Homework 4

Theory Part

Q1 Consider a layer in CNN that takes in a single channel input of 64 × 64, and has 96 filters. In each of the following cases, compute the number of parameters that are learned in this layer. We assume that bias is present for each weight.

[1] A convolution layer with filters of same size as the input.

$$Param = (ksize^2 + 1) * channel$$
$$Param = (64^2 + 1) * 96$$
$$Param = 393312$$

In [1]:
```
(64**2 + 1)* 96
```
Out[1]:
```
393312
```

[2] A convolution layer with 8 × 8 filters with stride of 4

$$Param = (ksize)^2 * channel + bias$$
$$Param = (8^2 + 1) * 96$$
$$Param = 6240$$

In [2]:
```
(8**2 + 1) * 96
```
Out[2]:
```
6240
```

[3]. A convolution layer with 1 × 1 filter and a stride of 1

$$Param = (ksize)^2 * channel + bias$$
$$Param = (1^2 + 1) * 96$$
$$Param = 192$$

Q2 Suppose you would have a neuron which has an RBF kernel as activation function (remember the evil wolf? Drop your linear style of thoughts. Circumferential thoughts can be nice too.)

$$y = exp(-(x_1^2 + x_2^2)) + b$$

with parameter b. *What would be the shapes realized by the set of points* $\{(x1, x2) : y((x1, x2)) = 0\}$ *as a function of b ? Explain in at most 2 sentences and/or a little math.*

$$0 = exp(-(x_1^2 + x_2^2)) + b$$

$$-b = exp(-(x_1^2 + x_2^2))$$

$$-ln(-b) = x_1^2 + x_2^2$$

Therefore, it is a circle centered around the origin with radius $\sqrt{-ln(-b)}$. Obviously this is only valid when $-1 < b < 0$.

Supposed now we add weight,

$$y = exp(-(w_1 x_1^2 + w_2 x_2^2)) + b$$

what shapes can we realize now? Explain in at most 5 sentences and/or a little math. You can make references to publicly available in the internet materials to explain.

$$-ln(-b) = w_1 x_1^2 + w_2 x_2^2$$

$$-ln(-b) = \frac{x_1^2}{w_1^{-1}} + \frac{x_2^2}{w_2^{-1}}$$

$$1 = \frac{x_1^2}{-ln(-b) \cdot w_1^{-1}} + \frac{x_2^2}{-ln(-b) \cdot w_2^{-1}}$$

$$1 = \left(\frac{x_1}{\sqrt{-ln(-b) \cdot w_1^{-1}}}\right)^2 + \left(\frac{x_2}{\sqrt{-ln(-b) \cdot w_2^{-1}}}\right)^2$$

hence it is an ellipse, centered around the origin with radius $\sqrt{-ln(-b) \cdot w_1^{-1}}$ along the x_1 axis and radius $\sqrt{-ln(-b) \cdot w_2^{-1}}$ along the x_2 axis.

Q3 Suppose you have five linear neurons neurons n1, . . . , n5, realizing above decision boundaries as shown in Figure 1. That is: for every decision boundary we have outputs are = 0.5 in the zones marked with red plusses, and = 0.2 in the zones marked with the blue minuses.

figure1

As you know, each neuron is realized by:

$$n_i = 0.3H(w_1^{(i)} x_1 + w_2^{(i)} x_2 + b^{(i)}) + 0.2, \quad H(z) \in 0, 1$$

where H is the threshold activation function. You want to predict positive values in a shape marked in green in Figure 1. You want to achieve this prediction by combining these neurons using a threshold neuron H:

$$y = H(\sum_i v_i^* n_i + b^*)$$

[1] _what do you have to do with the weights of n_5 so that you can move the decision boundary of n_5 so that you can realize the shape in green shown above (in the sense of having positive values inside and negative values outside.)? Give a qualitative description. Note: Give a qualitative description in 3 sentences at most. Note that there is an x- and an y-axis, which helps you to express vectors qualitatively._

The position of the decision boundary of n_5 depends on its weight and biases. Particularly, the ratio between w_1 and w_2 determines the slant of the boundary, while the ratio between the b and w_2 determines its offset from origin. As the desired position is a shift upwards (given that the boundary continues infinitely), we want to decrease b so that the boundaries shift upwards.

