Pricing Asian Options

By Jovan Allan, Joseph Leong

What Is an Asian Option?

Also known as an Average Price Option

There are many variations of asian optioins, we will be looking at this particular example where

the payoff for asian call option is
$$ C_a = Max\left(\frac{1}{n} \sum_{i=1}^{n} S_d(i) - K,0\right) $$

Compared to European Call
$$ C_e = Max\left(S(t) - K,0\right) $$

Advantages

- Reduces the risk of Market manipulation
- Less Volatile than European options

Monte Carlo Simulation

This is a statistical technique used to model probabalistic events.

Idea:

- generate many values of outputs from a predefined range of inputs
- aggregate the outputs to get a sample average.
- sample average will converge to expected value as # of outputs increase
- sample average will have an approximatly Normal Distribution as # of outputs increases

Applying this idea to Asian Option Prices

First, generate daily asian share prices using Risk-Neutral GBM model $${S_d(0),S_d(1),...S_d(n)} $$

Second, calculate the present value of the Asian Option Payoff
$$ Y=e^{\frac{-rn}{N}} \left(\sum_{i}^{n} \frac{S_d(i)}{n} -K \right)^{+} $$

Third, Repeat to generate $k-1$ more simulations so that in total there are k simulated Asian Option payoffs
$$ Y_1,Y_2...Y_k $$

Last, average of k simulated Asian Option payoffs will yields the estimate of the Risk-Neutral valuation of the call. $$ \sum_{i}^{k} \frac{Y_i}{k} \rightarrow e^{\frac{-rn}{N}}E_{RN} \left(Payoff \right) $$

MONTECARLO SIMULATION Code

using the following parameters

s = 61 d = 126 (1/2 year) K = 63 r = 0.08 σ = 0.3

In [2]:
```
#Variable Selection
share = 61
day = 126
strike = 63
rate = 0.08
sigma = 0.3
```

In [40]:
```
#Creates a matrix of n simulations of stock realizations
asianOptions=function(s,D,K,r,sd,n) {
    xd=matrix(c(rnorm((D)*n,(r-(sd^2/2))/252,sd/sqrt(252))),n,D)
    sd=s*exp(t(apply(xd,1,cumsum)))
    avg=apply(sd,1,mean)
    callpv=pmax(avg-K,0)*exp(-r*D/252)
    return(mean(callpv))
}

asianOptions.V=function(s,D,K,r,sd,n) {
    xd=matrix(c(rnorm((D)*n,(r-(sd^2/2))/252,sd/sqrt(252))),n,D)
    sd=s*exp(t(apply(xd,1,cumsum)))
    avg=apply(sd,1,mean)
    callpv=pmax(avg-K,0)*exp(-r*D/252)
    return(var(callpv))
}

#Create Euro Call option
euroOption=function(s,K,r,sd,t) {
  w=((r+sd^2/2)*t/252-log(K/s))/(sd*sqrt(t/252))
  c=s*pnorm(w)-K*exp(-r*t/252)*pnorm(w-sd*sqrt(t/252))
  return(c)
}
```

How Does Asian Option Compare to Euro Option?

In [4]:
```
x=0;for(i in 1:1000) {
  x=x+asianOptions(share,day,strike,rate,sigma,1000)
}
print(x/1000)
```

```
[1] 2.626515
```

Compare to European option price?

In [4]:
```
euroOption(share,strike,rate,sigma,day)
```

5.37134528172943

Visulization of Asian Option

In [5]:
```
realization = function(s,D,K,r,sd,n){
    xd=matrix(c(rnorm((D)*n,(r-(sd^2/2))/252,sd/sqrt(252))),n,D)
    sd=s*exp(t(apply(xd,1,cumsum)))
    return(sd)
}
```

