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# External imbalances between China and the United States: A dynamic analysis with a life-cycle model

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ABSTRACT

In this paper I use a life-cycle model to study the role of population aging and pension income as drivers of China's persistent trade surplus vis-à-vis the United States. China's rapid increase in life expectancy coupled with its relatively low pension expenditures may help to explain the country's high savings rate, persistent trade surpluses and accumulation of a sizable net foreign asset position. Although China's high productivity growth has a strong negative impact on its trade balance, the model predicts a positive net foreign asset position and trade balance for China for most years in the simulation period.



## 1. Introduction

China and the United States, the world's largest economies, have experienced large and persistent external imbalances over recent decades. China has run a trade surplus since 1995, accumulating a stock of net foreign assets by 2015 that amounted to 15% of its GDP (red dashed lines in Fig. 1). The United States (US) has run a trade deficit since 1976, amassing net foreign debt equal to 40% of GDP by 2015 (blue solid lines in Fig. 1). US-China bilateral trade also has long been in a state of imbalance.

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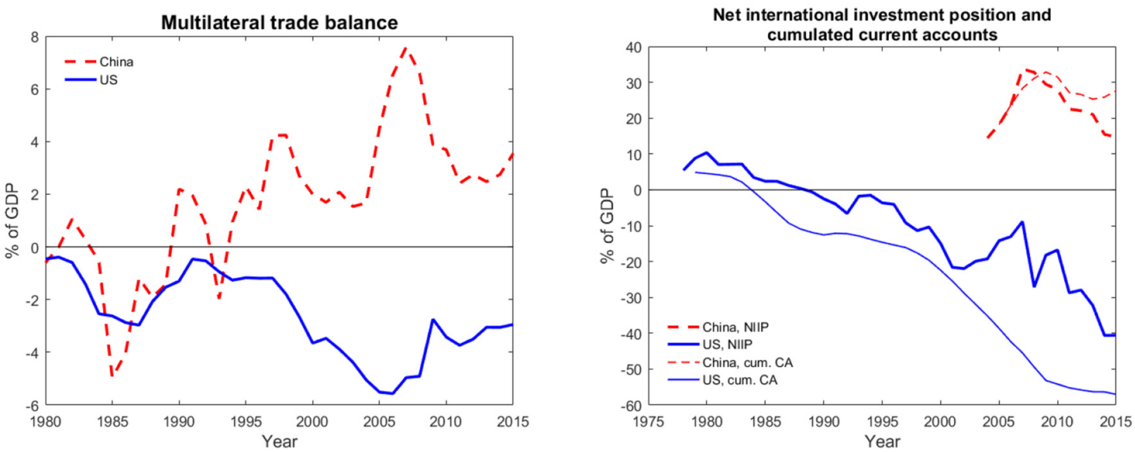


**Fig. 1.** Left panel: External balance on goods and services 1980–2015 for China and the US. Source: World Bank, World Development Indicators. Right panel: Net international investment position and cumulative current account balance. Source: IMF Balance of Payment Statistics and author's calculations.

US imports from China have exceeded US exports to China since the mid-1980s, and the bilateral trade deficit has increased at an accelerating rate until quite recently.[1]

This paper suggests that demographic trends and the disparity in the level of social security between China and the US have impacted household consumption and savings decisions and, therefore, on the observed external imbalances. During the period from 1980 to 2015, China experienced a rapid increase in life expectancy. Yet, while Chinese life expectancy approached that of the US, China's old-age social security expenditures remained significantly lower. When analyzed with a two-country life-cycle model, these trends indicate that the Chinese population has prepared for a longer retirement by increasing savings. This has, in turn, improved China's current account balance and contributed to the build-up of the country's foreign asset position. The impact of this channel appears quantitatively large, and the model predicts a positive trade balance for China for most of the simulation period.

The main contribution of this paper is to provide a complementary explanation for the persistent external imbalances between China and the US. Early research on external imbalances focuses on the industrialized world and struggles to explain the capital flows from fast-growing emerging economies to industrialized countries. This is in line with neoclassical theory, which predicts that capital flows into countries with rapid productivity growth (e.g., the discussion on the "capital allocation puzzle" in Gourinchas and Rey (2014)). A more recent strand of the literature considers external imbalances between emerging and industrialized economies (see e.g., Caballero et al. (2008), Mendoza et al. (2009), Song et al. (2011), and Coeurdacier et al. (2015)). It emphasizes the importance of financial markets in driving the capital outflows from the emerging world, suggesting, among other things, that capital flight from emerging markets signals both underdevelopment and an inability to supply safe assets. Apart from Coeurdacier et al. (2015), however, most authors of this strand place less emphasis on explaining the observed high household savings of Chinese or their low propensities to consume.

I take a different approach, using a two-country life-cycle model to quantify the importance of social security, demographic transition, productivity growth and government expenditures in shaping the dynamics of the bilateral trade balance and the net foreign asset position between the US and China during 1980–2015. The model is close to that of Ferrero (2010) in that it is a two-country model featuring a Gertler (1999) life-cycle structure. However, I extend this model to include a pay-as-you-go social security system, endogenous labor supply, and distortionary taxation.

The results suggest that demographic transition and the level of social security expenditures have a quantitatively significant impact on household consumption and saving decisions, and that this is reflected on the current account, trade balance and net foreign asset positions between China and the US. In the model, the aging of the population boosts savings rates as households prepare for the long retirement by accumulating financial wealth. Because of China's rapid demographic transition, the impact on its current account is positive. The relatively low social security expenditures in China relative to the US (approximately 2% of output in China and 6% of output in the US between 1980 and 2015) have a quantitatively large impact on the household savings. When both social security and demographic transition are accounted for, the model predicts a trade surplus for China for most of the simulation period. Even after controlling for high productivity growth and its strong negative impact on the trade balance, the model still predicts a positive net foreign asset position and trade balance for China for most years in the simulation period. Using a three-country extension of the model in which the rest of the world is calibrated to match the main trading partners of China and the US, the results of the paper remain qualitatively unchanged.

The rest of the paper is organized as follows. Section 2 discusses related literature. Section 3 presents the model. Section 4 presents the results of the static and dynamic analysis. Section 5 concludes.

[1] The current account and bilateral trade balance are plotted in Fig. 15 (Appendix A). The US current account has been negative since 1982, while China has run a current account surplus since 1994.

## 2. Literature review and contribution

The paper is related to studies that use life-cycle and overlapping generations models (OLG) to analyze questions related to population aging, social security, and external imbalances. More generally, it relates to the recent literature on the external imbalances between emerging and industrialized economies.

The core modeling framework in this study is the life-cycle model proposed by Gertler (1999). It allows modeling an economy with realistic average lengths of life, working time, and time spent in retirement. This type of model is thus well suited to studying the economic implications of the demographic transition and social security.

Relevant to this discussion, the Gertler (1999) framework has been used by Kilponen et al. (2006), Fujiwara and Teranishi (2008), Ferrero (2010) and Carvalho et al. (2016). Kilponen et al. (2006) relax Gertler's assumption of a stationary demographic structure to analyze the implications of population aging and social security in a small open economy (Finland). Fujiwara and Teranishi (2008) analyze the welfare implications of monetary and fiscal policy on different age groups in a closed economy. Carvalho et al. (2016) study the impact of population aging on the real interest rate in a closed economy, considering the role of social security as well. Distinct from Kilponen et al. (2006), Fujiwara and Teranishi (2008), and Carvalho et al. (2016), my focus is on external imbalances between two large economies. Unlike Kilponen et al. (2006) and Fujiwara and Teranishi (2008), I also abstract from nominal frictions as the focus is on the low-frequency dynamics. Furthermore, departing from Carvalho et al. (2016), my model features endogenous labor supply together with distortionary taxes so that the quantitative findings are robust to the effects of social security through the labor supply channel. Indeed, Ferrero (2010) is, to my knowledge, the only paper that uses the Gertler (1999) model to a two-country world, i.e., the US and six other industrialized countries, to analyze the dynamics of the external balance and the real interest rate. Ferrero (2010) shares the above-mentioned difficulty of explaining capital flows between emerging and industrialized countries as the model's predictions for China are counterfactual. Here, I extend the Ferrero (2010) model by introducing a pay-as-you-go (PAYG) social security system together with variable labor supply and distortionary labor taxes.

Examples of studies with multi-period OLG models include Saarenheimo (2005), Domeij and Floden (2006), and Attanasio et al. (2016), who analyze questions related to external imbalances, population aging, and social security in multi-country settings.[2] China is included as a region in Saarenheimo (2005) and Attanasio et al. (2016), who make projections for global variables including external imbalances and the real interest rate. These studies focus on projecting the future, rather than explaining observed dynamics, i.e., the results are driven by forecasts, not observed values of variables related to demographic, technological or fiscal development. In contrast to these papers, I also take into account the impact of population aging on external imbalances through the labor supply channel.

Two-country OLG models motivated specifically by China's external surplus include Eugeni (2015) and Coeurdacier et al. (2015). Eugeni (2015) analyzes the impact of social security on external imbalances and the real interest rate, finding that the absence of a social security system (PAYG pension system) leads to higher savings and capital exports. This, in turn, suggests that social security could play a role in explaining capital outflows from countries such as China. In contrast to Eugeni (2015), this paper features a rich demographic structure that can be used to analyze the effects of social security amidst demographic change. Coeurdacier et al. (2015) study external imbalances with a 3-period OLG model, in which external imbalances are driven by the growth differentials between countries and differences in the credit constraints faced by households in different economies. In contrast to Coeurdacier et al. (2015), I abstract from financial frictions to focus on the impact of demographics and social security arising in a frictionless financial market.

Chinese savings have also been studied with dynastic models by İmrohoroğlu and Zhao (2018a and 2018b). The impact of demographic change on household savings is different in a dynastic model than in life-cycle and OLG models. In a dynastic model, individuals derive utility from the well-being of their ancestors and future generations, not just their own well-being. Because the young save for themselves and to provide for the retired elderly, and the old save to leave bequests to the young, the aggregate effect of demographic change depends on which of the different savings motives dominates. As in the life-cycle model used here, İmrohoroğlu and Zhao (2018a) determine that population aging results in higher aggregate savings, while İmrohoroğlu and Zhao (2018b) find social security to be negatively associated with household saving.

Finally, several recent papers, including Caballero et al. (2008), Mendoza et al. (2009), Angeletos and Panousi (2011), Song et al. (2011), and Maggiori (2017), analyze the impact of financial markets on external imbalances between emerging and industrialized countries. Caballero et al. (2008) develop a model that rationalizes international capital flows as a global equilibrium outcome driven by differences in the abilities of different regions to supply financial assets. Mendoza et al. (2009) argue that when countries with less developed financial markets and low enforceability of financial contracts integrate with the international financial system, capital tends to flow towards more developed financial markets. Song et al. (2011) propose a model for the Chinese economy in which capital outflows occur because productive firms, unlike the less productive ones, have limited access to financial markets. Angeletos and Panousi (2011) explain capital flows from China to United States as an outcome of high

[2] Multi-period OLG models can account for the potential effects of heterogeneity within broader age groups such as the working-age population. In the Gertler (1999) model, working-age population is assumed to be homogenous. While an analysis with hetereogeneity within the working-age population presents a relevant extension to this paper, the challenges related to obtaining Chinese cohort-specific data on demographic development, labor supply, savings, and wages would first need to be overcome. Saarenheimo (2005) includes China as an area in the analysis, but abstracts from age- and cohort-specific life expectancies. While Attanasio et al. (2016) take into account age-specific fertility rates (without considering migration) and survival probabilities (not cohort-specific), they assume a fixed retirement age for all cohorts and regions, as well as a fixed labor supply for all age groups.

entrepreneurial risk, limited insurance possibilities, and precautionary savings. Maggiori (2017) explains the emergence of global imbalances and the role of United States as a provider of safe assets to the rest of the world as a result of higher level of US financial sector development. In this paper, I assume no frictions in financial markets in order to focus on the potential explanatory power of the life-cycle aspects and social security systems.

## 3. The model

The model is a symmetric two-country model with a life-cycle structure as in Gertler (1999).[3] The population in each country at time $t$ consists of two groups of individuals: workers, whose total number is $N_t^w$, and retirees, whose number equals $N_t^r$. All agents enter the economy as workers at the age of 20. They remain workers with probability $\omega_{t,t+1}$ or retire with probability $1 - \omega_{t,t+1}$. Retirees face the probability $\gamma_{t,t+1}$ of surviving to the next period. These parameters are calibrated to match the observed average lengths of life, time spent at work and time spent in retirement.

### 3.1. Households

The preferences of households are given by a constant elasticity of substitution (CES) non-expected utility function of the form

$$V_t^z = \left\{ \left[ (C_t^z)^{\upsilon} (1 - l_t^z)^{1-\upsilon} \right]^{\rho} + \beta_{t,t+1}^z \left[ E_t (V_{t+1}|z)^{\mu} \right]^{\frac{\rho}{\mu}} \right\}^{\frac{1}{\rho}}, z = \{w, r\} \tag{1}$$

where $C_t^z$ is consumption and $l_t^z$ the fraction of time allocated to work at time $t$ by a person of type $z$ (retiree if $z = r$ and worker if $z = w$). Each individual has one unit of time per period to use for either work or leisure. Parameter $\beta_t^z$ is the subjective discount factor, and $E_t(V_{t+1}|z)$ is the expectation of the value function in the next period of the person of type $z$. The Epstein-Zin preferences allow the separation of income risk aversion from aversion to intertemporal substitution. Parameter $\rho$ captures the intertemporal elasticity of substitution, which is given by $\sigma = 1/(1 - \rho)$. Parameter $\mu$ captures attitudes towards income risk. Risk neutrality (i.e. $\mu = 1$) is assumed to yield consumption decisions that are linear in wealth, which facilitates the aggregation of the model. The only source of income risk is the exogenous probability of the worker's retirement, so the effect of income risk aversion is reasonable.

#### 3.1.1. Retirees

A retiree's expectation of the value function is $E_t(V_{t+1}|r) = V_{t+1}^r$. As he or she takes into account the probability of dying before the next period, the effective discount factor is $\beta_{t,t+1}^r = \beta\gamma_{t,t+1}$. A retiree, born in period $j$ and retired in period $i$, chooses consumption-saving allocation and leisure to maximize

$$V_t^{jr}(i) = \left\{ \left[ \left( C_t^{jr}(i) \right)^{\upsilon} (1 - l_t^r)^{1-\upsilon} \right]^{\rho} + \beta\gamma_{t,t+1} \left( V_{t+1}^{jr}(i) \right)^{\rho} \right\}^{\frac{1}{\rho}} \tag{2}$$

subject to

$$A_{t+1}^{jr}(i) = \frac{R_t A_t^{jr}(i)}{\gamma_{t-1,t}} + W_t \xi l_t^{jr} (1 - \tau_t) + S_t^{jr} - C_t^{jr}(i). \tag{3}$$

Retirees consume out of their non-human wealth $A_t^r$, labor income $W_t \xi l_t^{jr}$, where $W_t$ is the wage per unit of labor, net of distortionary labor income taxes $\tau_t$ and lump sum social security transfer $S_t^{jr}$. The productivity of a unit of labor provided by retirees is $\xi \in (0,1)$ times that of a worker, which leads to a lower labor supply by retirees in the equilibrium. Retirees participate in a perfect annuities market that provides insurance against the uncertainty of the time of death such that each retiree receives a gross return on wealth of $R_t/\gamma_{t-1,t}$. $R_t$ is the real interest rate. The pension scheme is a public pay-as-you-go (PAYG) pension system in which the pension income is a transfer from taxpayers to retirees.