[2] _after moving the decision boundary of n5 appropriately, the green shape looks a bit like an logical AND-combination of the +-zones for every neuron. How to choose the weights v_i^* and the bias b^* in $y = H(\sum_i v_i^* n_i + b^*)$ so that you can realize the green shape (in the sense of having positive values inside and negative values outside that shape)? Note: n_i gives out values either 0.5 or 0.2_

Lets say that function H has a threshold $h = 0$, such that

$$H(z) = 1[z > h] = 1[z > 0]$$

If we were to take green area as {+1}, for the threshold neuron to fire $+1$, we will need all neurons n_i to fire 0.5,

$$1 = H(\sum_i 0.5v_i^* + b^*)$$

$$1 = 1[(\sum_i 0.5v_i^* + b^*) > 0]$$

$$(\sum_i 0.5v_i^* + b^*) > 0 \quad -- \quad (1)$$

If we were to take non-green area as {-1}, for the threshold neuron to fire -1, we will need at least one neuron n_j to fire 0.2,

$$0 = H(\sum_{i-1} v_i^* n_i + v_j^* n_j + b^*)$$

$$0 = H(\sum_{i-1} 0.5v_i^* + 0.2v_j^* + b^*)$$

$$0 = 1[\sum_{i-1} 0.5v_i^* + 0.2v_j^* + b^* > 0]$$

$$\sum_{i-1} 0.5v_i^* + 0.2v_j^* + b^* <= 0 \quad -- \quad (2)$$

For simplicity, we set $v_i^* = 1$ for all i. Finding b,

$$0.5 \cdot 5 + b^* > 0 \quad -- \quad (1)$$

$$2.5 + b^* > 0$$

$$b^* > -2.5$$

$$0.5 \cdot 4 + 0.2 + b^* <= 0 \quad -- \quad (2)$$

$$2.2 + b^* <= 0$$

$$b^* <= -2.2$$

$$therefore \quad -2.5 < b <= -2.2$$

we can pick any b within this range, e.g. $b = -2.4$ with our $v_i^* = 1$, such that

$$H(\sum_i 0.5 - 2.4) = H(2.5 - 2.4) \quad for\ all\ n_i = 0.5$$
$$= H(0.1) = 1[0.1 > 0] = 1 \quad (green)$$

$$H(\sum_{i-1} 0.5 + 0.2 - 2.4) = H(2.2 - 2.4) \quad for\ four\ n_i = 0.5\ and\ one\ n_j = 0.2$$
$$= H(-0.2) = 1[-0.2 > 0] = 0 \quad (outside)$$

Coding - Part 1

For this section, there is a script called `hw4_code.py` that will hold some definitions e.g. the `FlowerDataset` class and `train_model` function.

Below we will do some sanity check on the dataset.

In [3]:

```
from hw4_code import *
import matplotlib.pyplot as plt
from torchvision import transforms

# testing dataset
flower_dataset = FlowerDataset('..\\datasets\\flowersstuff\\102flowers\\flowers_data', mode='train')
flower_dataset_val = FlowerDataset('..\\datasets\\flowersstuff\\102flowers\\flowers_data', mode='val')

for i in range(3):
    flower1 = flower_dataset[i]
    print('label:',flower1['label'])
    image = transforms.ToPILImage()(flower1['image'])

    invTrans = transforms.Compose([ transforms.Normalize(mean = [ 0., 0., 0. ],
                                                         std = [ 1/0.229, 1/0.224, 1/0.225 ]),
                                    transforms.Normalize(mean = [ -0.485, -0.456, -0.406 ],
                                                         std = [ 1., 1., 1. ]),
                                    transforms.ToPILImage(),
                                   ])
    clear_image = invTrans(flower1['image'])

    plt.figure()
    plt.subplot(121)
    plt.imshow(clear_image)
    plt.title('original')
    plt.subplot(122)
    plt.imshow(image)
    plt.title('normalized')
```

```
label: 45
label: 87
label: 20
```

We will then use resnet models to classify our flowers.

