| | Lecture 6 - Linear Systems + Fundamental Solutions |
|-------------------|--|
| rions | Consider The homogeneous linear system (|
| en de | (S x'= AL+) x AL+) ∈ C'([a,b] → (R"×") toe[a,b] |
| | $\sqrt{\hat{x}}(b) = \hat{x}$, $x \in \mathbb{R}^N$ |
| | |
| eranio Service | and the hinhomogeneous system $\bigotimes \hat{\vec{X}} = A(t)\hat{\vec{x}} + f(t)$ as above with $\hat{\vec{X}}(t_0) = \hat{\vec{X}}_0$ $f \in C^1[a,b]$ |
| | $(\tilde{X}(t_0) = \tilde{X}_0) \qquad \text{fe } C^1[a_ib]$ |
| | [2] 그리는 얼마 되었습니다. 그리는 내가 하는 사람들이 되는 사람들이 되는 사람이 하고 있는데 다른 |
| e | By Picard Iteration/Contraction mapping both @ and @ have unique solutions on all [a, b] Principle of superposition: if yer & yer satisfy @, then so does a,y(1) + a,z,y(2)(1) |
| wa | unique solutions on all [a, b] |
| 0 | Principle of superposition: if y(1) & y(2) satisfy &, then so does |
| | a,y(1) + a, y(2)(t) |
| 0 | We define a fundamental solution of YIt) EGG (R) |
| design | as a matrix whose columns are linearly independent solutions to to |
| | ar a matrix whose columns are linearly independent solutions to & Y= [y(1)(t) y(2)(t) y(1)(t)] |
| | |
| | Note than that $\frac{dA}{dt} = A(t)Y$ |
| | |
| | Question: Our Does there always exist such a fundamental solution |
| | matrix 4/4)? Clearly, we can find a linearly |
| aut | independent set of vectors at t-to. The question is |
| | Whether they remain independent \$t \(\in \Ga \). |
| | |
| | Answer: Such a 414) exists. |
| | Proof Saply contradiction: |
| | Suppose there exist y(1)(t),, y(n)(t) that are linearly independent |
| | at t=to but dependent at t=t. |
| - | Then Fa. a s.t Za: 4(i)(t.)=0 Lut Sa: 4(i)/+)+0 |
| - | Then Fa,, an s.t Zaig(i)(t)=0 but Zaig(i)(to) to |
| | Consider the init's value problem & with you in itial cond y (t) =0. |
| The second second | Then it has two solutions y=0 and y= Zaiy(i)(t). |
| - | These are different at t=t, violating uniqueners |
| | |

| | Lecture 6-2 |
|--|--|
| | An alternate proof is to show that W(t) = det Y(t), |
| egy yayaa kaasa uu ya ahaa ahaa ahaa ahaa ahaa ahaa ah | aho called the Wronskian of the so heteois |
| WHI PARKASANATA CANASA LANGASANATA MANA | g(y(1)(t),,y(n)(t) is never you |
| s consistent antenen sirante a consistentino | [발발에 다시마일 요리마리 기사, 발문으로 모르고 있었다. 하는 그리고 보고 보고 하는 사람들이 없는 사람 수로 보다. |
| | Lidville's Theorem: dtwlt) = (tr Alt))w(t) |
| | (note this implies W(t) = e to W(s)) ds (W(o) 70) |
| | P-1 1/4 1/2 1/4/1 1/4 1/4 1/4 1/4 1/4 1/4 1/4 1/4 1 |
| The second se | Proof: $w(t+h) + w(t) = w(t+h) = det(y(t+h))$ $= det(y(t) + \frac{dy}{dt}h) + o(h)$ |
| | = clet (Y(t) + A(t)Y(t) &) + o(b) |
| of the state of th | = det Y(t) det (I+A(t)L) + o (-h) |
| | now it is easy to see that clet (I+hA) = 1+h Tr(A) + O(h2) |
| | So W(t+h) = W(t)(1+hTr(A)) + o(h) |
| | |
| e cinita anche e accione e accione de casa de la casa d | therefore $\lim_{h\to 0} \frac{W(t+h)-W(t)}{h} = \frac{(1+h)W(t)-W(t)}{h} = \frac{1}{h}W(t)$ |
| | |
| | As ide on with order systems |
| ang sanganaga malaki ina atau maka apasamba | As ide on with order systems let $\chi^{(n)} + p_{n-1} \chi^{(n-1)} + \dots + p_1 \chi' + p_0 \chi = 0$ (***) |
| te a maritima esta la maiorita na cata de material de mari | |
| e in the second sec | then by the usual trick u=x, u=x',, u=x(n-i) |
