Vehicle Trajectory Prediction by Integrating Physics- and Maneuver-Based Approaches Using Interactive Multiple Models

Guotao Xie, Hongbo Gao, Lijun Qian, Bin Huang, Keqiang Li, and Jianqiang Wang

***Abstract*—Vehicle trajectory prediction helps automated vehicles and advanced driver-assistance systems have a better understanding of traffic environment and perform tasks such as criticality assessment in advance. In this study, an integrated vehicle trajectory prediction method is proposed by combining physics- and maneuver-based approaches. These two methods were combined for the reason that the physics-based trajectory prediction method could ensure accuracy in the short term with the consideration of vehicle running dynamic parameters, and the maneuver-based prediction approach has a long-term insight into future trajectories with maneuver estimation. In this study, the interactive multiple model trajectory prediction (IMMTP) method is proposed by combining the two predicting models. The probability of each model in the interactive multiple models could recursively adjust according to the predicting variance of each model. In addition, prediction uncertainty is considered by employing unscented Kalman filters in the physics-based prediction model. To the maneuver-based method, random elements for uncertainty are introduced to the trajectory of each maneuver inferred by using the dynamic Bayesian network. The approach is applied and analyzed in the lane-changing scenario by using naturalistic driving data. Comparison results indicate that IMMTP could achieve a more accurate prediction trajectory with a long prediction horizon.**

***Index Terms*—Automated vehicles (AVs) and advanced driver-assistance systems, interactive multiple models (IMMs), physics-based and maneuver-based approaches, vehicle trajectory prediction.**

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I. Introduction

AUTOMATED vehicles (AVs) and advanced driver-assista nce systems have received extensive research interest because they show great potential for use in a more efficient, safer, and cleaner transportation system [1]. Developments in the field will evidently increase in both quality and importance with time [2]. However, an issue to be addressed is why AVs are not widely used at present. Technically speaking, one of the crucial reasons is the increasingly complex and highly uncertain traffic environment [3]. To solve this problem, similar to human driving skills, AVs should be able to predict the traffic environment for its future change. This approach could improve the ability of AVs to understand the traffic environment and helps to make the trajectory planning [4] and tracking [5]. With the development of communication technology [6], vehicle-to-vehicle (V2V) communication devices could help to obtain additional information to predict vehicle trajectories in complex traffic environments.

Physics-based prediction is one of the common techniques to predict vehicle trajectories in the short term [7], [9]–[12], [20]. Huang and Tan [7] explored vehicle future trajectory prediction and examined the possible methodologies for trajectory prediction based on the differential global positioning system. In this paper, different dynamic trajectory prediction models were compared; results indicated that one of the main error causes is the changes in driver's intention. Sorstedt *et al.* [10] considered the driver control input parameter to obtain better predictions. In [9], collision risk estimation between vehicles was realized on the basis of communication tools by predicting the trajectories of the surrounding vehicles. Polychronopoulos *et al.* proposed a model-based description of the traffic environment for an accurate prediction of vehicle's path, which is situation adaptive and calculation dynamic [11]. The dynamic prediction model using a Kalman filter (KF) was employed [12]. The KF could obtain the optimal solution to predict in the linear Gaussian environment. To deal with the nonlinear problems, unscented Kalman filters (UKFs) were employed to predict in the constant turn rate and acceleration (CTRA) model [20]. In [19], extensive

Kalman filters (EKFs) and Monte Carlo methods for the nonlinear problems were introduced and compared. However, this model can only predict with the sustainable results in the short term.

For a longer prediction horizon, maneuver-based approaches are used to predict the vehicle trajectory at a higher level [13]–[15], [29]. Liu *et al.* [13] developed a trajectory prediction approach for lane change considering high-level driving status. This approach was based on driving behavior estimation and classification using hidden Markov models. Gindele *et al.* [14] presented a filter based on the dynamic Bayesian network that could estimate the driving behaviors and anticipate the future trajectories. In [15] and [29], the authors developed an integrated approach to predict the trajectories based on maneuver estimation. In this paper, the driving maneuvers were inferred for each vehicle by employing a Bayesian network. Probabilistic trajectory models were proposed to predict the vehicle configuration forward in time. The maneuver-based trajectory prediction approach could predict vehicle trajectories at a higher level for longer prediction. Additionally, some time-series models learning from data could also predict trajectories such as artificial neural networks [16] or autoregressive integrated moving average models [17]. However, these did not consider the vehicle motion models, which could predict more accurately in the short term.

Some methods integrate short- and long-term vehicle trajectory prediction approaches [8], [18]. Houenou *et al.* proposed a new trajectory prediction approach that combines a trajectory prediction based on the constant yaw rate and acceleration motion model and a maneuver recognition-based trajectory prediction [8]. A cubic spline function was defined as the weight function to obtain the final predicted trajectory. Also, in [18], a hierarchical multiresolution framework called prediction in a dynamic environment was presented to strengthen the results of another prediction algorithm. The above-mentioned methods could hardly adapt to different environments and different prediction accuracies.

With the aim to acquire accurate trajectory predictions in a long horizon, it is crucial to consider both the dynamic motion with physical laws in the short term and the high-level driving patterns with maneuver estimations in the long term. In other words, the trajectory prediction model adapting to different prediction horizons, namely short-term prediction horizon and long-term prediction horizon, could predict the vehicle trajectory accurately in a long term.

The objective of this study is to build an integrated approach by combining physics- and maneuver-based prediction methods, which could predict accurately in the long term. In this study, the uncertainty is considered in the nonlinear dynamic prediction model using the unscented transform (UT). In the maneuver-based prediction model, the dynamic Bayesian network is used to estimate driving behaviors by considering sequential information. Random elements for uncertainty are introduced to the maneuver-based prediction models. Moreover, the interactive multiple model trajectory prediction (IMMTP) method is proposed by integrating the advantages of both prediction models, which could predict accurately in the long term. This integration is shown in Fig. 1.

