1042-2587

Computable Entrepreneurship

Roger Koppl

The mathematics of "computable economics" proves that entrepreneurship policy is unlikely to succeed if it presumes policy makers can replace the unplanned results of the entrepreneurial market process with *ex ante* judgments about which enterprises are best. It is mathematically impossible for policy makers or their assignees to make the required computations of opportunity costs. Some business professors dream of finding a grand algorithm that will allow them to guide entrepreneurial decisions and to judge in advance which decisions are good and which bad. The logic of computable economics, however, reveals this dream to be a form of magical thinking.

Introduction

When supporting policies promoting entrepreneurship, policy makers have some expectation regarding the outcome of those policies. While the desire to control outcomes may be a good thing in some policy contexts, in the case of entrepreneurship policy, it is likely to be a bad thing. The problem is that the policy may not produce its intended result. While policies can go awry and achieve only unintended results for many reasons, I will pay attention to one important reason that seems to have been neglected by scholars of entrepreneurship policy: Some entrepreneurship policies require policy makers to perform impossible computations.

To control outcomes, policy makers must substitute their own decisions for those of entrepreneurs. In the case of entrepreneurship policy, that substitution means replacing the outcome of the entrepreneurial market process (described below) with the outcome of a computation made by a policy maker. Instead of waiting to see what result the market produces, the policy maker decides who the winners should be. The Georgia Research Alliance (GRA), for example, directly finances start-ups (PRNewswire, 2008), thus substituting its judgment for that of credit markets by directing tax money to favored start-ups. Such policies are not likely to have their intended effects, because the policy-making body cannot figure out what it needs to know to make its choices. Arguments borrowed from the mathematics of "computable economics" help to show why.

The entrepreneurial market process consists of the daily decision making of many independently acting entrepreneurs, each striving to establish, maintain, or develop an enterprise. Each entrepreneur responds principally to the business environment consisting of rivals, input suppliers, and output demanders. The overall result of the process is generated by the distributed decisions of many entrepreneurs (and others). The overall result is the unintended consequence of all those decisions, unintended because the actors

Please send correspondence to: Roger Koppl, tel.: 1 973 443 8846; e-mail: koppl@fdu.edu.

have not gotten together ahead of time to coordinate their actions. The aggregate level and type of entrepreneurial activity emerges as the unintended consequence of the actions taken by all independent entrepreneurs in the attempt to seize profit opportunities.

The process is decentralized and therefore unplanned, even though each individual entrepreneur plans. Some entrepreneurs plan more carefully, others less. Each entrepreneur makes a computation of prospective profit, whether the computation is explicit, precise, and sophisticated, or implicit and approximate. Thus, the entrepreneurial market process involves plenty of planning and computation. But the overall process itself is not planned. Its outcome, therefore, is not computed ahead of time. Hayek characterized the entrepreneurial market process as a "discovery process" because we need the process to discover what each of the different actors in the system should do. The entrepreneurial market process does not know where it is going until it gets there, nor does it need to. A government wishing to control the outcome of the entrepreneurial market process, however, must do what the market does not do: plan the overall result of the process in advance. It must compute outcomes. A government wishing to control outcomes must predict the results of the process ahead of time. But, as we shall see, such prediction is not always possible.

The notion that markets are hard to predict may be familiar to most readers, and it is a commonplace that it is hard to outguess financial markets. It may be unfamiliar, however, to link the idea to relatively abstract branches of mathematics. Making such a link clarifies that the difficulty of predicting the outcome of the entrepreneurial market process is not merely a question of the delicacy of the issue or the need to apply the proper skills and knowledge. It is not a question of difficulties that need to be overcome. When entrepreneurship policy requires policy makers to predict the entrepreneurial market process, it sets policy makers a truly impossible task.

