### HowMuchData

0.9.1

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# **Contents**

1	Obs	ervatio	ns on Taleb's "How much data do you need"	1
2	Hier	archica	I Index	5
	2.1	Class	Hierarchy	5
3	Clas	ss Index		7
	3.1	Class	List	7
4	File	Index		9
	4.1	File Lis	st	9
5	Clas	ss Docu	mentation	11
	5.1	expone	ential_distribution< RealType > Struct Template Reference	11
		5.1.1	Detailed Description	12
	5.2	Kappa	Result Struct Reference	12
		5.2.1	Detailed Description	13
		5.2.2	Constructor & Destructor Documentation	14
			5.2.2.1 KappaResult()	14
		5.2.3	Member Function Documentation	14
			5.2.3.1 calc_kappa()	14
			5.2.3.2 initialize()	14
			5.2.3.3 update_conf_int()	14
			5.2.3.4 update_dev()	15
		5.2.4	Member Data Documentation	15
			5.2.4.1 mad relierr	15

ii CONTENTS

		5.2.4.2	ns	15
5.3	Kappa	Results St	ruct Reference	16
	5.3.1	Detailed	Description	16
	5.3.2	Construc	etor & Destructor Documentation	16
		5.3.2.1	KappaResults() [1/2]	16
		5.3.2.2	KappaResults() [2/2]	17
	5.3.3	Member	Data Documentation	17
		5.3.3.1	ns	17
		5.3.3.2	param_label	17
		5.3.3.3	taleb_offset	18
5.4	lognori	mal_distrib	oution< RealType > Struct Template Reference	18
	5.4.1	Detailed	Description	20
	5.4.2	Construc	tor & Destructor Documentation	20
		5.4.2.1	lognormal_distribution()	20
	5.4.3	Member	Function Documentation	20
		5.4.3.1	cdf()	20
		5.4.3.2	cf_fourier()	21
		5.4.3.3	cf_fourier_lnx()	21
		5.4.3.4	cf_fourier_mixed()	22
		5.4.3.5	cf_lambert_w()	22
		5.4.3.6	cf_series()	22
		5.4.3.7	cfprime_fourier()	23
		5.4.3.8	cfprime_fourier_lnx()	23
		5.4.3.9	cfprime_fourier_mixed()	24
		5.4.3.10	cfprime_lambert_w()	24
		5.4.3.11	cfprime_series()	24
		5.4.3.12	characteristic_function()	25
		5.4.3.13	characteristic_function_prime()	25
		5.4.3.14	mad2()	26
		5.4.3.15	max()	26

CONTENTS

		5.4.3.16 min()	26
		5.4.3.17 pdf()	26
		5.4.3.18 quantile()	26
5.5	norma	_switch_mean< RealType > Class Template Reference	27
	5.5.1	Detailed Description	28
	5.5.2	Constructor & Destructor Documentation	28
		5.5.2.1 normal_switch_mean()	28
	5.5.3	Member Function Documentation	28
		5.5.3.1 cdf()	28
		5.5.3.2 pdf()	28
		5.5.3.3 quantile()	29
5.6	norma	_switch_stddev< RealType > Class Template Reference	29
	5.6.1	Detailed Description	30
	5.6.2	Constructor & Destructor Documentation	30
		5.6.2.1 normal_switch_stddev()	30
	5.6.3	Member Function Documentation	31
		5.6.3.1 cdf()	31
		5.6.3.2 pdf()	31
		5.6.3.3 quantile()	31
5.7	Numpu	unct Struct Reference	32
	5.7.1	Detailed Description	32
5.8	pareto	_distribution< RealType >::param_type Class Reference	32
	5.8.1	Constructor & Destructor Documentation	33
		5.8.1.1 param_type()	33
	5.8.2	Member Function Documentation	33
		5.8.2.1 alpha()	33
		5.8.2.2 mu()	33
		5.8.2.3 sigma()	34
	5.8.3	Friends And Related Function Documentation	34
		5.8.3.1 operator"!=	34

iv CONTENTS

		5.8.3.2	operator<<	34
		5.8.3.3	operator==	34
		5.8.3.4	operator>>	35
5.9	pareto_	_distributio	on < RealType > Class Template Reference	35
	5.9.1	Detailed	Description	37
	5.9.2	Construc	tor & Destructor Documentation	37
		5.9.2.1	pareto_distribution()	37
	5.9.3	Member	Function Documentation	37
		5.9.3.1	alpha()	37
		5.9.3.2	max()	37
		5.9.3.3	min()	38
		5.9.3.4	mu()	38
		5.9.3.5	param() [1/2]	38
		5.9.3.6	param() [2/2]	38
		5.9.3.7	sigma()	38
	5.9.4	Friends A	And Related Function Documentation	38
		5.9.4.1	operator>>	39
5.10	Pinelis <sup>7</sup>	TalebInteg	rand< Dist > Class Template Reference	39
	5.10.1	Detailed	Description	39
	5.10.2	Construc	tor & Destructor Documentation	39
		5.10.2.1	PinelisTalebIntegrand()	39
5.11	Pinelis <sup>7</sup>	TalebInteg	rand< exponential_distribution<>> Class Template Reference	40
	5.11.1	Detailed	Description	40
	5.11.2	Construc	tor & Destructor Documentation	40
		5.11.2.1	PinelisTalebIntegrand()	40
5.12	Pinelis <sup>7</sup>	TalebInteg	rand< pareto_distribution<>> > Class Template Reference	41
	5.12.1	Detailed	Description	41
	5.12.2	Construc	tor & Destructor Documentation	41
		5.12.2.1	PinelisTalebIntegrand()	41
5.13	student	t_t_distribu	ution< RealType > Struct Template Reference	42
	5.13.1	Detailed	Description	43