In [4]:

```
# Getting pretrained resnet
from torchvision import models

def get_pretrained_resnet(use_gpu=True):
    model = models.resnet18(pretrained=True)
    if use_gpu:
        model = model.cuda(0)
    return model

resnetmodel = get_pretrained_resnet()
resnet_dict = resnetmodel.state_dict()
# for k in resnet_dict.keys():
#     if 'bn' not in k and 'down' not in k:
#         # printing any statedict without batch-norm and downsample
#         print(k)
```

In [5]:
```
# training with various learn rate
for lr in [0.1, 1, 10]:
    print("\n\nWith learning rate {},\n".format(lr))
    learnrate = lr
    optimizer = optim.SGD(resnetmodel.parameters(), lr=learnrate, momentum=0.9)

    trainedmodel = train_model(flower_dataset, resnetmodel, optimizer,
                               epoch=1, mode='train', use_gpu=True, print_every=1)
    trainedmodel = train_model(flower_dataset_val, trainedmodel, optimizer,
                               epoch=1, mode='val', use_gpu=True, print_every=1)
```

```
With learning rate 0.1,

[train] - Epoch 0..
      >> Epoch loss 1.84155 accuracy 0.032                      in 446.3549s
[val] - Epoch 0..
      >> Epoch loss 1.62162 accuracy 0.088                      in 28.9867s


With learning rate 1,

[train] - Epoch 0..
      >> Epoch loss 0.68266 accuracy 0.462                      in 437.4866s
[val] - Epoch 0..
      >> Epoch loss 0.20868 accuracy 0.795                      in 33.7598s


With learning rate 10,

[train] - Epoch 0..
      >> Epoch loss 0.44307 accuracy 0.559                      in 438.3159s
[val] - Epoch 0..
      >> Epoch loss 0.15011 accuracy 0.837                      in 28.1213s
```

In [6]:
```
# now lets do the same thing, but train with an empty resnet

def get_empty_resnet(use_gpu=True):
    model = models.resnet18(pretrained=False)
    if use_gpu:
        model = model.cuda(0)
    return model

emptymodel = get_pretrained_resnet()
empty_dict = resnetmodel.state_dict()
# for k in empty_dict.keys():
#     if 'bn' not in k and 'down' not in k:
#         # printing any statedict without batch-norm and downsample
#         print(k)
```

In [7]:
```
# training with various learn rate
for lr in [0.1, 1, 10]:
    print("\n\nWith learning rate {},\n".format(lr))
    learnrate = lr
    optimizer = optim.SGD(emptymodel.parameters(), lr=learnrate, momentum=0.9)

    trainedmodel = train_model(flower_dataset, emptymodel, optimizer,
                               epoch=1, mode='train', use_gpu=True, print_every=1)
    trainedmodel = train_model(flower_dataset_val, trainedmodel, optimizer,
                               epoch=1, mode='val', use_gpu=True, print_every=1)
```

```
With learning rate 0.1,

[train] - Epoch 0..
      >> Epoch loss 1.84189 accuracy 0.033                      in 437.1789s
[val] - Epoch 0..
      >> Epoch loss 1.61854 accuracy 0.087                      in 28.1587s


With learning rate 1,

[train] - Epoch 0..
      >> Epoch loss 0.68072 accuracy 0.458                      in 439.9016s
[val] - Epoch 0..
      >> Epoch loss 0.20165 accuracy 0.789                      in 27.8535s


With learning rate 10,

[train] - Epoch 0..
      >> Epoch loss 0.43625 accuracy 0.569                      in 435.8893s
[val] - Epoch 0..
      >> Epoch loss 0.15598 accuracy 0.828                      in 27.9031s
```