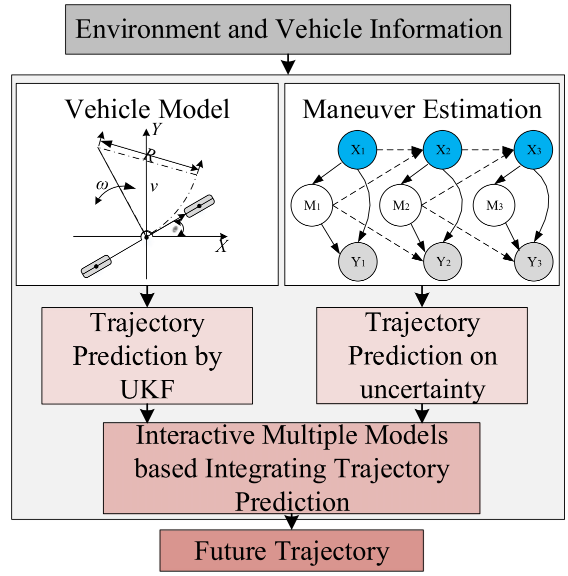

Fig. 1. Integration of physics- and maneuver-based trajectory prediction models.

The remainder of this paper is organized as follows. Section II compares and introduces the different physics-based models and methods that deal with uncertainty. The maneuver-based approach is described in Section III. Section IV analyzes the integration approach. The integrating approach is applied in the vehicle lane-change scenario, and the prediction results are compared and analyzed by using naturalistic driving data in Section V. Section VI presents concluding remarks.

II. Physics-Based Model for Trajectory Prediction

This section introduces various physics-based motion models as well as filtering and prediction algorithms with consideration of uncertainty. The physics-based models assume that some running parameters such as velocity and acceleration are constant in the near future [19]. In this study, the kinematic models are considered in the physics-based prediction model. Various kinematic motion models exists, such as the constant acceleration model, the CTRA model, and the constant curvature and acceleration model [20]. Some methods, such as KFs, EKFs, UKFs, and particle filters (PF), are employed to deal with uncertainties [21].

A. Vehicle Kinematic Model

For more accurate prediction, the CTRA model is used in this study as the physics-based prediction model. The CTRA model assumes that the turn rate and acceleration are constant. The state space of the CTRA model can be expressed as follows:

$$\vec{x} = (x, y, \theta, v, a, \omega) \tag{1}$$

where (x, y) means the position of the vehicle, θ means the rotation angle of the vehicle, ω is the angle velocity, and v and a represent the velocity and acceleration of the vehicle in the running direction, respectively. With the aim to predict trajectories of other vehicles, the V2V communication technology could be used to get the information.

Assume the state transition equation as follows:

$$\vec{x}(t+\Delta t) = \Delta f(t) + \vec{x}(t). \tag{2}$$

$\Delta f(t)$ in the state transition equation is given by (3), shown at the bottom of this page, where Δt means the sampling time. In this study, the sampling time is 0.1 s.

B. Trajectory Prediction Considering Uncertainty

Uncertainty should be considered in the information recursive update and trajectory prediction because of measurement noise and processing noise. Various methods could be used to handle uncertainty online and recursively, such as KF, EKF, UKF, and PF [21].

In this study, the filtering problem considering uncertainty could be described as the following equations:

$$\vec{x}(t+\Delta t) = f(\vec{x}(t), t) + q(t) \tag{4}$$

$$\vec{y}(t) = h(\vec{x}(t), t) + r(t) \tag{5}$$

where f is the motion function, q is the system noise defined as the Gaussian noise in this study, $\vec{y}$ is the observation parameters, h is the observation function, and r is the observation noise. The prediction could be realized by iterating the system motion function.

The KF assumes that the posterior density is Gaussian at every time step and can obtain the optimal solution to the tracking problems in the linear Gaussian environment. However, it cannot handle nonlinear problems. For example, the KF could not obtain the optimal solution for the CTRA motion model because the dynamic process exits nonlinear equations, as shown in (3).

To solve the nonlinear problem, extensions of the KF can be used by defining Gaussian approximations to the joint distribution. One of the extensions is called EKF based on Taylors series theory [22]. However, as shown in (3), Jacobian matrices with the first-order filters or Hessian matrices for the second-order filters, in this case, can be difficult and complex processes to calculate [23]. In addition, it has been indicated that the linearized EKF performs nonlinear propagations of probability distributions less accurately than the UKF [24]. Moreover, with the high nonlinear (3), the EKF can be sometimes inefficient when the error propagation cannot be approximated based on the first or second order of Taylor series and would even diverge after some recursion. Another extension of the KF is the UKF, which is based on the UT for the approximation of joint distribution. The PF is one of the solutions to nonlinear problems by simulating enough particles [25]. However, simulating many particles is time-consuming. In this study, the UT is employed to handle the uncertainty of the nonlinear system. In the UT, a fixed number of points from the original distribution are chosen to estimate the variables transformed from the nonlinear function. It can be used to approximate the joint distribution of variables, which captures higher order of moments for the nonlinear function than the standard EKF [26].

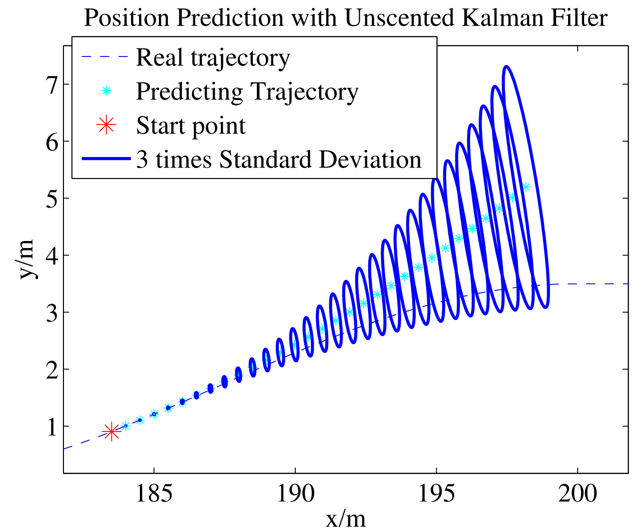

Fig. 2. Position prediction with a UKF.

