

# Charge to Mass Ratio of the Electron

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**Abstract**—For an electron moving in a circular path in a magnetic field, if we know the magnetic field strength, accelerating voltage, and radius of the electron's trajectory, then we can make an estimation of the electron's charge to mass ratio. We calculated an average charge to mass ratio of  $2.08 \times 10^{11} \pm 1.81 \times 10^8$  Coulombs per kilogram.

## I. THEORY

TO come up with a procedure to measure the charge to mass ratio of an electron, we had to find a way to relate the two quantities to each other mathematically. The first important relationship is between an electron's kinetic energy  $K$  and total energy  $E$ .

$$K = \frac{1}{2}mv^2 \quad (1)$$

$$E = eV \quad (2)$$

$m$  is the electron's mass in kilograms,  $v$  is the electron's velocity in meters per second,  $e$  is the electron's charge in Coulombs, and  $V$  is the accelerating voltage in Volts.

When an electron is in motion, all of its energy is kinetic, and we can relate eq. 1 and eq. 2 in:

$$\frac{1}{2}mv^2 = eV \quad (3)$$

When the moving charged particle is subjected to a magnetic field, it experiences a force that is always perpendicular to its velocity  $v$ . The magnitude of this force  $F$  can be calculated in:

$$F = evB \quad (4)$$

$B$  is the strength of the magnetic field in Teslas. Because this force is always perpendicular to the particle's motion, it causes the particle to travel in a circular path. The force on a particle in a circular path can also be computed in:

$$F = \frac{mv^2}{R} \quad (5)$$

$R$  is the radius of the circular path in meters. By setting eq. 4 and eq. 5 equal to each other, we find the following relationship:

$$evB = \frac{mv^2}{R} \quad (6)$$

where we can solve for velocity  $v$ .

$$v = \frac{eBR}{m} \quad (7)$$

Substituting this value of  $v$  into eq. 3 allows us to find a way to directly calculate the charge to mass ratio of the electron.

$$\frac{e}{m} = \frac{2V}{R^2B^2} \quad (8)$$

Therefore, all we need to know to find the charge to mass ratio of the electron undergoing circular motion in a magnetic field is the magnetic field strength  $B$ , the accelerating voltage  $V$ , and the radius of the electron's path  $R$ .

## II. APPROACH

The approach we used to measure the charge to mass ratio of the electron was designed after the experiment of Bainbridge [1]. The apparatus used consists of a vacuum tube supported between Helmholtz coils. A filament contained inside an anode with a single slit releases a thin beam of electrons into the vacuum tube, whose paths are made visible by mercury vapor in the vacuum tube. A power supply provides voltage to the anode and a DC Voltmeter attached across is used to record it. An Adjust-A-Volt is attached to the filament to control the current running through the filament. A separate power supply is used to power the Helmholtz coils.

When the filament is heated by running current through it, electrons are evaporated out into a negatively charged "cloud" via thermionic emission. The anode, which is kept at a positive potential relative to the filament, attracts the electrons and accelerates them away from the filament. The potential difference between the anode and the filament provides the accelerating potential  $V$  from eq. 8.

Once the electrons have escaped the anode, they are subjected to a magnetic field provided by the Helmholtz coils and travel in a circular path until they collide with a mercury atom. If the electron has enough kinetic energy, one of the mercury's electrons is ejected. When another electron takes its place, the excess energy given off is visible as blue light and makes it possible to see the electron beam. If the pressure is too high in the vacuum tubes, the electrons will only travel too small a distance before colliding with a particle to make a full beam. If the pressure is too low, too few collisions will take place to see the electron beam. At a pressure of about  $10^{-4}$  atmospheres, the electrons can travel about 8 to 10 centimeters before colliding with another particle, and enough collisions occur to see the electron beam.

Once the electron leaves the anode, all of its energy is kinetic and can be computed in eq. 1. This allows us to use the relationship between kinetic energy and total energy shown in eq. 3.

