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Dynamics Analysis of 2PPPPS-R-2PPPPS Serial-Parallel Mechanism

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Abstract. In this paper, the stress condition of each hinge point of the new 2PPPPS-R-2PPPPS serial-parallel mechanism is analyzed comprehensively by establishing dynamic equations and Euler equations. Given the stress at output terminal of the mechanism that has given spatial position and pose, the movement parameters of principal axis, the movement parameters of connecting rod and the movement parameters of horizontal and vertical moving sliders, we can solve the inverse dynamics solution of the mechanism, and through computer programming, we can calculate and draw the intuitive and effective results.

Introduction

The series robots have low stability [1]. But it is easy to solve the dynamics forward solution, the kinematic coupling degree of each joint is smaller, and the control is simple; the working space of robot is larger than that of machine tool. Serial robots are more complex and costly. The parallel robot has the advantages of high stiffness, large load capacity, compact structure and high position accuracy [2], which are complementary with the tandem robot. However, it is difficult to solve the dynamics forward solution, the movement branches are easy to be coupled, the working space is relatively small [3], and the application in the actual production activity is less. Based on the advantages and disadvantages of the serial robot and the parallel robot, the 2PPPPS-R-2PPPPS serial-parallel robot is constructed (as shown in Fig.1),it has the advantages of serial and parallel robot [4,5]. In the form of practical examples, this paper calculate and draw the intuitive and effective results by using computer programming on the study object[6].

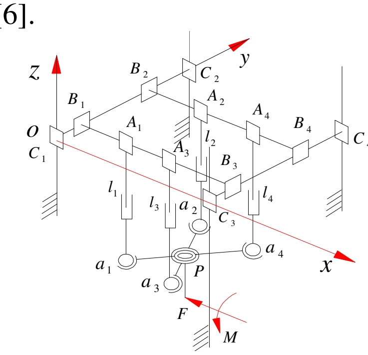

Figure 1. The series-parallel mechanism and its spatial force diagram

Dynamic Analysis

When the spatial position of each hinge point of the mechanism and the spatial position of the output spindle are known (shown in Fig.1)[7], the force and torque at the end of the output spindle of the mechanism can be expressed as F and M respectively, according to Newton's law of motion and Euler equation [8], the dynamics inverse solution of the mechanism shown in Figure 1 is solved[8].

Suppose that the length of the output moving platform branch a_1a_4 and a_2a_3 is d, because of the special structure of the mechanism, the moving platform branch chain $A_1l_1a_1a_4l_4A_4$ is always in the plane P_{14}, the Euler equation of the moving platform branch a_1a_4 in this plane with a_1 as the fulcrum (as shown in Fig. 2) can be obtained, as follows.

$$\begin{aligned} & F_{a_4}^x \cdot \cos\alpha_{P_{14}}^x \cdot \sin\beta_{a_4}^x \cdot d + F_{a_4}^y \cdot \cos\alpha_{P_{14}}^y \cdot \sin\beta_{a_4}^y \cdot d \cdot + F_{a_4}^z \cdot d \cdot \cos\alpha_{a_1a_4}^{l_4} \\ & F_P^x \cdot \cos\alpha_{P_{14}}^x \cdot \sin\beta_{P_{14}}^x \cdot \frac{d}{2} + F_P^y \cdot \cos\alpha_{P_{14}}^y \cdot \sin\beta_{P_{14}}^y \cdot \frac{d}{2} + F_P^z \cdot \frac{d}{2} \cdot \cos\alpha_{a_1a_4}^{l_4} \cdot \\ & = J_{a_1a_4} \cdot \omega_{a_1a_4}^{n_{14}} \end{aligned} \tag{1}$$

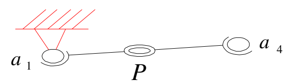

Figure 2. The moment balance analysis of moving platform branch a_1a_4 with a_1 as the fulcrum