Computable Economics

The literature on "computable economics" (Velupillai, 2005) has helped to make social scientists aware of the sort of impossible tasks I refer to. Computable economics looks at economics from the point of view of what can and cannot be computed. In traditional economics, everyone is able to do all sorts of complicated computations. You may literally see pages of complicated mathematics describing what the economist thinks entrepreneurs and others are doing. Common sense suggests that economists often assume too much about what mathematical feats can be achieved by the economic actors of their theory. Computable economics goes beyond this commonsense insight to identify cases in which traditional economists have assumed that people can perform feats of computation that are mathematically impossible.[1]

Mathematicians use the words "computable" and "noncomputable" to distinguish possible from impossible mathematical feats. Mathematicians say that a function is not "computable" if you cannot program a computer to solve it. They require only that the computer has a finite memory and the program has a finite length, since it is not possible to build a computer with infinite memory or write an infinitely long program.[2] If no combination of computer and program can solve a well-formulated mathematical problem, then the problem is "undecidable." To show that a problem is undecidable, a mathematician must show that no possible combination of program and computer will get

1. In an article-length review of Velupillai's volume, Koppl (2008) provides a reader-friendly survey of the mathematical tools of computable economics.
2. See Koppl and Rosser (2002, p. 344) for a somewhat more technical account of computability.

Figure 1

Matching Pennies

		Player Two	
		Heads	Tails
Player One	Heads	0,1	1,0
	Tails	1,0	0,1

Each player must choose heads or tails. The first number in each box is the payoff to Player One, who chooses the row. The second number is the payoff to Player Two, who chooses the column. A payoff of 1 is preferred to a payoff of 0. If both players pick the heads or both pick tails, then Player Two wins. If one picks heads and the other picks tails, then Player One wins.

you the answer. In the strict mathematical sense, when a function is not computable or a problem not decidable, it is not just that we cannot hope to solve it realistically. It is not merely hard to solve; it is impossible to solve. As I will argue, the existence of such impossible-to-solve problems is relevant for entrepreneurship policy.

The literature on computability and decidability goes back to Kurt Gödel's famous incompleteness proof of 1931. Gödel (1931/1967) showed that mathematics is radically incomplete because there are true theorems that cannot be proved. He showed that there is literally no end to the list of true-but-not-provable theorems. Thus, he put his finger on a big problem, namely, that there are whole regions of mathematical truth that we just cannot get to. Before Gödel, many mathematicians implicitly assumed we could compute anything, at least in principle, and rigorously prove any true mathematical theorem. After Gödel, it has become an ongoing challenge to work out what we can and cannot compute, what is and is not mathematically possible.

A simple example from game theory may help to suggest what noncomputability is all about and why this branch of mathematics may carry lessons for entrepreneurship policy.[3] Figure 1 illustrates the game of matching pennies. Because the two players must make their choices simultaneously and independently, neither one can make a reliable computation of the other player's action and cannot, therefore, "react" properly to it. If player 1 picks heads, for example, then player 2 should "react" by picking tails. But if player 2 picks tails, then player 1 should "react" by picking tails, which would cause player 2 to "react" by picking heads. And so on. Each "reaction" on the first side causes a "reaction" on the other side that induces further "reaction" on the first side. In the game of matching pennies, it is mathematically impossible to compute the right strategic choice (Koppl & Rosser, 2002). Thus, any decision a real person might make in this situation would have an element of arbitrary choice in it. You cannot compute your way to a correct strategy choice.[4] Note that the problem is not missing data. Each player is fully informed, and yet each may be unable to compute a correct strategy choice.

3. The example from Koppl and Rosser (2002) merely clarifies and highlights some of Binmore (1987).

4. Experts in game theory might object that one can easily compute the "Nash equilibrium in mixed strategies." Koppl (forthcoming) explains the error in this objection, which is that no one has a logically

This example shows that computability issues are not just about abstract problems in mathematics. They can crop up in social situations because each person needs to anticipate the actions of others in order to compute his or her best path forward. In the entrepreneurial market process, entrepreneurs need to anticipate the actions of their rivals, of suppliers of their inputs, of demanders of their output, and of any policy makers whose decisions may affect the markets in which they operate. It is generally impossible to anticipate all such actions by means of some master computation, because the different players involved are independent actors trying to make their own best paths forward.