The first order condition with respect to leisure is

$$l_t^{jr}(i) = 1 - \frac{C_t^{jr}(i)\varsigma}{W_t \xi (1 - \tau_t)} \tag{4}$$

where $\varsigma = \frac{1-\upsilon}{\upsilon}$. The retiree's decision rule for consumption is given by

$$C_t^{jr}(i) = \varepsilon_t \pi_t \left( \frac{R_t A_t^{jr}(i)}{\gamma_{t-1,t}} + H_t^{jr}(i) + P_t^{jr}(i) \right) \tag{5}$$

[3] The life-cycle structure is described in Appendix C.

where $\varepsilon_t\pi_t$ is the marginal propensity to consume (mpc) out of wealth and $H_t^{jr}(i)$ and $P_t^{jr}(i)$ are the present discounted values of a retiree's lifetime human wealth and pension benefits. The mpc evolves according to the nonlinear difference equation

$$\varepsilon_t\pi_t = 1 - \frac{\varepsilon_t\pi_t}{\varepsilon_{t+1}\pi_{t+1}}\gamma_{t,t+1}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho\sigma(1-v)}\beta^\sigma(R_{t+1})^{\rho\sigma} \tag{6}$$

*3.1.2. Workers*

A worker's expectation of the value function in the next period is $E_t(V_{t+1}|w) = \omega_{t,t+1}V_{t+1}^w + (1-\omega_{t,t+1})V_{t+1}^r$ and the effective discount factor is $\beta_{t,t+1}^w = \beta$. A worker born in period j chooses consumption-saving allocation and leisure to maximize

$$V_t^{jw} = \left\{\left[\left(C_t^{jw}\right)^v\left(1-l_t^{jw}\right)^{1-v}\right]^\rho + \beta\left[\omega_{t,t+1}V_{t+1}^{jw} + \left(1-\omega_{t,t+1}\right)V_{t+1}^{jr}\right]^\rho\right\}^{\frac{1}{\rho}} \tag{7}$$

subject to

$$A_{t+1}^{jw} = R_tA_t^{jw} + W_tl_t^{jw}(1-\tau_t) - C_t^{jw} \tag{8}$$

As retirees, workers consume out of non-human wealth and wage income net of labor income taxes. Workers and retirees in both countries consume a single (numeraire) good that can be traded internationally.

The first-order condition with respect to labor is

$$l_t^{jw} = 1 - \frac{C_t^{jw}\varsigma}{W_t(1-\tau_t)} \tag{9}$$

The consumption Euler equation for workers is

$$C_t^{jw}\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-v)}\beta R_{t+1}\Omega_{t+1}\right]^\sigma = \omega_{t,t+1}C_{t+1}^{jw} + (1-\omega_{t,t+1})\varepsilon_{t+1}^{\frac{\sigma}{1-\sigma}}\chi C_{t+1}^{jr} \tag{10}$$

where $\chi = \xi^{-(1-v)}$ and $\Omega_t \equiv \omega_{t-1,t} + (1-\omega_{t-1,t})\varepsilon_t^{\frac{1}{1-\sigma}}\chi$ is an additional discount factor in the workers' value function. The worker's mpc, $\pi_t$, evolves according to

$$\pi_t = 1 - \frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho\sigma(1-v)}\beta^\sigma(R_{t+1}\Omega_{t+1})^{\sigma-1} \tag{11}$$

The worker's decision rule for consumption is

$$C_t^{jw} = \pi_t\left(R_tA_t^{jw} + H_t^{jw} + P_t^{jw}\right) \tag{12}$$

where $H_t^{jw}$ is the present discounted value of a worker's human wealth net of taxation and $P_t^{jw}$ is the present discounted value of a worker's pension benefits once retired.

*3.1.3. Aggregation*

Because marginal propensities to consume for both retirees and workers do not depend on individual characteristics, aggregate consumption can be expressed as

$$C_t = \pi_tA_tR_t(\varepsilon_t\lambda_t + 1 - \lambda_t) + \pi_t(H_t^w + P_t^w) + \varepsilon_t\pi_t(H_t^r + P_t^r) \tag{13}$$

where $\lambda_t$ denotes the share of financial assets held by retirees such that $\lambda_t = \frac{A_t^r}{A_t}$, where $A_t$ is aggregate financial wealth and $A_t^r$ retirees' financial wealth. $H_t^w$ and $H_t^r$ are the present discounted values of workers' and retirees' aggregate human wealth and $P_t^w$ and $P_t^r$ the present discounted values of social security wealth. The asset distribution evolves according to

$$\lambda_{t+1} = \omega_{t,t+1}\left(R_t\lambda_t\frac{A_t}{A_{t+1}}(1-\varepsilon_t\pi_t) + \frac{W(1-\tau_t)L_t^r\xi + S_t - \varepsilon_t\pi_t(H_t^r + P_t^r)}{A_{t+1}}\right) + (1-\omega_{t,t+1}) \tag{14}$$

where $S_t$ are the aggregate social security transfers paid to retirees in period $t$. The present discounted value of retirees' aggregate human wealth is

$$H_t^r = \xi L_t^r W_t(1-\tau_t) + \gamma_{t,t+1}\frac{\psi_t}{\psi_{t+1}}\frac{H_{t+1}^r}{(1+n_{t,t+1})R_{t+1}}, \tag{15}$$

where $\psi_t$ is the old age dependency ratio, i.e. $\psi_t = \frac{N_t^r}{N_t^w}$, and $n_{t,t+1}$ is the growth rate of the number of workers between periods $t$ and $t+1$. $L_t^r$ is retirees' aggregate labor supply. The discount rate of the present value of total human wealth for current retirees is augmented by the growth rate of the retired labor force $\left(\frac{\psi_t}{\psi_{t+1}}\right)$. Similarly, the present discounted value of workers' aggregate human wealth is

$$H_t^w = L_t^w W_t(1-\tau_t) + \omega_{t,t+1}\frac{H_{t+1}^w}{(1+n_{t,t+1})R_{t+1}\Omega_{t+1}} + (1-\omega_{t,t+1})\frac{H_{t+1}^r \varepsilon_{t+1}^{\frac{1}{1-\sigma}}\chi}{\psi_{t+1}(1+n_{t,t+1})R_{t+1}\Omega_{t+1}}, \tag{16}$$

where $L_t^w$ is workers' aggregate labor supply. Present discounted value of retirees' aggregate pension benefits at time t is

$$P_t^r = S_t + \gamma_{t,t+1}\frac{\psi_t}{\psi_{t+1}}\frac{P_{t+1}^r}{R_{t+1}} \tag{17}$$

and the present discounted value of social security for workers is

$$P_t^w = \omega_{t,t+1}\frac{P_{t+1}^w}{(1+n_{t,t+1})R_{t+1}\Omega_{t+1}} + (1-\omega_{t,t+1})\frac{P_{t+1}^r \varepsilon_{t+1}^{\frac{1}{1-\sigma}}\chi}{\psi_{t+1}(1+n_{t,t+1})R_{t+1}\Omega_{t+1}} \tag{18}$$

### 3.2. Firms

The goods market is competitive and the representative firm produces consumption goods with constant returns to scale Cobb-Douglas production technology. Aggregate output is given by $Y_t = (X_t L_t)^\alpha K_t^{1-\alpha}$, where $X_t$ is the level of exogenous labor augmenting productivity at time $t$,$L_t$ is the aggregate effective labor force, $K_t$ is physical capital, and $\alpha \in (0,1)$ is the labor share.[4]

The aggregate effective labor force consists of the effective labor input by the two agent types such that $L_t = L_t^w + \xi L_t^r$. Capital depreciates at rate $\delta \in (0,1)$ and quadratic adjustment cost makes investing new capital costly. The size of the adjustment cost is given by $\phi > 0$. Productivity $X_t$ grows at rate $x_t$, which follows an AR(1) process given by $x_t = (1-\theta)x_{ss} + \theta x_{t-1} + u_t^x$.

### 3.3. Government

Government consumes $G_t$ in each period and pays retirees a total amount of $S_t$ of social security benefits. Expenditures are financed with tax revenues $T_t$ and by issuing one-period debt $B_{t+1}$. Government spending and social security are assumed to be exogenously determined fractions of output: $G_t = g_t Y_t$ and $S_t = s_t Y_t$. The government's per period budget constraint is

$$B_{t+1} = R_t B_t + G_t + S_t - T_t \tag{19}$$

and the intertemporal budget constraint is

$$R_t B_t = \sum_{v=0}^{\infty}\frac{T_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty}\frac{G_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty}\frac{S_{t+v}}{\prod_{z=1}^{v} R_{t+z}} \tag{20}$$

The government's per-period discount rate equals the world interest rate $R_t$ and is lower than the households' per-period discount rates. With elastic labor supply, total tax revenue is

$$T_t = \tau_t W_t L_t. \tag{21}$$

As in Leeper (1991), labor tax $\tau_t$ is adjusted according to the fiscal policy rule

$$\tau_t = \tau_{t-1} + \theta\left[b_t - \overline{b}_t\right] \tag{22}$$

so that in the long run, the debt-to-output ratio $b_t = \frac{B_t}{Y_t}$ converges to an exogenously determined steady state value $\overline{b}_t$.

[4] See Appendix F for further details on the firm.

### 3.4. A competitive world equilibrium

A competitive world equilibrium is a sequence of quantities and prices such that in each country (i) households maximize utility subject to their budget constraints, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance an exogenous level of total spending, and (iv) all markets clear.

In each economy, households' aggregate non-human wealth equals the aggregate capital stock, government bonds, and net foreign assets $F_t$ such that

$$A_t = K_t + B_t + F_t \tag{23}$$

The net foreign asset position evolves according to

$$F_{t+1} = R_t F_t + NX_t \tag{24}$$

while the trade balance $NX_t$ is determined by the aggregate resource constraint

$$NX_t = Y_t - C_t - I_t - G_t. \tag{25}$$

The current account $CA_t$ consists of the trade balance and net interest payments on foreign assets

$$CA_t = NX_t + (R_t - 1)F_t \tag{26}$$

Return $R_t$ is equalized across the two countries, and in equilibrium, foreign asset positions in the two countries sum up to 0, i.e., $F_t + F_t^* = 0$ so that the internationally traded asset is in zero net supply. The law of one price is assumed to hold.

## 4. Quantitative analysis

Having described the model, I now analyze the drivers of the bilateral external imbalances between US and China during 1980–2015. During this 35-year period, China and the US experienced persistent external imbalances both multilaterally and vis-à-vis each other, accumulating large foreign asset positions of opposite signs. The period coincides with China's transition towards a market economy which started in 1978 and made China an active participant in international trade.[5]

In the following discussion, I introduce several potential drivers of the external imbalances as exogenous variables into the two-country life-cycle model presented in Section 3, and simulate the transition dynamics of the model during 1980–2015. The potential drivers include demographic changes and differences in pension systems, fiscal policy, and productivity growth. The dynamic transition of the model economy is first simulated with a standard deterministic simulation, and then with a deterministic simulation with updates in which the paths of exogenous variables are updated in every period. In the second simulation, agents revise their expectations according to the latest available information. The method, also known as the extended path method, was originally proposed by Fair and Taylor (1983).

Section 4.1 discusses the data and evolution of the exogenous variables in both US and China between 1980 and 2015. Section 4.2 presents the calibration of the model and a comparative static analysis. Section 4.3 presents the results of a deterministic simulation and section 4.4 a deterministic simulation with updates. Section 4.5 discusses the role of the labor supply in the model.

### 4.1. Demographics, social security, fiscal policy, and technological progress in the US and China during 1980–2015

The change in the countries' demographic structures is the largest long-term trend included in the simulation. During the simulation period, population growth rates fell and life expectancies increased in both countries, leading to an increase in of the old-age dependency ratios (the ratio of population aged 65+ to population aged 20–64 years, see Figs. 2 and 3). This trend is projected to continue for several more decades. However, both the demographic structures and the pace of change have differed between the countries. In 1980, there were almost twice as many elderly in the US for every working-age individual as in China, but towards the end of the 20th century, the difference in old-age dependency ratios declined. This was mainly due to the relatively rapid growth in life expectancy in China, which grew by eight years (from 67.3 to 75.4 years) between 1980 and 2015, in comparison to six years (from 72.9 to 78.9 years) in the US. The convergence of the old-age dependency ratios was dampened by the fact that the Chinese population growth rate remained higher than in the US over the entire period (and despite a rapid decline amplified by the introduction of the one-child policy in 1979).

Based on the United Nations population forecast revisions, this demographic transition does not appear to have been fully anticipated. As estimation and forecasting methodologies have evolved over time and new census data has become available, the UN

[5] Given China's external trade openness during the entire period, it is worthwhile to study the dynamics of the trade balance since the early 1980s, i.e., even when many of the features of China's planned economy were still in place. Analysis of the effects of China's political system, however, is beyond the scope of this paper and must be left for future research.

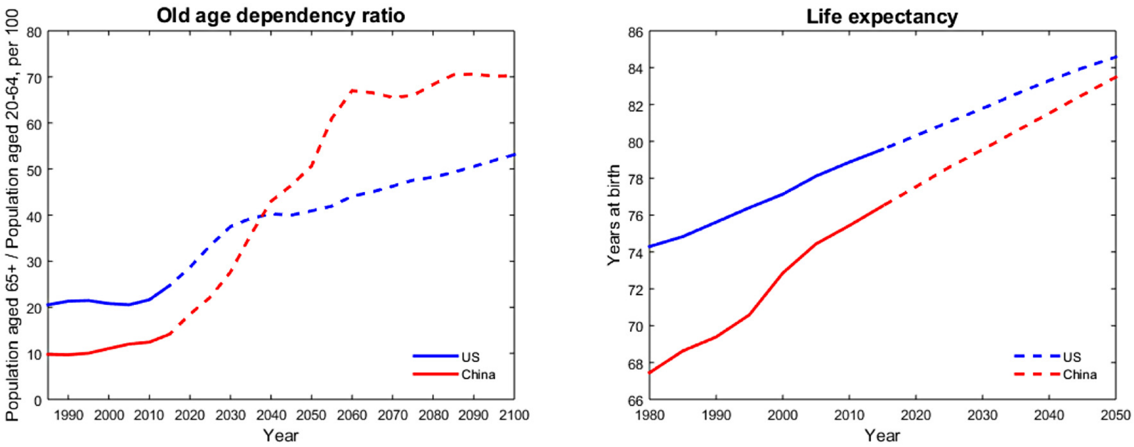


**Fig. 2.** Left panel: Old-age dependency ratio in the US and China (data frequency: 5 years) between 1985 and 2015 and projections until 2100. Right panel: Life expectancy at birth in the US and China in 1980–2015 and projections (medium variant) until 2050. Source: United Nations World Population Prospects: The 2015 Revision.

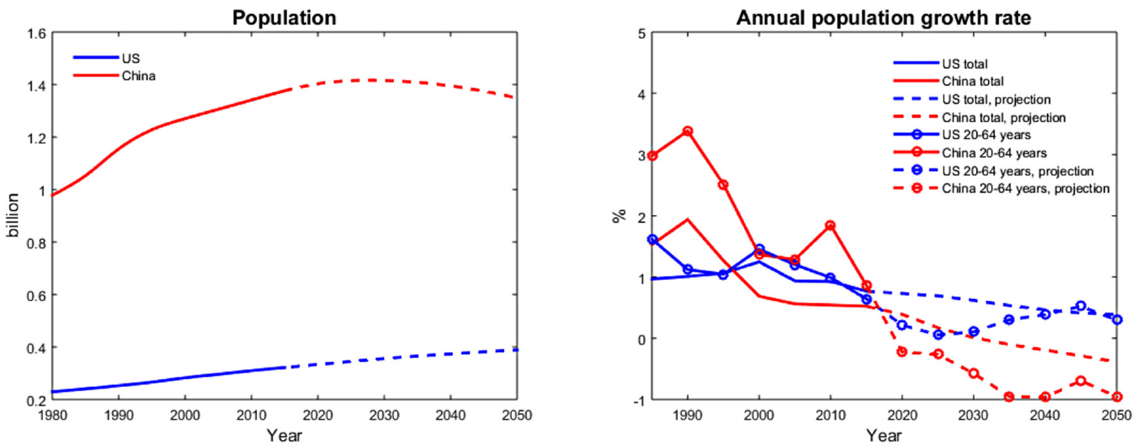


**Fig. 3.** Left panel: Total population in the US and China in 1980–2015 and projections until 2050. Right panel: The annual population growth rate in the given year and five preceding years 1985–2015 and projections to 2050. Source: United Nations World Population Prospects: The 2015 Revision.

forecasts of life expectancy at birth have been revised upwards in nearly every revision round (see Fig. 17, Appendix A.3), and the forecasts of population growth rates have been revised upwards for the US and downwards for China several times each (see Fig. 18, Appendix A.3). The time horizon of the demographic projections has also increased over time. Up to 1994, the projections were made until 2025. During 1994–2008, they were extended to 2050. Since 2010, they have targeted 2100. In the simulation with updates (in section 4.4), the paths of exogenous demographic variables are adjusted in each period so that the demographic variables in the model match the values of the most recent figures in the UN World Population Prospects database.