In the formula (1), $F_{a_4}^x$, $F_{a_4}^y$, $F_{a_4}^z$ means the three stress components of the moving platform hinge point a_4 that are respectively along the x, y and z axis of the space fixed coordinate system. F_P^x, F_P^y, F_P^z means the three stress components of the moving platform center point P that are respectively along the x, y and z axis of the space fixed coordinate system. $\alpha_{P_{14}}^x$, $\alpha_{P_{14}}^y$ means the angle between the plane P_{14} and the x and y axis of the space fixed coordinate system. $\alpha_{a_1a_4}^{l_4}$ means the angle between the connecting rod l_4 (the z axis of the fixed coordinate system) and the moving platform branch a_1a_4. $\beta_{a_4}^x$, $\beta_{a_4}^y$ means the angle between the moving platform branch a_1a_4 and the projection of $F_{a_4}^x$, $F_{a_4}^y$ in plane P_{14}. $J_{a_1a_4}$ means the moment of Inertia the moving platform branch a_1a_4 around the point a_1 that parallel to the normal n_{14} of the plane P_{14}. $\omega_{a_1a_4}^{n_{14}}$ means the angular acceleration component of the moving platform branch a_1a_4 around hinge point a_1 that parallel to the normal n_{14} of the plane P_{14}.

Similarly, the moving platform branch chain $A_2l_2a_2a_3l_3A_3$ is always in the plane P_{23}, so that the Euler equation of the moving platform branch a_2a_3 in this plane with a_2 as the fulcrum (as shown in Fig. 3) can be obtained as follows.

$$\begin{aligned} & F_{a_3}^x \cdot \cos\alpha_{P_{23}}^x \cdot \sin\beta_{a_3}^x \cdot d + F_{a_3}^y \cdot \cos\alpha_{P_{23}}^y \cdot \sin\beta_{a_3}^y \cdot d \cdot + F_{a_3}^z \cdot d \cdot \cos\alpha_{a_2a_3}^{l_3} \\ & F_P^x \cdot \cos\alpha_{P_{23}}^x \cdot \sin\beta_{P_{23}}^x \cdot \frac{d}{2} + F_P^y \cdot \cos\alpha_{P_{23}}^y \cdot \sin\beta_{P_{23}}^y \cdot \frac{d}{2} + F_P^z \cdot \frac{d}{2} \cdot \cos\alpha_{a_2a_3}^{l_3} \cdot \\ & = J_{a_2a_3} \cdot \omega_{a_2a_3}^{n_{23}} \end{aligned} \tag{2}$$

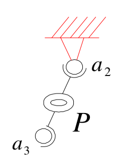

Figure 3. The moment balance analysis of moving platform branch a_2a_3 with a_2 as the fulcrum

In the formula (2), $F_{a_3}^x$, $F_{a_3}^y$, $F_{a_3}^z$ means the three stress components of the moving platform hinge point a_3 that are respectively along the x, y and z axis of the space fixed coordinate system. F_P^x, F_P^y, F_P^z means the three stress components of the moving platform center point P that are respectively along the x, y and z axis of the space fixed coordinate system. $\alpha_{P_{23}}^x$, $\alpha_{P_{23}}^y$ means the angle between the plane P_{23} and the x and y axis of the space fixed coordinate system. $\alpha_{a_2a_3}^{l_3}$ means the angle between the connecting rod l_3 (the z axis of the fixed coordinate system) and the moving platform branch a_2a_3. $\beta_{a_3}^x, \beta_{a_3}^y$ means the angle between the moving platform branch and the projection of $F_{a_3}^x$, $F_{a_3}^y$ in plane P_{23}. $\beta_{P_{23}}^x$, $\beta_{P_{23}}^y$ means the angle between the moving platform branch a_2a_3 and the projection of F_P^x, F_P^y in plane P_{23}. $J_{a_2a_3}$ means the moment of Inertia of the moving platform branch a_2a_3 around the point a_2 that parallel to the normal n_{23} of the plane P_{23}. $\omega_{a_2a_3}^{n_{23}}$ means the angular acceleration component of the moving platform branch a_2a_3 around hinge point a_2 that the normal n_{23} of the plane P_{23}.