The example of matching pennies invokes the childhood logic of "you think that I think that you think." Player 1 picks tails if he thinks that player 2 will pick heads. Player 1 picks heads, however, if he thinks that player 2 thinks that he thinks that player 2 will pick heads. But if he thinks that player 1 thinks that he thinks . . . In this situation, "there is exhibited an endless chain of reciprocally conjectural reactions and counter-reactions. This chain can never be broken by an act of knowledge but always only through an arbitrary act—a resolution . . . The paradox still remains no matter how one attempts to twist or turn things around" (Morgenstern, 1935/1976, p. 174). The paradox cannot be resolved because, as Koppl and Rosser (2002) have shown, it is mathematically impossible to compute a best strategy.

The same problem of interlocking expectations crops up in the more complex interactions of the entrepreneurial market process. This problem imposes a constraint on what entrepreneurship policy can successfully aim at. Policies that can succeed only if the policy maker correctly computes the outcome of the entrepreneurial process are doomed to failure because it is, in general, mathematically impossible to make such computations.

Da Costa and Doria (2005) draw out an important policy implication of the computability problems we have been reviewing. The "determination of equilibrium prices in a competitive market" is "formally equivalent" to solving games whose solution may be mathematically impossible to compute. "So, the main argument in favor of a planned economy clearly breaks down." And yet, they report, "the equilibrium point of the market is eventually reached while we cannot in general compute it beforehand" (pp. 38–39).[5]

Well-meaning policy makers are overambitious when they try to substitute policy plans for the entrepreneurial market process. Markets have a crucial advantage over such overambitious policy makers: markets do not have to know where they are going. Markets can achieve their equilibria because the overall results are not planned and need not be computed in advance. Policies that require policy makers to compute the results of the entrepreneurial market process ahead of time give policy makers an impossible task.

Entrepreneurship Policy

Policies ensuring institutional transparency, predictable taxation, and secure property rights do not require policy makers to compute specific outcomes in order to achieve their

---

compelling reason to go along with this supposed "solution" to the game. Koppl and Rosser (2002) consider mixed strategies. The problem is not that no pure-strategy Nash equilibrium exists. As Koppl (forthcoming) explains, the problem is that participants may not be able to compute a best-reply strategy even when observers can compute the Nash equilibrium.

5. The result of Da Costa and Doria is quite radical and even shocking to mathematicians in this field. Da Costa and Doria point out that "finite games" may not be computable if the description of the game is not fully explicit. Their result shows that problems of computability are ubiquitous, cropping up even in contexts we had mistakenly thought of as safe from such problems.

intended goal of promoting entrepreneurial ventures. Such policies create a reliable set of rules that entrepreneurs can play by. Policy makers cannot predict which entrepreneurs will be winners and which will be losers, but they can predict that the game will go on in an orderly way and produce favorable outcomes. Policies that attempt to control outcomes, however, are not likely to succeed. Policies that, for example, directly support start-ups that are expected to contribute the most to "jobs growth" or some other end cannot succeed unless policy makers perform the mathematically impossible feat of predicting the future.

Velupillai (2007) has made the point in a valuable paper on "The Impossibility of an Effective Theory of Policy in a Complex Economy." He argues that, in the informal summary of Salzano and Colander (2007), "ultimately there is an undecidability of policy in a complex economy" (p. xvii). More precisely, he shows that "an *effective* theory of economic policy is impossible for such an economy" (p. 273). Velupillai is very careful to give "effective" a precise mathematical meaning (p. 273, n. 1). Rosser (1939) expressed the basic idea as meaning, "essentially that an effective method of solving a certain set of problems exists if one can build a machine which will then solve any problem of the set with no human intervention beyond inserting the question and (later) reading the answer" (p. 56). Thus, Velupillai (2007) has shown that if the economy is complex, then you cannot program a computer to predict the specific outcome of a policy.[6]