With the use of the UT, the prediction results could be obtained by iterating the motion function with uncertainty. One example of the prediction results in the lane-changing case is shown in Fig. 2. The prediction results indicate that the physics-based prediction model could predict accurately in the short term. However, in the long term, the prediction is unsustainable with a significantly large covariance.

III. Maneuver-Based Trajectory Prediction

In this section, the long-term trajectory prediction will be introduced based on the maneuver estimation. First, the driving behavior awareness (DBA) model will be presented based on the dynamic Bayesian network to infer the driving maneuvers. Then, the maneuver-based trajectory prediction will be introduced by considering the random elements for trajectory uncertainty.

A. Driving Behavior Awareness

In this study, maneuvers such as following road and lane changing are estimated by the proposed DBA model. Three layers exist in this DBA model, namely observation, hidden,

$$\Delta f(t) = \begin{pmatrix} \frac{v+a\Delta t}{\omega}\sin(\theta+\omega\Delta t) + \frac{a}{\omega^2}\cos(\theta+\omega\Delta t) - \frac{v}{\omega}\sin(\theta) - \frac{a}{\omega^2}\cos(\theta) \\ -\frac{v+a\Delta t}{\omega}\cos(\theta+\omega\Delta t) + \frac{a}{\omega^2}\sin(\theta+\omega\Delta t) + \frac{v}{\omega}\cos(\theta) - \frac{a}{\omega^2}\sin(\theta) \\ \omega\Delta t \\ a\Delta t \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

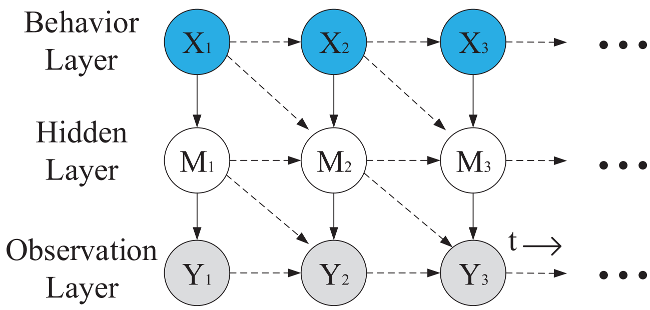

Fig. 3. DBA network. The network is composed of observation, hidden, and behavior layers. Y represents the observation parameters, X means behavior parameters, and M is the hidden parameter, which will be optimized. t reflects the time causal flow.

and behavior layers. The network is shown in Fig. 3. On the basis of dynamic Bayesian theory, the DBA model is defined as a directed acyclic graph, composed of a prior Bayesian network and a two-slice temporal Bayes net according to the first-order Markov assumption. The nodes in the network representing variables, such as driving behaviors, are connected by arcs and parameters and expressed as conditional probabilistic distributions.

In this study, the parameters of a certain structure for the DBA network are learned from realistic driving data by applying expectation maximization. However, the structure of the DBA network is difficult to optimize even with the genetic algorithm. In general, searching the global solution accurately in a limited time is impossible [27]. In this research, a distributed genetic algorithm, which enhances the rate of DBA evolution and the performance of the DBA model, is employed to obtain the optimized structure. This algorithm was discussed in detail in [28].

B. Trajectory Prediction

On the basis of the DBA model, the long-term vehicle trajectory can be predicted. In this study, environment information, such as a road map, is used as evidence for prediction. In this part, the vehicle trajectory predictions of driving behaviors, namely, lane keeping and lane changing, are introduced.

The prediction trajectory based on maneuver estimation could be expressed as follows:

$$p\left(X_{k+k_p}|Z_{1:k}\right) = \sum_{B_0} p\left(X_{k+k_p}|B=B_0\right) p\left(B=B_0|Z_{1:k}\right) \tag{6}$$

where $Z_{1:k}$ is the observed information sequence, X_{k+k_p} is the predicting parameters, such as position and velocity, k is the starting time of prediction, k_p is the length of prediction, B denotes the maneuvers, and B_0 represents one of the exact maneuvers such as lane changing.

1) Lane Keeping: When the driving behavior is road following, the prediction can be based on the road evidence and initial dynamic parameters. The longitudinal parameters along the road $X^{\mathrm{long}R}$ are predicted by the discrete Wiener process acceleration model [29], [30]

$$X^{\mathrm{long}R} = \left(x^{\mathrm{long}R}, v^{\mathrm{long}R}, a^{\mathrm{long}R}\right)^T \tag{7}$$

$$X_{k+1}^{\mathrm{long}R} = A{*}X_k^{\mathrm{long}R} + B{*}\omega_a \tag{8}$$

$$A = \begin{pmatrix} 1 & \Delta t & \frac{1}{2}\Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \\ 1 \end{pmatrix} \tag{9}$$

where Δt is the time period, $X^{\mathrm{long}R}$ includes longitudinal position along the road $x^{\mathrm{long}R}$, longitudinal velocity along the road $v^{\mathrm{long}R}$, and acceleration $a^{\mathrm{long}R}$, and ω_a is the process noise scalar. The covariance is given by

$$Q_k = B\sigma_{\Delta a}^2 B^T = \begin{pmatrix} \frac{1}{4}\Delta t^4 & \frac{1}{2}\Delta t^3 & \frac{1}{2}\Delta t^2 \\ \frac{1}{2}\Delta t^3 & \Delta t^2 & \Delta t \\ \frac{1}{2}\Delta t^2 & \Delta t & 1 \end{pmatrix} \sigma_{\Delta a}^2. \tag{10}$$