In [75]:
```
x = seq(1:126)
y = realization(share,day,strike,rate,sigma,10)
plot(x,y[1,], main = "Plot a single Simulation of Stock prices over 126 days"
,
     xlab = "Days",
     ylab = "Share Price $",
     xlim = c(0,126),
     ylim=c(20,100),
     type = 'l')
abline(h=strike)
print("Average Stock Price")
mean(y[1,])
print("Payoff of Asian Option")
max((mean(y[1,]) - strike),0)*exp(-rate*day/252)
```

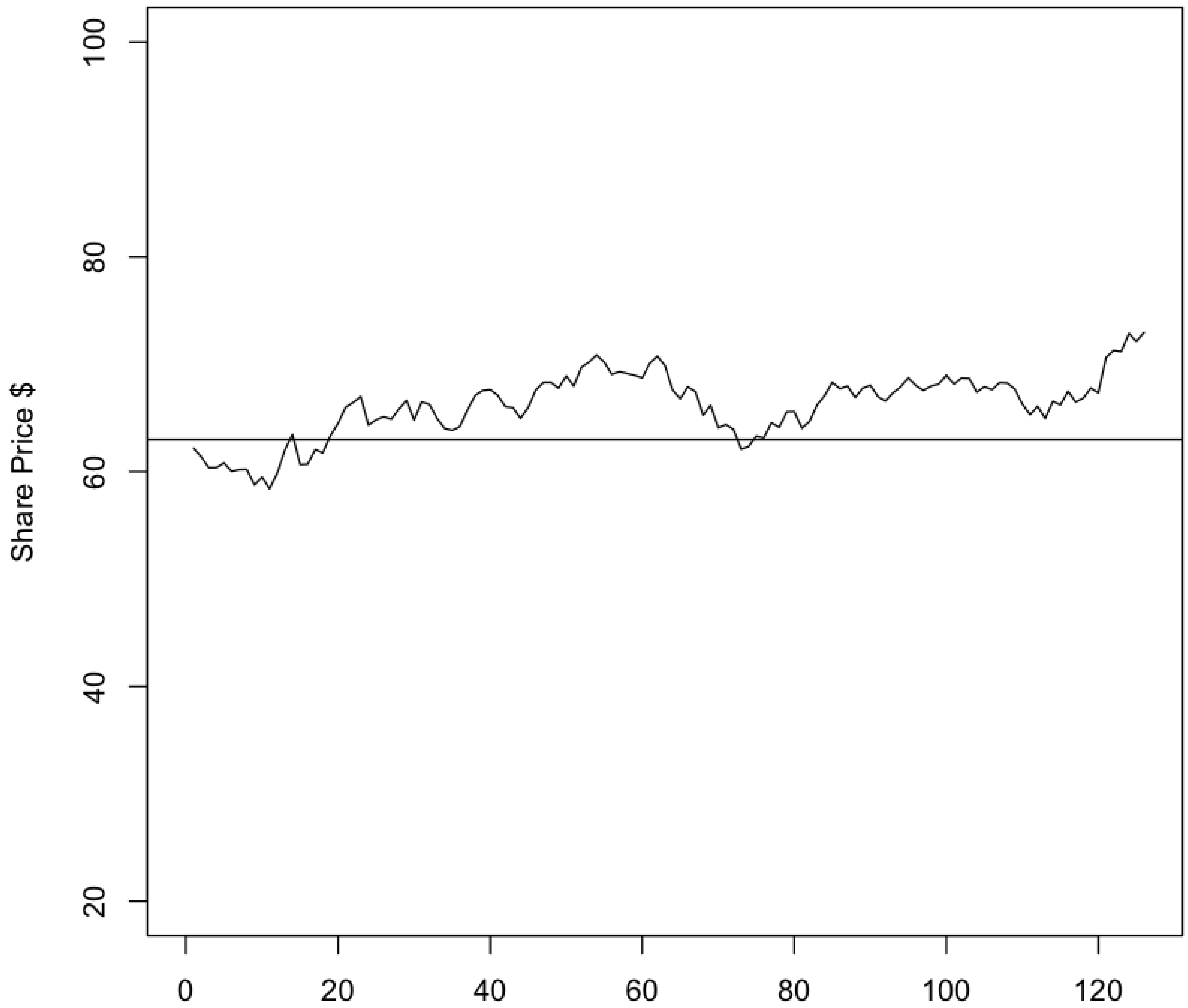
```
[1] "Average Stock Price"
```

66.1635730426605

```
[1] "Payoff of Asian Option"
```

3.03952756937519