Since the spatial position and pose of the output spindle of the mechanism is known, the Euler equation of the moving platform branch a_1a_4 with a_2 as the fulcrum around the output spindle axis n (as shown in Fig. 4) can be obtained, as follows.

$$\begin{aligned}&F_{a_1}^x\cdot\cos\alpha_{P_a}^x\cdot\sin\gamma_{a_1}^x\cdot\frac{d}{2}+F_{a_1}^y\cdot\cos\alpha_{P_a}^y\cdot\sin\gamma_{a_1}^y\cdot\frac{d}{2}+\\&F_{a_4}^x\cdot\cos\alpha_{P_a}^x\cdot\sin\gamma_{a_4}^x\cdot\frac{d}{2}+F_{a_4}^y\cdot\cos\alpha_{P_a}^y\cdot\sin\gamma_{a_4}^y\cdot\frac{d}{2}\\&=J_{a_1a_4}\cdot\omega_{a_1a_4}^n\end{aligned}. \tag{3}$$

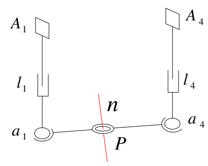

Figure 4. The moment balance analysis of moving platform branch a_1a_4 around the output spindle axis n

In the formula (3), $F_{a_1}^x$, $F_{a_1}^y$ means the two stress components of the moving platform hinge point a_1 that are respectively along the x and y axis of the space fixed coordinate system. $\alpha_{P_a}^x$, $\alpha_{P_a}^y$ means the angle between the x and y axis of the space fixed coordinate system and the plane P_a that include these moving platform hinge point a_1, a_2, a_3 and a_4. $\gamma_{a_1}^x$, $\gamma_{a_1}^y$ means the angle between the moving platform branch a_1a_4 and the projection of $F_{a_1}^x, F_{a_1}^y$ in plane P_a. $\gamma_{a_4}^x$, $\gamma_{a_4}^y$ means the angle between the moving platform branch a_1a_4 and the projection of $F_{a_4}^x$, $F_{a_4}^y$ in plane P_a. $J_{a_1a_4}$ means the moment of Inertia of the moving platform branch a_1a_4 around the moving platform center point P that parallel to the output spindle axis n. $\omega_{a_1a_4}^n$ means the angular acceleration component of the moving platform branch a_1a_4 around the moving platform center point P that parallel to the output spindle axis n.

According to the derivation process of formula (3), the Euler equation of the moving platform branch a_2a_3 with the moving platform center point P as the fulcrum around the output spindle axis n (as shown in Fig. 5) can be obtained, as follows,

$$\begin{aligned}&F_{a_2}^x\cdot\cos\alpha_{P_a}^x\cdot\sin\gamma_{a_2}^x\cdot\frac{d}{2}+F_{a_2}^y\cdot\cos\alpha_{P_a}^y\cdot\sin\gamma_{a_2}^y\cdot\frac{d}{2}+\\&F_{a_3}^x\cdot\cos\alpha_{P_a}^x\cdot\sin\gamma_{a_3}^x\cdot\frac{d}{2}+F_{a_3}^y\cdot\cos\alpha_{P_a}^y\cdot\sin\gamma_{a_3}^y\cdot\frac{d}{2}\\&=J_{a_2a_3}\cdot\omega_{a_2a_3}^n\end{aligned}. \tag{4}$$

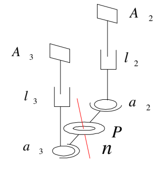

Figure 5. The moment balance analysis of moving platform branch a_2a_3 around the output spindle axis n

In the formula (4), $F_{a_2}^x$, $F_{a_2}^y$ means the two stress components of the moving platform hinge point a_2 that are respectively along the x and y axis of the space fixed coordinate system. $\gamma_{a_2}^x$, $\gamma_{a_2}^y$ means the angle between the moving platform branch a_2a_3 and the projection of $F_{a_2}^x$, $F_{a_2}^y$ in plane P_a. $\gamma_{a_3}^x$, $\gamma_{a_3}^y$ means the angle between the moving platform branch a_2a_3 and the projection of $F_{a_3}^y$, $F_{a_3}^x$ in plane P_a. $J_{a_2a_3}$ means the moment of Inertia of the moving platform branch a_2a_3 around the moving platform center point P that parallel to the output spindle axis n. $\omega_{a_2a_3}^n$ means the angular acceleration component of the moving platform branch a_2a_3 around the moving platform center point P that parallel to the output spindle axis n.