CONTENTS

6	File	Docum	entation		45
	6.1	/Users	/jdunn/Dod	cuments/XCode/how_much_data/include/exponential_distribution.h File Reference	45
	6.2	/Users	/jdunn/Dod	cuments/XCode/how_much_data/include/lognormal_distribution.h File Reference .	45
	6.3	/Users	/jdunn/Doo	cuments/XCode/how_much_data/include/normal_switch_mean.h File Reference .	46
	6.4	/Users	/jdunn/Dod	cuments/XCode/how_much_data/include/normal_switch_stddev.h File Reference .	46
	6.5	/Users	/jdunn/Dod	cuments/XCode/how_much_data/include/pareto_distribution.h File Reference	47
	6.6	/Users	/jdunn/Dod	cuments/XCode/how_much_data/include/student_t_distribution.h File Reference .	47
	6.7		•	cuments/XCode/how_much_data/monte_carlo_test/monte_carlo_test.cpp File Ref-	47
		6.7.1	Function	Documentation	49
			6.7.1.1	calc_kappa()	49
			6.7.1.2	calculate_kappa()	49
			6.7.1.3	operator<<()	50
			6.7.1.4	quantile()	50
			6.7.1.5	rel_err()	51
	6.8			cuments/XCode/how_much_data/pinelis_taleb_test/pinelis_taleb_test.cpp File Ref-	51
		6.8.1	Function	Documentation	52
			6.8.1.1	calculate_kappa()	52
			6.8.1.2	show_usage()	53
Inc	dex				55

### **Chapter 1**

# Observations on Taleb's "How much data do you need"

#### Introduction

Nassim Nicholas Taleb has recently released a paper entitled *How much data do you need? An operational metric for fat-tailedness*. to appear in the International Journal of Forecasting. The paper focuses on the preasymptotic behavior of the sum of independent identically distributed random variables that are governed by various fat-tailed distributions. He introduces a metric labeled "kappa" that's related to the growth rate of the mean absolute deviation (MAD) as the number of summands increases. Kappa is defined by the following formula.

kappa(n0,n) = 2 - (log(n)-log(n0))/(log(MAD(n) - MAD(n0))

Normally distributed variables have a kappa of zero. Thus the extent to which kappa is in excess of zero measures the "fat-tailedness".

#### **Purpose**

Taleb analyzes a number of distributions to estimate their kappa. He gives explicit formulae for kappa(1,2), which he was able to derive analytically, but he also includes tables of kappa(1,30) and kappa(1,100) for two distributions, namely the Pareto distribution and Student's t distribution. This package is my effort to replicate those tables and perhaps add others for the other distributions. It's a work in progress, as there are some items that I can't closely replicate. However, none of the differences are large enough to call into question Taleb's results.

#### What's included

This package contains two algorithms implemented in C++, one of which relies on monte carlo simulations, and a second one that takes advantage of an integral representation of the MAD given the derivative of characteristic function of the underlying distribution. I experimented with a third algorithm that used a discrete fourier transform, but it's been dropped from the package because it's much inferior to the integral representation.

The package includes several files developed for this project:

• monte\_carlo\_test.cpp. The main program to run the monte carlo test. It's bullet-proof but so slow that it's hard to obtain the required accuracy in a reasonable time frame even using multi-threading.

- pinelis\_taleb\_test.cpp. The main program implementing the integral representation of MAD. This is my current method of choice.
- pinelis taleb graph.R. An R script to graph the results of a run of pinelis taleb test.
- pareto\_distribution.h. Contains a class for the pareto distribution, modeled on the classes in boost::random and including items normally computed in boost::math::statistical distributions
- student\_t\_distribution.h. Similar to pareto but starts as a derived class from boost::random::student\_t\_←
  distribution
- exponential distribution.h. A derived class from boost::random::exponential distribution
- lognormal\_distribution.h. A derived class from boost::random::lognormal\_distribution. The class implementing the distribution includes several versions of the calculation of the characteristic function: some based on numerical integration from the definition, one based on a asymptotic series for small angular frequency and one based on a approximation using Lambert W functions.
- normal\_switch\_mean.h. A class implementing a 50/50 mixture of two normal distributions both with sigma=1, one of which has mean +d and the second of which has mean -d.
- normal\_switch\_stddev.h. A class implements a mixture of two normal distributions, the first of which has mean=0 and sigma=1 occurring with probability (1-p) and the second of which has mean=0 and sigma=a occurring with probability p.

In addition, i've included the apparatus I developed for adaptive integration, which seems to work much better that the Boost version that was originally used. The code is spread through the following files:

- · adaptive integration.h. The file that defines the interface to the adaptive integration routines.
- adaptive\_integration\_impl.h. Most of the actual implementation.
- gauss\_kronrad.h. Interface to the functions used to calculate the nodes and weights for the integration.
- gauss\_kronrod\_impl.h. The implementing code for gauss\_kronrod.
- myfloat.h. Some basic definitions allowing various types of multi-precision numbers. Multiprecision is not used in the current implementation of how\_much\_data.

Finally included is a test program.

• lognormal test.cpp. A program to test the various versions of the calculation of the characteristic function. .

In order to improve portability, a meson.build file is included, which allows an easy port to other systems once the needed packages are installed.

Update Feb. 15, 2019

I noticed that Taleb covered the same topic in N. N. Taleb, Statistical Consequences of Fat Tails. The procedure outlined in Section 6.5 of that monograph using results of Pinelis is of general applicability and is incorporated into pinelis\_taleb\_test.

#### **Further Documentation**

The code has been documented using the doxygen system. The result can be accessed in the doc subdirectory of the output directory.

#### **Observations**

So far my results are close to Taleb's. As Taleb mentions in his paper such distributions require huge amounts of data to produce reasonable estimates of the MAD and this fact is mirrored in the number of monte carlo runs or in the size the arrays used in the fast fourier transform. I'm pushing the limit of my computer's capability for the cases where alpha = 1.25 or alpha = 1.5. The convolution is limited by the size of available memory and the monte carlo approach is limited by the amount of time and the number of processors available. The pinelis\_taleb integral bypasses most of these problems, but it requires (1) a very accurate calculation of the characteristic function and its derivative, and (2) some ingenuity in choosing the right contour for integration.

I've experimented with other measures of scale, such as the 95% confidence interval spread, for which the amount of computation needed is much more modest, but these results may not be relevant if MAD is the measure which best characterizes the uncertainty.

#### Relationship with the Stable Distribution

Almost all of the distributions modeled have an associated stable distribution that is asymptotically equivalent to the modeled distribution. The kappa for stable distributions is always 2 - alpha. However, in most cases the MAD for the associated stable distribution is significantly different from the MAD for the modelled distribution and the divergence of kappa from 2 - alpha is associated with the rate of convergence of the MAD to the MAD of the associated stable distribution.

