{(A4)}\quad \hat{\rho} = \frac{1}{2} \pm \left(\frac{1}{4} - \frac{1}{4 + \left(\hat{\theta}_2^2 / \hat{\theta}_1 \right)} \right)^{1/2}.$$

Since $\hat{\sigma} = 1 + (2\hat{\rho} - 1)/[(1 - \hat{\rho})\hat{\theta}_2]$, choose the value for $\hat{\rho}$ in (A4) that will give $\hat{\sigma} > 1$ (i.e., choose $\hat{\rho} > \frac{1}{2}$ when $\hat{\theta}_2 > 0$, and $\hat{\rho} < \frac{1}{2}$ when $\hat{\theta}_2 < 0$). As $\hat{\theta}_2 \to 0$ then $\hat{\rho} \to \frac{1}{2}$ from (A4), and then the limiting value of $\hat{\sigma}$ is computed from the first equation in (A3).

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