To measure the charge to mass ratio of the electron, we need three measurements: the radius of the electrons' circular path

$R$ , the accelerating voltage  $V$  of the anode, and the magnetic field strength  $B$  provided by the Helmholtz coils.

The radius of the electrons' path is measured by sight using measurement posts attached to the vacuum tube. The accelerating voltage is set using the anode power supply and recorded using the DC Voltmeter attached across. The magnetic field strength of the Helmholtz coil is computed from the equation for a magnetic field around current-carrying coil.

$$B = \frac{8\mu NI}{\sqrt{125}a} \quad (9)$$

$\mu$  is the permeability of free space,  $4\pi \times 10^{-7}$  Tesla-meters per square meter.  $N$  is the number of turns in the Helmholtz coils, which in this case is 72.  $a$  is the radius of the coils, which in this case is 0.33 meters.  $I$  is the current running through the Helmholtz coils in Amperes, which is set by the Helmholtz power supply.

By manipulating the strength of the magnetic field with the current running through the Helmholtz coils, we can change the radius of the electrons' path. We took 5 measurements for each of 3 different values of accelerating voltage in the anode: 25V, 30V, and 35V. At each value, we recorded the current through the Helmholtz coils needed to produce a circular path with each of the five measurement posts inside the vacuum tube. In total, this gave us 15 different sets of data.

TABLE I  
INITIAL OBSERVATIONS

Anode Voltage $V$ (V)	Radius $R$ (cm)	Helmholtz Current $I$ (A)
24.9	3.20	2.50
24.9	3.90	2.08
24.9	4.50	1.72
24.9	5.20	1.51
24.9	5.70	1.30
29.9	3.20	2.78
29.9	3.90	2.27
29.9	4.50	1.94
29.9	5.20	1.66
29.9	5.70	1.45
35.0	3.20	3.03
35.0	3.90	2.51
35.0	4.50	2.09
35.0	5.20	1.78
35.0	5.70	1.58

We assumed no uncertainty for  $R$  as those values were given to us. For the uncertainty of  $V$  and  $I$ , we assumed a constant uncertainty for the values given to us by the DC Voltmeter and power supply.

$$\sigma V = \frac{0.1}{\sqrt{12}} V \quad (10)$$

$$\sigma I = \frac{0.1}{\sqrt{12}} A \quad (11)$$

Using the value of current through the Helmholtz coil at each data point, we then calculated the strength of the magnetic field using eq. 9 for each data point.

For our uncertainty of the magnetic field, we found a constant error with eq. 12.

TABLE II  
MAGNETIC FIELD STRENGTH AT EACH DATA POINT

$V$ (V)	$R$ (cm)	Magnetic Field Strength $B$ (G)
24.9	3.20	4.90
24.9	3.90	4.08
24.9	4.50	3.37
24.9	5.20	2.96
24.9	5.70	2.55
29.9	3.20	5.45
29.9	3.90	4.45
29.9	4.50	3.80
29.9	5.20	3.25
29.9	5.70	2.84
35.0	3.20	5.96
35.0	3.90	4.92
35.0	4.50	4.10
35.0	5.20	3.49
35.0	5.70	3.10

$$\sigma B = \left( \frac{8\mu N}{\sqrt{125}a} \right) \sigma I = 5.66 \times 10^{-3} G \quad (12)$$

With  $V$ ,  $R$ , and  $B$  recorded at each data point, we then calculated the charge to mass ratio and the uncertainty of the ratio at each data point.