#### **Lognormal Distribution**

As Taleb mentions in his paper, the lognormal distribution is a well behaved thin tail distribution for small sigma, but becomes a fat-tailed distribution for sigmas much in excess of one. The weakest part of the calculation here is associated with the large sigma runs, These have three widely divergent scales associated with them. For instance, when mu=0 and sigma=5, the mode is about 1.4e-11, the mean and MAD are about 2.7e+5 and the stardard deviation is about 7.2e+10. The MAD for the normal distribution is sqrt(2/pi) times it's standard deviation and therefore the the MAD normallized by  $n^{\wedge}$ .5 must go from 2.7e+5 to 5.7e+10 to reach it's asymptotic state.

#### Acknowledgements

- 1. The package makes heavy use of the Boost C++ headers available at boost.
- 2. The routines to calculate the Gauss/Kronrod nodes and weights use the Eigen headers. These are available at Eigen.
- 3. One of the calculations of the characteristic function for the lognormal distribution uses Lambert W functions. I've used the C++ code for the complex Lambert W function from Istvan Mezo's web page.
- 4. The routines in the adaptive\_integration routine started out life as machine C++ translations of Fortran routines in QUADPACK, which is part of SLATEC and therefore in the public domain (http://en. 
  wikipedia.org/wiki/QUADPACK). The routines were then heavily modified to take advantage of the C++ language.
- 5. One of the modifications made to QUADPACK is the addition of the ability to calculate the nodes and weights for the Gauss Kronrod integration on the fly. For this purpose Dirk Laurie's method is used kronrod. ← ps. The routines used here are C++ translations of Dirk Laurie's MATLAB code, which is included in Walter Gautschi's OPQ suite OPQ.

#### License

The code included here is covered by the MIT license.

# Chapter 2

# **Hierarchical Index**

### 2.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

1
2
6
8
7
9
2
2
5
9
0
1
2

6 Hierarchical Index

# **Chapter 3**

# **Class Index**

### 3.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

exponential_distribution< Rearrype >	
Instances of struct exponential_distribution generate random variates for exponential distribution	11
KappaResult	
Struct holding the resutls of a single alpha	12
KappaResults	
Sturcture holding the results from all runs	16
lognormal_distribution < RealType >	
Class with functions related to the lognormal distribution	18
normal_switch_mean< RealType >	
Class whose instances represent a 50/50 mixture of normal distriubtions with sigma = 1 and	
means +d and -d	27
normal_switch_stddev< RealType >	
Class whose instances represent a mixture of normal distriubtions with mu=0 and stddev of 1 w	
prob	29
Numpunct	
Sturcture passed by imbue to ostreams to use commas in numbers	32
pareto_distribution< RealType >::param_type	32
pareto_distribution< RealType >	
Instantiations of class template pareto_distribution model a Pareto Type 2 distribution	35
PinelisTalebIntegrand	
Functor used in the integratoin of the Pinelis Taleb result	39
PinelisTalebIntegrand< exponential_distribution<>>>	
Functor used in the integration of the Pinelis Taleb result for exp. dist	40
PinelisTalebIntegrand< pareto_distribution<>>>	
Functor used in the integration of the Pinelis Taleb result for pareto dist	41
student_t_distribution < RealType >	
Instances of class student_t_distribution give random variates for a student t distribution with	
parameter n = alpha	42

8 Class Index

# Chapter 4

# File Index

### 4.1 File List

Here is a list of all documented files with brief descriptions:

/Users/jdunn/Documents/XCode/how_much_data/include/exponential_distribution.h	45
/Users/jdunn/Documents/XCode/how_much_data/include/lognormal_distribution.h	45
/Users/jdunn/Documents/XCode/how_much_data/include/normal_switch_mean.h	46
/Users/jdunn/Documents/XCode/how_much_data/include/normal_switch_stddev.h	46
/Users/jdunn/Documents/XCode/how_much_data/include/pareto_distribution.h	47
/Users/jdunn/Documents/XCode/how_much_data/include/student_t_distribution.h	47
/Users/jdunn/Documents/XCode/how_much_data/monte_carlo_test/monte_carlo_test.cpp	47
/Users/jdunn/Documents/XCode/how_much_data/pinelis_taleb_test/pinelis_taleb_test.cpp	51

10 File Index

### **Chapter 5**

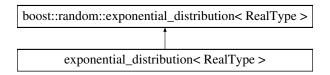
### **Class Documentation**

#### 5.1 exponential\_distribution < RealType > Struct Template Reference

instances of struct exponential\_distribution generate random variates for exponential distribution

```
#include <exponential_distribution.h>
```

Inheritance diagram for exponential\_distribution < RealType >:



#### **Public Member Functions**

- exponential\_distribution (RealType lambda)
  - constructor give lambda
- RealType cdf (RealType x, bool lower\_tail=true) const
  - return the cdf or the complement of the cdf
- RealType pdf (RealType x) const
  - return the pdf given x
- RealType quantile (RealType p) const
  - return the quantile give the probability p
- RealType lambda () const
  - return the lambda paramter of the distribution
- RealType alpha\_stable () const
  - return the alpha of the asymptotic stable distribution \*/
- RealType mean () const
  - Return mean of distribution.
- RealType mad () const
  - Return mean absolute deviation of the distribution.
- RealType mad2 () const
  - Return the mad of the square of the distribution.

• RealType ci (RealType level=RealType(.05)) const

Return the confidence interval of the distribution.

 complex < RealType > characteristic\_function (RealType omega) const return the characteristic function given real omega

 complex < RealType > characteristic\_function (complex < RealType > omega) const return the characteristic function given complex omega

 complex < RealType > characteristic\_function\_prime (RealType omega) const return the derivative of characteristic function given real omega

complex < RealType > characteristic\_function\_prime (complex < RealType > omega) const
return the derivative of characteristic function given complex omega

#### **Friends**

template < class charT, class traits >
 std::basic\_ostream < charT, traits > & operator << (std::basic\_ostream < charT, traits > &os, const
 exponential\_distribution &dist)

Write distribution to std::ostream.

#### 5.1.1 Detailed Description

```
template < class RealType = double > struct exponential_distribution < RealType >
```

instances of struct exponential\_distribution generate random variates for exponential distribution

Author

Created by Joseph Dunn on 1/3/19.