Differences in the level of social security expenditures are also included in the simulation as potential drivers of the external imbalances. In recent years, public pension spending in China as a share of GDP has been only one-third that of the US (2.1% versus 6.6%; see Fig. 4). The GDP share of public social security expenditures has not changed substantially in either country during the time for which the data is available. China's low GDP share of pension spending is not explained by its younger population structure alone. Had the pension spending been proportional to the old-age dependency ratio, the social security expenditures in China would have needed to be 3.3–4.0% of GDP to match the US level. In addition, the coverage of the pension scheme, defined as the share of population aged 15–65 covered by mandatory pension schemes, is only approximately 27.7% in China (2010), compared to 71.4% in the US (2005) (OECD, 2013). In the simulations, the pension system is assumed to have full coverage, which is likely to yield conservative results with regard to the impact of social security on households' savings.[6]

The simulation also takes into account the differences in government expenditures and their impact on the external imbalances. As shown in Fig. 4, government expenditures have been higher in the US (15.5% of GDP on average) than in China

[6] See Appendix D for a description of the old-age pension systems in China and the US.

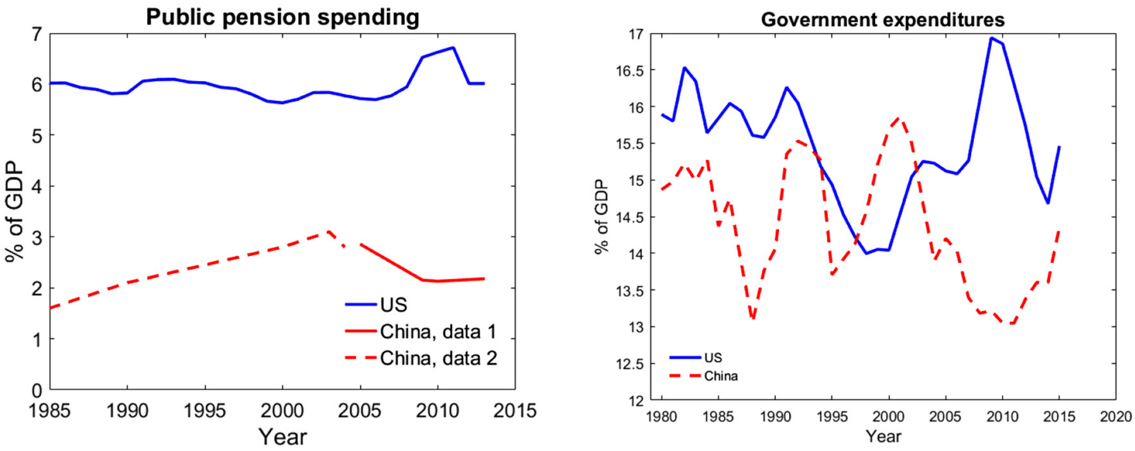


**Fig. 4.** Left panel: Public pension expenditures, % of GDP in the US and China. Source: OECD Data on Social Protection and Asian Development Bank: Social Protection Index Database (data 1); Naughton (2007) (data 2). Right panel: General government final consumption expenditures. Source: World Bank, World Development Indicators 2016.

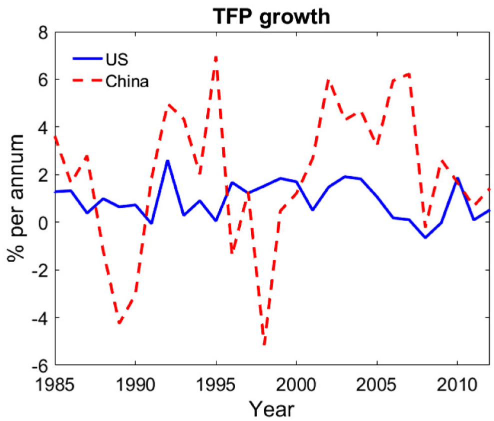


**Fig. 5.** Annual TFP growth in the US and China 1980–2012. Source: Penn World Table 9.0.

(14.4% of GDP on average) for most of the simulation period. Data for net government debt is not available for China, and therefore government debt is assumed to evolve according to the same fiscal rule in both countries.

Finally, productivity growth is included in the simulations as a potential driver of the external imbalances. The counterfactual implications about trade surplus and net foreign debt that several neoclassical models suggest for China in the earlier literature are driven by its high productivity growth during most of the simulation period. Fig. 5 shows the TFP growth rate for China and the US. On average, TFP grew by 1.9% in China and by 0.9% in the US every year between 1980 and 2012 according to data from the Penn World Table 9.0.

### *4.2. Calibration and the steady state*

To analyze the dynamics of the external imbalances between China and the US between 1980 and 2015, the time-varying exogenous variables, which include life expectancy, population growth rate, pension and other government expenditures, and total-factor productivity (TFP), are set to match the values directly observable in the data. Given the values of the time-varying exogenous variables, the model parameters are then set to match the means of the simulated GDP shares of the trade balance, consumption and investment, as well as the mean of the world real interest rate, with data during the simulation period.

Table 1 presents the values of exogenously determined variables related to demographics, fiscal policy, pensions, and TFP in the initial and final states. The dynamics are solved with two different simulation methods (with and without updates on the exogenous variables), and the table presents the values for both simulations.

In the first simulation (I), the model is calibrated so that in the initial state, the economies match the data in 1980, and converge to a new steady state in which the exogenous variables match the data in 2015. The economies are not in a steady state in 2015, even though the exogenous variables remain fixed from then onwards as it takes longer for several endogenous variables to

**Table 1**
Values of exogenous variables (technological growth rate, demographic variables, government debt and expenditures, and social security expenditures) in the initial state (1980), the final state (2015 in the deterministic simulation and 2100 in the simulation with updates, which is the final year of the United Nation's population forecast) and over time (mean in the period 1980–2015) in the data and the model. (I) Deterministic simulation (II) Simulation with updates.

| Exogenous variable | | Data | Model | | | |
|---|---|---|---|---|---|---|
| | | | I | | II | |
| | | | Initial state 1980 | Final state | Initial state 1980 | Final state |
| $x_1$, $x_2$ | technology growth rate (US, China) | 0.01 (mean 1980–2015 between China and the US) | 0.01 | 0.01 | 0.01 | 0.01 |
| $n_1$, $n_2$ | population growth rate (US, China) | 1980: 0.0215 (mean between China and the US) 2015: 0.0052100: 0.00 (projected mean in 2015) | 0.0215 | 0.005 | 0.0215 | 0.00 |
| $\gamma_1$ | probability to survive (if retired)/life expectancy at birth (US) | 1980: 72.9 years (estimate in 1980) 2015: 78.9 years (estimate in 2015) 2100: 88.7 years (projection in 2015) | 0.8758/72.9 | 0.9289/78.9 | 0.8758/72.9 | 0.9580/88.7 |
| $\gamma_2$ | probability to survive (if retired)/life expectancy at birth (China) | 1980: 67.3 years (estimate in 1980) 2015: 75.4 years (estimate in 2015) 2100: 85.2 years (projection in 2015) | 0.7506/67.3 | 0.9174/75.4 | 0.7506/67.3 | 0.9543/85.2 |
| $\omega_1$ | probability to stay in the labor force/effective retirement age (US) | 65 years | 0.9777/65 | 0.9777/65 | 0.9777/65 | 0.9777/65 |
| $\omega_2$ | probability to stay in the labor force/effective retirement age (China) | 63 years | 0.9769/63 | 0.9769/63 | 0.9769/63 | 0.9769/63 |
| $b_1$ | government debt, % of GDP (US) | | 0.2 | 0.2 | 0.2 | 0.2 |
| $b_2$ | government debt, % of GDP (China) | | 0.2 | 0.2 | 0.2 | 0.2 |
| $g_1$ | government spending, % of GDP (US) | 0.155 | 0.155 | 0.155 | 0.155 | 0.155 |
| $g_2$ | government spending, % of GDP (China) | 0.144 | 0.144 | 0.144 | 0.144 | 0.144 |
| $s_1$ | social security spending, % of GDP (US) | 0.066 | 0.060 | 0.070 | 0.060 | 0.060 |
| $s_2$ | social security spending, % of GDP (China) | 0.021 | 0.020 | 0.020 | 0.020 | 0.020 |

reach their new steady state values. In the initial and final steady states, productivity, and population growth rates ($x$ and $n^w$) are the same across the countries to prevent one economy from outgrowing the other. The annual productivity growth rate is 1% in both steady states and economies, which is the average of the observed growth rates between the countries between 1980 and 2015. The annual growth rate of working-age population decreases gradually from 2.15% in 1980 to 0.5% in 2015 and thereafter. The probability of staying in the labor force, $\omega$, is the same in the initial and final state, and it is slightly higher in the US than in China ($\omega_{US} = 0.9777$ and $\omega_{China} = 0.9769$), corresponding to an (effective) age of retirement of 63 years in China and and 65 years in the US. Life expectancy, captured by the survival probability $\gamma$, differs between the countries in both states. It corresponds to the observed values of 72.9 years (US) and 67.3 years (China) in 1980 and 78.9 years (US) and 75.4 years (China) 2015, respectively. Government spending and public pension spending are permanently at different levels, matching the average in the data between 1980 and 2015. Government debt in both countries is 20% of GDP (recall that no data on Chinese government net debt is available).

In the second simulation (II), the model is calibrated so that the economies match the 1980 data in the initial state and converge to a new steady state in which demographic variables match the values projected for 2100, which is the last year in the 2015 demographic forecast. However, the agents' information set regarding the values of exogenous variables during the transition and at the terminal point, as well as the date when the terminal point is reached, is updated in each period. In particular, the agents' information set regarding demographic variables consists of the most recent UN population forecast. Thus, as each new UN population forecast becomes available, the agents' information set is updated. In the final step of the simulation, the final steady state values match the predicted values for 2100 according to the 2015 UN forecast. The annual steady state growth rate of working-age population decreases gradually from 2.15% in 1980 to 0% in the final steady state, which is the average population growth rate projected for the countries for 2100. Life expectancy increases from 72.9 years (US) and 67.3 years (China) in 1980 to 88.7 years (US) and 85.2 years (China) in 2100. Other exogenous variables are set as in the first simulation.

Table 2 presents the values of time-invariant parameters. They are chosen to be the same across the countries so that the results are driven by the dynamics of the observables instead of differences in the unobservable fixed parameters. The labor share of output, $\alpha$, equals 2/3 and follows the convention in the literature (e.g., Cooley (1995) and Gertler (1999)).[7] The discount factor, $\beta$, equals 0.994 so that the net real interest rate matches the mean value of 3% over the sample period in the data. Consumption share in utility, $\upsilon$, and the relative productivity of retirees, $\xi$, are chosen to match employment rates among population aged 15–64 and 65 and over. The value for the consumption share in utility falls between the values used in related papers, including

[7] According to the International Labour Organization (2015), the labor share of output declined both in the US and in China over recent decades. Bai et al. (2006) estimate the labor share of output to be 50–54% in China. Analyzing the impact of different production technologies is beyond the scope of this study.

**Table 2**
Values of the time-invariant parameters in the simulation. The parameters are uniform across the countries.

| Parameter | | Value |
|---|---|---|
| $\alpha$ | labor share of output | $\frac{2}{3}$ |
| $\beta$ | discount factor | 0.994 |
| $\delta$ | depreciation rate | 0.2 |
| $\phi$ | investment adjustment cost factor | 3 |
| $\sigma$ | elasticity of intertemporal substitution | 0.55 |
| $\xi$ | productivity of a unit of labor, retiree to worker | 0.5 |
| $\upsilon$ | elasticity of period utility with respect to consumption | 0.7 |

Gertler (1999) ($\nu = 0.4$) and Kilponen et al. (2006) ($\nu = 0.8$).[8] There is no consensus in the literature about the value of the elasticity of intertemporal substitution, $\sigma$, the values ranging from 0.1 (Hall (1988)) to 2 (Gruber (2013)). A recent meta-study by Havranek et al. (2015) reports that, in a sample of 169 studies, the mean estimate for the elasticity of intertemporal substitution is 0.594 for the US and 0.530 for China; the value chosen here lies in this range. Depreciation rate, $\delta$, and the investment adjustment cost factor, $\phi$, are chosen so as to match the mean of the investment-to-output ratio over the sample period. Table 3, which compares the simulated moments with data, shows that the model matches the data reasonably well.

*4.2.1. Steady state implications of the model*

Before moving to the dynamic simulations, I analyze the steady state implications of the model. Throughout the simulation period, China has had a younger population and lower level of social security expenditures than the US. This subsection presents a comparative static analysis around the steady state to illustrate the effects of life expectancy and social security in the model.

Fig. 6 shows the steady state effects of life expectancy. The economies are assumed to be in a steady state in which the values of exogenous variables match the average between the countries in the data in 2015, apart from $\gamma$, the parameter determining life expectancy, which is allowed to differ. For the US, $\gamma_1$ is fixed at 0.9319, which corresponds to a life expectancy of 79.5 years. For China, $\gamma_2$ varies between 0.92 and 0.9319. At the dotted vertical line the life expectancy in China matches the projected value in 2020 ($\gamma_2$=0.9247 corresponding to a life expectancy of 76.5 years), when the difference in life expectancies between the countries is three years.

The model predicts that the country with higher life expectancy accumulates a positive net foreign asset position and satisfies its higher consumption demand by importing goods from abroad, thereby running a permanent trade deficit. High life expectancy increases the motivation of households to save for retirement and thereby hold more financial wealth. Household wealth is invested partly in foreign assets as the country with lower life expectancy is willing to borrow from abroad as the world interest rate is below its autarky rate. Even though the country with higher life expectancy runs a permanent trade deficit, it earns a sufficiently high interest rate on its foreign assets to be able to hold a permanent positive net foreign asset position. The marginal propensities to consume of its residents are lower, but its consumption is high because the residents are wealthier and a higher share of non-human wealth is held by retirees, whose mpc is higher than that of workers.

The observed difference in life expectancies between China and the US in 2020, three years, implies a positive trade balance of 0.002% of GDP and a negative net foreign assets position of 4% of GDP for China in the steady state. The higher the life expectancy in China (the larger $\gamma_2$), the smaller its trade surplus and net foreign debt. Moreover, the higher the life expectancy in China, the lower the world real interest rate as an increase in life expectancy raises household savings and the aggregate level of financial wealth in the world. The effect is qualitatively similar to that documented by Ferrero (2010).

Fig. 7 presents the comparative statics with public pension spending. Apart from social security, which is allowed to differ between the countries, the economies identical so that the values of exogenous variables correspond to the average of observed levels over the sample period. Public pension spending in the US is set at 6% of GDP, and varies from 2% to 6% in China.

The model predicts that, in the steady state, the country with lower social security expenditures holds a positive net foreign asset position and runs a trade deficit. Because the retirees in that country have lower pension income, households accumulate more assets before retirement, which pushes down the world real interest rate. The world interest rate is below the autarky rate of the country with higher social security expenditures, which causes capital to flow from the country with less social security (China) to the country with more social security (US). Due to the lower pension expenditures, the low-pension country can also maintain a lower tax rate, which leads to higher labor supply and level of human wealth. Because of its relatively high human and financial wealth, its aggregate consumption is higher. High consumption demand is partly satisfied with foreign imports, which are financed with interest payments on the net foreign assets held by the country. The effect of social security on external imbalances is quantitatively significant, and it seems likely that social security can be an important factor in understanding the high level of savings in China. The observed 4% difference in the share of GDP that is spent on public pensions implies a trade deficit

[8] Households' labor supply choice involves a labor force participation choice (the extensive margin, typically measured by the number of individuals at work), and a choice on the effective number of hours of work supplied (the intensive margin, typically measured by the hours worked by worker). In the model, the extensive margin is not explicitly modeled, so the pool of workers consists of the entire working-age population. I match the labor supply with data on employment rates in each age group, defined as share of employed to the size of population (the extensive margin) instead of hours worked per worker as it would overestimate labor supply of the working-age population.