In [8]:
```
# now lets do the same thing, but train with an empty resnet
import torch.nn as nn

def get_unfrozen_resnet(numcl, use_gpu=True):
    model = models.resnet18(pretrained=False)
    num_ftrs = model.fc.in_features
    model.fc = nn.Linear(num_ftrs, numcl)
    if use_gpu:
        model = model.cuda(0)
    return model

unfrozenmodel = get_pretrained_resnet()
unfrozen_dict = resnetmodel.state_dict()
# for k in unfrozen_dict.keys():
#     if 'bn' not in k and 'down' not in k:
#         # printing any statedict without batch-norm and downsample
#         print(k)
```

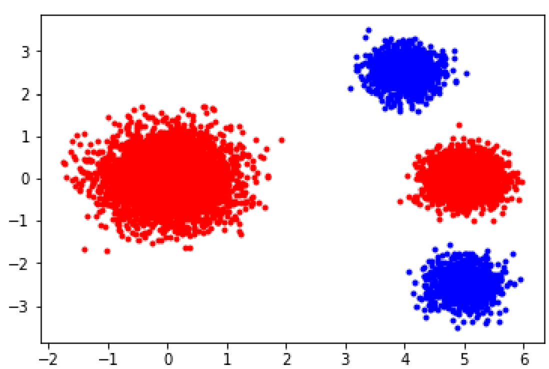
In [9]:
```
# training with various learn rate
for lr in [0.1, 1, 10]:
    print("\n\nWith learning rate {},\n".format(lr))
    learnrate = lr
    optimizer = optim.SGD(unfrozenmodel.parameters(), lr=learnrate, momentum=0.9)

    trainedmodel = train_model(flower_dataset, unfrozenmodel, optimizer,
                               epoch=1, mode='train', use_gpu=True, print_every=1)
    trainedmodel = train_model(flower_dataset_val, trainedmodel, optimizer,
                               epoch=1, mode='val', use_gpu=True, print_every=1)
```

```
With learning rate 0.1,

[train] - Epoch 0..
      >> Epoch loss 1.84166 accuracy 0.033                      in 437.0670s
[val] - Epoch 0..
      >> Epoch loss 1.62038 accuracy 0.092                      in 28.1243s


With learning rate 1,

[train] - Epoch 0..
      >> Epoch loss 0.68146 accuracy 0.461                      in 436.3131s
[val] - Epoch 0..
      >> Epoch loss 0.20818 accuracy 0.784                      in 27.8541s


With learning rate 10,

[train] - Epoch 0..
      >> Epoch loss 0.43975 accuracy 0.566                      in 435.7458s
[val] - Epoch 0..
      >> Epoch loss 0.18098 accuracy 0.817                      in 27.9208s
```

In [10]:
```
from hw4_code2 import *
import numpy as np

data, label = from_text('samplestr.txt')
true_mask = label.reshape((-1)).astype(bool)
false_mask = (1-label).reshape((-1)).astype(bool)

data_true = np.transpose(data[true_mask])
data_false = np.transpose(data[false_mask])

print('samples with positive class:', len(data_true[0]),
      '\nsamples with negative class:', len(data_false[0]))

plt.plot(data_true[0], data_true[1], '.b')
plt.plot(data_false[0], data_false[1], '.r')
```

```
samples with positive class: 2000
samples with negative class: 8000
```

Out[10]: [<matplotlib.lines.Line2D at 0x2000c26a0b8>]



In [11]:
```
'''
Creating a new Neural Network
'''
from torch import nn
import torch.nn.functional as F

class OurNet(nn.Module):
    def __init__(self):
        super(OurNet, self).__init__()
        self.fc = nn.Linear(2, 1)

    def forward(self, x):
        x = self.fc(x)
        return x

our_net = OurNet()
print(our_net)
```

```
OurNet(
  (fc): Linear(in_features=2, out_features=1, bias=True)
)
```