#### Copyright

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The documentation for this struct was generated from the following file:

• /Users/jdunn/Documents/XCode/how\_much\_data/include/exponential\_distribution.h

#### 5.2 KappaResult Struct Reference

the struct holding the resutls of a single alpha

#### **Public Member Functions**

- void initialize (double param\_in, const vector< int > &ns\_in)

initialize the structure

void update\_dev (size\_t m, size\_t m\_ci, const vector< double > &dev, const vector< double > &abs\_dev, const vector< double > > &x\_in)

update the deviations w result from one thread

void update\_conf\_int (double ci\_level)

calculate the confidence interval for all trials

KappaResult (vector< int > ns)

constructor

· void calc\_kappa ()

the kappa\_mad by duration

#### **Public Attributes**

· double param

the parameter for the run

· double mad rel err

the relative error of the mad vs theory

• size\_t m = 0

the number of trials run for mad

• size\_t m\_ci = 0

the number of trials for the conf. interval

vector< int > ns

the durations calculated

vector< double > sum\_dev

sum or raw deviation by duration

vector< double > sum\_abs\_dev

sum of abs deviations by duration

• vector< vector< double  $>> x_ci$ 

the results by trial and duration

double ci\_rel\_err

the relative error of the ci vs theory

vector< double > conf\_int

the calculated ci by duration

cpu\_times elapsed\_time

the elapsed time for the run

vector< double > mad

the mean absolute deviation by duration

vector< double > kappa\_mad

#### 5.2.1 Detailed Description

the struct holding the resutls of a single alpha

structure holding the results of a run for a single parameter value

#### 5.2.2 Constructor & Destructor Documentation

#### 5.2.2.1 KappaResult()

constructor

#### **Parameters**

ns the durations of the output

#### 5.2.3 Member Function Documentation

#### 5.2.3.1 calc\_kappa()

```
void KappaResult::calc_kappa ( ) [inline]
```

the kappa\_mad by duration

calculate the kappa from the mad and ci variables

#### 5.2.3.2 initialize()

initialize the structure

#### **Parameters**

param← _in	the alpha of the run
ns_in	the durations of the run

#### 5.2.3.3 update\_conf\_int()

calculate the confidence interval for all trials

#### **Parameters**

ci_level	the confidence level to use
----------	-----------------------------

#### 5.2.3.4 update\_dev()

update the deviations w result from one thread

#### **Parameters**

m	the number of trials for mad
m_ci	the number of trials for ci
dev	sum of deviation by duration
abs_dev	sum of abs dev by duration
x_in	the variate by trial

#### 5.2.4 Member Data Documentation

```
5.2.4.1 mad_rel_err
```

```
double KappaResult::mad_rel_err
```

the relative error of the mad vs theory

the relative error of mad vs theory

#### 5.2.4.2 ns

```
vector< int > KappaResult::ns
```

the durations calculated

the durations saved

The documentation for this struct was generated from the following files:

- /Users/jdunn/Documents/XCode/how\_much\_data/monte\_carlo\_test/monte\_carlo\_test.cpp
- /Users/jdunn/Documents/XCode/how\_much\_data/pinelis\_taleb\_test/pinelis\_taleb\_test.cpp

#### 5.3 KappaResults Struct Reference

sturcture holding the results from all runs

#### **Public Member Functions**

KappaResults (const vector< int > &ns, const vector< double > &params, const string param\_label, size\_t taleb\_offset)

constructor

- KappaResults (const vector < int > &ns, size\_t n\_params, string param\_label, size\_t taleb\_offset)
   constructor
- KappaResult & at (size\_t i)

return reference to a particular result

void dump\_results (ostream &os, string dist\_name)

#### **Public Attributes**

vector< int > ns

the durations calculated

string param label

a vector holding the results of each run the label for the parameter

vector< KappaResult > kr

the results of the runs by duration

· size\_t taleb\_offset

the offset into Taleb's table of results. =0 for none available.

mutex kr\_mutex

a mutex for writing results

#### 5.3.1 Detailed Description

sturcture holding the results from all runs

structure holding the results of all parameter values

#### 5.3.2 Constructor & Destructor Documentation

#### 5.3.2.1 KappaResults() [1/2]

constructor

#### **Parameters**

ns	the durations calculated	
params	the params for each run	
param_label	the label for the param	
taleb_offset	the column offset into the table of taleg's results. 0 for none	

#### **5.3.2.2 KappaResults()** [2/2]

#### constructor

#### **Parameters**

ns	the durations	
n_params	the # of params	
param_label the param_label		
taleb_offset	the column offset into Talebs table	

#### 5.3.3 Member Data Documentation

#### 5.3.3.1 ns

```
vector< int > KappaResults::ns
```

the durations calculated

the durations saved

#### 5.3.3.2 param\_label

```
string KappaResults::param_label
```

a vector holding the results of each run the label for the parameter

the name of the parameter

#### 5.3.3.3 taleb\_offset

```
size_t KappaResults::taleb_offset
```

the offset into Taleb's table of results. =0 for none available.

the column offset into Taleb's table

The documentation for this struct was generated from the following files:

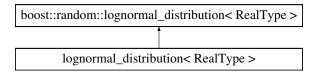
- /Users/jdunn/Documents/XCode/how\_much\_data/monte\_carlo\_test/monte\_carlo\_test.cpp
- /Users/jdunn/Documents/XCode/how\_much\_data/pinelis\_taleb\_test/pinelis\_taleb\_test.cpp

#### 5.4 lognormal\_distribution < RealType > Struct Template Reference

a class with functions related to the lognormal distribution.

```
#include <lognormal_distribution.h>
```

Inheritance diagram for lognormal\_distribution < RealType >:



#### **Public Member Functions**

• lognormal\_distribution (RealType mu, RealType sigma, IntegrationController< RealType > &cf\_ctl, int type=1)

the constructor for the distribution

 $\bullet \quad {\sf template}{<} {\sf typename} \; {\sf Engine} >$ 

RealType operator() (Engine &eng)

return a random number from the normalized distribution

RealType cdf (RealType x, bool lower\_tail=true) const

return the cumulative distribution function

RealType pdf (RealType x) const

return the probability density function

• RealType quantile (RealType p) const

return the quantile corresponding to a given propability

• RealType mu () const

return the mu parameter of the distribution

• RealType sigma () const

return sigma parameter of the distribution

• RealType alpha\_stable () const

return the alpha parameter of the asymptotically equivalent stable distribution

• RealType min () const

Returns the smallest value that the distribution can produce.