Velupillai's argument seems to have led him to an appreciation of Nobel laureate F. A. Hayek's "lifelong skepticism on the scope for policy in economies that emerge and form spontaneous orders" (p. 288).[7] Hayek, as I noted above, characterized the entrepreneurial market process as a "discovery process." The economic problem is not so much how to allocate given resources optimally as how to figure out what the "givens" really are. Hayek's insights suggest that entrepreneurship policy is the most important application of Velupillai's policy undecidability argument. The Army Corps of Engineers, for example, faces the same computational problems faced by private engineering firms. It is otherwise with the entrepreneurial market process. Each entrepreneur within that process attempts to compute prospective profit. Some succeed, others fail.[8] But the process does not require anyone to compute the final result ahead of time. In a very literal sense, the market does not know where it is going and does not need to know. Policy makers who wish to control or greatly influence the results of the entrepreneurial market process must know where they want to go. They must calculate outcomes ahead of time, which, Velupillai has shown, is not generally possible.[9]

Modern governments do not always recognize the sort of epistemic limits featured in computable economics. On January 17, 2008, for example, Georgia Governor Sonny

6. He carefully specifies "complex economy" to imply "a dynamical system capable of computation universality" (Velupillai, 2007, p. 280).

7. I am puzzled by Velupillai's appeal to "poetry . . . imagination and compassion" in policy making. While we want policymakers to have imagination and compassion—especially compassion—I do not see how these traits overcome our epistemic limits.

8. My argument does not depend on any narrow assumptions about entrepreneurial motives. Entrepreneurs compute prospective profit even when other ends motivate them. Sometimes profits are a way to keep score. Sometimes an entrepreneur is willing to sacrifice profits for other ends. In any event, however, an entrepreneur must compute prospective profits if only to avoid unwanted bankruptcy.

9. An anonymous referee asks what computable economics implies about the role of cost-benefit analysis in policy making. Once you have numerical values for costs and benefits, you can subtract the sum of costs from the sum of benefits. The problem is whether the costs and benefits you have listed are both correct and complete. In some cases, that question may be linked to computability issues. It is not generally possible to construct a reasonably complete and correct list of costs and benefits of a policy whose effects are not computable.

Perdue announced "the creation of a $40 million Georgia Research Alliance (GRA) Venture Capital Fund, which will allow the state to partner with the private sector and provide early-stage financing to startup companies based on ideas developed in Georgia's research universities" (PRNewswire, 2008). Governor Perdue seems to express confidence in his ability to predict market outcomes when he says, "I expect Georgia to lead the way as a global center for health, and helping our young technology and research companies grow and stay in Georgia is a key part of that effort" (PRNewswire, 2008). The computability problems I have pointed to, however, suggest that the Governor's apparent confidence is unwarranted. To know that health and not, say, ceramics is the right future for the Georgia economy, the Governor would have to be able to calculate the consequences of supporting each industry in order to deduce the superior benefits of promoting the health care industry.

Undecidability affects a policy's critics and proponents alike. Therefore, it is not generally possible to provide positive proof that a given policy takes the wrong course. It is impossible to prove, for example, that Governor Perdue is wrong and health care is not the best economic path for Georgia. But if the Governor is right, it would only be by a wild coincidence. You will never have solid evidence the policy was wrong unless the chosen policy is an obvious and abject failure. If ceramics happened to be the right path, then the value of Georgia's ceramics output would exceed the value of its healthcare output. The alternative output, the opportunity cost, would be higher than the chosen output. But if Velupillai's undecidability result is correct, then we cannot calculate that opportunity cost and we cannot therefore provide positive proof that the wrong course has been adopted.

