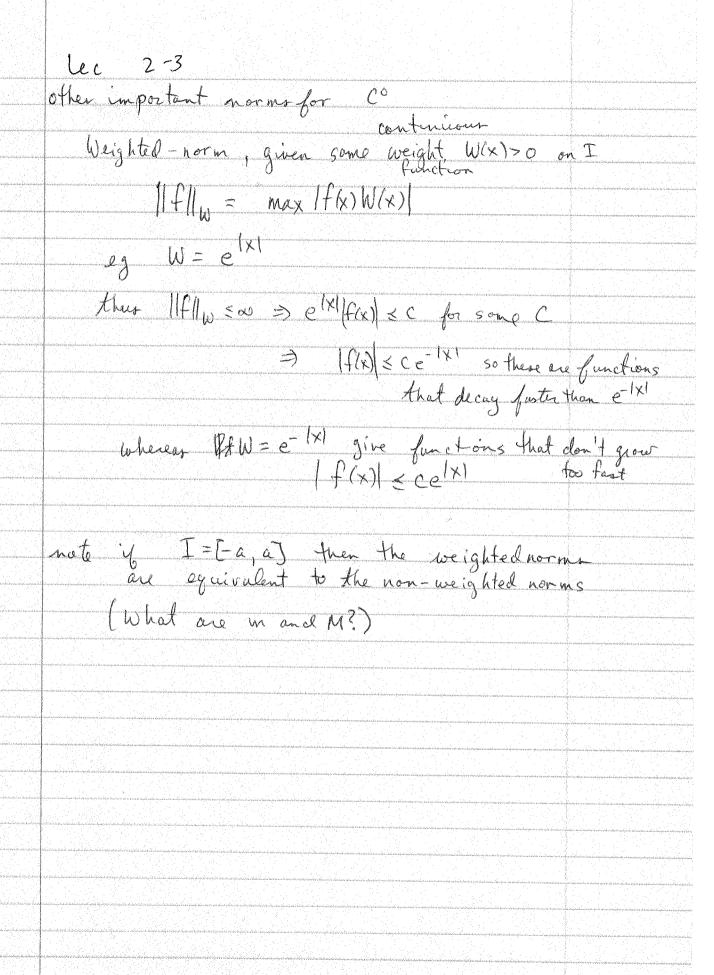
A FIXED - POINT THEOREM,	
LECTURE POR 3-1 (WILL BE USED TO SIMPLIFY	
EXISTENCE	WHIQUENESS
A FIXED-POINT THEOREM, LECTURE 2000 3-1 (WILL BE USED TO SIMPLIFY EXISTENCE- Review: What is a vector space?	
Closed us des addition and a tralication to	and the second s
Closed under addition and multiplication bill it is some space and xes, ye	S a LER
then av + bu E S (Note this sain die	that Shas
then ax+by E S (Note this implies a year	o elevent)
What is a norm? a norm 1/x/1 is a map from &	5 & Bt
& xxx Saturbuins (i) Inlax 11 = lat 11x11	rearrows are summer eministrated and the distribution of the properties of the summer
$(ii) x+a \leq x + a $	TRIANGLE
(i) \\\\	H.X = O REQUALTY
	AND AMERICAN STATE TO THE SERVICE SERVICE STATE SERVICE AND
What is a Banachy Space?	. THE EXCENSION AND AND AND EXCENSION AND AND AND AND AND AND AND AND AND AN
A complete normed linear dector space	Avera seminal militarian reconstruction for the confidence of the
A complete nor med linear dector space note the norm defined a distance between x &y d(x,y) = x-y) The sequence (um) > u = um-u -	TOO STATE THE PROPERTY OF THE
d(x,y) = x-y	Professional American and American and American and American and American and American Americ
I The seguence <um>> u = um-u -</um>	
A seguence (um) is Cauchy of 4870 JN>0 st.	n, n > N
2/ un- un 1/2	CONTRACTOR
A normed linear space B is a Banach space	-yit
A normed linear space B is a Banach space is complete it y each Cauchy sequence (um)	· -> u ues
Norms play the role of absolute values	
let 5 c B, 5 is closed if it contains all its limit point	to
Norms play the role of absolute values let 5° CB, 5° is closed if it contains all its limit point USING BANACH SPACE IDEAS, WE'LL RE-PROVE EXISTED	UCE-UNIQUENESS
the threateness are all the second and the second are all the second and the second and the second are all the second and the	UP CONTROL OF THE PROPERTY OF
SUPPOSE f(t,x) COMPONENTWISE CONTINUOUS IN IRM+1	es series proposes serviciones escretares estre asses estre asses de la confection de la confection de la confe
SUPPOSE f(t,x) COMPONENTWISE CONTINUOUS IN IR ⁿ⁺¹ in an (n+1) finil rectange Ra, b = \(\frac{3}{2} (\frac{4}{16}) \) \(\frac{1}{2} \) + \(\frac{1}{2} \)	a, 1x-xolxb}
	and description to the second
and If(t,x)-f(t,y) K x-y for	an tanan manan aga manan manan manan manan an
	anamanan maganamanan seringan menunan manaman menungan menungkan da
	and the second of the second