The output moving platform consists of branch a_1a_4 and a_2a_3 connected by rotating pair, the Euler equation with the hinge point a_1 and a_2 as the fulcrum around the axis a_1a_2 (as shown in Fig. 6) can be obtained, as follows,

$$\begin{aligned}
&(F_{a_3}^x \cdot \sin\alpha_{P_a}^x \cdot \sin\gamma_{a_3}^{a_1a_2} + F_{a_3}^y \cdot \sin\alpha_{P_a}^y \cdot \sin\beta_{a_3}^{a_1a_2} + F_{a_3}^z \cdot \sin\alpha_{P_a}^z \cdot \sin\xi_{a_3}^{a_1a_2}) \cdot d \cdot \sin\varphi_{a_2a_3}^{a_1a_2} + \\
&(F_{a_4}^x \cdot \sin\alpha_{P_a}^x \cdot \sin\gamma_{a_4}^{a_1a_2} + F_{a_4}^y \cdot \sin\alpha_{P_a}^y \cdot \sin\beta_{a_4}^{a_1a_2} + F_{a_4}^z \cdot \sin\alpha_{P_a}^z \cdot \sin\xi_{a_4}^{a_1a_2}) \cdot d \cdot \sin\varphi_{a_1a_4}^{a_1a_2} + \\
&(F_P^x \cdot \sin\alpha_{P_a}^x \cdot \sin\gamma_P^{a_1a_2} + F_P^y \cdot \sin\alpha_{P_a}^y \cdot \sin\beta_P^{a_1a_2} + F_P^z \cdot \sin\alpha_{P_a}^z \cdot \sin\xi_P^{a_1a_2}) \cdot \frac{d}{2} \cdot \sin\varphi_{a_1a_4}^{a_1a_2} + \\
&M_y \cdot \cos\phi_{a_1a_2}^y = J_a \cdot \omega_a^{a_1a_2}
\end{aligned} \quad (5)$$

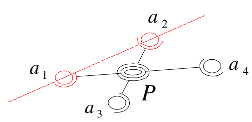

Figure 6. The moment balance analysis of moving platform with a_1 and a_2 as the fulcrum

In the formula (5), $F_{a_3}^z$, $F_{a_4}^z$, F_p^z means the stress components of the moving platform hinge point a_3, a_4 and center point P that are along the z axis of the space fixed coordinate system. $\alpha_{P_a}^z$ means the angle between the plane P_a and the z axis of the space fixed coordinate system. $\gamma_{a_3}^{a_1a_2}$, $\gamma_{a_4}^{a_1a_2}$, $\gamma_P^{a_1a_2}$ means the angle between the axis a_1a_2 and the projection of $F_{a_3}^x$, $F_{a_4}^x$, F_p^x in plane P_a. $\beta_{a_3}^{a_1a_2}$, $\beta_{a_3}^{a_1a_2}$, $\beta_{a_3}^{a_1a_2}$ means the angle between the axis a_1a_2 and the projection of $F_{a_3}^y$, $F_{a_4}^y$, F_p^y in plane P_a. $\xi_{a_3}^{a_1a_2}$, $\xi_{a_4}^{a_1a_2}$, $\xi_P^{a_1a_2}$ means the angle between the axis a_1a_2 and the projection of $F_{a_3}^z$, $F_{a_4}^z$, F_p^z in plane P_a. $\varphi_{a_2a_3}^{a_1a_2}$, $\varphi_{a_1a_4}^{a_1a_2}$ means the angle between the axis a_1a_2 and the moving platform branch a_1a_4 and a_2a_3. $\phi_{a_1a_2}^y$ means the angle between the axis a_1a_2 and the y axis of the space fixed coordinate system. M_y means the component of moment of couple M acting on the output moving platform around the y axis of the space fixed coordinate system. J_a means the moment of Inertia of the moving platform around the axis a_1a_2. $\omega_a^{a_1a_2}$ means the angular acceleration component of the moving platform around the axis a_1a_2.