The upshot of my computability argument is that entrepreneurship policy should not be built on the assumption that policy makers can second-guess the entrepreneurial market process. It would be a bit of a fluke if Georgia's investment in health care is worth the opportunity cost, i.e., the loss in alternative outputs. But because opportunity costs are impossible to calculate, it seems likely that the state of Georgia will be able to claim success for its investment in health care. It will probably be possible to attribute many healthcare jobs and much measured output to the state's action. This conjecture is supported by the website of the GRA.[10] The site says that the GRA "has evaluated the commercial potential of more than 250 inventions or discoveries at universities," which "has led to the formation of 66 early-stage companies that employ more than 430 people and have attracted $200 million in private equity investment." That is what is seen. What is not seen, as Frederic Bastiat (1850/1964) would have been quick to point out, is the alternative use for each resource. We see the current output, but not its opportunity cost. Thus, we should not think that policy makers can in fact second-guess the entrepreneurial market process when they point to the supposed successes of the past.

The point is underlined by J.S. Mill's comments on protectionism. When the government had supported an industry, Mill notes, "it would plume itself upon having enriched the country with a new branch of industry, would parade in statistical tables the amount of produce yielded and labour employed in the production, and take credit for the whole of this as a gain to the country." In fact, however, "any portion" of the gain they "had caused to be embarked in the newly-acquired branch of industry must have been withdrawn or withheld from some other; in which it gave, or would have given, employment to probably about the same quantity of labour which it employs in its new occupation" (Mill, 1870, pp. 95–96).

10. http://www.gra.org/ProgramsInitiatives/Commercialization/tabid/372/Default.aspx, accessed 12 June 2008.

If the state of Georgia parades statistical tables purporting to prove success for its healthcare venture, we should not consider the policy a true success and we should not imagine that Georgia policy makers have somehow succeeded in computing what is not computable.

The Organisation for Economic Co-operation and Development (OECD) report on National Innovation System provides another example. It encourages policy makers to "design" the "informal flows of knowledge and access to technical networks" and the "linkages and partnerships" involved in "innovative clusters," all "in the most efficient manner" (OECD, 1997, pp. 41–42). The computational task they invoke is certainly difficult, and probably impossible.

Although the OECD report is over 10 years old, its computational ambitions are, unfortunately, perfectly current. Ernst and Hart (2007), for example, call upon the "global community" to "anticipate" the negative externalities "that accompany the emergence of the global knowledge economy" (pp. 21–22). They call for "[g]lobal governance of the knowledge economy" that will "steer the emerging social capabilities for innovation in constructive directions and . . . build self-reinforcing momentum behind them" (p. 38).

My analysis points clearly in a promarket direction, but I wish to avoid the impression that promarket policy making is an easy business. First, little is said when we declare the superiority of "the free market" over "intervention." Any market is governed by formal and informal rules, and no one set of rules is uniquely able to render markets "free." It is easy enough to see the difference between Soviet-style socialism and Western democratic capitalism. It is not always easy to decide when a marginal change in the rules diminishes freedom. Second, computability problems affect policy makers who want to remove controls as well policy makers who want to impose them. The undecidability of policy carries dangers for deregulation, too.[11] Deregulation in the wholesale electric energy market created the California energy crisis of 2000 and 2001 because it "took the form of deregulating wholesale markets and prices while continuing to regulate retail prices at fixed hourly rates over the daily and seasonal cycles in consumption" (Smith, 2003, p. 472). Overambitious policy makers create dangers no matter the direction of their policies.

As we have seen, entrepreneurship policy can be overambitious. Some business professors dream of finding a grand algorithm that will allow them to guide entrepreneurial decisions and to judge in advance which decisions are good and which bad. The logic of computable economics, however, reveals this dream to be a form of magical thinking. We need entrepreneurs to make their decisions for themselves precisely because it is impossible for us to make those decisions for them. In other words, we need entrepreneurs, not functionaries. Entrepreneurship scholars should not forget this need when they give policy advice. Entrepreneurship scholars should counsel against policies that would require policy makers to perform the sorts of impossible computations discussed in this note. They should remind policy makers that we need entrepreneurs to help us discover through trial and error what we cannot compute with the aid of the best computers—namely, the future of the entrepreneurial market process.