**Table 3**
Comparison of the key moments (mean, minimum and maximum values and standard deviation) of variables in the data and in the model during the simulation period (1980–2015). (I) Deterministic simulation (II) Simulation with updates.

| Variable | | Value in the model (I) | Value in the model (II) | Value in data | Data source |
|---|---|---|---|---|---|
| | | mean min/max st. dev. | mean min/max st. dev. | mean min/max st. dev. | |
| R | World real interest rate | 1.03<br>1.03/1.07<br>0.02 | 1.02<br>1.01/1.04<br>0.01 | 1.03<br>1.00/1.08<br>0.02 | Nominal interest rate: IMF, International Financial Statistics. CPI inflation: IMF, World Economic Outlook 2015. |
| $\frac{NX_1}{Y_1}$ | Bilateral trade balance/output (US) | −0.01<br>−0.06/0.04<br>0.03 | −0.02<br>−0.05/0.01<br>0.01 | −0.01<br>−0.02/−0.01<br>0.00 | Bureau of Economic Analysis (BEA), U.S. Department of Commerce, author's calculation. Period: 1999–2015. |
| $\frac{NX_2}{Y_2}$ | Bilateral trade balance/output (China) | 0.00<br>−0.03/0.03<br>0.02 | 0.02<br>−0.01/0.03<br>0.01 | 0.06<br>0.03/0.09<br>0.02 | Bureau of Economic Analysis (BEA), U.S. Department of Commerce, author's calculation. Period: 1999–2015. |
| $\frac{NX_1}{Y_1}$ | Multilateral trade balance/output (US) | – | – | −0.03<br>−0.06/−0.00<br>0.02 | World Bank, World Development indicators. |
| $\frac{NX_2}{Y_2}$ | Multilateral trade balance/output (China) | – | – | 0.02<br>0.02<br>0.03 | World Bank, World Development indicators. |
| $\frac{F_1}{Y_1}$ | Net foreign assets/output (US) | −0.23<br>−0.47/−0.11<br>0.09 | −0.39<br>−0.83/−0.01<br>0.26 | −0.38<br>−1.72/1.57<br>0.79 | IMF Balance of Payment Statistics and World Economic Outlook |
| $\frac{F_2}{Y_2}$ | Net foreign assets/output (China) | 0.15<br>0.06/0.21<br>0.05 | 0.23<br>0.01/0.41<br>0.13 | 0.09<br>0.02/0.12<br>0.03 | IMF Balance of Payment Statistics and World Economic Outlook. Period: 2003–2015. |
| $\frac{C_1}{Y_1}$ | Consumption/output (US) | 0.53<br>0.50/0.56<br>0.02 | 0.53<br>0.51/0.56<br>0.01 | 0.65<br>0.60/0.69<br>0.02 | World Bank, World Development indicators. |
| $\frac{C_2}{Y_2}$ | Consumption/output (China) | 0.50<br>0.48/0.53<br>0.02 | 0.48<br>0.47/0.50<br>0.01 | 0.44<br>0.34/0.53<br>0.06 | World Bank, World Development indicators. |
| $\frac{I_1}{Y_1}$ | Investment/output (US) | 0.32<br>0.30/0.34<br>0.01 | 0.34<br>0.31/0.35<br>0.01 | 0.22<br>0.18/0.25<br>0.02 | IMF, World Economic Outlook 2015. |
| $\frac{I_2}{Y_2}$ | Investment/output (China) | 0.35<br>0.33/0.37<br>0.01 | 0.36<br>0.34/0.38<br>0.01 | 0.40<br>0.32/0.48<br>0.05 | IMF, World Economic Outlook 2015. |
| $\frac{L_1^w}{N_1^w}$ | Employment/population, ages 15–64 years (US) | 70.3<br>65.8/74.1<br>2.5 | 70.3<br>68.5/72.4<br>1.0 | 70.1<br>69.2/70.9<br>0.5 | OECD Labour Force Statistics 2016. |
| $\frac{L_2^w}{N_2^w}$ | Employment/population, ages 15–64 years (China) Note: Data available for 2000 and 2010 | 75.1<br>79.3/75.1<br>– | 72.5<br>70.1/73.7<br>0.9 | 73.1<br>72.8/73.5<br>0.2 | OECD Labour Force Statistics 2016. |
| $\frac{L_1^r}{N_1^r}$ | Employment/population, ages 65 + years (US) | 13.1<br>10.4/18.2<br>2.4 | 23.0<br>15.8/30.1<br>4.5 | 28.2<br>14.2/37.1<br>6.5 | OECD Labour Force Statistics 2016. |
| $\frac{L_2^r}{N_2^r}$ | Employment/population, ages 65 + years (China) Note: Data available for 2000 and 2010 | 20.9<br>25.0/20.9<br>– | 31.5<br>3.1/47.5<br>13.2 | 31.0<br>−5.2/47.1<br>11.7 | OECD Labour Force Statistics 2016. |

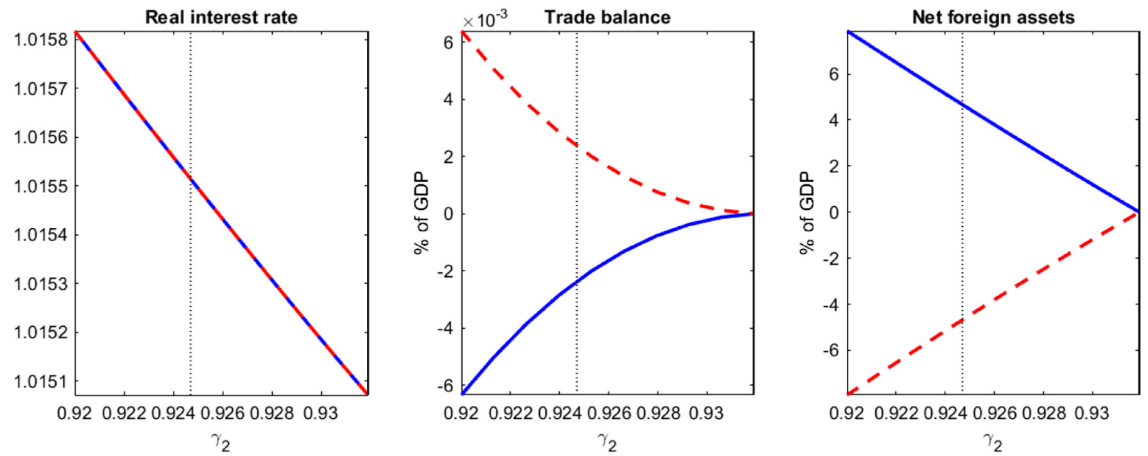


**Fig. 6.** Steady states for $0.92 \leq \gamma_2 \leq 0.9319$ and $\gamma_1 = 0.9319$ (blue line = US and red dotted line = China). At the leftmost vertical line, $\gamma_2 = 0.9247$, which corresponds to the projected life expectancy in China in 2020.

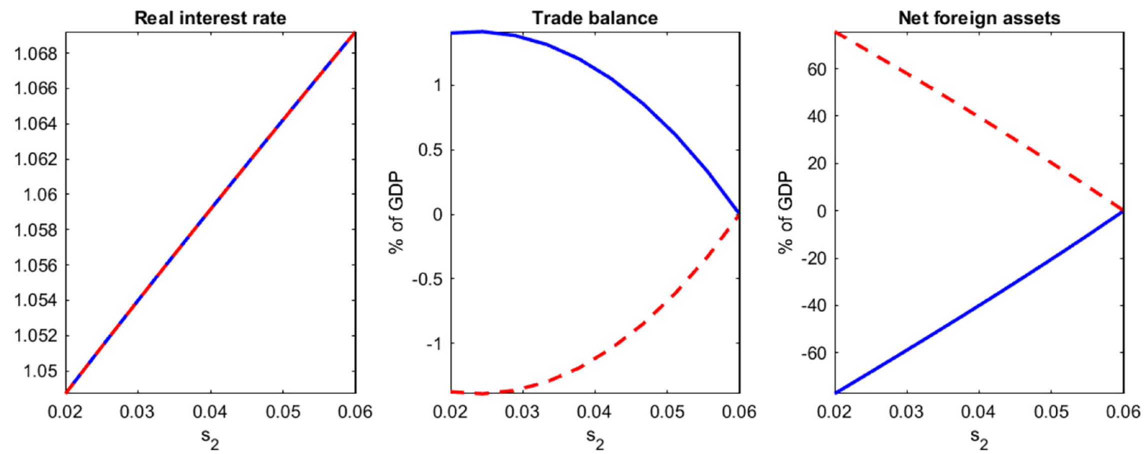


**Fig. 7.** Steady states for $0.02 \leq s_2 \leq 0.06$ and $s_1 = 0.06$ (blue line = US and red dotted line = China).

of 1.5% of GDP and a positive net foreign assets position of 70% of GDP for China in the steady state. The higher the social security spending in China, the smaller its net foreign asset position and larger its trade surplus in the steady state.

Other features that distinguish the economies are the relatively low effective retirement age and low level of government expenditure in China. Low retirement age predicts a negative net foreign asset position for the US in the steady state. Low government expenditures imply the opposite. In comparison to the effects of life expectancy and social security, however, the effects of retirement age and government expenditure on the steady state variables are small.

The steady state analysis suggests that the low level of social security expenditures may help explain China's trade surplus vis-à-vis the US. Given that life expectancy has increased more quickly in China, the steady state analysis further suggests that China's net foreign asset position has improved, implying that its trade balance has been elevated during the period of the demographic transition.

### 4.3. Deterministic simulation

In this subsection, I run a deterministic simulation to analyze the dynamic effects of the demographic transition, social security, government expenditures, and TFP fluctuations on the trade balance and net foreign asset position of the two countries. The economies are assumed initially to be in a state in which the exogenous variables match the data in 1980. They then converge to a new steady state in which the exogenous variables match the data in 2015. During the transition, the exogenous variables follow the path observed in the data. Using a sequential approach, I first introduce demographic transition, while keeping all other

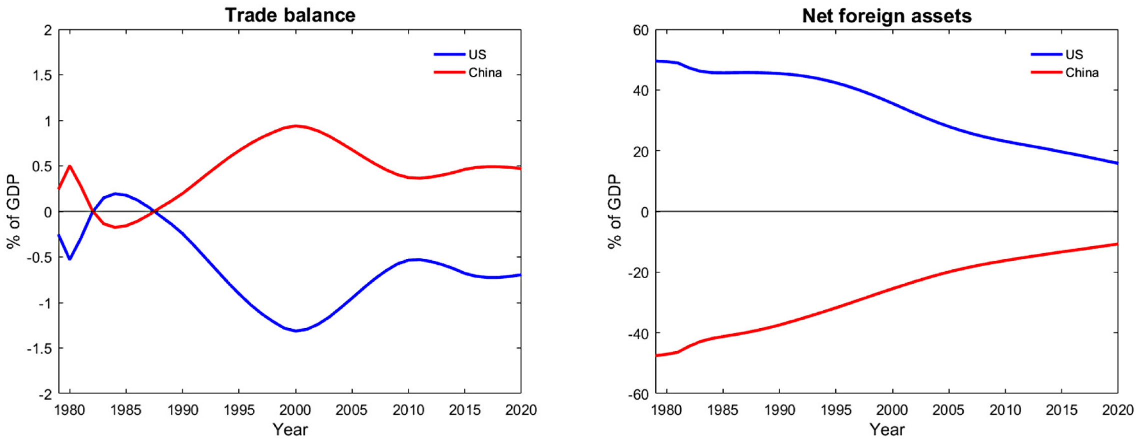


**Fig. 8.** Effects of permanent and temporary demographic changes on the trade balance (left panel) and the net foreign asset position (right panel).

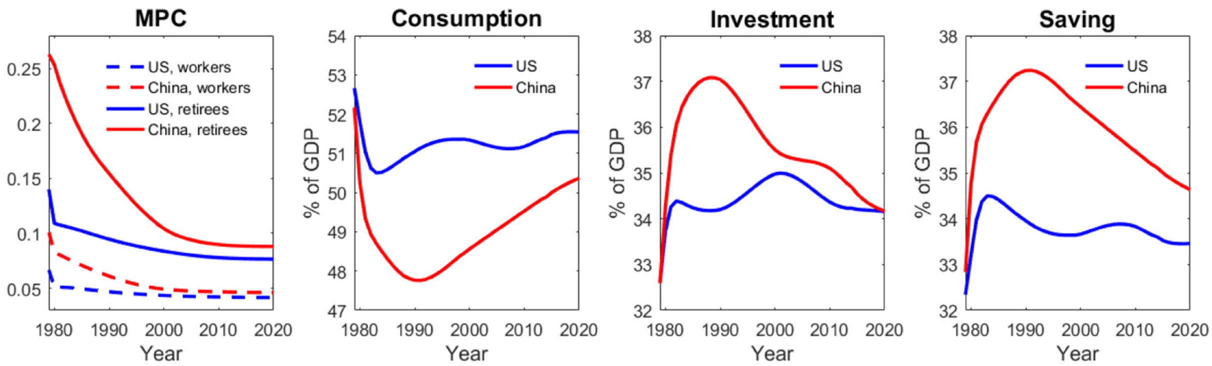


**Fig. 9.** Effects of demographic changes on the marginal propensities to consume and the GDP shares of consumption, investment, and saving.

factors constant. Next, I introduce fiscal policy shocks and differences in public social security spending. Finally, I incorporate temporary fluctuations in TFP growth rates.

*4.3.1. The dynamic effects of demographic transition*

Between 1980 and 2015, the growth rate of population aged 15 to 64 fell from 2.9 to 0.2% in China, and from 1.4 to 0.5% in the US. Life expectancy increased from 67.4 to 75.4 years in China, and from 74.3 to 78.9 years in the US over the same period (United Nations (2015)). In the simulation in this subsection, population growth rates and life expectancies during the period 1980–2015 match the data (see Figs. 2 and 3) exactly.[9] In the initial and final states, population growth rate is assumed to be the average of the observed growth rates (2.15% in 1980 and 0.5% 2015) between the countries.

The results of the dynamic simulation with demographic transition are shown in Figs. 8 and 9. Consistent with the steady state results, higher retirement age and life expectancy together imply a positive net foreign asset position and negative trade balance for the US in the initial state. For the US, the size of the net foreign asset position is approximately 50% of GDP, while the trade deficit is 0.25% of GDP. In the initial state, the US households hold more financial wealth than the Chinese households because of higher life expectancy. Thus, the US net foreign asset position is positive. China, in contrast, holds a negative foreign asset position and runs a trade surplus.

From the initial state, the economy faces transitory and permanent changes to the demographic structure as described above. As life expectancy increases permanently in both countries, the mpc's and consumption of both retirees and workers initially fall as households increase their savings due to their expectation of more years spent in retirement. The mpc's decline more in China because the increase in Chinese households' life expectancy is larger. As savings increase, the real interest rate falls, and investment rises as the economies start to adjust to a new steady state with higher capital stock.

The net effect of the demographic shock on the trade balance is positive for China as the increase in savings exceeds the rise in investment. It is negative for the US, where the effect on investments is larger. The reason is that the increase in Chinese savings

[9] The frequency of the United Nations data is five years. In the simulation, the value for each five-year period in the UN data is interpreted as the value in the last year of that period. The values in between are linearly interpolated.

pushes the real interest rate below the autarky level of the US interest rate, and therefore the change in investments exceeds the change in domestic savings caused by the aging of its domestic population. As the US trade balance weakens, and as the fall in the interest rate causes the returns on its foreign assets to fall, the US stock of foreign assets starts to decline and China's stock of foreign assets begins to climb.

As the age structures between the countries become more similar, the external imbalances gradually become smaller. As the capital stock approaches the new steady state level, investments decline, with a positive impact on the trade balance both in the US and in China. However, the trade surplus of China becomes smaller because of the simultaneous rise in the aggregate consumption share. The rise in consumption share is driven by rising consumption among the retirees, which, in turn, was caused by the increase in aggregate wealth held by the retirees and their rising share in the population (given that the mpc of retirees is higher than that of workers).

The fluctuations in the simulated trade balance are caused by population growth fluctuations during the period. In periods of relatively high population growth in China, investments need to increase to maintain an equal rate of return across countries. This temporarily weakens the trade balance. Overall, the relatively rapid population aging in China causes a slow, continuous decline in the net foreign asset position of the US that resembles the downward trend observed in the data, even though the implications on the sign of the net foreign asset position is counterfactual.