In [12]:
```
# testing the Dataset subclass from hw4_code2

train_set = ImbaDataset('.', mode='train')
print('train:',train_set[2])
test_set = ImbaDataset('.', mode='test')
print('test:', test_set[2])
```

```
train: {'data': tensor([ 0.7959,  0.3896], dtype=torch.float64), 'label': tensor([ 0.], dtype=torch.float64)}
test: {'data': tensor([-0.5324, -0.7683], dtype=torch.float64), 'label': tensor([ 0.], dtype=torch.float64)}
```

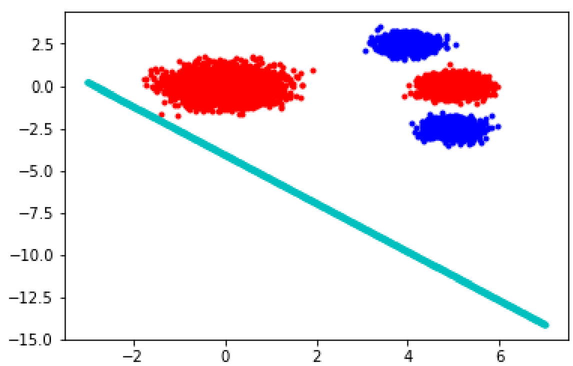
In [13]:
```
# simple training
import torch
from torch.utils.data import DataLoader
import torch.optim as optim
from torch.autograd import Variable
from torch.utils.data.sampler import WeightedRandomSampler

our_net = OurNet().double()
optimizer = optim.SGD(our_net.parameters(), lr=0.1)
criterion = nn.BCEWithLogitsLoss()

def train_ournet(dataset, model, optimizer, criterion, batch_sampler=None,
                 mode='train', positive_weight=1):
    '''
    positive_weight : weight of the accuracy of positive classes compared to negative classes
                      e.g. 2 means positive classes has 2x the weight of negative classes
    '''
    print_truecount = True
    if batch_sampler:
        loader = DataLoader(dataset, batch_sampler=batch_sampler)
    else:
        loader = DataLoader(dataset, batch_size=128)
    running_loss = 0
    running_corrects = 0
    total_data = 0

    for inputdata in loader:
        data = Variable(inputdata['data'])
        labels = Variable(inputdata['label']).view(-1,1)
        if print_truecount:
            print('True count:', labels.sum().item(), 'out of', len(labels),
                  "({}%)".format(labels.sum().item() * 100 / len(labels)))
            print_truecount = False

        outputs = model.forward(data)
        _, predictions = outputs.max(dim=1)
        predictions = predictions.view(-1,1)

        loss = criterion(outputs, labels) / len(dataset)
        if mode == 'train':
            loss.backward()
            optimizer.step()

        # balanced stuff
        total_data = len(labels) + (positive_weight - 1) * labels.sum().item()
        corrects = (predictions.double().cpu() == labels.cpu()).double()
        weights = (positive_weight - 1) * labels.cpu() + 1

#         print(corrects, weights)

        running_corrects += (corrects * weights).sum().item() #/ float(128*100)
        running_loss += loss.item()


    running_corrects /= float(128*100)
    if mode == 'train':
        print("Trained with accuracy {} and loss {}\n".format(running_corrects, running_loss))
    elif mode == 'test':
        print("Tested with accuracy {} and loss {}\n".format(running_corrects, running_loss))
    return model
```