RealType max () const

Returns the largest value that the distribution can produce.

• RealType mean () const

Return mean of distribution.

RealType mad () const

Return mean absolute deviation of the distribution.

• RealType mad2 () const

Return the mad of the square of the distribution.

• RealType ci (RealType level=RealType(.05)) const

Return the confidence interval of the distribution.

complex< RealType > cf\_series (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1 ←
 \_norm=nullptr, int \*\_neval=nullptr)

cf via asymptotic series for small omega

complex< RealType > cfprime\_series (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1\_norm=nullptr, int \*\_neval=nullptr)

derivative of cf via asymptotic series for small omega

complex< RealType > cf\_lambert\_w (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1\_norm=nullptr, int \*\_neval=nullptr)

Much faster than the fourier integrals but has problems when sigma > 1.

complex< RealType > cfprime\_lambert\_w (complex< RealType > omega, RealType \*\_error=nullptr, Real
 —
 Type \*\_l1\_norm=nullptr, int \*\_neval=nullptr)

return the approximate derivative of char.

complex< RealType > cf\_fourier (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1 ←
 \_norm=nullptr, int \*\_neval=nullptr)

Return the characteristic function along a designer countour.

complex< RealType > cfprime\_fourier (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1\_norm=nullptr, int \*\_neval=nullptr)

Return the derivative of the char. fun. wrt omega using designer contour.

complex< RealType > cf\_fourier\_lnx (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1\_norm=nullptr, int \*\_neval=nullptr)

Return the characteristic function via Fourier integral in In(x) domain.

complex< RealType > cfprime\_fourier\_Inx (complex< RealType > omega, RealType \*\_error=nullptr, Real
 —
 Type \*\_I1\_norm=nullptr, int \*\_neval=nullptr)

Return the derivative of char. function via Fourier integral in ln(x) domain.

complex< RealType > cf\_fourier\_mixed (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1\_norm=nullptr, int \*\_neval=nullptr)

Return the char. fun. using mixed Lambert and shifted contour.

• complex< RealType > cfprime\_fourier\_mixed (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1\_norm=nullptr, int \*\_neval=nullptr)

The derivative of the char. fun.

• complex< RealType > characteristic\_function (complex< RealType > omega, RealType \*\_error=nullptr, RealType \*\_l1\_norm=nullptr, int \*\_neval=nullptr)

return the approximate characteristic function

• complex< RealType > characteristic\_function\_prime (complex< RealType > omega, RealType \*\_← error=nullptr, RealType \*\_I1\_norm=nullptr, int \*\_neval=nullptr)

return the derivative of the characteristic function

#### **Static Public Member Functions**

static void print\_fourier\_integrand (ostream &os, int n, complex < RealType > omega, RealType sigma)
 print integrand of CFIntegrand

#### **Friends**

template < class charT, class traits >
 std::basic\_ostream < charT, traits > & operator << (std::basic\_ostream < charT, traits > &os, const lognormal\_distribution &dist)

Write distribution to std::ostream.

#### 5.4.1 Detailed Description

```
template < class RealType = double > struct lognormal_distribution < RealType >
```

a class with functions related to the lognormal distribution.

Describes a random variable distributed as  $exp(sigma \ X + mu)$  adjusted to zero mean where X is normally distributed.

#### 5.4.2 Constructor & Destructor Documentation

#### 5.4.2.1 lognormal\_distribution()

the constructor for the distribution

#### **Parameters**

in	ти	the mu parameter	
in	sigma	the sigma parameter	
	cf_ctl	[in] reference controller	
in	type	type of cf calculaiton 1 ln(x), 2 bespoke, 3 w, 4 mixed	

#### 5.4.3 Member Function Documentation

#### 5.4.3.1 cdf()

```
template<class RealType = double>
RealType lognormal_distribution< RealType >::cdf (
```

```
RealType x,
bool lower_tail = true ) const [inline]
```

return the cumulative distribution function

#### **Parameters**

in	X	the quantile variable
in	lower_tail	flag indicating which tail to use

#### 5.4.3.2 cf\_fourier()

Return the characteristic function along a designer countour.

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
out	_neval	the # of evaluations

#### 5.4.3.3 cf\_fourier\_lnx()

Return the characteristic function via Fourier integral in ln(x) domain.

#### **Parameters**

	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the
out	neval	the # of evaluations

#### 5.4.3.4 cf\_fourier\_mixed()

Return the char. fun. using mixed Lambert and shifted contour.

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
out	_neval	the # of evaluations

#### 5.4.3.5 cf\_lambert\_w()

Much faster than the fourier integrals but has problems when sigma > 1.

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
out	_neval	the # of evaluations

#### 5.4.3.6 cf\_series()

```
RealType * _11_norm = nullptr,
int * _neval = nullptr ) [inline]
```

cf via asymptotic series for small omega

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
out	_neval	the # of evaluations

#### 5.4.3.7 cfprime\_fourier()

Return the derivative of the char. fun. wrt omega using designer contour.

#### **Parameters**

	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the
	_neval	the number of evaluations

#### 5.4.3.8 cfprime\_fourier\_lnx()

Return the derivative of char. function via Fourier integral in ln(x) domain.

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
out	neval	the # of evaluations

#### 5.4.3.9 cfprime\_fourier\_mixed()

The derivative of the char. fun.

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
out	_neval	the # of evaluations

#### 5.4.3.10 cfprime\_lambert\_w()

return the approximate derivative of char.

function using Lambert W function Much faster than the fourier integrals but has problems when sigma > 1

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
out	_neval	the # of evaluations

#### 5.4.3.11 cfprime\_series()

```
RealType * _error = nullptr,
RealType * _l1_norm = nullptr,
int * _neval = nullptr ) [inline]
```

derivative of cf via asymptotic series for small omega

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
out	_neval	the # of evaluations

#### 5.4.3.12 characteristic\_function()

return the approximate characteristic function

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the I1 norm
	_neval	{out} the # of evaluations

#### 5.4.3.13 characteristic\_function\_prime()

return the derivative of the characteristic function

#### **Parameters**

in	omega	the angular frequency
out	_error	the integration error
out	_l1_norm	the
out	_neval	the # of evaluations

Generated by Doxygen

#### 5.4.3.14 mad2()

```
template<class RealType = double>
RealType lognormal_distribution< RealType >::mad2 ( ) const [inline]
```

Return the mad of the square of the distribution.

this is the approximation used by Taleb

#### 5.4.3.15 max()

```
template<class RealType = double>
RealType lognormal_distribution< RealType >::max ( ) const [inline]
```

Returns the largest value that the distribution can produce.