#### *4.3.2. The effects of social security and fiscal policy*

Between 1980 and 2015, the output shares of social security expenditures and general government expenditures were usually higher in the US than in China (see Fig. 4). In this subsection, I analyze the dynamics by introducing government expenditures and social security spending in addition to the demographic transition described in the previous subsection. Public pension spending is assumed to grow slowly from 6% to 7% of GDP in the US, and to remain at 2% of GDP in China over the simulation period. The GDP share of general government expenditures, which exactly matches the data, is assumed to be 15.5% for the US and 14.4% for China in the initial and final states. Government debt as a share of GDP is endogenously determined by the fiscal rule (Eq. (22)), and I assume it to be 20% in the initial and final states in both countries due to the lack of available data on Chinese government net debt.

The results of the dynamic simulation are shown in Fig. 10. The effect of public pension expenditures on the external imbalances is noticeable. Consistent with the steady state results, high public pension expenditures in the US have a negative effect on its initial net foreign asset position: whereas the demographic factors would counterfactually predict the US to hold a positive net foreign asset position of approximately 50% of GDP (recall from Fig. 9), when differences in social security expenditures are accounted for, the initial net foreign asset position is negative (right graph in Fig. 10). Despite life expectancy being lower in China, low pensions raise the aggregate level of non-human wealth in the economy, resulting in a positive net foreign asset position in the initial state.

Relatively high government expenditures have a small negative effect on the US' net foreign asset position in the steady state. They increase the tax rate, lowering the labor income of employees, which crowds out private consumption and savings. The impact on steady state trade balance is positive because of the negative wealth effect on consumption, but the net foreign asset position is weaker because of lower savings.

The dynamics of the trade balance and the net foreign assets are nevertheless mainly driven by the demographic changes, which cause a large increase in Chinese savings and result in an increase in its net foreign asset position vis-à-vis the US. Overall, when the differences in social security and government expenditures are also accounted for, the model dynamics qualitatively match the data well.

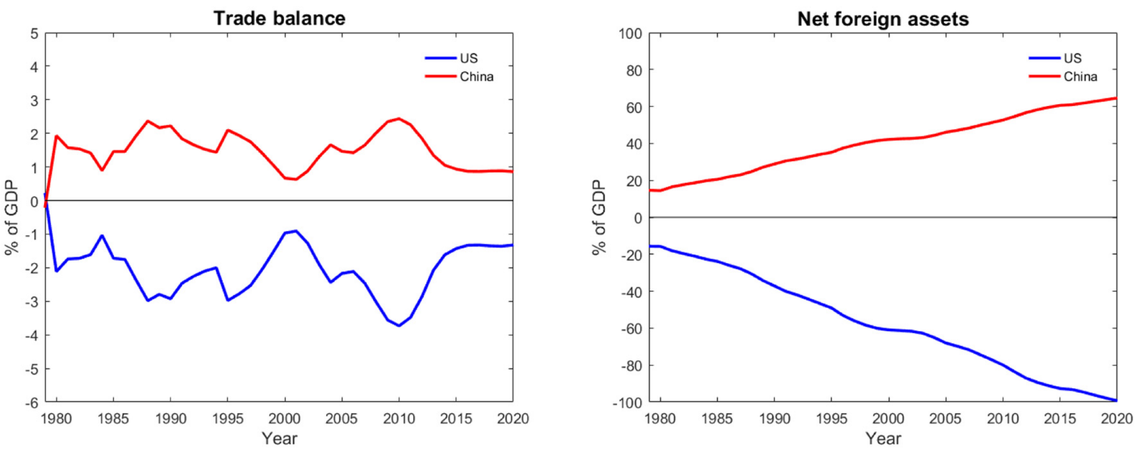


**Fig. 10.** Effects of a permanent demographic changes with social security and fiscal policy on the trade balance (left panel) and on the net foreign asset position (right panel).

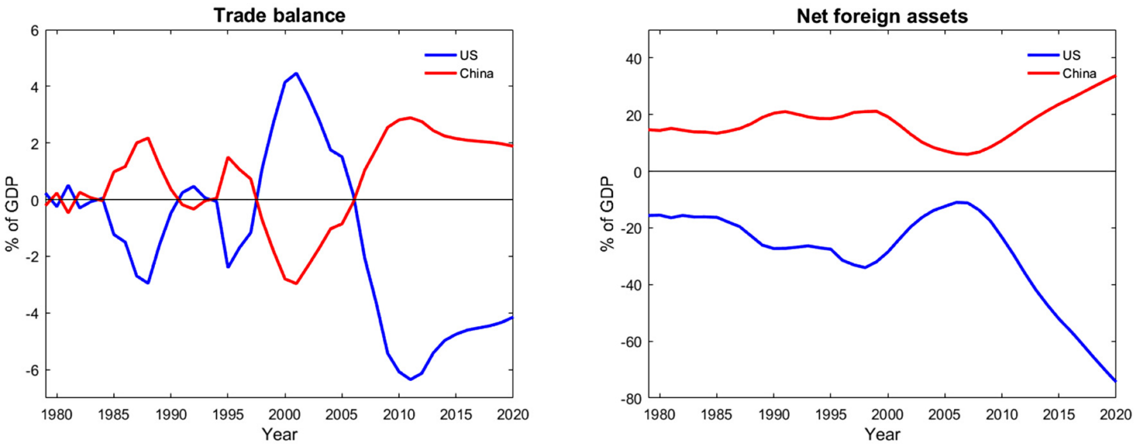


**Fig. 11.** Effects of a permanent demographic changes, social security, government expenditures, and TFP growth on the trade balance (left panel) and the net foreign asset position (right panel).

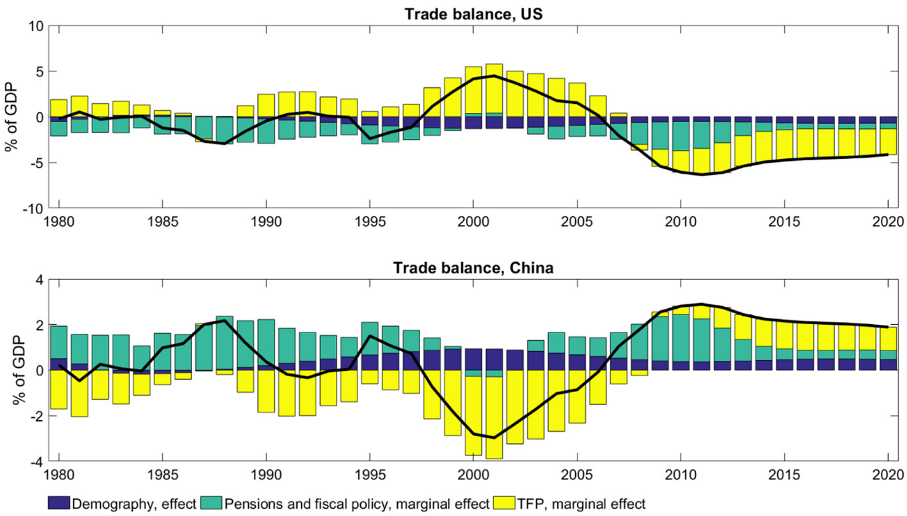


**Fig. 12.** Decomposition of the effects of demographic factors and marginal effect of pensions and fiscal policy and TFP on the dynamics of the trade balance.

#### *4.3.3. The effects of productivity growth fluctuations*

During 1980–2015, the average annual TFP growth rate was 1.9% in China and 0.9% in the US. In this subsection, I introduce temporary TFP fluctuations that exactly match the data (see Fig. 5), as well as to demographic, social security, and government expenditure shocks. In the initial and final states, the productivity growth rate is assumed to remain constant at 1%.

Fig. 11 reports the resulting dynamics. As expected, TFP shocks have a strong impact on the trade balance. Periods of relatively high productivity growth in China raise investment and consumption, which weaken its trade balance and the net foreign asset position. However, due to the underlying demographic trends and differences in social security and government expenditures between China and the US, the trend in the Chinese net foreign asset position is increasing. For most of the periods in the sample years, the model predicts a positive trade balance for China.

Fig. 12 shows the simulation decomposed into the effects of demographic transition and the marginal effects of pensions, fiscal policy, and TFP. Even though the negative impact of TFP shocks strengthens the US trade balance, especially in the early 2000s, the opposite effects of demographic factors, social security, and fiscal policy predict a trade deficit for the US, especially at the beginning of the simulation period. Social security is a key element in driving the results. Without it, the model would predict a counterfactual trade deficit and foreign debt for China for almost the entire simulation period.

### 4.4. Deterministic simulation with updates

The deterministic simulation used in the previous subsection assumes that the paths of the exogenous variables are known to the agents at the beginning of the simulation period. In this subsection, I present an alternative solution to the model in which a deterministic simulation is performed for every period of the simulation, but the shocks and paths of exogenous variables are allowed to change. In particular, the initial values of the exogenous and endogenous variables in each period are given by the simulation from the previous period, and new transition paths are solved based on the updated information. Unlike in the previous simulation, innovations in exogenous variables are unanticipated. This approach allows the agents to update their knowledge on the future paths of exogenous variables as they become available in the data. The transition paths are constructed as a compilation of values of different rounds of simulation.

The assumptions regarding the future paths of exogenous variables are as follows. In each period, the agents are assumed to know the paths of life expectancy and population growth as given in the respective vintage of the UN population forecast (plotted for selected years in Figs. 17 and 18). TFP growth rate and government expenditures are assumed to follow AR(1) processes known by the agents, who estimate the autoregressive parameter given past data in each period. The agents observe the current exogenous realization of TFP and government expenditures, and forecast the future values given the AR(1) process. The agents thus assume that, after the temporary shock has occurred, these variables slowly converge to their steady state values. In each period there is a shock to TFP and government expenditures so that the compiled series of these variables match the data exactly (plotted in Figs. 4 and 5). Therefore, unlike in the deterministic simulation, the agents are not assumed to know the entire path of, say, productivity growth at the beginning of the simulation. Pension expenditures are assumed to be constant over time. With respect to the endogenous variables, I assume that the economies are otherwise in a steady state implied by the exogenous variables apart from net foreign assets and trade balance, which are assumed to be zero at the beginning of the period.

The results of the simulation are presented in Fig. 13. The simulated paths of the trade balance and the net foreign asset position match the data better than those obtained with the deterministic simulation. The Chinese trade balance is positive for almost the entire simulation period, apart from a sharp deterioration caused by a sharp unexpected downward revision in life expectancy in the 1982 population forecast. Periods such as the mid-2000s, when the TFP grew at rates up to 6%, cause the trade balance to decline. After a plunge in early 1980s, however, it remains positive for the entire simulation period. With the simulation with updates, China is predicted to have a positive net foreign asset position that averages 23% of GDP during the simulation period. For the US as well, the simulated trade balance and the negative and deteriorating foreign asset position match the data well.

### 4.5. Life expectancy and the labor supply channel

The previous results are driven by a demographic shock that increases life expectancy and savings. However, if labor supply is assumed to be fixed as in Ferrero (2010), the impact of the demographic shock could be unrealistically large and lead to counterfactual excessive savings by the young, as well as a dynamically inefficient world interest rate. Endogeneity of labor supply is needed to have a better match between the model and the data as it allows the households to adjust their labor supply in response to the shock and therefore the increase in savings by the working-age population is more modest. As a result, the share of wealth held by the elderly does not decline, which is in line with the data. In the US, the share of wealth held by the elderly has in fact increased over the past 30 years (see Fig. 16). The interest rate also declines less and remains dynamically efficient.

Fig. 14 compares the effects of demographic change (simulation in section 4.3.1 with the exception that population growth rate is assumed to remain constant) with fixed and variable labor supply.[10]

With both fixed and variable labor supply, an increase in life expectancy lowers the marginal propensities to consume in both countries, which leads to a long-run decline in consumption if not offset by an increase in the present value of lifetime wealth. With variable labor supply, the decline in consumption and the increase in the marginal utility of consumption require an increase in labor supply for a given wage and tax rate for the intratemporal labor supply optimality conditions to hold. The increase in labor supply in retirement raises the present discounted value of workers' human wealth. As workers anticipate higher labor income during their retirement, the negative wealth effect of aging on their consumption is smaller than under fixed labor supply.[11]

Fig. 14 shows that retirees' labor supply is higher in the US in the initial state due to a higher life expectancy that raises the present value of workers' human wealth. Similarly, workers' labor supply is initially lower than in China. The increase in longevity increases labor supply in retirement so much that workers' labor supply declines in both countries over the long run. The increase in expected labor income dampens the negative wealth effect of aging. Savings by workers increase less with variable labor supply, and the share of financial wealth held by the retirees does not fall in either economy. The increase in aggregate financial wealth is also smaller with variable labor supply, and the world real interest rate remains higher, leading to lower investment

[10] The preference parameters are calibrated as in Table 2. Social security expenditures, the level of government expenditures, and the steady state value of government debt are assumed to be at the same level in both countries (3%, 15% and 20% of GDP, respectively). Growth rate of working rate population is fixed (2.15%). The simulation abstracts from temporary TFP and government expenditure shocks. With fixed labor supply, retirees are assumed not to work ($\xi = 0$).

[11] If the retirees' labor supply increases sufficiently, the workers react by reducing their labor supply and increasing consumption. Thus the increase of life expectancy may result in a decline in the world aggregate financial wealth and an increase in the real interest rate. For further discussion, see Fujiwara and Teranishi (2008) on the non-monotonic effects of life expectancy in the Gertler (1999) framework.

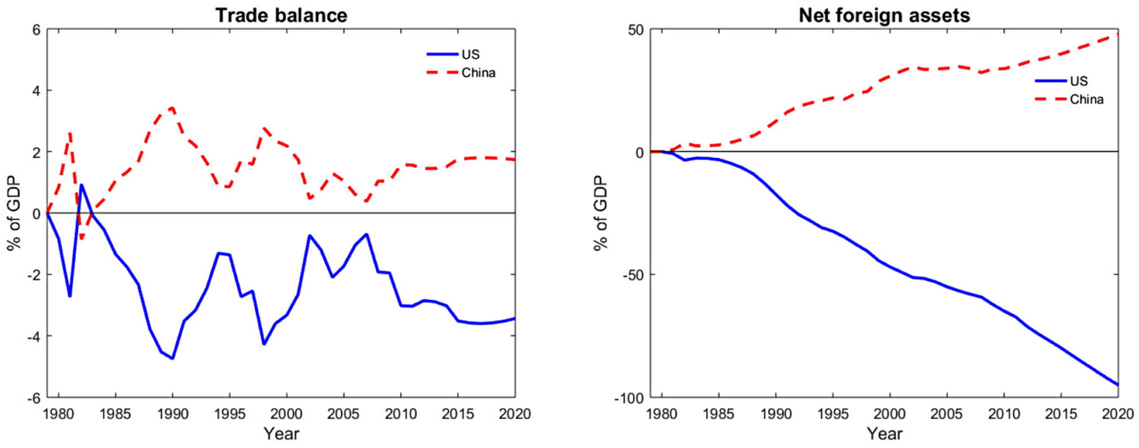


**Fig. 13.** Effects of permanent demographic changes, social security, government expenditures, and TFP growth on the trade balance (left panel) and on the net foreign asset position (right panel), simulation with updates.