In [14]:
```
our_net_trained = train_ournet(train_set, our_net, optimizer, criterion)
our_net_trained = train_ournet(train_set, our_net_trained, optimizer, criterion, mode='test')
our_net_trained = train_ournet(test_set, our_net_trained, optimizer, criterion, mode='test')
print(our_net_trained.state_dict())
```

```
True count: 0.0 out of 128 (0.0%)
Trained with accuracy 0.625 and loss 0.00496864125207587

True count: 0.0 out of 128 (0.0%)
Tested with accuracy 0.625 and loss 0.004945526858552439

True count: 0.0 out of 128 (0.0%)
Tested with accuracy 0.625 and loss 0.004952957972913324

OrderedDict([('fc.weight', tensor([[-0.1962, -0.1364]], dtype=torch.float64)), ('fc.bias', tensor([-0.5553],
dtype=torch.float64))])
```

In [15]:
```
w = our_net_trained.state_dict()['fc.weight']
b = our_net_trained.state_dict()['fc.bias']
w1 = w[0,0].item()
w2 = w[0,1].item()
b1 = b[0].item()

m = -w1/w2
c = -b/w2

line_x = [ i/100 for i in range(-300, 700)]
line_y = [ m*i + c for i in line_x]

plt.plot(data_true[0], data_true[1], '.b')
plt.plot(data_false[0], data_false[1], '.r')
plt.plot(line_x, line_y, '.c')
```

Out[15]: [<matplotlib.lines.Line2D at 0x2000a1886a0>]



To achieve 50-50 on minibatches, we need a Sampler subclass.

In [16]:
```
class OurBatchSampler(object):
    r"""
    Special batch sampler class that ensures a 50-50
    dataset classes.
    """

    def __init__(self, dataset, batch_size, iteration):
        self.dataset = dataset
        self.batch_size = batch_size
        self.iteration = iteration

        self.idx_plus = [idx for idx in range(len(dataset)) if dataset[idx]['label'].item() > 0]
        self.idx_minus = [idx for idx in range(len(dataset)) if idx not in self.idx_plus]

    def __iter__(self):
        batch = []
        for i in range(self.iteration):
            idplus = np.random.choice(self.idx_plus, self.batch_size // 2).tolist()
            idminus = np.random.choice(self.idx_minus, self.batch_size // 2).tolist()
            batch += idplus + idminus

            yield batch
            batch = []

    def __len__(self):
        return self.batch_size * self.iteration

train_sampler = OurBatchSampler(train_set, 128, 100)
print('train sampler built')
test_sampler = OurBatchSampler(test_set, 128, 100)
print('test sampler built')
```

```
train sampler built
test sampler built
```

In [17]:
```
our_net_trained = train_ournet(train_set, our_net, optimizer, criterion,
                               batch_sampler=train_sampler, positive_weight=4)
our_net_trained = train_ournet(train_set, our_net_trained, optimizer,
                               criterion, batch_sampler=train_sampler, mode='test', positive_weight=4)
our_net_trained = train_ournet(test_set, our_net_trained, optimizer,
                               criterion, batch_sampler=train_sampler, mode='test', positive_weight=4)
print(our_net_trained.state_dict())
```

```
True count: 64.0 out of 128 (50.0%)
Trained with accuracy 0.5 and loss 0.009352104866691635

True count: 64.0 out of 128 (50.0%)
Tested with accuracy 0.5 and loss 0.008281729356746472

True count: 64.0 out of 128 (50.0%)
Tested with accuracy 0.5 and loss 0.00829769013872341

OrderedDict([('fc.weight', tensor([[-0.0777, -0.1335]], dtype=torch.float64)), ('fc.bias', tensor([-0.5483],
dtype=torch.float64))])
```

In [18]:
```
w = our_net_trained.state_dict()['fc.weight']
b = our_net_trained.state_dict()['fc.bias']
w1 = w[0,0].item()
w2 = w[0,1].item()
b1 = b[0].item()

m = -w1/w2
c = -b/w2

line_x = [ i/100 for i in range(-300, 700)]
line_y = [ m*i + c for i in line_x]

plt.plot(data_true[0], data_true[1], '.b')
plt.plot(data_false[0], data_false[1], '.r')
plt.plot(line_x, line_y, '.c')
```

Out[18]:
```
[<matplotlib.lines.Line2D at 0x2000a103b38>]
```