According to [30], the lateral position along the road can be described according to the road information by using the Ornstein–Uhlenbeck process, which can be given by

$$\dot{y}_t^{\mathrm{lat}R} = \alpha\left(\mu - y_t^{\mathrm{lat}R}\right) + \omega_t, \alpha > 0 \tag{11}$$

where $y_t^{\mathrm{lat}R}$ means the lateral position along the road, μ is the long-term mean, which could be the middle of the lane, ω_t is the white Gaussian noise, and α models the average speed at which the vehicle returns to the middle of the lane. Considering the autocorrelation function of ω_t as follows:

$$E\left[\omega_t\left(t\right)\omega_t\left(t+\tau\right)\right] = 2\alpha\sigma_{y_{\mathrm{lat}R}}^2 \delta\left(\tau\right) \tag{12}$$

and an exponentially decaying autocovariance function of $y_t^{\mathrm{lat}R}$ as follows:

$$E\left[\left(y_t^{\mathrm{lat}R}\left(t\right) - \overline{y_t^{\mathrm{lat}R}}\left(t\right)\right)\left(y_t^{\mathrm{lat}R}\left(t+\tau\right) - \overline{y_t^{\mathrm{lat}R}}\left(t+\tau\right)\right)\right] = \sigma_{y_{\mathrm{lat}R}}^2 e^{-\alpha|\tau|} \tag{13}$$

the process could be discretized as

$$y_{k+1}^{\mathrm{lat}R} - y_k^{\mathrm{lat}R} = \left(1 - e^{-\alpha\Delta t}\right)\left(\mu - y_k^{\mathrm{lat}R}\right) + \omega^{\mathrm{lat}R} \tag{14}$$

$$\omega^{\mathrm{lat}R} = \sigma_{y_{\mathrm{lat}R}}^2 \left(1 - e^{-2\alpha\Delta t}\right) \tag{15}$$

where $\overline{y_t^{\mathrm{lat}R}}\left(t\right) = E[y_t^{\mathrm{lat}R}(t)]$; $T_c = 1\alpha$ is the time constant that influence the delay rate of the mean evolution; the long term mean is supposed to be the middle of the lane width w_L, $\mu = 0.5w_L$; and $\sigma_{(y_{\mathrm{lat}R})}^2$ is the limiting value of variance, which is also related to the lane width.

The yaw angle of vehicles can be modeled as the deviation of the middle line of the road as follows:

$$p\left(\varphi_k^{FR}\right) = \mathcal{N}\left(\varphi_k^R, \sigma_\varphi^2\right) \tag{16}$$

where φ_k^R is the road orientation and σ_φ^2 is the variance of the predicted yaw rate angle.

2) Lane Changing: When the vehicle is estimated to change lanes, the trajectory could be predicted by a sine function according to the road. In the road coordinates, the trajectory of

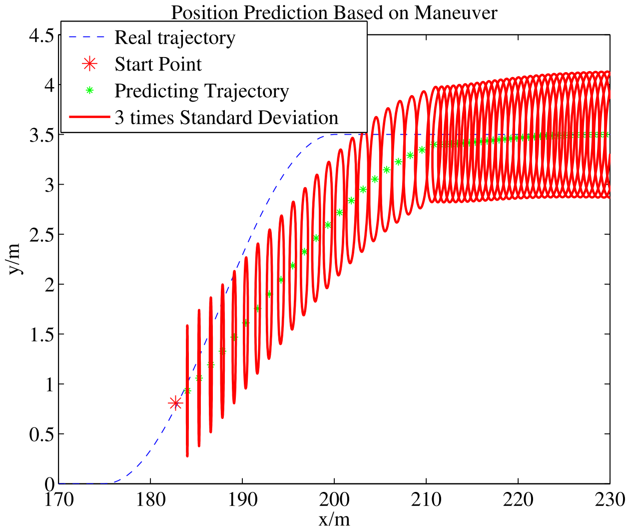

Fig. 4. Position prediction based on lane changing.

vehicles can be represented as the following functions:

$$y^{\mathrm{lat}R}\left(\Delta x^{\mathrm{long}R}\right) = -\frac{w_L}{2}\cos\left(\frac{\pi}{l_R}\Delta x^{\mathrm{long}R}\right), \quad \forall \Delta x^{\mathrm{long}R} \in [0, l_R] \tag{17}$$

where $\Delta x^{\mathrm{long}R}$ is the relative longitudinal distance from the maneuver start point, w_L is the lane width, and l_R is the length of the lane-change maneuver. Uncertainty can be introduced by modeling the start point of the maneuver as

$$p\left(y^{\mathrm{lat}R}\left(\Delta x^{\mathrm{long}R} = 0\right)\right) = \mathcal{N}\left(\left(\frac{w_L}{2}\right), \sigma^2_{y_s^{\mathrm{lat}R}}\right). \tag{18}$$

The yaw angle φ_k^{LC} is given by

$$\varphi_k^{\mathrm{LC}} = \arctan\left(\frac{\pi w_L}{2 l_R}\sin\left(\frac{\pi}{l_R}\Delta x^{\mathrm{long}R}\right)\right). \tag{19}$$

Thus, at every estimation period, the trajectory could be adapted to the current lateral position $y_k^{\mathrm{lat}R}$ and yaw angle φ_k^{LC}. Then, the remaining trajectory of the maneuver can be obtained by solving l_R and $\Delta x^{\mathrm{long}R}$ as follows:

$$l_R = \frac{\pi w_L}{2\tan\varphi_k^{\mathrm{LC}}}\sin\left(\arccos\left(-\frac{2y_k^{\mathrm{lat}R}}{w_L}\right)\right) \tag{20}$$

$$\Delta x^{\mathrm{long}R} = \frac{\arccos\left(-\frac{2y_k^{\mathrm{lat}R}}{w_L}\right)}{\pi} l_R. \tag{21}$$

When the prediction reaches the end of the lane change, the trajectory will be predicted based on the road following behavior [29]. The uncertainty along the curve could be modeled as the discrete Wiener process acceleration model as shown in the trajectory prediction based on lane keeping.