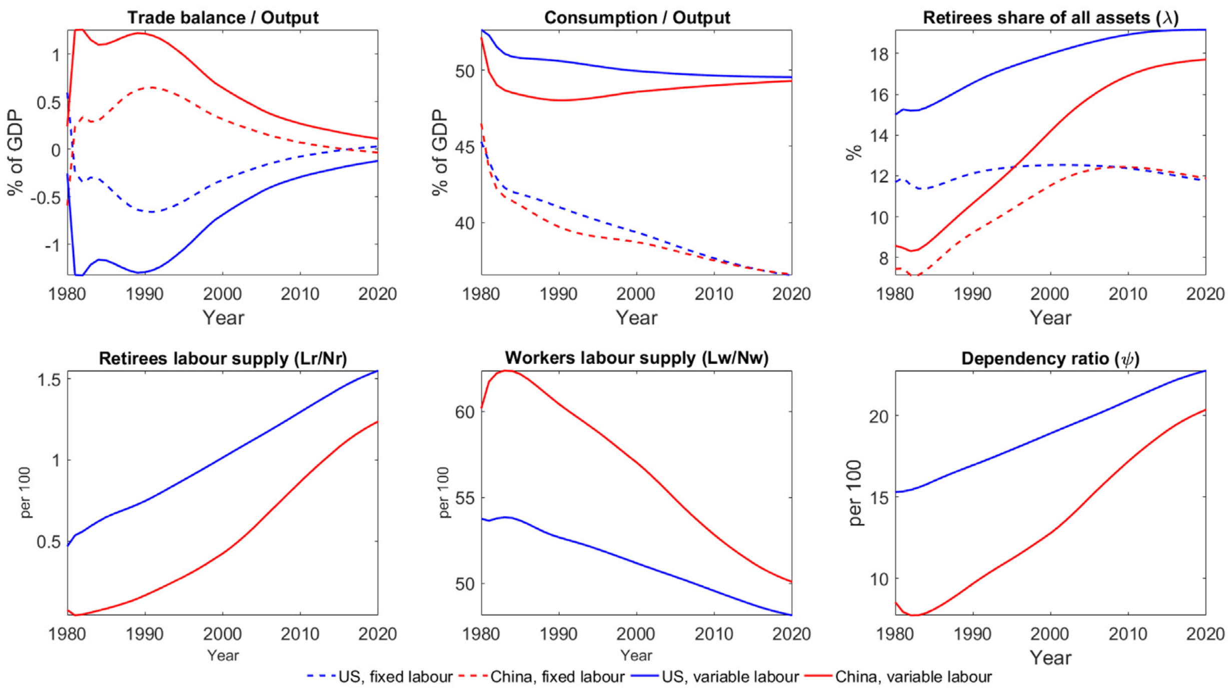


**Fig. 14.** Impact of population aging with variable and fixed labor supply.

and higher consumption than with the fixed labor supply model. The impact on the external balance is qualitatively similar as higher levels of consumption that occur with variable labor supply are coupled with lower investment-to-output ratios. The labor supply dynamics of the model match the data qualitatively (see Fig. 19 Section A.2 in the Appendix).

## 5. Conclusions

This paper examined whether the relatively rapid demographic transition in China, combined with its low level of social security, may help explain China's accumulation of a large positive net foreign asset position over recent decades and its persistent positive trade balance vis-à-vis the US. The analysis is performed with a model that features a life-cycle structure and a pay-as-you-go pension system, as well as endogenous labor supply and distortionary taxation.

The results suggest that demographic transition and disparities in the level of social security elucidate the observed external imbalances between the countries. The model predicts a long-lasting improvement in China's net foreign asset position and a positive trade balance for most of the simulation period. The rapid increase in life expectancy increases Chinese savings and generates current account and trade surpluses, but the role of low social security income in explaining the observed pattern is crucial. Even though life expectancy grew faster in China, it remained lower than in the US for the entire simulation period. Thus, without social security, the model would counterfactually predict a negative trade balance and net foreign asset position for China.

Temporary TFP fluctuations remain important drivers of the trade balance, but even after controlling for the effects of productivity growth, the model predicts a positive trade balance and net foreign asset position for China for most of the simulation period. The results also hold in a three-country extension of the model in which the rest of the world is calibrated to match the countries' main trading partners.[12] TFP growth rate fluctuations cause more volatility in the trade balance when the model is solved deterministically than with the alternative method in which temporary shocks are not known in advance and agents learn gradually. Therefore, the impact of the TFP fluctuations is smaller with the latter method and the results better aligned with data.

Finally, the analysis omits the effects of the Chinese central bank policies, including foreign exchange policies, capital controls, trade policies, and financial market imperfections – all topics that provide interesting opportunities for future research.

## Data availability

replication folder (Original data) (Mendeley Data)


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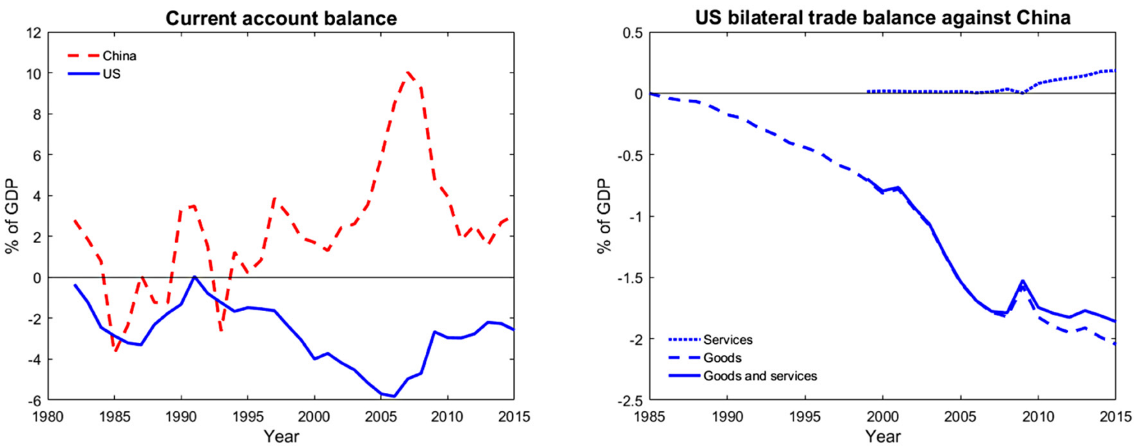
## Appendix

## A. Figures

### A.1. Current account balance and bilateral trade balances




[12] Results for the three-country model are available upon request.

**Fig. 15.** Left: Current account balance. Source: IMF Balance of Payment Statistics. Right: US bilateral trade balance against China 1985–2015. Sources: US Census Bureau (goods) and Bureau of Economic Analysis (goods and services since 1999).

### A.2. Wealth distribution

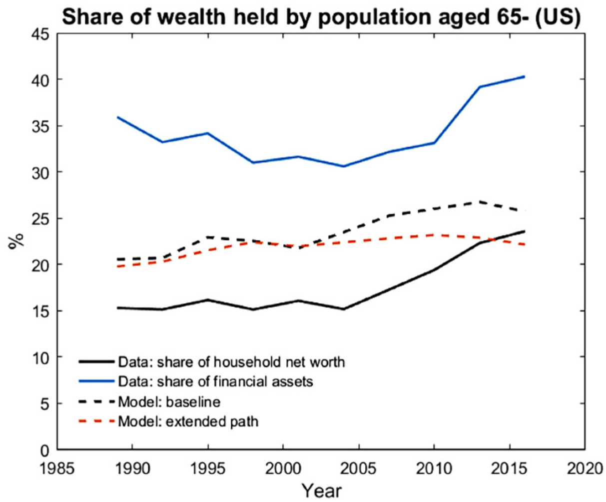



**Fig. 16.** Share of wealth held by population aged 65 and over. Source: Federal Reserve, Survey of Consumer Finances (SFC) 1989–2016 and author's calculations.

### A.3. Projections from United Nations Population Prospects

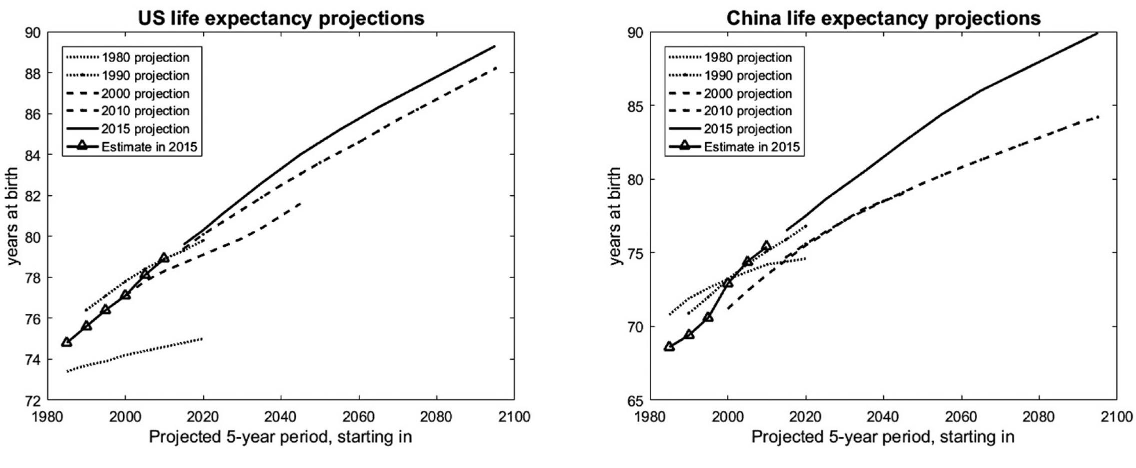



**Fig. 17.** Left panel: Life expectancy at birth, US. Right panel: Life expectancy at birth, China. Source: United Nations World Population Prospects: 1980–2015 Revisions.

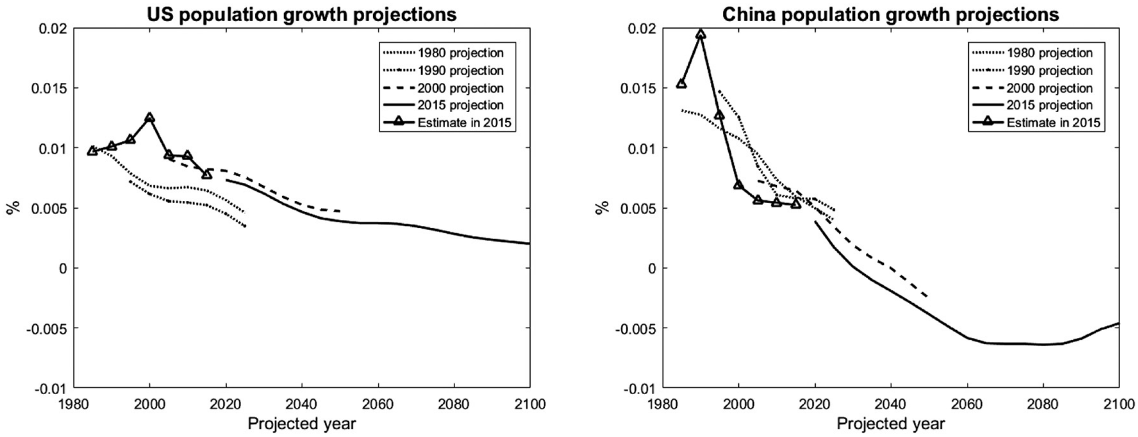



**Fig. 18.** Left panel: Population growth rate, US. Right panel: Population growth rate, China. Source: United Nations World Population Prospects: 1980–2015 Revisions.

## B. Labor force statistics

On the extensive margin, the employment to population ratio has been historically higher in China than in the US, both among populations aged 15–64 and populations aged 65 and over (right panel in Fig. 19), which is also qualitatively captured by the model (see Table 3). On the intensive margin, hours worked per worker have also been consistently higher in China than in the US (see left panel in Fig. 19). Hours worked show persistent declines in both countries. In the model, labor supply among the working-age population declines as a result of population aging.

According to the 2010 population census, labor income was a primary source of support for 28% of the rural population aged 65 and over, and for 4% of the urban population in China.[13] In the US, the employment-to–population ratio among the population aged 65 and over has displayed an upward trend since the early 1980s (see right panel in Fig. 19), suggesting that retirees' role in income provision is important.

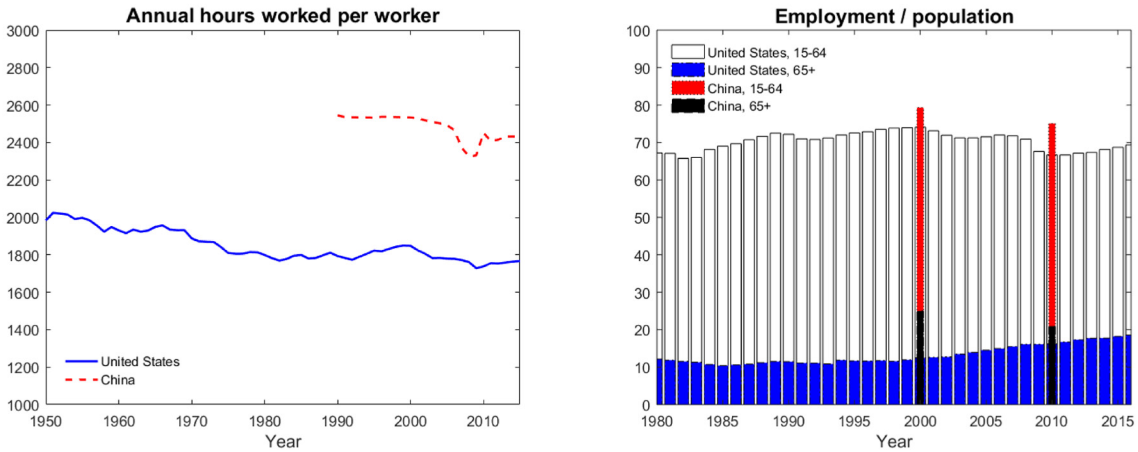



**Fig. 19.** Left panel: Annual hours worked per worker. Source: The Conference Board Total Economy Database 2015. Right panel: Employment to population ratio on age groups 15–64 and 65 and over. Source: OECD Labor Force Survey 2018.

## C. Life-cycle structure

The population of a country at time t consists of two groups of individuals: workers, whose total number is $N_t^w$, and retirees, whose number equals $N_t^r$. All agents enter the economy as workers at the age of 20 and remain workers with probability $\omega_{t,t+1}$

[13] Original source: National Bureau of Statistics of China (2012), cited by Jiang et al. (2016). In 2004, working was the primary source of income 8% of urban men and 4% of urban women, and 43% of rural men and 23% of rural women. Original source: National Bureau of Statistics of China (2005), cited by Naughton (2007).

and retire with probability $1 - \omega_{t,t+1}$. During every period $(1 - \omega_{t,t+1} + n_{t,t+1})N_t^w$, new workers are born. As a result, the number of workers grows each period at rate $n_{t,t+1}$ and the law of motion for aggregate labor force is

$$N_{t+1}^w = (1-\omega_{t,t+1} + n_{t,t+1})N_t^w + \omega_{t,t+1}N_t^w = (1 + n_{t,t+1})N_t^w \tag{27}$$

At time $t$, probability of a retiree to survive to the next period is $\gamma_{t,t+1}$. The law of motion for the number of retirees is

$$N_{t+1}^r = (1-\omega_{t,t+1})N_t^w + \gamma_{t,t+1}N_t^r \tag{28}$$

The ratio of number of retirees to the number of workers, dependency ratio, is given by $\psi_t = N_t^r/N_t^w$ and can be solved to evolve according to

$$(1 + n_{t,t+1})\psi_{t+1} = (1-\omega_{t,t+1}) + \gamma_{t,t+1}\psi_t \tag{29}$$

## D. Pension systems

**China** In the early 1980s, the Chinese pension system covered mostly urban workers in the public sector and state-owned enterprises, and until the early 1990s, all of China's pension liabilities were unfunded (Naughton, 2007). After the beginning of the 1980s, the Chinese economy and demographic structure began to undergo changes that necessitated pensions system reform. The targets were to increase the pension coverage to urban employees of private enterprises and, eventually, to rural residents, and tackle the challenge posed by the aging population by setting up a partly funded system.

The current two-tier pension system was introduced in 1998 and revised in 2006 (OECD, 2015). It consists of a basic public pension funded on a pay-as-you-go basis, and mandatory employee contributions to a second-tier plan. The coverage of the pension system is still largely limited to urban workers. The second-tier plan, which is a funded individual-account system, was de facto functional in only 11 out of the 33 Chinese provinces.The pension scheme covers 27.7% of the population aged 15–65 and 33.5% of the labor force. Depending on individual earnings, the gross replacement rate of the basic pensions system, defined as the pension benefits as a share of individual lifetime average earnings, varies between 30% and 50% (OECD, 2015). With the defined contribution pillar, the gross replacement rate is approximately 75%.

**The US** Social Security, the US public pension system, originated with the Social Security Act of 1935. It is a defined-benefit, earnings-based public pension scheme. A means-tested old-age pension benefit provides additional income for low-income pensioners. Social security covered 71.4% of the population aged 15 to 65 and 92.2% of the labor force in 2010 (OECD, 2015). The gross replacement rate of the social security varies between 25% and 45%, which is low in comparison to the OECD average (OECD, 2015). Private pension funds play an important role in the American scheme.