Plot a single Simulation of Stock prices over 126 days



Days

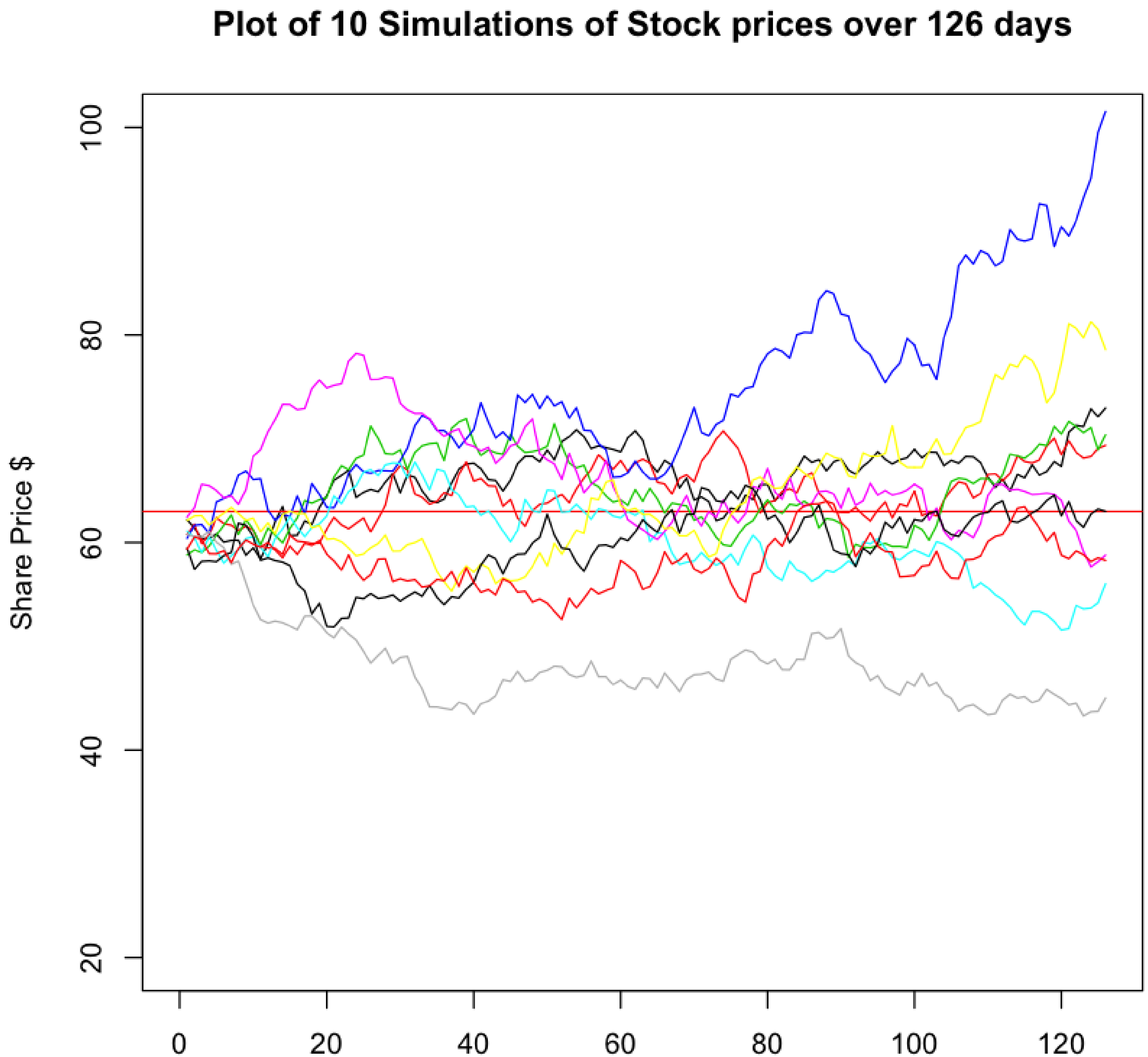
In [77]:
```
plot(x,y[1,], main = "Plot of 10 Simulations of Stock prices over 126 days",
     xlab = "Days",
     ylab = "Share Price $",
     xlim = c(0,126),
     ylim=c(20,100),
     type = 'l')
abline(h= strike, col='red' )
for( i in 2:10){
    lines(x,y[i,], col = i)
}
avg=apply(y,1,mean)

print("Average Stock Price")
avg
print("Payoff of Asian Option")
mean(pmax((avg - strike),0)*exp(-rate*day/252))
```

```
[1] "Average Stock Price"
```