The prediction result based on the lane-change maneuver is shown in Fig. 4; the result indicated that the maneuver-based trajectory prediction could predict in the long term. The predicting variance does not increase considerably as the prediction time advances, unlike in the dynamic prediction model. However, the short-term prediction accuracy is worse than that of the dynamic prediction models because maneuver-based trajectory prediction does not consider dynamic vehicle motions.

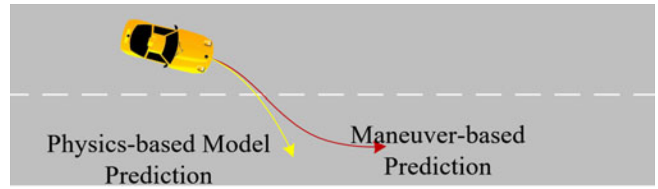

Fig. 5. Vehicle trajectory prediction comparison between physics- and maneuver-based prediction.

IV. Integrating Physics- and Maneuver-Based Trajectory Prediction

As described above, physics- and maneuver-based prediction models have different advantages. Physics-based prediction models could predict the trajectory accurately only in a short time, whereas maneuver-based prediction has a higher level ability to look ahead over a longer time; this difference is shown in Fig. 5. In the long term, the physics-based model predicts the trajectory poorly, which is shown as the yellow line.

The proposed IMMTP method in this study combined the physics- and maneuver-based trajectory prediction methods to predict accurately in the short term and look ahead at a high-level for long terms. In the IMMTP method, the trajectory prediction results of physics- and maneuver-based models, including the predicting uncertainty, were interacted, mixed, combined, and updated to predict the long-term vehicle trajectory. As far as the authors' awareness, this was seldom researched to integrate the different predicting horizon results using the interactive multiple model (IMM) method.

This section introduces the algorithm to integrate both trajectory prediction models for more accurate results in long-term prediction. This section introduces the proposed IMMTP using IMMs [31].

A. Integrated Trajectory Prediction Method

Fusing and integrating information from multiple models is a trend because no perfect model exists for a certain problem [34], [35]. There are various approaches to combine multiple models mentioned above. The IMM method used in [32] and[33] has the ability to combine advantages of different models. In this study, an integrated trajectory prediction method called IMMTP is proposed on the basis of the IMM method. The probability of each predicting model in the IMM could recursively adjust according to the predicting variance of each model.

IMMTP is shown as in Fig. 6. This figure shows four main parts, namely interaction and mode probability prediction, the physics-based prediction model, the maneuver-based prediction model, and combination and model probability update. In this model, $\hat{x}_k^i$ and P_k^i are the mean and covariance, respectively, of input information from the last prediction results of Model i at time k. $\hat{x}_k^{*i}$ and P_k^{*i} are the mean and covariance, respectively,

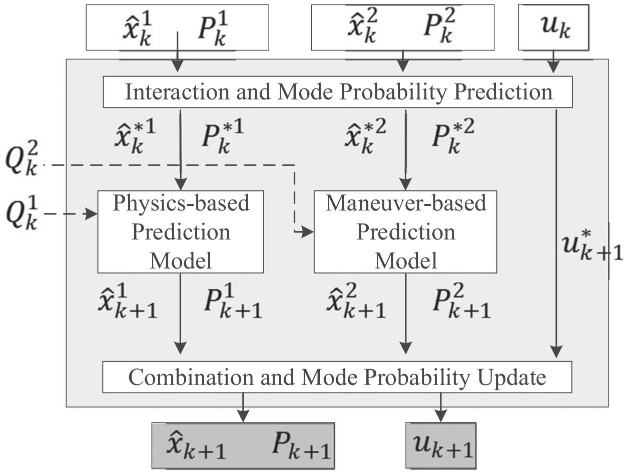

Fig. 6. IMMTP approach.

of input information for the prediction model after interaction and mode probability prediction. μ_k represents the weight coefficient vector for the two single models at the prediction time k. Also, μ^*_{k+1} is the weight coefficient vector after interaction and mode probability prediction. $\hat{x}_{k+1}$, P_{k+1}, and μ_{k+1} are the final outputs, namely the mean, the covariance, and the weight, respectively. Moreover, the uncertainty of each model is considered by adding Q^1_k and Q^2_k, which represent the uncertainty of the two single models.

In this study, the integrated method combines the physics-based model (M^1) and the maneuver-based model (M^2). Thus, the system can be considered as a discrete set of two models, which can be represented as

$$M = \{M^1, M^2\}. \tag{22}$$

Each model is assumed to have its own prior probability $\mu^i_0 = P(M^i_0), i = 1, 2$. The varying system could be seen as Markov switching systems [36], [37]. The transition probability is denoted by

$$p_{ij} = P\left(M^j_k | M^i_{k-1}\right) \tag{23}$$

where k means the prediction time and p_{ij} represents the probability of the transition from Model i to Model j. In other words, it is a first-order Markov chain system. The transition matrix is time invariant [38].

The integrating algorithm includes interaction and mixing, prediction, and combination.

1) Interaction and Mixing: The individual predicting results are mixed on the basis of the predicted model probability and the model transition probability. The interaction and mixing processes could be expressed as follows:

$$\bar{c}_j = \sum_{i=1,2} p_{ij}\mu^i_k \tag{24}$$

$$\mu^{i|j}_k = \frac{1}{\bar{c}_j} p_{ij}\mu^i_k \tag{25}$$

where $\bar{c}_j$ is the normalization factor and μ^i_k is the probability of Model i. Then, the mixed mean and covariance for each model are as follows:

$$\hat{x}^{*j}_k = \sum_{i=1,2} \mu^{i|j}_k \hat{x}^i_k \tag{26}$$

$$P^{*j}_k = \sum_{i=1,2} \mu^{i|j}_k \left(P^i_k + (\hat{x}^i_k - \hat{x}^{*j}_k)\left(\hat{x}^i_k - \hat{x}^{*j}_k\right)^T\right) \tag{27}$$

where P^i_k and $\hat{x}^i_k$ are the final covariance and mean for a single model i, respectively.