Social Security is partly funded. Almost all wage income is subject to FICA (Federal Insurance Contributions Act) taxes, which are administered by the Social Security Trust Funds. The Federal Old-Age and Survivors Insurance (OASI) fund holds the accumulated old-age pension fund assets and pays the benefits (The Board of Trustees, 2016). Any excess income is deposited into the fund and invested in special US government bonds (Special Issue Securities). The OASI funds reserves at the end of 2015 amounted to $2.78 trillion. Income in 2015 was $798 billion (of which 85% consisted of payroll taxes) and costs $776 billion (of which 99% were benefit payments) (The Board of Trustees, 2016).

## E. Parameter sensitivity

**Table 4**

| | | $NX_t/Y_t$ | $NX^*_t/Y^*_t$ | $F_t/Y_t$ | $F^*_t/Y^*_t$ | $R_t$ |
|---|---|---|---|---|---|---|
| Baseline simulation | | | | | | |
| | mean | −0.02 | 0.02 | −0.39 | 0.23 | 1.02 |
| | min/max | −0.05/0.01 | −0.01/0.03 | −0.83/−0.01 | 0.01/0.41 | 1.01/1.04 |
| | standard deviation | 0.01 | 0.01 | 0.26 | 0.13 | 0.01 |
| β discount factor | | | | | | |
| $\beta = 0.990$ | mean | −0.02 | 0.01 | −0.38 | 0.23 | 1.03 |
| | min/max | −0.05/0.01 | −0.01/0.03 | −0.81/−0.01 | −0.01/0.40 | 1.01/1.04 |
| | standard deviation | 0.01 | 0.01 | 0.25 | 0.13 | 0.01 |
| $\beta = 0.998$ | mean | −0.03 | 0.02 | −0.41 | 0.24 | 1.02 |
| | min/max | −0.05/0.01 | −0.01/0.04 | −0.85/−0.01 | 0.01/0.42 | 1.00/1.03 |
| | standard deviation | 0.01 | 0.01 | 0.26 | 0.13 | 0.01 |
| σ elasticity of intertemporal substitution | | | | | | |
| $\sigma = 0.45$ | mean | −0.02 | 0.01 | −0.29 | 0.17 | 1.03 |
| | min/max | −0.04/0.02 | −0.02/0.03 | −0.68/0.02 | −0.02/0.34 | 1.02/1.05 |
| | standard deviation | 0.01 | 0.01 | 0.22 | 0.12 | 0.01 |
| δ depreciation rate | | | | | | |
| $\delta = 0.15$ | mean | −0.02 | 0.01 | −0.28 | 0.16 | 1.04 |
| | min/max | −0.04/0.02 | −0.02/0.03 | −0.67/0.01 | −0.01/0.33 | 1.02/1.06 |
| | standard deviation | 0.01 | 0.01 | 0.21 | 0.11 | 0.01 |
| ϕ investment adjustment cost factor | | | | | | |
| $\phi = 0.2$ | mean | −0.03 | 0.02 | −0.35 | 0.20 | 1.02 |
| | min/max | −0.07/0.03 | −0.03/0.05 | −0.81/0.08 | −0.07/0.40 | 1.01/1.05 |
| | standard deviation | 0.02 | 0.02 | 0.27 | 0.15 | 0.01 |
| $\phi = 10$ | mean | −0.02 | 0.02 | −0.43 | 0.26 | 1.02 |
| | min/max | −0.04/−0.00 | 0.00/0.04 | −0.55/−0.02 | 0.02/0.43 | 1.01/1.04 |
| | standard deviation | 0.01 | 0.01 | 0.25 | 0.12 | 0.01 |
| ξ productivity of a unit of labor, retiree to worker | | | | | | |
| $\xi = 0.6$ | mean | −0.02 | 0.01 | −0.33 | 0.20 | 1.03 |
| | min/max | −0.04/0.01 | −0.01/0.03 | −0.73/−0.00 | 0.05/0.36 | 1.01/1.04 |
| | standard deviation | 0.01 | 0.01 | 0.23 | 0.12 | 0.01 |
| v elasticity of period utility with respect to consumption | | | | | | |
| $v = 0.75$ | mean | −0.02 | 0.01 | −0.38 | 0.23 | 1.03 |
| | min/max | −0.04/0.01 | −0.01/0.03 | −0.80/−0.01 | 0.01/0.40 | 1.01/1.04 |
| | standard deviation | 0.01 | 0.01 | 0.25 | 0.13 | 0.01 |

Table 4 presents the sensitivity of the mean, minimum, maximum, and standard deviation of the simulated trade balances ($NX_t/Y_t$ and $NX^*_t/Y^*_t$), net foreign asset positions ($F_t/Y_t$ and $F^*_t/Y^*_t$) and the real interest rate ($R_t$) to changes in single parameter values during the period 1980–2015 in the simulation with updates. The remaining parameters are calibrated as in the baseline simulation (Table 2).

## F. Technical appendix

### The model

The appendix presents derivations of retiree's and worker's problems, the aggregation result, the problem of the firm and definition of the equilibrium and the external sector.

### *F.1. Households*

### Retirees

A retiree born in period $j$ and retired in period $i$ chooses consumption-saving allocation and labor input to maximize

$$V_t^{jr}(i) = max\left\{\left[\left(C_t^{jr}(i)\right)^{\upsilon}\left(1-l_t^r\right)^{1-\upsilon}\right]^{\rho} + \beta\gamma_{t,t+1}\left(V_{t+1}^{jr}(i)\right)^{\rho}\right\}^{\frac{1}{\rho}} \tag{30}$$

subject to

$$A_{t+1}^{jr}(i) = \frac{R_t A_t^{jr}(i)}{\gamma_{t-1,t}} + W_t \xi l_t^{jr}(1-\tau_t) + S_t^{jr} - C_t^{jr}(i) \tag{31}$$

$R_t$ is the world interest rate that clears the international capital market and $\gamma_{t,t+1}$ (henceforth, $\gamma_{t,t+1} \equiv \gamma_{t+1}$) is the retirees' probability to survive from one period to the next. $W_t \xi l_t^{jr}(1-\tau_t)$ is the net wage income and $S_t^{jr}$ is the pension income from the government. The proportional income tax $\tau_t$ is paid both by the workers and the retirees.

The first-order condition with respect to asset accumulation is

$$\upsilon\left(C_t^{jr}(i)\right)^{\upsilon\rho-1}\left(1-l_t^{jr}\right)^{\rho(1-\upsilon)} = \beta\gamma_{t+1}\left(V_{t+1}^{jr}(i)\right)^{\rho-1}\frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)} \tag{32}$$

The envelope condition is

$$\frac{\partial V_t^{jr}(i)}{\partial A_t^{jr}(i)} = \left(V_t^{jr}\right)^{1-\rho}\upsilon\left(C_t^{jr}(i)\right)^{\upsilon\rho-1}\left(1-l_t^{jr}\right)^{(1-\upsilon)\rho}\frac{R_t}{\gamma_t} \tag{33}$$

The first-order condition with respect to labor is

$$l_t^{jr}(i) = 1 - \frac{C_t^{jr}(i)\varsigma}{W_t\xi(1-\tau_t)} \tag{34}$$

where $\varsigma = \frac{1-\upsilon}{\upsilon}$.

Combining the first-order conditions and the lagged envelope condition $\left(\frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)}\right)$, and noting that $\sigma = \frac{1}{1-\rho}$ gives the following Euler equation for retirees:

$$C_{t+1}^{jr}(i) = C_t^{jr}(i)\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)}\beta R_{t+1}\right]^{\sigma}. \tag{35}$$

Guess that consumption is a fraction of total lifetime wealth:

$$C_t^{jr}(i) = \varepsilon_t\pi_t\left(\frac{R_t A_t^{jr}(i)}{\gamma_t} + H_t^{jr}(i) + P_t^{jr}(i)\right) \tag{36}$$

where $H_t^{jr}$ is the present discounted value of a retiree's lifetime human wealth and $P_t^{jr}(i)$ is the present discounted value of a retiree's lifetime pension benefits. The present discounted value of a retiree's human wealth can be written as

$$\begin{aligned} H_t^{jr}(i) &= W_t(1-\tau_t)\xi l_t^{jr}(i) + \frac{W_{t+1}(1-\tau_{t+1})\xi l_{t+1}^{jr}(i)}{R_{t+1}/\gamma_{t+1}} + \frac{W_{t+2}(1-\tau_{t+1})\xi l_{t+1}^{jr}(j)}{(R_{t+1}/\gamma_{t+1})(R_{t+2}/\gamma_{t+2})} + \cdots \\ &= \sum_{v=0}^{\infty}\frac{W_{t+v}(1-\tau_{t+v})\xi l_{t+v}^{jr}(i)}{\prod_{s=1}^{v}(R_{t+s}/\gamma_{t+s})} = W_t(1-\tau_t)\xi l_t^{jr}(i) + \frac{H_{t+1}^{jr}(i)}{R_{t+1}/\gamma_{t+1}} \end{aligned}. \tag{37}$$

Similarly, the present discounted value of a retiree's pension benefits can be written as

$$P_t^{jr}(i) = S_t^{jr} + \frac{P_{t+1}^{jr}(i)}{R_{t+1}/\gamma_{t+1}} \tag{38}$$

Combining the Euler eq. (35) with the guess (36) gives a law of motion for the marginal propensity to consume as follows. First, combine the budget constraint (31) with the guess (36) to obtain the following expression for consumption:

$$C_t^{jr}(i) = \varepsilon_t\pi_t\left(\frac{R_t}{\gamma_t} * \frac{\gamma_t}{R_t}\left(A_{t+1}^{jr}(i) - W_t\xi l_t^{jr}(i)(1-\tau_t) - S_t^{jr} - C_t^{jr}(i)\right) + H_t^{jr}(i) + P_t^{jr}(i)\right) \tag{39}$$

Second, substitute this expression (39), a lagged expression of the guess (36) and the expressions for present value of human (37) and social security (38) wealth into the Euler eq. (35) to obtain the following expression

$$\begin{aligned} &\varepsilon_{t+1}\pi_{t+1}\left(\frac{R_{t+1}A^{jr}_{t+1}(i)}{\gamma_{t+1}}+H^{jr}_{t+1}(i)+P^{jr}_{t+1}(i)\right) \\ &=\left\{\varepsilon_t\pi_t\left(A^{jr}_{t+1}(i)+\frac{H^{jr}_{t+1}(i)+P^{jr}_{t+1}(i)}{R_{t+1}/\gamma_{t+1}}+C^{jr}_t(i)\right)\right\}*\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)}\beta R_{t+1}\right]^{\sigma} \end{aligned} \tag{40}$$

Dividing both sides of the equation by the LHS (left-hand-side) of itself and using the Euler equation and the conjecture again, a retiree's mpc can be solved to evolve according to

$$\varepsilon_t\pi_t=1-\frac{\varepsilon_t\pi_t}{\varepsilon_{t+1}\pi_{t+1}}\gamma_{t+1}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho\sigma(1-\upsilon)}\beta^{\sigma}(R_{t+1})^{\rho\sigma}. \tag{41}$$

**Workers**

A worker born in period $j$ chooses consumption-saving allocation and labor input to solve

$$V^{jw}_t=max\left\{\left[\left(C^{jw}_t\right)^{\upsilon}\left(1-l^{jw}_t\right)^{1-\upsilon}\right]^{\rho}+\beta\left[\omega_{t+1}V^{jw}_{t+1}+(1-\omega_{t+1})V^{jr}_{t+1}\right]^{\rho}\right\}^{\frac{1}{\rho}} \tag{42}$$

where $\omega_{t,t+1}$ (henceforth, $\omega_{t,t+1}\equiv\omega_{t+1}$) is the worker's probability to remain a worker, subject to

$$A^{jw}_{t+1}=R_tA^{jw}_t+W_tl^{jw}_t(1-\tau_t)-C^{jw}_t \tag{43}$$

The first-order condition with respect to asset accumulation is

$$\upsilon\left(C^{jw}_t\right)^{\upsilon\rho-1}\left(1-l^{jw}_t\right)^{\rho(1-\upsilon)}=\beta\left(\omega_{t+1}V^{jw}_{t+1}+(1-\omega_{t+1})V^{jw}_{t+1}\right)^{\rho-1}\left[\omega_{t+1}\frac{\partial V^{w}_{t+1}}{\partial A^{w}_{t+1}}+(1-\omega_{t+1})\frac{\partial V^{jr}_{t+1}}{\partial A^{jw}_{t+1}}\right] \tag{44}$$

The envelope conditions are

$$\frac{\partial V^{jw}_t}{\partial A^{jw}_t}=\left(V^{w}_t\right)^{1-\rho}\upsilon\left(C^{w}_t\right)^{\upsilon\rho-1}\left(1-l^{w}_t\right)^{(1-\upsilon)\rho}R_t \tag{45}$$

and

$$\frac{\partial V^{jr}_t(i)}{\partial A^{jw}_t}=\frac{\partial V^{jr}_t(i)}{\partial A^{jr}_t(i)}\frac{\partial A^{jr}_t(i)}{\partial A^{jw}_t}=\frac{\partial V^{jr}_t(i)}{\partial A^{jr}_t(i)}=\left(V^{r}_t\right)^{1-\rho}v\left(C^{r}_t\right)^{\upsilon\rho-1}\left(1-l^{r}_t\right)^{(1-\upsilon)\rho}R_t \tag{46}$$

The first-order condition with respect to labor is

$$l^{w}_t(j)=1-\frac{C^{w}_t(i)\varsigma}{W_t(1-\tau_t)} \tag{47}$$

Combining the first-order conditions and the envelope conditions gives the Euler equation for the workers. First, substitute into the first-order condition for asset accumulation (44) the envelope conditions, taking into account the assumption of no compensation for the risk of death:

$$\upsilon(^{C^{jw}_t}(1^{-}=\beta(^{\omega_{t+1}}*[\omega_{t+1}(V^{jw}_{t+1})^{1-\rho}\upsilon(C^{jw}_{t+1})^{v\rho-1}(1-l^{jw}_{t+1})^{(1-\upsilon)\rho}R_{t+1}+(1-\omega_{t+1})(V^{jr}_{t+1}(i))^{1-\rho}\upsilon(C^{jr}_{t+1}(i))^{v\rho-1}(1-l^{jr}_{t+1}(i))^{(1-\upsilon)\rho}R_{t+1}].$$

Then substitute the labor FOC (47) to obtain

$$\begin{aligned} &C^{jw}_t\left(\frac{W_{t+1}(1-\tau_{t+1})}{W_t(1-\tau_t)}\right)^{\frac{\rho(1-\upsilon)}{\rho-1}}(\beta R_{t+1})^{\rho} \\ &=\left(\omega_{t+1}V^{jw}_{t+1}+(1-\omega_{t+1})V^{jw}_{t+1}\right)\left[\omega_{t+1}\left(V^{jw}_{t+1}\right)^{1-\rho}\left(C^{jw}_{t+1}\right)^{\rho-1}+(1-\omega_{t+1})\left(V^{jr}_{t+1}(i)\right)^{1-\rho}\upsilon\left(C^{jr}_{t+1}(i)\right)^{\rho-1}\xi^{(\upsilon-1)\rho}\right]^{-\sigma} \end{aligned} \tag{48}$$

Conjecture that similarly as for retirees, the value function is linear in consumption:

$$V_t^{jw} = \Delta_t^{jw} C_t^{jw} \left( \frac{\varsigma}{W_t(1-\tau_t)} \right)^{1-\upsilon} \tag{49}$$

In addition, define the adjustment term to be $\Omega_t \equiv \omega_{t-1,t} + (1-\omega_{t-1,t})\varepsilon_t^{\frac{1}{1-\sigma}}\chi$ and define $\chi = \xi^{-(1-\upsilon)}$. Substitute the conjectures for the value functions into eq. (48). The resulting Euler equation takes the form

$$C_t^{jw}\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)} \beta R_{t+1}\Omega_{t+1}\right]^{\sigma} = \omega_{t+1}C_{t+1}^{jw} + (1-\omega_{t+1})\left(\frac{\Delta_{t+1}^{r}}{\Delta_{t+1}^{w}}\right)\chi C_{t+1}^{jr}(i) \tag{50}$$

which equals

$$C_t^{jw}\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)} \beta R_{t+1}\Omega_{t+1}\right]^{\sigma} = \omega_{t+1}C_{t+1}^{jw} + (1-\omega_{t+1})(\varepsilon_{t+1})^{\frac{\sigma}{1-\sigma}}\chi C_{t+1}^{jr}(i).$$