66.1635730426605 64.7027101012699 65.0614325373428 74.4280104141445
60.0866543796086 66.5024792453617 64.7272195595935 48.2245928004413
59.8177080588912 58.1886026979329

```
[1] "Payoff of Asian Option"
```

2.26606271621985


Plot of 10 Simulations of Stock prices over 126 days
Share Price $
100
80
60
40
20
0
20
40
60
80
100
120

Days

Expected Value and Variance

for 100, 100 000 , 1 000 000 simulations ran 10 times

In [78]:
```
numofsim = 100
for(i in 1:10){
    print(c(asianOptions(share,day,strike,rate,sigma,numofsim),asianOptions.V
(share,day,strike,rate,sigma,numofsim)))
    }
```

```
[1]  2.612068 28.534359
[1]  2.896276 26.756216
[1]  2.15570 18.95884
[1]  1.837812 25.745627
[1]  3.279394 21.071770
[1]  2.401967 26.215206
[1]  3.635624 14.438151
[1]  2.838711 22.412249
[1]  2.791585 18.892563
[1]  2.984013 17.974606
```

n = 1000000 [1] 2.632722 20.175960 [1] 2.629572 20.107152 [1] 2.617941 19.956537 [1] 2.625505 20.098192 [1] 2.624909 20.016488 [1] 2.615629 19.977494 [1] 2.628922 20.132319 [1] 2.619424 20.036418 [1] 2.623533 20.027172 [1] 2.632191 20.116323

n = 10000000 [1] 2.623967 20.082084 [1] 2.625328 20.081116 [1] 2.622219 20.046961 [1]

2.623188 20.043647 [1] 2.622652 20.065704 [1] 2.625139 20.087812 [1] 2.622739 20.044615 [1] 2.623389 20.073185 [1] 2.624379 20.077573 [1] 2.625775 20.083400

To be accurate within 0.01 cent
$$ 0.01<\sqrt{VAR(\bar{Y}}) = \sqrt{\frac{VAR(Y)}{k}} $$

Since $VAR(Y)$ is unknown, use the sample variance $\widehat{VAR(Y)}$ instead
$$ \begin{align} 0.01 &< \sqrt{\frac{\widehat{VAR(Y)}}{k}} \\ k &< \frac{\widehat{VAR(Y)}}{0.01^2} \\ &< \frac{(\approx 20)}{0.01^2} \\ &< 200000 \end{align} $$

To be accurate within 0.001 $$ \begin{align} 0.001 &< \sqrt{\frac{\widehat{VAR(Y)}}{k}} \\ k &< \frac{\widehat{VAR(Y)}}{0.001^2} \\ &< \frac{(\approx 20)}{0.001^2} \\ &< 2000000 \end{align} $$

How does the parameters affect the cost of asian option?

- Convex in s and K

Discussion

- Slow due to need to simulated lot of realizations
- Probabalistic results as oppose to determinstic