2) Prediction: In this part, the predicting results for the next step are computed, and the probability of each model could be calculated. For each model, the prediction can be performed as

$$\left(\hat{x}^i_{k+1}, P^i_{k+1}\right) = \text{Pred}_i\left(\hat{x}^{*i}_k, P^{*i}_k\right) \tag{28}$$

in which Pred_i is the function of the prediction model i.

The likelihood of the prediction for each model can be expressed as the function of the covariance of each model. In this study, the function of the likelihood for each model is designed as follows:

$$\Lambda^i_{k+1} = 1/\left(P\left(x^i_{k+1}\right) + P\left(y^i_{k+1}\right)\right) \tag{29}$$

where $P(x^i_{k+1})$ is the prediction variance for x with the only model i, and $P(y^i_{k+1})$ is the prediction variance for y with the only model i.

Thus, the probability of each model at step $k+1$ is

$$c = \sum_{i=1,2} \Lambda^i_{k+1}\bar{c}_i \tag{30}$$

$$\mu^i_{k+1} = \frac{1}{c}\Lambda^i_{k+1}\bar{c}_i. \tag{31}$$

3) Combination: Now, the prediction for the next step could be obtained on the basis of the updated probability of each model. The final predicted mean $\hat{x}_{k+1}$ and covariance P_{k+1} can be computed as

$$\hat{x}_{k+1} = \sum_{i=1,2} \mu^i_{k+1}\hat{x}^i_{k+1} \tag{32}$$

$$P_{k+1} = \sum_{i=1,2} \mu^i_{k+1}(P^i_{k+1} + (\hat{x}^i_{k+1} - \hat{x}_{k+1})(\hat{x}^i_{k+1} - \hat{x}_{k+1})^T). \tag{33}$$

V. Application and Result Analysis in Lane-Change Scenarios

In this section, the integrating approach will be employed in the lane-change scenario. In this part, the platform of naturalistic driving data collection will be introduced. Then, the results of different trajectory prediction models in the lane-change scenario will be compared and analyzed.

A. Naturalistic Driving Data Collection

To estimate driving behaviors, naturalistic driving data were collected in Beijing. Some cases that include the information sequence are used to analyze the performance of the trajectory prediction methods.

Fig. 7. Vehicle platform (the vehicle platform includes a 360° laser LIDAR, four external cameras, one internal camera, the CAN information, and data storing computer).

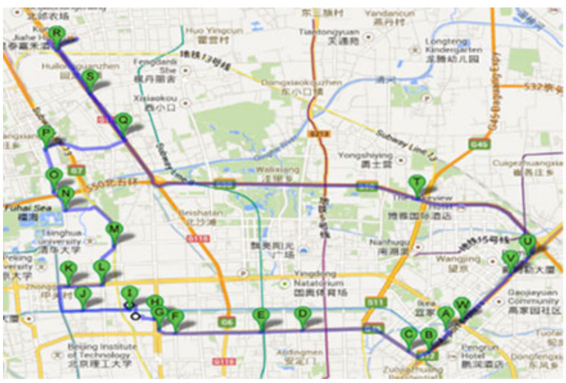

Fig. 8. Test route (the route includes a highway, ring road, airport express lane, and a normal city road, with approximately 18 km).

In this experiment, the platform is equipped with a 360° laser LIDAR that can detect neighboring vehicles in the environment. Four external cameras and one internal camera are equipped to obtain video information including drivers operation and traffic environment. Moreover, driving information such as the velocity of the instrumented vehicle, yaw rate, lateral acceleration, and steering angle could be obtained from the controller area network (CAN). The vehicle platform is shown in Fig. 7.

Fifty common participants were asked to drive the vehicle platform with their own usual driving styles. The driving route includes a highway, ring road, airport express lane, and a normal city road in Beijing, covering a distance of approximately 18 km. The route is shown in Fig. 8.

To process information such as data filtering and label the cases such as the lane-change cases and some other target traffic cases, the processing and labeling software was developed. The information from LIDAR and cameras is displayed by using this software. Also, the serial information, including the steering angle and lateral velocity, is represented by different color curves. These help to determine the start and the end points of the lane-change cases and other target cases during the manual labeling process.

B. Trajectory Prediction Result Analysis and Comparison

In this part, with the aim to analyze and compare the prediction results, one case from the naturalistic driving database is analyzed in detail and the average predicting results based on the database are presented.

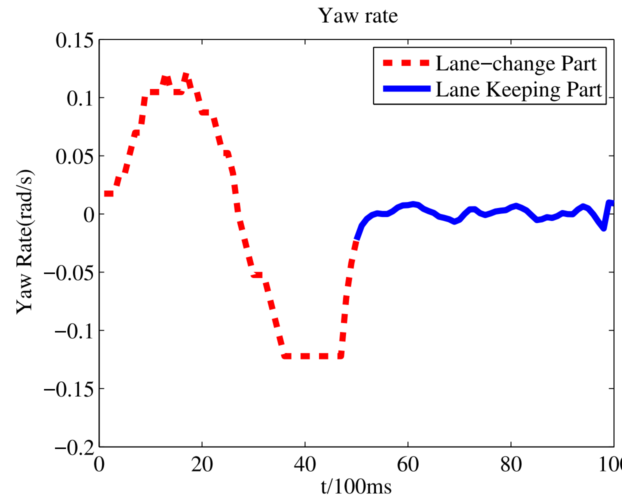

Fig. 9. Yaw rate in the sequential information given in two parts, namely the lane changing (shown in red dotted line) and lane keeping (shown in blue solid line).