| sagasti en interpretario estra compresenta en francia espera de sentra e distra | we can rewrite this as du 101 |
| an ann aire ann an taoine an aire an an taon an | $\frac{du}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ |
| erana dana materialian dan materialian dan materialian dan materialian dan materialian dan materialian dan dan | -6-P: |
| s entrancial en es alle reconnecteur à en entrança | y,,,, y, are solutions to (4), the Wronskian is defined as |
| | W6= [4, 42 i 4,] |
| | $W(t) = \begin{bmatrix} y_1 & y_2 & \dots & y_n \\ y_1 & y_2 & \dots & y_n \end{bmatrix}$ |
| | (y(n-1) y(n-1) (y(n-1) |
| enestein tain toenestein automorphismistein footsti besti | This is yes \Leftrightarrow y, (t) yn(t) are linearly dependent |
| nga nghandigandiga nahananahanananahannan nahan kenadarah | |

| | Lecture 6-3 |
|--|---|
| etippore grant for the second and th | Applying Liouville's Theorem to the matrix form of (**), we find |
| | $\frac{dW}{dt} = -P_{n-1}(t)W(t)$ |
| | $\Rightarrow W(t) = e^{-(p_{n-1}(s)) + s} W(t_0)$ |
| | [인생님 - 그리스 그렇는데는 어느 아니다는 이 집안의 보고를 들는 그들이 하는 아니는 그렇게 하고 있는 아니트 이렇는 것 같아 하는 것 같아요? 그는 이 나를 하는 것 같아요? |
| | this result is known as Abel's formula and tells wo that |
| | y, (t),, y, (t) are either always dependent or always independent |
| e na produciona que a respecta de monte en en en en en entre la mesta de mesta de la competita de la competita | EXAMPLE X"+2x'+x=0 & has -Pn-1 = 2 so should get W = Ce-zt |
| | this has exact solutions $y = e^{-t}$ and $y = te^{-t}$ |
| | $w(t) = \begin{cases} e^{-t} & te^{-t} \\ -e^{-t} & e^{-t} - te^{-t} \end{cases} = e^{-2t} \begin{cases} 1 & t \\ -1 & 1-t \end{cases} = e^{-2t} (1-t+t) = e^{-2t}$ |
| | $\begin{bmatrix} -e^{-t} & e^{-t} - te^{-t} \end{bmatrix} = \begin{bmatrix} -1 & 1-t \end{bmatrix}$ |
| | |
| | Back to Fundamental Solutions |
| | Suppose 41t) and ZIt) are both fundamental solling |
| | C=AY and Z=AZ |
| and the second s | lt wit = Ylt) Ylto-121to |
| and a construction of the | than W = 4 4(to)-1 2(to) |
| en a productiva de la concepció de la concepci | = A Y(t) Y(t) 7 2(t) |
| | = Aω(€) |
| | So W is also a fundamental Solution. |
| | Forther Wlto) = Ylto)Ylto)-12lto)=Zlto) |
| | By uniqueness then W(t)=2(t) |
| | Lá C= Y(to)-1/Z(to) |
| Salah salah kiladi masak kinad malah angan and najarak kinad malah salah salah salah salah salah salah salah s | Conclusion Z(t)= Y(t) a. Any two fundamental solutions are identical up |
| | to right-multiplication by a constant matrix |
| the second sec | Further Y(t)Y(to)" = Z(t)Z(to)", and note both = I at t=to |
| aansta aanakan oo arar aha sha aha ta'aya baaba qaa ahaan sa | Define Ylt, to) as the fundamental solution to @ such that Ylto, to) = I |
| and a fine of a state of the st | Then the general solution to 60 is X(t) = Y(t, to) x(to). |
| n francosia na la consecución cia la sistema de la consecución dela consecución de la consecución dela consecución de la | Lemma Ylt, to)=Ylt, s)Yls, to) |
| animaine parties | proof: for fixed 5, Ylt, 5) is a fundamental solution matrix |
| and produced free production, so with containing and production of the second | and YGS, ED) is an invertible matrix |
| and produce of the standard and representations of the standard and the st | therefore Y(t,5)Y(5, to) is a fundamental solution matrix |
| | 들마족() 그는 문제 나는 맛있는 경향이 목표였는 말라고 있는 말라고 있는 그리를 하였다. 그 국민들이 그리는 데 아니라 하는 다른 작품이 되어 있는 데 얼마를 다 다른 사람이 모 |

```
Lecture 6-4
  Further: At t = s
     LHS |_{t=s} = Y(s, t_0) RHS |_{t=s} = Y(s,s)Y(s, t_0) = Y(s, t_0)
  So the two sides of the egn are equal at t=5 and both Solve &.
  By uniqueness they are equal the [a, b] o
The inhomogeneous problem &
                                         X' = A(t)x + f(t)
                                         XIt=t= Xo
 to solve let x(t) = Y(t, to)y(t), then x(to) = Y(to, to)y(to) = y(to).
     further dx = dy (t, to) y + Y(t, to) dy at
                       = Alt) # (t, to) y + Y(t, to) # =
                       = A(t)x + Y(t,to) 提 = A(t)x+f(t) by ()
                               Y(t, ts) = f(+)
                                   * y'= Y'(t, to)f(t)
  So by quadratures y=x0+ fty-'(s, to) f(s) ds
                          Y''(t,t_0)x(t) = x_0 + \int_{t_0}^{t} Y''(s,t_0)f(s)ds
                   \chi(t) = Y(t,t_0) \left( x_0 + \int_0^t v^{-1}(s,t_0) f(s) ds \right)
      The Variation of Parameters Formula or Duhand's Formula
 All, so for in great generality. When Alt) is constant, you have all seen Ylt, to) many times.

It is Ylt, to) = e Alt-to)
 (Note that it is tempting but very wrong when A depends on ting to write A = e^{\int_0^t A(s) ds})
  let's review the theory of matrix exponentials
  For the next of the lecture
                                     A is an uxu real matrix
\begin{cases} \frac{dx}{dt} = Ax \\ X |_{0} = X_{0} \end{cases}
```

| Lecture 6-5 | |
|--|---------------------------|
| We look for solutions $\vec{x} = e^{\lambda t} v$ and find that $\lambda v = Av$ | |
| (A-XI)v=0 has solutions only if det(A-XI)=0 | and amadem |
| $(A-\lambda T)v=0$ has solutions only if $dit(A-\lambda T)=0$ i.e. if λ are eigenvalue & v its associated eigenvalue | lox |
| 보통 화물이 하는 이렇게 보고 있는데 하고 있는데 이번 이 나는데 하는데 되었다. 그는데 이렇게 되었다. | |
| 4 A has a linearly independent eigenvectors $\vec{v}_1,, \vec{v}_n$ | |
| Y A has a linearly independent eigenvectors $\vec{v}_1,, \vec{v}_n$ with corresponding eigenvalues $\lambda_1,, \lambda_n$ | ************ |
| Then we may write $\vec{x}_0 = \sum_{k=1}^{\infty} a_k \vec{v}_k$ and $\vec{x}(t) = \sum_{k=1}^{\infty} a_k \vec{v}_k e^{\lambda_k t}$ | nginana hangi isaliga kal |
| Letting $D = [\vec{v}_1 \cdots \vec{v}_n]$ and $A = diag(\lambda_1, \dots, \lambda_n)$ | andronomika kapina pina |
| $AD = D\Lambda \Rightarrow A = D\Lambda D^{-1}$ | ropeitennelen let synt |
| $\frac{1}{H} = \frac{D}{D} \frac{1}{\chi}$ | karaginari penerah dig |
| Letting $y = D^{-1}x$ then $\frac{dy}{dt} = D^{-1}\frac{dx}{dt} = D^{-1}D\Lambda D^{-1}x$ | |
| h particular dy: Airy: 50 y= [entenzt] So for the diagonalized problem Ylt, to)= [entenzt] ient] | |
| So for the diagraphized problem Ylt.to = [eht exet] | acapananana ka |
| ight | |
| | |
| D-1X= ext | idiomapos |
| D-1 x = e>rt D-1 x | prostation sa |
| | |
| x = D[evis evin] D-1 x | |
| 50 for a Diagonalizable matrix, the fundamental solution matrix | |
| So for a Diagonalizable matrix, the fundamental solution mutrix is ent = D[exit. exit] D-1 | |
| Note that from this defer construction, we it is difficult to see that the | |

| | 용하게 하는 시간 같은 그의 경기를 하는 사람이다. 그는 이 사는 것으로 하는 것을 보고 하는 것이 되었다. 그런 것이 되었다. 그런 것은 것 물로 하는 것은 것이 되었다. 그를 한 것이었는 것이 되었다고 있어요? 그는 것은 것이 되었다. 그는 것이 되었다. |
|---|--|
| | Lecture 6 - G |
| | Recall if It has man different eigenvalues, and fewer than in |
| oo's an aissa's soor an aban seed was a seed weeking. | linearly independent eigenvectors. Then there exist eigenvalues he |
| sagana pagalana ana araga an magana araga araga | with multiplicity 1/4>1. Comesponding to lik in an eigenvector vik |
| | 5. t (A- hx I)Vx and generalized eigen vectors |
| | $w_1, \dots, w_{n_k}, s. \in (A - \lambda_k I) w_1 = v_k$ |
| | and $(A-\lambda_k I)W_j = W_{j-1}$ $j=1,,N_{k-1}$ |
| | not $(A-\lambda_k I)^{n_k} \omega_k^{\gamma} = 0$ for all $v \in \text{span } \{v_k, \omega_l, \ldots, \omega_{n_k-1}\}$ |
| | the generalized eigenspace of he |
| | |
| tandisaran anggita a manamanan anggaranggita. | In this Case, A can't be diagonalized A can only be put in Jordan |
| | Canonical Form A = DJD-1 where J is composed of ukxuk blocks |
| | of the Form J [] |
| | of the Form Ti - Die Lill (ne by ne) |
| | |
| ecencia no emperario, e a despera del filme de del proportionem | |
| | Our one tool to solve (XXX) is Picard iteration |
| ington ground on ground the system of the first and system of the first of the standard of the standard of the | $X_{N+1} = X_{0} + \int_{0}^{t} A x_{N}(s) ds \Rightarrow x_{1} = X_{0} + \int_{0}^{t} A x_{0} ds = X_{0} \text{ (If } At) \times 0$ |
| erant et protosia protos da el compaña e astrologica e de | |
| raina kerikara dan mendahan disiri dipendaha | $X_{ij} = \sum_{k=0}^{\infty} \frac{A^{ik} + k}{k!}$ (exactly Taylor sories started of exponential |
| | k=0 #: |
| alaysin katan diga kunin salama diga kata jiha diga katan diga katan diga katan diga katan diga katan diga kat | C. III II II II O ON TO ME |
| and the first and the state of | So we define the matrix exponential $e^{M} = \sum_{k=0}^{\infty} \frac{nk}{k!}$ |
| amin'ny kaodim-ny paositra dia kaodim-na dipina amin'ny | Then ent = Zi (AE) converges uniformly on any compact interval |
| agaman da da karanga karanga da da karanga da da da karanga da | Then e = Le Converges uniformly away compact intorval |
| eritationis missos anches in missionis mentiones (i.e., i.e., i.e., i.e., i.e., i.e., i.e., i.e., i.e., i.e., | |
| | |
| ha that a charles ha i the scenario in a consul an fact | Claim eAt = Ylt, 0) (for this we will need to we stracted |
| ala di barbar hara fasta satura peranjang ang pang bara fasa. | that AB=BA => en+B=eneB. (Ala) |
| and a second contract of the second contract of | the lack of this identify is why Y(t, 0) # e30 |
| i dinasi summandan katulah dinasi | Claim eAt=Ylt,0) (for this we will need to word served that AB=BA => eA+B=eAeB. the lack of this identify is why Ylt,0) = e e e e e e e e e e e e e e e e e e |
| | |
| maritime promises | Two parts: Clearly eAO = I so they agree at t=0. |
| ani providi tamanasi inspansi imani inginis | |
| | Also \$ eAlt+W = eAh eAt so weath - eAt = weath = eAh I) eAt |
| | -VAN-ZMAN-ZMAN-ZMAN-ZMAN-ZMAN-ZMAN-ZMAN-ZM |

now were the limit as h->0 (and note the two gide commute)

to get to ent = Me Mt So eAt is a fundamental solution D if A is diagonalizable, we can reproduce our earlier result 4 A = DAD-1 42= DND-1 . DND-1 = DN2D-1 induction => An = DAND-1 So ent = D(E (NE)K) D which = D(exit exut) D-1 as before EXPONENTIALS OF JORDAN BLOCKS example A = [89] has double eigenvalue x=2 with eigenvector $v = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ le generalized eigenvector $v_2 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ let $D = \begin{bmatrix} -3 & -2 \\ 2 & 1 \end{bmatrix}$ -than D-1 = [12] Then $A = D \wedge D^{-1} = \begin{bmatrix} -3 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$ We can check that for a 2×2 Jordan block $J = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$, $J^n = \begin{bmatrix} \lambda^n & n\lambda^{n-1} \\ \lambda^n \end{bmatrix}$ (e^{It}), = (e^{It})₂₂= e^{xt} en byone $(e^{3t})_{12} = 0 + t + \frac{1}{2! \cdot 2} + \frac{1}{3! \cdot 3!^2} + \frac{1}{3! \cdot 3!^2} + \cdots$ = $t(1+\frac{1}{2})+\frac{1}{2}(3+12+...)= te^{kt}$ est = elt[t]

| | | vidyos nass, nais santengaissaget amerifica | in and a linear services and a conjugate of a consistency of the consi | | na nyaéna nanonan'i manaisa napanan se | ra i Kirahanaka ninga mananana mina mina m | essenne griffet voor som oon oor oor | en en el comercio de la comercia de |
|--|--|---|--|---|--|---|--|--|
| General | Jordan block | egas no sociolo de la principa de consecutación de la consecutación de la consecutación de la consecutación de | an iya e gerina a gaze waxa kara iya isa isa | | wana pour junto en manda con | | | et nastijas tina sprasičinja nastijaji nastijaji saiti |
| lst | A be an a | ×n Jorda | an bloc | k | s salineon way naganiwa na manaka wa | (m. 15-140) (m. 1440) (m. | | or gan the contraction of |
| | $\Lambda = \lambda I + D$ | | | | Si, k-1 | has or | es in m | th cliage |
| | | | | | | above the i | nain +2 | eros el |
| | | his Du.m | notatio | n i | not-stan | ideral) | construores de l'indicat faccacia de la | . mangalanana ancada a consa |
| | In this note | tion D" | '° = I , | Duim | =0 'G | mzn | | |
| ppa a anno ago anno arriva ago a anigano arriva anis rivorir rivorir rivorir rivorir rivorir rivorir rivorir r | | | | | | | | inner para e presidente |
| Not | ie that (D | w, 1) b = t | on, b | nd th | at I | onnutes | with D | 1, m |
| | in , | o ₁ (a – 1) | ossinga managa managa sa | executavos estanas as es encena | arter expansive communication conserved | | anaman wanaman makama makama | |
| the | $ \begin{array}{ccc} $ | (k) | / p- K D h | k | en primingo quantina primingo que de la constanción de la constanción de la constanción de la constanción de l | o consessiones a reconstructural de la consessione de la consessione de la consessione de la consessione de la | termina managan na man | ni oi en en en den en e |
| horizon leenee konsaksin engikanip kan | manaka manana manana manana ke c | | IK DIL | | | one a su sus entre estre es se es gaz en circo estre es | ne vanoan varianta ante | g y ot marines nome access desc |
| Then w | $ \begin{array}{ccc} $ | = elt [| <u> </u> | = | t t/2 | t³/3 · · | - (n-1)! | |
| | namente en | minimum minimum minimum kes | | onimatritainain | 1 F | _ | | |
| | o est en esperante en el en esta esta esta esta esta esta esta esta | | | | (menamanan imaa maan interne | | | |
| | | ing memorial and an apple and they are | | | | | Ł | |
| | od patanga ang ang ang ang ang ang ang ang ang | | opou v acetami pou pou opou | | sanahana da sanata nannya | *************************************** | 1 ^t | J |
| | mentranska egina men en eksperansk kortige fan av heferenskyl | in a series and the series of | · | *************************************** | 3100mmanananananan | | | |
| COMPLEX | EIGENVALUES | <u>a naje</u> nia mada koon sa mala on najenia | reconstantinas, nelistante anna nivel este este nive | oosensis saansasseenseens | ************************* | | | enperator estencimiento en encon |
| Note th | t all thin go | esth oug | sh uncl | ianged | for con | nplex eig | envalu | <u>u </u> |
| | power series | | | | | | | |
| Using th | e ligen-de com | position w | ould req | uire | comple | x arithr | metic. | |
| This a | an be avoid | ed using - | the B | | eradoren o esta era era en en esta en entra en | | 21 | |
| | and a constant and special | kkan dikadan saka makamani melanja | ekanaga ana anata finatinan | | | | | |
| REAL JO | PDAN FORM: | If A | is a | 2×2 и | uatrix c | of eigen | alve & | tiB |
| Hen | Freal matr | ix D s. | £ | رم | - B T | | timoduceriverymickel generalis | |
| lended a margin agrif som granneng vid agrand den gelt kom agranne de stade f | A = DI g the power S | 19 ⁻¹ a | nd 1 | 1= LB | | | -sia Bt 1 | |
| | atta | erila on | e find | s e' | nt= ed | I Sin Bt | cosst | |
| us m | y we power s | | U | | | | 10 kg 1 kg 1 kg 1 kg 1 | |
| | o he powers el Jordan for | erdadina aprolej servernik disesses se vez sende | internal internal control of the con | | | | | dirantapa di kalanda dipada alaha da |

and the second of the second o

| and the second s | Letue 6-9 | | | | | | | | |
|--|---|--|--|--|--|--|--|--|--|
| anne and a second se | 1) Diagonal, corresponding to real eigenvalues with comple | £., | | | | | | | |
| and a state of the process of the state of the | Sets of eigenvectors | | | | | | | | |
| | 2) Non-trivial real Jordan blocks (rank deficient real eigenv | | | | | | | | |
| | 3) 2×2 blocks (\$i si) corresponding to complex eigenv | alue- | | | | | | | |
| umana ny positra dia dia kaominina dia kaominina dia mandra ny fisiana dia mandra ny fis | w/ complète eigenspaces | | | | | | | | |
| | 4) Nontrivial complex Jordan blocks | un en se senem a se que dos a que de ser estrança de como despora non interna de transferencia. | | | | | | | |
| | Bi I Where Ri = [ac -Bi] | y y y gant a gant ang py dan alp a mala ay paga agaman ny digy an ang banaga ana ang taong da anag ta | | | | | | | |
| | Bi I blee Bi = [oci - si] | ayan karang dan sada matanda da da manada matanda da da manada da | | | | | | | |
| annan an shin dha dha dhannan an dha ta tao tao an dha bha dha dha tha ta ta tha an ta | B€) | ensing apagagi pengilik pendang pendangan kalaman apara melakat binakan pendan | | | | | | | |
| | t 11 | on ete que a construero para la mesta con presente en este mentra de la complexión de construer de este vers ve | | | | | | | |
| nigo enque contingua asone ano escreto describe de escribera en como de escribera de escribera de escribera de | Together, these show that the matrix exponential consists of sums of terms like the ext cospt and the singt | and a second contract of the c | | | | | | | |
| ara, sang pangganang panggan karah sanggan panggan panggan panggan panggan panggan panggan panggan panggan pan | sunct of terms some x e cospi and a x c sue se | kantun pierroni karan tokupa ataun da ilipunkaran ti nitu adapata kantun india di d | | | | | | | |
| A supervision and provide the executive first and are required to the first an existence and a contract of the | | enantimate disease to compare haben the street an empire provincial and accompared a compared and accompared a | | | | | | | |
| erren er en en en felle ett i det er er erren er och en trope å en felle til ett et er er tropen en år felle | | el matemati e si di aspeci elibera di manchi cha manesti ne aspeci esti aspeciali matemati e aspeciale di di | | | | | | | |
| construction control of the control | | Gustan dan dan saka dan dan dan dan dan dan dan dan dan da | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| tannan panaha ia marantani in madahamatani inga kata kata kata kata kata kata kata ka | | operatition plans de als citations being a said to acceptance as a catalogue of a principle, the | | | | | | | |
| Antonia de prima de la prima de la constante d | | | | | | | | | |
| | | fan en betaan de gene sûnske fei hêter op fat te en skeit het en skeit op de gene fei op fat en fat en fat en | | | | | | | |
| particular and a second and a se | | | | | | | | | |
| ganasining karawasa sa sa katalan lada sa saga di basasa katala | | di an ing sa isang a kangia gi anakana ang manang mining kang ang ang ang | | | | | | | |
| italian kasifa dia mana malai pama malaifa kasifa kasifa kasifa kasifa kasifa kasifa kasifa kasifa kasifa kasi | | | | | | | | | |
| A realism and a realist place are specified and appropriate place are an expense. | | | | | | | | | |
| allinational periodicide distribution and confidence to a configuration of the consistence of the confidence of the conf | | | | | | | | | |
| ise su aprili dia transferio strancia di anticolori di ancienti ancienti di ancienti di ancienti di ancienti d | | i anakini adaminya malyin ingidi malinkarakan karakan ingidi | | | | | | | |
| rander takak kecamatan kan ang mahana ang kananda kan sa kananda kananda kananda kananda kananda kananda kanan Kananda kananda kanand | | kada maja kanamigi marana alam anara jama kanami kanami kanami kanami kanami kanami kanami kanami kanami kanam | | | | | | | |
| espirales aporturação mediatorio de la terresida por terresida en estado | | ka kantan halimustan da adan ngabah salamangan tanan angan angan angan tanan salaman tanan tanan tanan tanan t | | | | | | | |
| aan aan ta'i ga istabii da kasaa ka k | | de production de la compact de maintenant de viron propriée antien de la colonidad de la colon | | | | | | | |
| si atau taga na ana ana ana ana | | the figure at the first the transfer and a profession of the conference of the conference of the conference of | | | | | | | |
| apang saaning dirapan aha dalam na mang manggunan dalam panggan dalam na panggan dalam na panggan dalam na pan | | a agus agus a ceangagas a agus agus agus agus an agus an an agus agus agus agus agus agus agus agus | | | | | | | |
| and a state of the part of the state of the | | | | | | | | | |