TABLE III  
CHARGE TO MASS RATIO AT EACH POINT

$V$ (V)	$R$ (cm)	$e/m$ (C/kg)	$\sigma e/m$ (C/kg)
24.9	3.20	$2.01 \times 10^{11}$	$5.18 \times 10^8$
24.9	3.90	$1.97 \times 10^{11}$	$5.93 \times 10^8$
24.9	4.50	$2.16 \times 10^{11}$	$7.69 \times 10^8$
24.9	5.20	$2.10 \times 10^{11}$	$8.40 \times 10^8$
24.9	5.70	$2.36 \times 10^{11}$	$1.08 \times 10^9$
29.9	3.20	$1.97 \times 10^{11}$	$4.51 \times 10^8$
29.9	3.90	$1.99 \times 10^{11}$	$5.40 \times 10^8$
29.9	4.50	$2.04 \times 10^{11}$	$6.39 \times 10^8$
29.9	5.20	$2.09 \times 10^{11}$	$7.54 \times 10^8$
29.9	5.70	$2.28 \times 10^{11}$	$9.34 \times 10^8$
35.0	3.20	$1.93 \times 10^{11}$	$3.99 \times 10^8$
35.0	3.90	$1.90 \times 10^{11}$	$4.65 \times 10^8$
35.0	4.50	$2.06 \times 10^{11}$	$5.94 \times 10^8$
35.0	5.20	$2.13 \times 10^{11}$	$7.12 \times 10^8$
35.0	5.70	$2.25 \times 10^{11}$	$8.42 \times 10^8$

For the uncertainty of the charge to mass ratio, we used the following equation at each data point.

$$\sigma \frac{e}{m} = \sqrt{\left( \frac{d(\frac{e}{m})}{dB} \right)^2 \sigma B^2 + \left( \frac{d(\frac{e}{m})}{dV} \right)^2 \sigma V^2} \quad (13)$$

$$\sigma \frac{e}{m} = \sqrt{\left( \frac{4V}{B^3 R} \right)^2 \sigma B^2 + \left( \frac{2}{B^2 R^2} \right)^2 \sigma V^2} \quad (14)$$

We calculated an average charge to mass ratio of  $2.08 \times 10^{11} \pm 1.81 \times 10^8$  Coulombs per kilogram.

### III. CONCLUSION

The accepted value for the charge to mass ratio of the electron is  $1.76 \times 10^{11}$  Coulombs per kilogram compared to our value of  $2.08 \times 10^{11} \pm 1.81 \times 10^8$  Coulombs per kilogram. The accepted value was not within the uncertainty of our calculated value. Our value was 15.3 percent away from the accepted value.

I think our data was inaccurate partially because the equipment used was old and thus less precise. This experiment was heavily dependent upon measurements taken from the DC Voltmeter and the operation of the power supplies and Adjust-A-Volt. I think if we retried the experiment with newer, more precise equipment, our data would have been closer to the accepted value.

I also think our data was inaccurate because we used the equation for Newtonian kinetic energy in this experiment (eq. 1) rather than using relativistic energy. According to special relativity, the actual total energy of a particle with mass is:

$$E = \gamma mc^2 \quad (15)$$

in which  $\gamma$  is defined as:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (16)$$

$c$  is the speed of light,  $3.00 \times 10^8$  meters per second, and  $v$  is the velocity of the particle measured by a stationary observer. As  $v$  approaches  $c$ , the value of  $\gamma$  approaches infinity.

For a particle at rest,  $\gamma$  simplifies to 1. That means that the energy of a particle at rest is given by:

$$E = mc^2 \quad (17)$$

Then, the kinetic energy of a particle is given by the total energy minus the rest energy:

$$K = (\gamma - 1)mc^2 \quad (18)$$

Thus, if the velocity of the electrons in the vacuum tube is sufficient, this true value of their kinetic energy could deviate from the value of the energy given by eq. 1. This factor must be taken into account as a reason our measured value of the charge to mass ratio is different from the accepted value.

I also think our data was inaccurate because we took too few measurements. If we retried the experiment with more data points, our measured value would be more accurate.

#### IV. ACKNOWLEDGEMENTS

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#### REFERENCES

- [1] Romblom, Dave and Robert Pizzi. *Physics 3L/4L Laboratory Manual*. Hayden-McNeil Publishing, USA, 4th edition, 2016.