Let 2/1 2-2 We talked about R" as a vector space k as a normed vector space - all major norms ene. equivalent => they have the same Couchy sequences Of for any interval I, CO(I) is a vector space. If we want to Here the norm we use is important (note, a normal space co is all continuous functions w/ funte norm) eg the Lo norm IIfI= sup If(x) L2 norm ||f||2 = [S||f(x)|24] 1/2 standa 1 (x) = 2. x - 1 = 1 -> 0 So Afrit of ar an element of L' Important Theorem (C°(I) is complete under the Loc norm (note convergence in Lo is equivalent to unform convergence, and it is a basis cheorem that a uniform limit of continuous functions is continuous)



LECTURE 3-2	
let T be a vector valued map defined on a subset \$	
SCB, which maps 5 into itself ie	
ue's > Tues	
	The state of the s
T called contracting if I/ = st	
T called contracting if I/3 st 1/Tu-Tv 1/ 1/14-V/) for u, v & S**,</td <td></td>	
	1
CONTRACTION MAPPING THM	5< B,
if B a Banach space and I a contracting map fro	m 8 5
if B a Banach space and T a contracting map fro 5 closed, to itself then U=Tu has a unique solution in	· · · · · · · · · · · · · · · · · · ·
	The contract of the contract o
PROOF: PICARD ITERATION	The second secon
let un+1 = Tun	and the control of th
EXISTENCE then un+1 - un = Tun - Tun-1 < N un= un-	- Company of the content of the cont
by induction un+' - un < / u' - uo	
J	no orazina anama a mengelejaren basa a sari sa cristo esta entre entre en secultor este escelutor e
Note this implies $\langle u^n \rangle$ cauchy	· · · · · · · · · · · · · · · · · · ·
WENT MAN N>M >N 00	
$\ u^{n}-u^{m}\ < \sum_{k=m}^{n} \ u^{k-1}-u^{k}\ < \sum_{k=n}^{n} \ u^{k-$	u *
	[n'-no /1
<u> </u>	
So un - su u E S since 5 closed	
Thus u a fixed point in 5	Chinary and the second of the
by passing to linit of ant = Tun	MATPOCIDATES NOTES A PROGRAM FOR CONTRACT
UNIQUENESS SUPPOSE USV fixed points	jykkrypenkkaakkopsisisi kojennilikkry
$ u-v = Tu-Tv \le \wedge u-v $	PASTONIANIANAS ILLE ENTRALIZAÇÃO AND ANTONIA ENTRALIZAÇÃO
$u \neq v \Rightarrow \wedge > 1$ CONTRADICTION	

2-5 is pick N large enough s.t p(fn,f*) < E Un>N then p [[f*], f*) < p(T(f*), fn+1) + p(fn+1, f*) Steplat for = p(f(f*), T(fu)) + p(fu+, f*) < c ((f*, fn)) + p (fn., f*) S. CETEsuppose for afd gave 2 fixed pts $\rho(T(f), T(gs) = \rho(f, g)$ $p(f(f),T(g)) \leq c p(f,g) < p(f,g)$ contradiction un futy possible y p(f,g)=0 example let X = C°(S') where S is the circle of circumpence 1 i.e. continuous functions of period 1 clearly a contraction w/ c=1/2 $\int dt T(f(x)) = \frac{1}{2}f(2x)$ What is its fixed point? For any fo, to the sequence for = T(fu) > f* tala f = Sin 2 FX f. = = sin 4xx $f_n = \frac{1}{2^n} \sin \left(2^{n+1} \pi x \right)$ $||f_n|| = \frac{1}{2}n \rightarrow f_n \rightarrow 0$ in sup-norm example X = C°(S') (could also use Wiverstrass T(f(x)) = Cos 2 Trx + = f(zx) M-test again a contraction w/ c= = start w/ fo = sin 2 Tx $f_1 = \frac{1}{2} \cos 2\pi \times \frac{1}{2} \sin 4\pi \times$ f2 = cos 211x + 2 cos 417x + 4 sin 811x f = 2 = cos(2minx) + 1 sin (2it nx) By contraction and

i su norm

Lipschitz functions	a de la composição de propos de designa incomença de população de contrator de como de como de população de co
불역 사람은 가하고 하고 향하는 1년 일반학 가수 가고 있는데 하는 1년 회에 가는 [변화되고 있는데]	teritaria de marcialis actividas de la constitución de la constitución de la constitución de la constitución d
let ECR" be open, f: E->R" in Lips if Varytel, 3 K s.t from tx	ichitz
y Waryter, 3K s.t frag. YX	19 6 E
f(x) - f(y) < K ×	
	ante entre entre entre entre entre la maja de maja de la contra de calenda en partir a la calenda de la calenda
Lipschitz is stronger than continuity b	at we also than
differentiability	
eg f(x) = Jx x E= IR+ in continuous,	but not lipschitz
on different	but not Lipschitz
f(x)= (X X X 0 is continuous +	Lipschitz,
f(x)= { X X x 0 is continuous of 2 2 x X > 0 not differen	Hieble
	ar sa na na nana anna na na na na na na na n
Lemma: any lipschitz function is unifo	- mly continuing
(review definition).	
of choose any X, then for any E ch	\$ = \& \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
(review definition): pf choose any x , then for any ε of then $ x-y < \varepsilon^{\frac{\varepsilon}{2}} \Rightarrow f(x) - f(y) < \kappa$.	
	De la companya da la companya da companya
Lømma amy differentiable furtation in lips convex set A, then f in Lipschitz	etaite for a compact
convex set A, then f in Lipschitz	w/ K= max IDf(x5)
	manusa
Proof since A convey the ots on a line	between 2 sts X+4 EA
	ptxeAE JN=x
definition f is locally lipschitz on E is for	any open set NCE
definition f is locally lipschitz on E & for f is Lipschitz on N	ooni waxaani waxaani waxaa waxaani waxaa ahaa ahaa ahaa waxaa in waxaa in waxaa iyo waxaa iyo waxaa ahaa waxaa
	and a second contract of the second contract
Now we'll show every different able function	in locally lipschitz
lemmaz lot FEC'(A) where A compact + c	
than f lipschitz w/ const K= max IID xe A	fall
in the quantity can an approximate the properties of the propertie	and a second contraction of accommon contraction and accommon contraction of the second contract
proof since A convex, if x, y c S and & connecting them &= 54+(1-5)	on the line
connecting them &= 514+11-07	** = A 0<551

2-17 so $f(y)-f(y) = \int \frac{d}{ds}f(\xi(s))ds$ $\left(\frac{d\xi}{ds}\right) = y-x$ $=\int Df(\xi(s))(y-x)ds$ $||f(y)-f(y)|| \leq ||y-x|| \int ||Df(\varepsilon(s))|| ds$ Corollary 3 if f C'on E open, it is locally lipschitz Pf for any xeE, JE>O st BE(K) CE BE(X) in compact + convex > Locally Lipschitz chrollery 4 let ECIRM and AGE w/ A compact

y of locally lipschitz on E then of lipschitz on

pf y A compact then A cambe covered by Diagram f(x) = 1x in Lipschitz but not C! g(x)= Jx in locally lipschitz but hot lipschifz eg if y=420 | f(x)-f(y)| = J42-JE = E1x-91 = 31E 3 = X so |f(x)-f(y) \$\\ \frac{1}{35\varepsilon} |x-y| So need K \ 35 not bounded for Small & h(x)=x² |x²-y²|= |x+y||x-y| Locally XE (R) Lipschitz of kipschitz if x < (|x+|y|)|x-y|| 1 yes Lipschitz not kipschitz