REFERENCES

Bastiat, F. (1964). What is seen and what is not seen. In S. Cain (Trans.), *Selected essays on political economy* (pp. 1–50). New York: D. Van Nostrand Company, Inc. (Original work published 1850).

11. In Koppl (forthcoming) I use complexity theory, including computable economics, to defend "Humean status quo bias." I quote Hume warning that "a regard to liberty, though a laudable passion, ought commonly to be subordinate to a reverence for established government" (Hume, 1983, Vol. VI, p. 533).

Binmore, K. (1987). Modeling rational players I. *Economics and Philosophy*, *3*, 9–55.

Da Costa, N.C.A. & Doria, F.A. (2005). Computing the future. In V. Velupillai (Ed.), *Computability, complexity and constructivity in economic analysis* (pp. 15–50). Oxford: Blackwell.

Ernst, D. & Hart, D.M. (2007). Governing the global knowledge economy: Mind the gap! Available at: http://mason.gmu.edu/~dhart/GGKE.doc, accessed 18 May 2008.

Gödel, K. (1967). On formally undecidable propositions of *principia mathematica* and related systems I. In J. van Heijenoort (Ed.), *From Frege to Gödel: A source book in mathematical logic, 1879–1931* (pp. 582–591). Cambridge, MA: Harvard University Press. (Original work published 1931).

Hume, D. (1983). *The history of England from the invasion of Julius Caesar to the revolution in 1688*. Indianapolis, IN: Liberty Fund.

Koppl, R. (2008). Thinking impossible things: A review essay. *Journal of Economic Behavior and Organization*, *66*, 837–847.

Koppl, R. (forthcoming). Complexity and Austrian economics. In R.J. Barkley (Ed.), *Handbook on complexity research*. Cheltenham, UK: Edward Elgar.

Koppl, R. & Rosser, J.B., Jr. (2002). All that I have to say has already crossed your mind. *Metroeconomica*, *53*(4), 339–360.

Mill, J.S. (1870). *Principles of political economy, Vol 1*. New York: D. Appleton and Company.

Morgenstern, O. [1935] (1976). Perfect foresight and economic equilibrium. In A. Schotter (Ed.), *Selected Economic Writings of Oskar Morgenstern*. New York: New York University Press.

Organisation for Economic Co-operation and Development (OECD). (1997). *National innovation systems*. Paris: OECD Publications. Available at: http://www.oecd.org/dataoecd/35/56/2101733.pdf, accessed 17 May 2008.

PRNewswire. (2008, January 17). $40 million Georgia Research Alliance venture capital fund announced. Available at: http://www.reuters.com/article/pressRelease/idUS227397+17-Jan-2008+PRN20080117, accessed 17 May 2008.

Rosser, J.B. (1939). An informal exposition of proofs of Godel's theorems and Church's theorem. *The Journal of Symbolic Logic*, *4*(2), 53–60.

Salzano, M. & Colander, D. (2007). Preface. In M. Salzano & D. Colander (Eds.), *Complexity hints for economic policy* (pp. v–xvii). Milan: Springer.

Smith, V. (2003). Constructivist and ecological rationality in economics. *American Economic Review*, *93*(3), 465–508.

Velupillai, V. (Ed.). (2005). *Computability, complexity and constructivity in economic analysis*. Oxford: Blackwell.

Velupillai, V. (2007). The impossibility of an effective theory of policy in a complex economy. In M. Salzano & D. Colander (Eds.), *Complexity hints for economic policy* (pp. 273–290). Milan: Springer.

Roger Koppl is Professor of Economics and Finance at the Silberman College of Business of Fairleigh Dickinson University and Director of the university's Institute for Forensic Science Administration.

I thank Maria Minniti and two anonymous referees for helpful comments.