Similarly, the Euler equation with the hinge point a_1 and a_3 as the fulcrum around the axis a_1a_3 (as shown in Fig. 7) can be obtained, as follows:

$$\begin{aligned}
&(F_{a_2}^x \cdot \sin\alpha_{P_a}^x \cdot \sin\gamma_{a_2}^{a_1a_3} + F_{a_2}^y \cdot \sin\alpha_{P_a}^y \cdot \sin\beta_{a_2}^{a_1a_3} + F_{a_2}^z \cdot \sin\alpha_{P_a}^z \cdot \sin\xi_{a_2}^{a_1a_3}) \cdot d \cdot \sin\varphi_{a_2a_3}^{a_1a_3} + \\
&(F_{a_4}^x \cdot \sin\alpha_{P_a}^x \cdot \sin\gamma_{a_4}^{a_1a_3} + F_{a_4}^y \cdot \sin\alpha_{P_a}^y \cdot \sin\beta_{a_4}^{a_1a_3} + F_{a_4}^z \cdot \sin\alpha_{P_a}^z \cdot \sin\xi_{a_4}^{a_1a_3}) \cdot d \cdot \sin\varphi_{a_1a_4}^{a_1a_3} + \\
&(F_P^x \cdot \sin\alpha_{P_a}^x \cdot \sin\gamma_P^{a_1a_3} + F_P^y \cdot \sin\alpha_{P_a}^y \cdot \sin\beta_P^{a_1a_3} + F_P^z \cdot \sin\alpha_{P_a}^z \cdot \sin\xi_P^{a_1a_3}) \cdot \frac{d}{2} \cdot \sin\varphi_{a_1a_4}^{a_1a_3} + \\
&M_x \cdot \cos\phi_{a_1a_3}^x = J_a \cdot \omega_a^{a_1a_3}
\end{aligned} \quad (6)$$

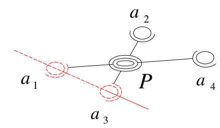

Figure 7. The moment balance analysis of moving platform with a_1 and a_3 as the fulcrum

In the formula (6), $\gamma_{a_2}^{a_1a_3}$, $\gamma_{a_4}^{a_1a_3}$, $\gamma_{P}^{a_1a_3}$ means the angle between the axis a_1a_3 and the projection of $F_{a_2}^{x}$, $F_{a_4}^{x}$, F_{p}^{x} in plane P_a. $\beta_{a_2}^{a_1a_3}$, $\beta_{a_4}^{a_1a_3}$, $\beta_{P}^{a_1a_3}$ means the angle between the axis a_1a_3 and the projection of $F_{a_2}^{y}$, $F_{a_4}^{y}$, F_{p}^{y} in plane P_a. $\xi_{a_2}^{a_1a_3}$, $\xi_{a_4}^{a_1a_3}$, $\xi_{P}^{a_1a_3}$ means the angle between the axis a_1a_3 and the projection of $F_{a_2}^{z}$, $F_{a_4}^{z}$, F_{p}^{z} in plane P_a. $\varphi_{a_2a_3}^{a_1a_3}$, $\varphi_{a_1a_4}^{a_1a_3}$ means the angle between the axis a_1a_3 and the moving platform branch a_1a_4 and a_2a_3. $\phi_{a_1a_2}^{x}$ means the angle between the axis a_1a_3 and the x axis of the space fixed coordinate system. M_x means the component of moment of couple M acting on the output moving platform around the x axis of the space fixed coordinate system. J_a means the moment of Inertia of the moving platform around the axis a_1a_3. $\omega_a^{a_1a_3}$ means the angular acceleration component of the moving platform around the axis a_1a_3.