Guess the consumption to be a fraction of total lifetime wealth

$$C_t^{jw} = \pi_t\left(R_{W,t}A_t^{jw} + H_t^{jw} + P_t^{jw}\right) \tag{51}$$

where $H_t^{jw}$ is the present discounted value of a worker's human wealth net of taxation and $P_t^{jw}$ is the present discounted value of a worker's pension benefits that he or she can assume to receive once retired. Substitute the per-period budget constraint (43) to get the following expression:

$$C_t^{jw} = \pi_t\left(A_{t+1}^{jw} - W_t l_t^{jw}(1-\tau_t) + C_t^{jw} + H_t^{jw} + P_t^{jw}\right)$$

For a retiree who has just quit the labor force, the consumption is guessed to be proportional to the assets they had as workers which they carry to their first period of retirement and human and social security as retirees:

$$C_t^{r|w} = \varepsilon_t \pi_t\left(R_t A_t^{jw} + H_t^{jr}(i) + P_t^{jr}(i)\right) \tag{52}$$

with the budget constraint being

$$A_{t+1}^{jr|w} = R_t A_t^{jw} + W_t l_t^{jw}(1-\tau_t) - C_t^{jw} \tag{53}$$

Substitute the budget constraint (53) into the conjecture (52) to obtain

$$C_t^{r|w} = \varepsilon_t \pi_t\left(A_{t+1}^{jr|w}(i) - W_t l_t^{jw}(1-\tau_t) + C_t^{jw} + H_t^{jr}(i) + P_t^{jr}(i)\right)$$

Combining the Euler eq. (50) with the guesses (51) and (52) and the budget constraints (43)and (53) gives a law of motion for the mpc as follows. First, substitute the guesses into the Euler equation to obtain

$$\begin{aligned}&\pi_t\left(R_t A_t^{jw} + H_t^{jw} + P_t^{jw}\right)\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)} \beta R_{t+1}\Omega_{t+1}\right]^{\sigma} \\ &\quad = \omega_{t+1}\pi_{t+1}\left(R_{t+1}A_{t+1}^{jw} + H_{t+1}^{jw} + P_{t+1}^{jw}\right) + (1-\omega_{t+1})\left(\frac{\Delta_{t+1}^{r}}{\Delta_{t+1}^{w}}\right)\chi\varepsilon_{t+1}\pi_{t+1}\left(R_{t+1}A_{t+1}^{jw} + H_{t+1}^{jr}(i) + P_{t+1}^{jr}(i)\right)\end{aligned} \tag{54}$$

and substitute the per-period budget constraints (43) and (53) to obtain

$$\begin{aligned}&\pi_t\left(R_{W,t}A_t^{jw} + H_t^{jw} + P_t^{jw}\right)\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)} \beta R_{t+1}\Omega_{t+1}\right]^{\sigma} \\ &\quad = \omega_{t+1}\pi_{t+1}\left(R_{W,t,t+1}\left(R_{W,t}A_t^{jw} + W_t l_t^{jw}(1-\tau_t) - C_t^{jw}\right) + H_{t+1}^{jw} + P_{t+1}^{jw}\right) \\ &\quad + (1-\omega_{t+1})\left(\frac{\Delta_{t+1}^{r}}{\Delta_{t+1}^{w}}\right)\chi\varepsilon_{t+1}\pi_{t+1}\left(R_{t+1}\left(R_{W,t}A_t^{jw} + W_t l_t^{jw}(1-\tau_t) - C_t^{jw}\right) + H_{t+1}^{jr} + P_{t+1}^{jr}\right)\end{aligned} \tag{55}$$

Substituting again the guess for $C_t^{jw}$, collecting terms and using the definition of the adjustment term, collecting terms with $A_t^{jw}$ on the LHS and dividing the equality by $\Omega_{t+1}\pi_{t+1}R_{t+1}$, this expression becomes

$$\begin{aligned} R_t A_t^{jw} * &\left[1-\pi_t-\frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)\sigma}\beta^\sigma\left(R_{t+1}\Omega_{t+1}\right)^{\sigma-1}\right] \\ &= \left[\pi+\frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)\sigma}\beta^\sigma\left(R_{t+1}\Omega_{t+1}\right)^{\sigma-1}\right]\left(H_t^{jw}+P_t^{jw}\right)-W_t l_t^{jw}(1-\tau_t) \\ &-\frac{\omega_{t+1}}{R_{t+1}\Omega_{t+1}}\left(H_{t+1}^{jw}+P_{t+1}^{jw}\right)-\frac{(1-\omega_{t+1})}{R_{t+1}\Omega_{t+1}}\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)\chi\varepsilon_{t+1}\left(H_{t+1}^{jr}(i)+P_{t+1}^{jr}(i)\right) \end{aligned}$$

Conjecturing that $H_t^{jw} = W_t l_t^{jw}(1-\tau_t) + \frac{\omega_{t+1}H_{t+1}^{jw}}{\Omega_{t+1}R_{t+1}} + \frac{(1-\omega_{t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)\chi\varepsilon_{t+1}H_{t+1}^{jr}(i)}{\Omega_{t+1}R_{t+1}}$ and $P_t^{jw} = \frac{\omega_{t+1}P_{t+1}^{jw}}{\Omega_{t+1}R_{t+1}} + \frac{(1-\omega_{t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)\chi\varepsilon_{t+1}P_{t+1}^{jr}(i)}{\Omega_{t+1}R_{t+1}}$, this can be written as

$$\left(R_t A_t^{jw}+H_t^{jw}+P_t^{jw}\right)*\left[1-\pi_t-\frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-\upsilon)\sigma}\beta^\sigma\left(R_{t+1}\Omega_{t+1}\right)^{\sigma-1}\right]=0$$

which implies that the workers' mpc can be solved to evolve according to

$$\pi_t = \left[1-\frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho\sigma(1-\nu)}\beta^\sigma\left(R_{t+1}\Omega_{t+1}\right)^{\sigma-1}\right] \tag{56}$$

### F.2. Aggregation

**Total assets**

Because of the perfect annuity market, $R_t A_{t-1}^r$ of the assets of retirees of period $t-1$ is carried to the next period $t$. Of workers who retire between periods $t-1$ and $t$, $(1-\omega_{t-1,\,t})(R_{W,t-1}A_{t-1}^w + W_{t-1}l_{t-1}^w(1-\tau_{t-1}) - C_{t-1}^w)$ adds to the retirees' assets in the beginning of period $t$ Thus, retirees' aggregate assets evolve according to

$$A_{t+1}^r = R_t A_t^r + (1-\omega_{t+1})\left(R_{t+1}A_t^w + W_t l_t^w(1-\tau_t)N_t^w - C_t^w\right) \tag{57}$$

Workers' aggregate assets evolve according to

$$A_{t+1}^w = \omega_{t+1}\left(R_{t+1}A_t^w + W_t l_t^w(1-\tau_t)N_t^w - C_t^w\right) \tag{58}$$

**Consumption**

Retirees' and workers' marginal propensities to consume do not depend on individual characteristics. The aggregate consumption for each group is thus total wealth times the groups' marginal propensities to consume. Retirees' aggregate consumption is

$$C_t^r = \varepsilon_t\pi_t\left(R_t A_t^r + H_t^r + P_t^r\right) \tag{59}$$

and workers' aggregate consumption is

$$C_t^w = \pi_t\left(R_t A_t^w + H_t^w + P_t^w\right) \tag{60}$$

Total aggregate consumption is

$$C_t = C_t^r + C_t^w = \varepsilon_t\pi_t\left(R_t A_t^r + H_t^r + P_t^r\right) + \pi_t\left(R_t A_t^w + H_t^w + P_t^w\right) \tag{61}$$

Denoting the share of assets held by retirees $\lambda_t = \frac{A_t^r}{A_t}$, total aggregate consumption is given by

$$C_t = \pi_t A_t R_{W,t}(\varepsilon_t\lambda_t + 1 - \lambda_t) + \pi_t\left(H_t^w + P_t^w\right) + \varepsilon_t\pi_t\left(H_t^r + P_t^r\right). \tag{62}$$

**Human wealth**

The present discounted value of retirees' aggregate human wealth is

$$H_t^r = \left(1 - \frac{L_t^w}{L_t}\right)\alpha Y_t(1-\tau_t) + \gamma_{t+1}\frac{\psi_t}{\psi_{t+1}}\frac{H_{t+1}^r}{(1+n_{t,t+1})R_{t+1}} \tag{63}$$

and the present discounted value of workers' aggregate human wealth is

$$H_t^w = \frac{L_t^w}{L_t}\alpha Y_t(1-\tau_t) - t_t^w Y_t + \omega_{t+1}\frac{H_{t+1}^w}{(1+n_{t,t+1})R_{t+1}\Omega_{t+1}} + (1-\omega_{t+1})\frac{H_{t+1}^r \varepsilon_{t+1}^{\frac{1}{1-\sigma}}\chi}{\psi_{t+1}(1+n_{t,t+1})R_{t+1}\Omega_{t+1}} \tag{64}$$

**Social security wealth**

With $S_t = S_t^{jr} N_t^r$, the present discounted value of retirees' aggregate pension benefits at time t is

$$P_t^r = S_t + \gamma_{t,t+1}\frac{P_{t+1}^r}{\left(1+n_{t,t+1}^r\right)R_{t+1}} \tag{65}$$

In the current period, workers receive no social security payments, but expect to receive social security once retired. The total value of social security payments for the working force at time t equals

$$P_t^w = \omega_{t,t+1}\frac{P_{t+1}^w}{\left(1+n_{t,t+1}^w\right)R_{t+1}\Omega_{t+1}} + (1-\omega_{t,t+1})\frac{P_{t+1}^r \varepsilon_{t+1}^{\frac{1}{1-\sigma}}\chi}{\psi_{t+1}(1+n_{t,t+1})R_{t+1}\Omega_{t+1}} \tag{66}$$

**Labor supply**

Because the individual labor supply for retirees (34) is linear in consumption which is linear in wealth (36), the aggregate labor supply by retirees is

$$L_t^r = N_t^r - \frac{\varsigma}{\xi W_t(1-\tau_t)} C_t^r \tag{67}$$

Similarly, aggregate labor supply by workers is

$$L_t^w = N_t^w - \frac{\varsigma}{W_t(1-\tau_t)} C_t^w \tag{68}$$

**Distribution of wealth**

Substituting expressions (58) and (60) into (57), the distribution of wealth between workers and retirees can be shown to evolve as follows:

$$\lambda_{t+1} = \omega_{t,t+1}\left(R_{W,t}\lambda_t\frac{A_t}{A_{t+1}}(1-\varepsilon_t\pi_t) + \frac{N_t^r W_t(1-\tau_t) l_t^r \xi}{A_{t+1}} + \frac{S_t - \varepsilon_t\pi_t(H_t^r + P_t^r)}{A_{t+1}}\right) + (1-\omega_{t+1}). \tag{69}$$

*F.3. Firms*

Firms operate in a competitive market and employ labor, capital, and investment to maximize present discounted value of profits, which is given by

$$V(I_{t-1}, K_t) = \max\left[(X_t(L_t^w + \xi L_t^r))^\alpha K_t^{1-\alpha} - W_t(L_t^w + \xi L_t^r) - I_t + \frac{V(I_t, K_{t+1})}{R_{t+1}}\right] \tag{70}$$

subject to the law of motion of capital

$$K_{t+1} = (1-\delta)K_t + \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_t\right)^2\right] I_t. \tag{71}$$

The term $\mu_t$ ensures that along the balanced growth path, investment adjustment costs equal zero. The capital is owned by the firm and wage is paid to the effective labor force. The aggregate effective labor force consists of the effective labor input by the two agent types such that $L_t = L_t^w + \xi L_t^r$. Capital depreciates at rate $\delta \in (0,1)$ and quadratic adjustment cost makes investing new capital costly. The size of the adjustment cost is determined by $\phi > 0$. The term $\mu_t$ ensures that along the balanced growth path, investment adjustment costs equal zero. Productivity $X_t$ grows at rate $x_t$, which follows an AR(1) process given by $x_t = (1-\theta)x_{ss} + \theta x_{t-1} + u_t^x$. $\alpha$ is the labor share. The Lagrangian is

$$\mathcal{L}(X_t(L_t^w + \xi L_t^r))^\alpha K_t^{1-\alpha} - W_t(L_t^w + \xi L_t^r) - I_t + \frac{V(I_t, K_{t+1})}{R_{t+1}} - q_t\left[(1-\delta)K_t + \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_t\right)^2\right]I_t - K_{t+1}\right]$$

and the first-order conditions with respect to labor force, capital, and investment solve

$$W_t = \alpha \frac{Y_t}{L_t^w + \xi L_t^r} \tag{72}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = -1 + \frac{\partial V(I_t, K_{t+1})}{\partial I_t} - q_t - q_t\left(\frac{I_t}{I_{t-1}} - \mu_t\right)^2 + q_t \phi \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - \mu_t\right) = 0$$

and

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \frac{\partial V(I_t, K_{t+1})}{\partial K_{t+1}} - q_t = 0$$

The envelope conditions are

$$\frac{\partial V(I_{t-1}, K_t)}{\partial I_{t-1}} = q_t \phi \left(\frac{I_t}{I_{t-1}}\right)^2 \left(\frac{I_t}{I_{t-1}} - \mu_t\right)$$

and

$$\frac{\partial V(I_{t-1}, K_t)}{\partial K_t} = (1-\alpha)\frac{Y_t}{K_{tt}} + (1-\delta)q_t$$

Substituting the envelope conditions into the first-order conditions gives the following first-order conditions for capital and investment:

$$q_t = \frac{1}{R_{t+1}}\left[(1-\alpha)\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)q_{t+1}\right] \tag{73}$$

and

$$q_t\left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_{t-1,t}\right)^2 - \phi\left(\frac{I_t}{I_{t-1}} - \mu_{t-1,t}\right)\frac{I_t}{I_{t+1}}\right] = 1 - \frac{\phi q_{t+1}}{R_{t+1}}\left(\frac{I_{t+1}}{I_t} - \mu_{t,t+1}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{74}$$

*F.4. A competitive world equilibrium and the external sector*

A competitive world equilibrium is a sequence of quantities and prices such that in each country (i) households maximize utility subject to their budget constraints, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance exogenous level of total spending, and (iv) all markets clear.

In each economy, total assets are the sum of capital stock, government bonds, and net foreign assets $A_t = K_t + B_t + F_t$. Net foreign asset position evolves according to $F_{t+1} = R_{t+1}F_t + NX_t$, where $NX_t$ is the trade balance $NX_t = Y_t - (C_t + I_t + G_t)$. Return $R_t$ is equalized across the two countries because of the clearing of the international asset markets, which implies that internationally traded assets are in zero net supply so that $F_t + F_t^* = 0$. Denoting the relative size of the economies by $RS_t = \frac{X_t N_t}{X_t^* N_t^*}$, the relative size of the economies evolves according to

$$\frac{X_{t+1}^* N_{t+1}^*}{X_{t+1} N_{t+1}} = \frac{X_t^* N_t^* (1 + x_{t,t+1}^* + n_{t,t+1}^*)}{X_t N_t (1 + x_{t,t+1} + n_{t,t+1})} \tag{75}$$

Given an initial relative size $\overline{RS}$ in period 1, the size in period $T$ is

$$\begin{aligned} RS_T &= \overline{RS}(1 + x_1 + n_1)(1 + x_2 + n_2)\ldots(1 + x_T + n_T) \\ &= \overline{RS} \prod_{s=1}^{T-1} (1 + x_{1+s} + n_{1+s}) \end{aligned} \tag{76}$$

Exogenous policy variables, the government-spending-to-output ratio $g_t$ and social-security-to-output ratio $s_t$, follow first-order autoregressive processes as follows:

$$g_t = (1-\theta_t)g^{ss} + \theta_t g_{t-1} + u_t^g \tag{77}$$

and

$$s_t = (1-\theta_t)s^{ss} + \theta_t s_{t-1} + u_t^s \tag{78}$$

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