#### 5.4.3.16 min()

```
template<class RealType = double>
RealType lognormal_distribution< RealType >::min ( ) const [inline]
```

Returns the smallest value that the distribution can produce.

#### 5.4.3.17 pdf()

return the probability density function

#### **Parameters**

```
x the quantile variable
```

#### 5.4.3.18 quantile()

return the quantile corresponding to a given propability

#### **Parameters**

p the target probability

The documentation for this struct was generated from the following file:

· /Users/jdunn/Documents/XCode/how much data/include/lognormal distribution.h

#### 5.5 normal\_switch\_mean < RealType > Class Template Reference

class whose instances represent a 50/50 mixture of normal distriubtions with sigma = 1 and means +d and -d

```
#include <normal_switch_mean.h>
```

#### **Public Member Functions**

- normal\_switch\_mean (RealType d)
- template<typename Engine >

RealType operator() (Engine &eng)

return a random number from the normalized distribution

- RealType min () const
- RealType max () const
- RealType d () const

return the half difference betwee the means

RealType cdf (RealType x, bool lower\_tail=true) const

return the cdf

• RealType pdf (RealType x) const

return the pdf

• RealType quantile (RealType p, bool lower\_tail=true) const

return the quantile given the probability p

• RealType alpha stable () const

return the alpha parameter of the asymptotically equivalent stable distribution

complex < RealType > characteristic\_function (RealType omega) const

return the char. fun. for real arguments

complex < RealType > characteristic\_function\_prime (RealType omega) const

return the derivative of the char. fun. for real arguments

- · RealType mean () const
- RealType mad () const
- · RealType mad2 () const
- RealType ci (RealType level=RealType(.05)) const

Return the confidence interval of the distribution.

#### **Friends**

ostream & operator << (ostream &os, normal\_switch\_mean &dist)</li>
 Write distribution to std::ostream.

#### 5.5.1 Detailed Description

```
template < typename RealType = double > class normal_switch_mean < RealType >
```

class whose instances represent a 50/50 mixture of normal distriubtions with sigma = 1 and means +d and -d

#### 5.5.2 Constructor & Destructor Documentation

#### 5.5.2.1 normal\_switch\_mean()

#### **Parameters**

#### 5.5.3 Member Function Documentation

#### 5.5.3.1 cdf()

#### return the cdf

#### **Parameters**

in	X	the desired quantile
in	lower_tail	flag for tail

#### 5.5.3.2 pdf()

return the pdf

#### **Parameters**

in )	(	the desired quantile
------	---	----------------------

#### 5.5.3.3 quantile()

return the quantile given the probability p

#### **Parameters**

in	р	the disired probability
in	lower_tail	flag for tail

The documentation for this class was generated from the following file:

• /Users/jdunn/Documents/XCode/how\_much\_data/include/normal\_switch\_mean.h

## 5.6 normal\_switch\_stddev < RealType > Class Template Reference

class whose instances represent a mixture of normal distributions with mu=0 and stddev of 1 w prob.

```
#include <normal_switch_stddev.h>
```

#### **Public Member Functions**

- normal\_switch\_stddev (RealType a, RealType p)
- template<typename Engine >
   RealType operator() (Engine &eng)

return a random number from the normalized distribution

- · RealType min () const
- RealType max () const
- RealType a () const

return the second stddev

RealType p () const

return the prob. of a switch to a

• RealType cdf (RealType x, bool lower tail=true) const

return the cdf

• RealType pdf (RealType x) const

return the pdf

• RealType quantile (RealType pp, bool lower\_tail=true) const

return the quantile given the probability p

• RealType alpha\_stable () const

return the alpha parameter of the asymptotically equivalent stable distribution

 complex < RealType > characteristic\_function (RealType omega) const return the char. fun. for real arguments

• complex < RealType > characteristic\_function\_prime (RealType omega) const return the derivative of the char. fun. for real arguments

- RealType mean () const
- RealType mad () const
- RealType mad2 () const
- RealType ci (RealType level=RealType(.05)) const

Return the confidence interval of the distribution.

#### **Friends**

ostream & operator<< (ostream &os, normal\_switch\_stddev &dist)</li>
 Write distribution to std::ostream.

## 5.6.1 Detailed Description

```
template < typename RealType = double > class normal_switch_stddev < RealType >
```

class whose instances represent a mixture of normal distriubtions with mu=0 and stddev of 1 w prob.

(1-p) and a with prob. p

#### 5.6.2 Constructor & Destructor Documentation

#### 5.6.2.1 normal\_switch\_stddev()

#### **Parameters**

in	а	the means are +- d from 0
in	p	the prob. of sigma=a

## 5.6.3 Member Function Documentation

#### 5.6.3.1 cdf()

#### return the cdf

#### **Parameters**

in	X	the desired quantile
in	lower_tail	flag for tail

#### 5.6.3.2 pdf()

## return the pdf

#### **Parameters**

in X	the desired quantile
------	----------------------

#### 5.6.3.3 quantile()

## return the quantile given the probability p

## **Parameters**

in	pp	the disired probability
in	lower_tail	flag for tail

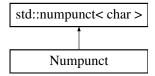
The documentation for this class was generated from the following file:

• /Users/jdunn/Documents/XCode/how\_much\_data/include/normal\_switch\_stddev.h

## 5.7 Numpunct Struct Reference

sturcture passed by imbue to ostreams to use commas in numbers

Inheritance diagram for Numpunct:



#### **Protected Member Functions**

- virtual char do\_thousands\_sep () const
- virtual std::string do\_grouping () const

#### 5.7.1 Detailed Description

sturcture passed by imbue to ostreams to use commas in numbers

The documentation for this struct was generated from the following file:

• /Users/jdunn/Documents/XCode/how\_much\_data/pinelis\_taleb\_test/pinelis\_taleb\_test.cpp

## 5.8 pareto\_distribution < RealType >::param\_type Class Reference

#### **Public Types**

typedef pareto\_distribution distribution\_type

#### **Public Member Functions**

- param\_type (RealType alpha\_arg, RealType mu\_arg=RealType(0.0), RealType sigma\_arg=RealType(1.0))

  Constructs the parameters of a pareto distribution.
- RealType alpha () const

Returns the "alpha" parameter of the distribution.

• RealType mu () const

Returns the "mu" parameter of the distribution.

• RealType sigma () const

Returns the "sigma" parameter of the distribution.

#### **Friends**

template < class charT, class traits >
 std::basic\_ostream < charT, traits > & operator << (std::basic\_ostream < charT, traits > &os, const
 param\_type &parm)

Writes the parameters to a std::ostream.

template < class charT , class traits >
 std::basic\_istream < charT, traits > & operator >> (std::basic\_istream < charT, traits > &is, param\_type
 &parm)

Reads the parameters from a std::istream.

• bool operator== (const param\_type &lhs, const param\_type &rhs)

Returns true if the two sets of parameters are equal.

bool operator!= (const param\_type &lhs, const param\_type &rhs)

Returns true if the two sets of parameters are different.

#### 5.8.1 Constructor & Destructor Documentation

#### 5.8.1.1 param\_type()

Constructs the parameters of a pareto\_distribution.

## 5.8.2 Member Function Documentation

### 5.8.2.1 alpha()

```
template<class RealType = double>
RealType pareto_distribution< RealType >::param_type::alpha ( ) const [inline]
```

Returns the "alpha" parameter of the distribution.

#### 5.8.2.2 mu()

```
template<class RealType = double>
RealType pareto_distribution< RealType >::param_type::mu ( ) const [inline]
```

Returns the "mu" parameter of the distribution.

#### 5.8.2.3 sigma()

```
template<class RealType = double>
RealType pareto_distribution< RealType >::param_type::sigma ( ) const [inline]
```

Returns the "sigma" parameter of the distribution.

#### 5.8.3 Friends And Related Function Documentation

#### 5.8.3.1 operator"!=

Returns true if the two sets of parameters are different.

#### 5.8.3.2 operator <<

Writes the parameters to a std::ostream.

## 5.8.3.3 operator==

Returns true if the two sets of parameters are equal.

#### 5.8.3.4 operator>>

Reads the parameters from a std::istream.

The documentation for this class was generated from the following file:

• /Users/jdunn/Documents/XCode/how much data/include/pareto distribution.h

## 5.9 pareto\_distribution < RealType > Class Template Reference

Instantiations of class template pareto\_distribution model a Pareto Type 2 distribution.

```
#include <pareto_distribution.h>
```

#### **Classes**

· class param\_type

#### **Public Types**

typedef RealType result\_type

#### **Public Member Functions**

• pareto\_distribution (RealType alpha\_arg, RealType mu\_arg=RealType(0.0), RealType sigma\_arg=Real ← Type(1.0))

Constructs a pareto\_distribution.

pareto\_distribution (const param\_type &parm)

Constructs a pareto distribution from its parameters.

• RealType alpha () const

Returns the alpha parameter of the distribution.

• RealType mu () const

Returns the mu parameter of the distribution.

• RealType sigma () const

Returns the sigma parameter of the distribution.

• RealType alpha stable () const

Return the alpha of the asymptotic stable distribution.

RealType min () const

Returns the smallest value that the distribution can produce.

• RealType max () const

Returns the largest value that the distribution can produce.

param\_type param () const

Returns the parameters of the distribution.

void param (const param\_type &parm)

Sets the parameters of the distribution.

RealType cdf (RealType x, bool lower\_tail=true) const

the cdf of the distribution

RealType pdf (RealType x) const

return the pdf of the distribution

• RealType quantile (RealType p) const

the quantile for a given probability

• RealType mean () const

Return the mean of the distribution.

RealType mad () const

Return the MAD of the distribution.

• RealType mad2 () const

Return the MAD of the square of the distribution.

complex< RealType > characteristic\_function (RealType omega) const

return the characteristic function of the distribution

complex< RealType > characteristic\_function (complex< RealType > omega) const

return the characteristic function of the distribution given complex argument

complex < RealType > characteristic\_function\_prime (RealType omega) const

return the derivative of th characteristic function of the distribution

complex < RealType > characteristic\_function\_prime (complex < RealType > omega) const

return the derivative of th char. fnct. of the dist. given complex arg.

RealType ci (RealType level=RealType(.05)) const

Return the 95% confidence interval.

· void reset ()

Effects: Subsequent uses of the distribution do not depend on values produced by any engine prior to invoking reset.

template < class Engine >

result\_type operator() (Engine &eng) const

Returns a random variate distributed according to the Pareto distribution.

template < class Engine >

result\_type operator() (Engine &eng, const param\_type &parm)

Returns a random variate distributed according to the Pareto distribution with parameters specified by param.

#### **Friends**

template < class charT, class traits >
 std::basic\_ostream < charT, traits > & operator << (std::basic\_ostream < charT, traits > &os, const pareto\_distribution & dist)

Write distribution to std::ostream.

template < class charT, class traits >
 std::basic\_istream < charT, traits > & operator >> (std::basic\_istream < charT, traits > &is, pareto\_distribution
 &dist)

Reads the parameters from a std::istream.

bool operator== (const pareto\_distribution &lhs, const pareto\_distribution &rhs)

Returns true if the two distributions will produce identical sequences of values given equal generators.

bool operator!= (const pareto\_distribution &lhs, const pareto\_distribution &rhs)

Returns true if the two distributions may produce different sequences of values given equal generators.

## 5.9.1 Detailed Description

```
template<class RealType = double>
class pareto_distribution< RealType >
```

Instantiations of class template pareto\_distribution model a Pareto Type 2 distribution.

Such a distribution produces random numbers with  $1 - F(x) = (1 + \frac{x - \mu}{\sigma})^{-\alpha}$  for  $x > \mu$ .

#### 5.9.2 Constructor & Destructor Documentation

#### 5.9.2.1 pareto\_distribution()

Constructs a pareto\_distribution.

alpha mu and sigma are the parameters of the distribution.

#### 5.9.3 Member Function Documentation

#### 5.9.3.1 alpha()

```
template<class RealType = double>
RealType pareto_distribution< RealType >::alpha ( ) const [inline]
```

Returns the alpha parameter of the distribution.

## 5.9.3.2 max()

```
template<class RealType = double>
RealType pareto_distribution< RealType >::max ( ) const [inline]
```

Returns the largest value that the distribution can produce.

```
5.9.3.3 min()
```

```
template<class RealType = double>
RealType pareto_distribution< RealType >::min ( ) const [inline]
```

Returns the smallest value that the distribution can produce.

```
5.9.3.4 mu()
```

```
template<class RealType = double>
RealType pareto_distribution< RealType >::mu ( ) const [inline]
```

Returns the mu parameter of the distribution.

```
5.9.3.5 param() [1/2]
```

```
template<class RealType = double>
param_type pareto_distribution< RealType >::param ( ) const [inline]
```

Returns the parameters of the distribution.

```
5.9.3.6 param() [2/2]
```

Sets the parameters of the distribution.

```
5.9.3.7 sigma()
```

```
template<class RealType = double>
RealType pareto_distribution< RealType >::sigma ( ) const [inline]
```

Returns the sigma parameter of the distribution.

#### 5.9.4 Friends And Related Function Documentation

#### 5.9.4.1 operator>>

Reads the parameters from a std::istream.

The documentation for this class was generated from the following file:

• /Users/jdunn/Documents/XCode/how much data/include/pareto distribution.h

## 5.10 PinelisTalebIntegrand < Dist > Class Template Reference

Functor used in the integratoin of the Pinelis Taleb result.

#### **Public Member Functions**

- PinelisTalebIntegrand (Dist &dist, const int n)
   construct the integrand
- double operator() (double x)

return the value of the Pinelis Taleb integrand

void operator() (std::vector< double > &xs)

#### 5.10.1 Detailed Description

```
template<typename Dist>
class PinelisTalebIntegrand< Dist>
```

Functor used in the integratoin of the Pinelis Taleb result.

#### 5.10.2 Constructor & Destructor Documentation

#### 5.10.2.1 PinelisTalebIntegrand()

construct the integrand

#### **Parameters**

in	dist	the distribution
in	n	the duration

The documentation for this class was generated from the following file:

/Users/jdunn/Documents/XCode/how much data/pinelis taleb test/pinelis taleb test.cpp

## 5.11 PinelisTalebIntegrand < exponential\_distribution <> > Class Template Reference

Functor used in the integration of the Pinelis Taleb result for exp. dist.

#### **Public Member Functions**

- PinelisTalebIntegrand (const exponential\_distribution<> &dist, const int n)
   construct the integrand
- double operator() (double x) const return the value of the Pinelis Taleb integrand for exp distribution Uses an adjusted contour of integration
- void operator() (std::vector< double > &xs) const

#### 5.11.1 Detailed Description

```
\label{lem:lemplate} \mbox{template} <> \\ \mbox{class PinelisTalebIntegrand} < \mbox{exponential\_distribution} <>> > \\
```

Functor used in the integration of the Pinelis Taleb result for exp. dist.

## 5.11.2 Constructor & Destructor Documentation

#### 5.11.2.1 PinelisTalebIntegrand()

construct the integrand

#### **Parameters**

in	dist	the distribution
in	n	the duration

The documentation for this class was generated from the following file:

• /Users/jdunn/Documents/XCode/how\_much\_data/pinelis\_taleb\_test/pinelis\_taleb\_test.cpp

## 5.12 PinelisTalebIntegrand < pareto\_distribution <> > Class Template Reference

Functor used in the integration of the Pinelis Taleb result for pareto dist.

#### **Public Member Functions**

- PinelisTalebIntegrand (pareto\_distribution<> &dist, const int n)
   construct the integrand
- double operator() (double x) const
   return the value of the Pinelis Taleb integrand for pareto distribution Uses an adjusted contour of integration
- void operator() (std::vector< double > &xs) const

#### 5.12.1 Detailed Description

```
\label{lem:lemplate} \mbox{template} <> \\ \mbox{class PinelisTalebIntegrand} < \mbox{pareto\_distribution} <>> > \\
```

Functor used in the integration of the Pinelis Taleb result for pareto dist.

#### 5.12.2 Constructor & Destructor Documentation

## 5.12.2.1 PinelisTalebIntegrand()

construct the integrand

#### **Parameters**

in	dist	the distribution
in	n	the duration

The documentation for this class was generated from the following file:

• /Users/jdunn/Documents/XCode/how\_much\_data/pinelis\_taleb\_test/pinelis\_taleb\_test.cpp

## 5.13 student\_t\_distribution < RealType > Struct Template Reference

Instances of class student t\_distribution give random variates for a student t distribution with parameter n = alpha.

```
#include <student_t_distribution.h>
```

 $Inheritance\ diagram\ for\ student\_t\_distribution < RealType >:$ 

```
boost::random::student_t_distribution< RealType >

student_t_distribution< RealType >
```

#### **Public Member Functions**

• student\_t\_distribution (RealType alpha)

construct an instance give alpha

RealType cdf (RealType x, bool lower\_tail=true) const

return the cdf or the complement of the cdf

RealType pdf (RealType x) const

return the probability density function at x

• RealType quantile (RealType p) const

return the quantile for probability p

RealType alpha () const

return the alpha = n of the distribution

• RealType alpha stable () const

return the alpha of the asymptotic stable distribution

• RealType mean () const

Return mean of distribution.

• RealType mad () const

Return mean absolute deviation of the distribution.

• RealType mad2 () const

Return the mad of the square of the distribution.

• RealType ci (RealType level=RealType(.05)) const

Return the confidence interval of the distribution.

RealType characteristic\_function (RealType omega) const

return the characteristic function of the distribution at omega

RealType characteristic\_function\_prime (RealType omega) const

return the derivative of the characteristic function at omega

## Friends

template < class charT, class traits >
 std::basic\_ostream < charT, traits > & operator << (std::basic\_ostream < charT, traits > &os, const student\_t\_distribution & dist)

Write distribution to std::ostream.

## 5.13.1 Detailed Description

template < class RealType = double > struct student\_t\_distribution < RealType >

Instances of class student  $\underline{t}$  distribution give random variates for a student  $\underline{t}$  distribution with parameter  $\underline{n}$  = alpha.

The documentation for this struct was generated from the following file:

 $\bullet \ / