12-3a

b) suppose $\dot{V} < 0 \quad \forall x \in E \setminus \{x_0\}$. Define $N_\varepsilon(0)$, N_δ as above

let $\tilde{x} \in N_\delta$ then by (a) $\varphi_t(\tilde{x}) \in N_\varepsilon \subset \bar{N}_\varepsilon \quad \forall t > 0$

So for any sequence $\{t_k\} \to \infty$, there exists a subsequence $\{t_n\}$ s.t. $x_n = \varphi_{t_n}(\tilde{x})$ converges

let $y_0 = \lim_{n\to\infty} x_n$

<u>Claim</u> $y_0 = 0$

Given the claim we have $\lim_{n\to\infty} V(\varphi_{t_n}(\tilde{x})) = 0$

and since $\dot{V} < 0$ this means $V(\varphi_{t_k}(\tilde{x})) \xrightarrow[k\to\infty]{} 0$

for any sequence $t_k \to \infty$, which implies $V(\varphi_t(\tilde{x})) \to 0$

and thus $\varphi_t(\tilde{x}) \to 0$

Assume $y_0 \neq 0$.

<u>Proof of claim</u> Since V strictly decreasing along trajectories, it follows that

$$V(\varphi_{t_n}(\tilde{x})) > V(y_0) \qquad \cancel{\forall t > 0} \qquad (1)$$

by continuity of V, it then follows

$$V(\varphi_t(\tilde{x})) > V(y_0) \quad \forall t > 0 \qquad (1)$$

if $y_0 \neq 0$ then $V(\varphi_s(y_0)) < V(y_0)$

and $\exists$ nbhd st y suff close to $y_0 \Rightarrow V(\varphi_s(y)) < V(y_0)$

since $\varphi_{t_n}(\tilde{x}) \to y_0$, $\exists N$ s.t. $n > N \Rightarrow V(\varphi_s(\varphi_{t_n}(\tilde{x}))) < V(y_0)$

$$V(\varphi_{s+t_n}(\tilde{x})) < V(y_0)$$

contradicting (1)

$\Rightarrow y_0 = 0$ and the origin is asymptotically stable.