Similarly, the Euler equation with the hinge point a_3 and a_4 as the fulcrum around the axis a_3a_4 (as shown in Fig. 8) can be obtained, as follows:

$$
\begin{aligned}
&(F_{a_1}^{x}\cdot\sin\alpha_{P_a}^{x}\cdot\sin\gamma_{a_1}^{a_3a_4}+F_{a_1}^{y}\cdot\sin\alpha_{P_a}^{y}\cdot\sin\beta_{a_1}^{a_3a_4}+F_{a_1}^{z}\cdot\sin\alpha_{P_a}^{z}\cdot\sin\xi_{a_1}^{a_3a_4})\cdot d\cdot\sin\varphi_{a_1a_4}^{a_3a_4}+\\
&(F_{a_2}^{x}\cdot\sin\alpha_{P_a}^{x}\cdot\sin\gamma_{a_2}^{a_3a_4}+F_{a_2}^{y}\cdot\sin\alpha_{P_a}^{y}\cdot\sin\beta_{a_2}^{a_3a_4}+F_{a_2}^{z}\cdot\sin\alpha_{P_a}^{z}\cdot\sin\xi_{a_2}^{a_1a_3})\cdot d\cdot\sin\varphi_{a_2a_3}^{a_3a_4}+.\\
&(F_{P}^{x}\cdot\sin\alpha_{P_a}^{x}\cdot\sin\gamma_{P}^{a_3a_4}+F_{P}^{y}\cdot\sin\alpha_{P_a}^{y}\cdot\sin\beta_{P}^{a_3a_4}+F_{P}^{z}\cdot\sin\alpha_{P_a}^{z}\cdot\sin\xi_{P}^{a_3a_4})\cdot\frac{d}{2}\cdot\sin\varphi_{a_1a_4}^{a_3a_4}+\\
&M_{y}\cdot\cos\phi_{a_3a_4}^{y}=J_a\cdot\omega_a^{a_3a_4}
\end{aligned}
\tag{7}
$$

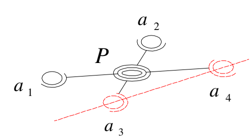

Figure 8. The moment balance analysis of moving platform with a_3 and a_4 as the fulcrum

In the formula (7), $\gamma_{a_1}^{a_3a_4}$, $\gamma_{a_2}^{a_3a_4}$, $\gamma_{P}^{a_3a_4}$ means the angle between the axis a_3a_4 and the projection of $F_{a_1}^{x}$, $F_{a_2}^{x}$, F_{p}^{x} in plane P_a. $\beta_{a_1}^{a_3a_4}$, $\beta_{a_2}^{a_3a_4}$, $\beta_{P}^{a_3a_4}$ means the angle between the axis a_3a_4 and the projection of $F_{a_1}^{y}$, $F_{a_2}^{y}$, F_{p}^{y} in plane P_a. $\xi_{a_1}^{a_3a_4}$, $\xi_{a_2}^{a_3a_4}$, $\xi_{P}^{a_3a_4}$ means the angle between the axis a_3a_4 and the projection of $F_{a_1}^{z}$, $F_{a_2}^{z}$, F_{p}^{z} in plane P_a. $\varphi_{a_1a_4}^{a_3a_4}$, $\varphi_{a_2a_3}^{a_3a_4}$ means the angle between the axis a_3a_4 and the moving platform branch a_1a_4 and a_2a_3. $\phi_{a_3a_4}^{y}$ means the angle between the axis a_3a_4 and the y axis of the space fixed coordinate system. J_a means the moment of Inertia of the moving platform around the axis a_3a_4. $\omega_a^{a_3a_4}$ means the angular acceleration component of the moving platform around the axis a_3a_4.

Similarly, the Euler equation with the hinge point a_2 and a_4 as the fulcrum around the axis a_2a_4 (as shown in Fig. 9) can be obtained, as follows:

$$
\begin{aligned}
&(F_{a_1}^{x}\cdot\sin\alpha_{P_a}^{x}\cdot\sin\gamma_{a_1}^{a_2a_4}+F_{a_1}^{y}\cdot\sin\alpha_{P_a}^{y}\cdot\sin\beta_{a_1}^{a_2a_4}+F_{a_1}^{z}\cdot\sin\alpha_{P_a}^{z}\cdot\sin\xi_{a_1}^{a_2a_4})\cdot d\cdot\sin\varphi_{a_1a_4}^{a_2a_4}+\\
&(F_{a_3}^{x}\cdot\sin\alpha_{P_a}^{x}\cdot\sin\gamma_{a_3}^{a_2a_4}+F_{a_3}^{y}\cdot\sin\alpha_{P_a}^{y}\cdot\sin\beta_{a_3}^{a_2a_4}+F_{a_3}^{z}\cdot\sin\alpha_{P_a}^{z}\cdot\sin\xi_{a_3}^{a_2a_4})\cdot d\cdot\sin\varphi_{a_2a_3}^{a_2a_4}+.\\
&(F_{P}^{x}\cdot\sin\alpha_{P_a}^{x}\cdot\sin\gamma_{P}^{a_2a_4}+F_{P}^{y}\cdot\sin\alpha_{P_a}^{y}\cdot\sin\beta_{P}^{a_2a_4}+F_{P}^{z}\cdot\sin\alpha_{P_a}^{z}\cdot\sin\xi_{P}^{a_2a_4})\cdot\frac{d}{2}\cdot\sin\varphi_{a_1a_4}^{a_2a_4}+\\
&M_{x}\cdot\cos\phi_{a_2a_4}^{x}=J_a\cdot\omega_a^{a_2a_4}
\end{aligned}
\tag{8}
$$

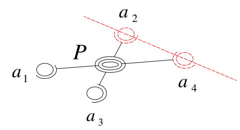

Figure 9. The moment balance analysis of moving platform with a_2 and a_4 as the fulcrum

In the formula (8), $\gamma_{a_1}^{a_2a_4}$, $\gamma_{a_3}^{a_2a_4}$, $\gamma_{P}^{a_2a_4}$ means the angle between the axis a_2a_4 and the projection of $F_{a_1}^{x}$, $F_{a_3}^{x}$,F_{p}^{x} in plane P_a. $\beta_{a_1}^{a_2a_4}$, $\beta_{a_3}^{a_2a_4}$, $\beta_{P}^{a_2a_4}$ means the angle between the axis a_2a_4 and the projection of $F_{a_1}^{y}$, $F_{a_3}^{y}$,F_{p}^{y} in plane P_a. $\xi_{a_1}^{a_2a_4}$, $\xi_{a_3}^{a_2a_4}$, $\xi_{P}^{a_2a_4}$ means the angle between the axis a_2a_4 and the projection of $F_{a_1}^{z}$, $F_{a_3}^{z}$,F_{p}^{z} in plane P_a. $\varphi_{a_2a_3}^{a_2a_4}$, $\varphi_{a_1a_4}^{a\ a_4}$ means the angle between the axis a_2a_4 and the moving platform branch a_1a_4 and a_2a_3. $\phi_{a_2a_4}^{x}$ means the angle between the axis a_2a_4 and the x axis of the space fixed coordinate system. J_a means the moment of Inertia of the moving platform around the axis a_2a_4. $\omega_{a}^{a_2a_4}$ means the angular acceleration component of the moving platform around the axis a_2a_4.

From the formula (1) to (8), we can see that there are 12 unknown variables $F_{a_i}^{x}$, $F_{a_i}^{y}$, $F_{a_i}^{z}$ $(i = 1,2,3,4)$ in these eight expressions, due to the special structure of the mechanism, the relationship between some variables is the following,

$$F_{a_1}^{z} = F_{a_2}^{z} = F_{a_3}^{z} = F_{a_4}^{z} = F_{P}^{z} / 4. \tag{9}$$

Thus it can be known that the formula (1) to (8) contains only 8 unknown variables, so formula (1) to (8) can be expressed in matrix form, as follows:

$$\mathbf{J} \cdot X = b. \tag{10}$$

The force Jacobi matrix J in formula (10) is a matrix composed of the coefficients of 8 unknown variables in formula (1) to (8), the X is a column vector composed of variables $[F_{a_1}^{x}\ F_{a_1}^{y}\ F_{a_2}^{x}\ F_{a_2}^{y}\ F_{a_3}^{x}\ F_{a_3}^{y}\ F_{a_4}^{x}\ F_{a_4}^{y}]^T$, the b is a column vector consisting of known items of formula (1) to (8) without 8 unknown variables. The unique definite solution can be obtained. When the space position and posture of mechanism are given, the stress F and moment of couple M of the output spindle are known, the Jacobi matrix J is nonsingular matrix.

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