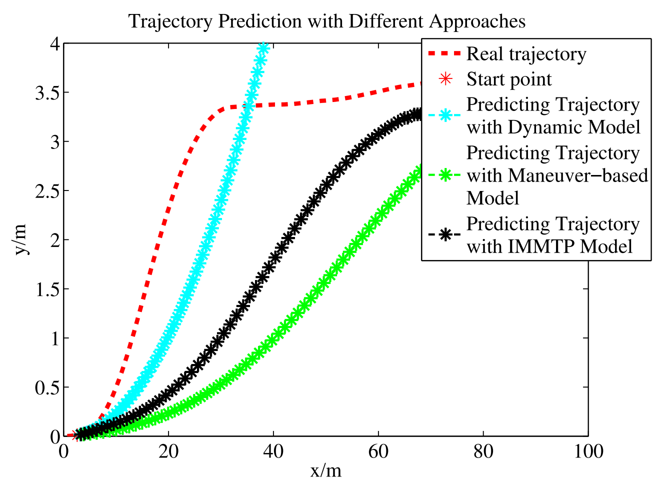

Fig. 10. Trajectory prediction with different approaches (the prediction time starts at 0.4 s after the start of lane change) (although the predicting results of physics-based model are better than that of IMMTP in the short term, the results of IMMTP are much better in the long term).

1) One Case Analysis: The trajectory sequential information, including lane-changing and lane-keeping maneuvers in one certain case from the naturalistic driving database, is used in the trajectory prediction comparison and result analysis. As shown in Fig. 9, the yaw rate in the sequential information is displayed. In this case, two parts exist, namely the lane-changing and lane-keeping parts. The average velocity is 7.9 m/s and the predicting horizon is 8 s.

The result of vehicle trajectory prediction based on IMMTP in the lane-change scenario is shown in Figs. 10 and 11. The results indicate that the physics-based prediction method could evidently obtain better prediction results in the short term, whereas the maneuver-based approach performs much better in long-term prediction. The proposed IMMTP approach can integrate the advantages of the two prediction models. In other words, the IMMTP model ensures prediction accuracy in the short term by taking advantage of the dynamic motion model and also predicts at a higher level for long-time-horizon predictions. Moreover,

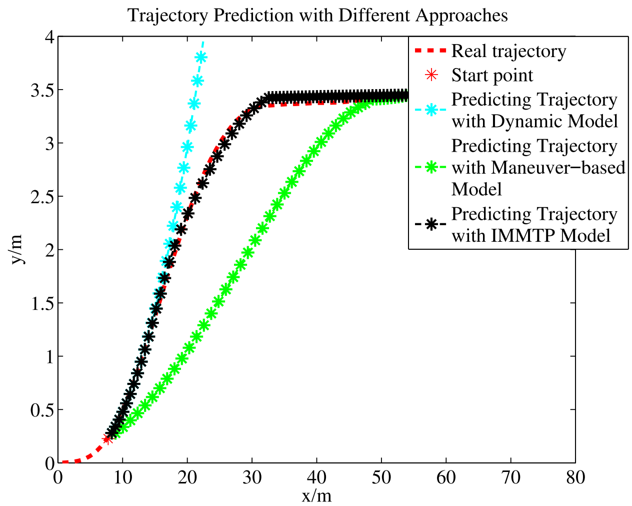

Fig. 11. Trajectory prediction with different approaches (the prediction time starts at 1.4 s after the start of lane change).

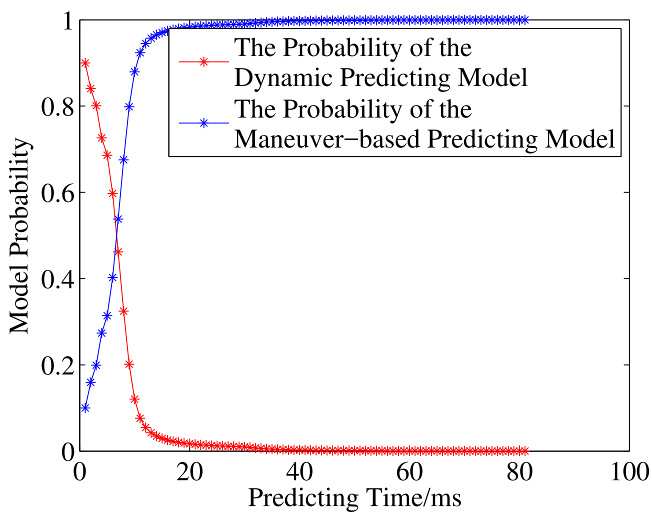

Fig. 12. Probability of each model in the IMMTP approach (the probability at time 1 ms is the original start value defined based on experience and the prediction time starts at 0.4 s after the start of lane change).

the two figures clearly show that different prediction results may be obtained with the proceeding of lane changing. At the beginning of lane changing, the prediction results are worse than the later ones because the lateral motion information such as yaw rate is not obvious at that time.

The probability of each model in the IMMTP approach is shown in Figs. 12 and 13. As we can see, the probability of the physics-based prediction model decreases with the prediction time, and the probability of the maneuver-based model increases as the prediction time advances. This situation coincides with our everyday experience. The two figures also clearly show that the functions of the weight coefficient differ with each case.

The predicting uncertainty in the results of different models is also analyzed and compared in Table I, in which the predicting uncertainty is estimated using three times standard deviation. The predicting uncertainty is analyzed in terms of the average three times standard deviation in the prediction horizon using

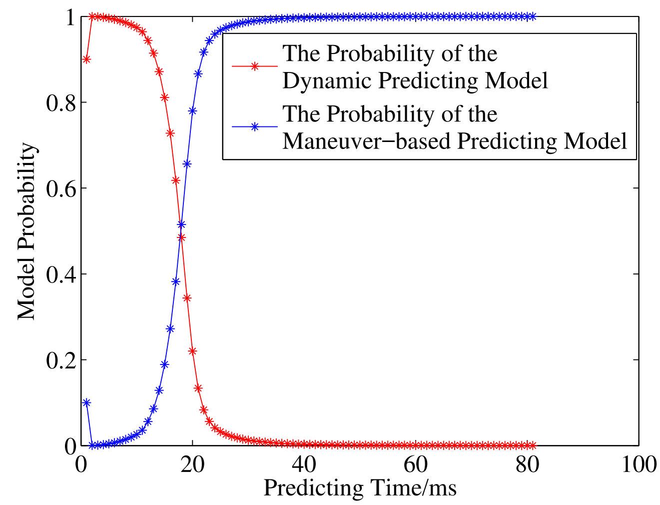

Fig. 13. Probability of each model in the IMMTP approach (the probability at time 1ms is the original start value defined based on experience and the prediction time starts at 1.4 s after the start of lane change).

TABLE I
ANALYSIS AND COMPARISON OF PREDICTION UNCERTAINTY USING THREE TIMES STANDARD DEVIATION (M)

Prediction model	Prediction time(s) 0.5	1	3	5	8
Physics-based model	0.28	0.96	7.82	20.89	49.81
Maneuver-based model	0.53	0.66	0.82	0.93	1.08
IMMTP model	0.23	0.46	0.75	0.88	1.05

the case of prediction after 1.4 s of the lane change. The table indicates that the predicting uncertainty of physics-based model increases largely with the increase of the prediction horizon, especially in the long-term prediction horizon. In the short-term prediction horizon like that less than 0.5 s, the predicting uncertainty of the maneuver-based model is much larger than that of the physics-based model. In the IMMTP model, the predicting uncertainty does not expend much in the long-term prediction and stays relatively small in the short-term prediction horizon, which means that the IMMTP model could predict the vehicle trajectory more confidently than the other two single models.

2) Average Predicting Results: The accuracy of the prediction results of the physics-based, maneuver-based, and the IMMTP models is analyzed and compared. The prediction results of the IMMTP approach are compared with those of an invariable coefficient model. The invariable coefficient model integrates the physics- and maneuver-based models with a fixed coefficient function [8]. The prediction results can be measured by the average error of prediction position shown as follows:

$$\mathrm{E} = \frac{1}{N}\sum_{i=1}^{N}\sqrt{{\lambda_1}^2(x_{i,p}-x_{i,r})^2+{\lambda_2}^2(y_{i,p}-y_{i,r})^2} \quad (34)$$

where N is the number of prediction points and λ_1 and λ_2 are the coefficients of the prediction errors. $x_{i,p}$ and $y_{i,p}$ are the

TABLE II
EVALUATION AND COMPARISON OF PREDICTION RESULTS IN TERMS OF AVERAGE ERROR (M)

Prediction model	Prediction time (s) 1	3	5	8
Physics-based model	0.05	0.48	1.72	4.80
Maneuver-based model	0.75	1.92	2.49	2.59
Invariable coefficient model	0.12	1.18	2.00	2.28
IMMTP model	0.14	0.69	1.13	1.55

prediction results of the position. $x_{i,r}$ and $y_{i,r}$ are the real value of the prediction points.

In this experiment, the 40 lane-change cases are used to compute the average error of prediction position, and the average error of the invariable coefficient model [8] is studied and compared with that of the models mentioned in this paper. As shown in Table II, with the increase in the prediction time, the average error of the physics-based model increases considerably. In other words, the physics-based model performs poorly in long-term prediction. Also, the prediction results from the invariable coefficient model and the IMMTP model indicate that the proposed IMMTP model could obtain better prediction results in the long term. In addition, the average computing time of the IMMTP method on an Intel Core i7-8.00 GB at 2.2 GHz is 78 ms when the prediction horizon is 8 s, which proves that IMMTP could be implemented in real time.

VI. CONCLUSION AND CONTRIBUTIONS

In this study, the physics- and maneuver-based trajectory prediction models were established to predict vehicle trajectories. The uncertainty in the dynamic trajectory prediction model was considered by using UT, which could solve nonlinear estimation and prediction problems. In the maneuver-based trajectory prediction approach, the maneuvers were estimated by employing the dynamic Bayesian network. The random elements were also included in the maneuver-based trajectory predicting method. Then, the two prediction methods were compared, and it is indicated in this paper that the physics-based trajectory predicting model could predict accurately with the vehicle motion model in the short term. On the contrary, the maneuver-based trajectory prediction model can predict the vehicle trajectory in a higher horizon for a long time. The IMMTP approach was proposed to predict accurately in the short term and anticipate with a higher horizon in the long term, combining the advantages of these two approaches. The probability of each model in the IMMTP approach could be recursively adapted according to the prediction variance of each model.

Finally, the IMMTP approach was applied and analyzed in the lane-changing scenarios by using the naturalistic driving data collected in Beijing. The average prediction error was proposed as the evaluation index for the prediction models. The comparison indicates that the IMMTP model could predict more accurately than the single maneuver-based prediction model in the long run. Also, this integrated method has the ability to predict in the long term with fewer prediction errors than the physics-based prediction model. Moreover, the prediction results from the invariable coefficient and the IMMTP models indicate that the proposed IMMTP model could obtain better prediction results by recursively adapting weight coefficients.

In future work, the road geometry would be the additional information to predict. Moreover, other vehicles or road users in the traffic will be considered, and their interacting influences could be taken into account to predict vehicle trajectories.

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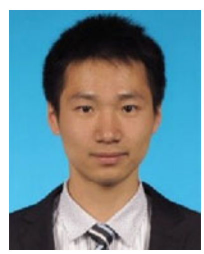

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