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Computer Architecture Homework #1: Starting Up with Matrices

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Abstract

One common task for computers is multiplication of matrices; it also serves as an excellent example for learning how a computer actually executes programs. This is the first in a series of homeworks that should make that clear.

This is the first of a string of three homeworks. There will be follow-on homework from this next week, so you must have this done by then!

Take the two matrices:

$$A = \begin{pmatrix} 1 & 0 & 3.14 & 2.72 \\ 2.72 & 1 & 0 & 3.14 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} \tag{1}$$

$$B = \begin{pmatrix} 1 & 1 & 0 & 3.14 \\ 0 & 1 & 3.14 & 2.72 \\ 0 & 1 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix} \tag{2}$$

Do the following:

1. Find the matrix product AB. Do this by hand, and show your work.
2. Count
   (a) the number of real (floating point) multiplications necessary, and
   (b) the number of real (floating point) additions necessary.
3. Express
   (a) the number of real (floating point) multiplications necessary, and
   (b) the number of real (floating point) additions necessary

   as a function of N for multiplying two $N \times N$ matrices.
4. Write *pseudocode* for a program to multiply two $N \times N$ matrices.
5. Write a matrix multiply routine in Python (do this yourself, not by using Numpy or some other library!)

(1)

$$AB = \begin{pmatrix} 1+0+0+10.88 & 1+0+3.14+8.16 & 0+0+3.14+5.44 & 3.14+0+0+2.72 \\ 2.72+0+0+12.56 & 2.72+1+0+9.42 & 0+3.14+0+6.28 & 8.5408+2.72+0+3.14 \\ 1+0+0+4 & 1+1+1+3 & 0+3.14+1+2 & 3.14+2.72+0+1 \\ 1+0+0+16 & 1+2+3+12 & 0+6.28+3+6 & 3.14+5.44+0+4 \end{pmatrix}$$

$$= \begin{pmatrix} 11.88 & 12.3 & 8.58 & 5.86 \\ 15.28 & 13.14 & 9.42 & 14.4008 \\ 5 & 6 & 6.14 & 6.86 \\ 17 & 18 & 15.28 & 12.58 \end{pmatrix}$$

(2) (a) multiplications : $4 \times 16 = 64$

(b) additions : $3 \times 16 = 48$

(3)

(a) multiplications : N^3

(b) additions : $(N-1)N^2 = N^3 - N^2$

Time complexity of Matrix multiplications : $O(N^3)$

(4)

```
func MatrixMutiply (A, B, N)
init res = list [N][N]
for (i ; 0, N-1)
    for (j; 0; N-1)
        for (k; 0; N-1)
            res [i][j] += A[i][k] * B[k][j]
return res
```

(5) Please see the attached file "matrix-hw-7234187.py"

Screenshot is attached to the PDF

```python
# Name: Xu Haoran  Student ID: 72344187  python 3.11.9

def matrix_multiply(A, B):
    N = len(A)
    # initialize an NxN result matrix
    C = [[0.0 for _ in range(N)] for _ in range(N)]
    for i in range(N):
        for j in range(N):
            for k in range(N):
                C[i][j] += A[i][k] * B[k][j]
    return C

# multiply A and B matrices from the problem
A = [
    [1, 0, 3.14, 2.72],
    [2.72, 1, 0, 3.14],
    [1, 1, 1, 1],
    [1, 2, 3, 4]
]

B = [
    [1, 1, 0, 3.14],
    [0, 1, 3.14, 2.72],
    [0, 1, 1, 0],
    [4, 3, 2, 1]
]

C = matrix_multiply(A, B)
for row in C:
    print(row)

# output
# [11.88, 12.3, 8.58, 5.86]
# [15.280000000000001, 13.14, 9.42, 14.400800000000002]
# [5.0, 6.0, 6.140000000000001, 6.86]
# [17.0, 18.0, 17.28, 12.58]
```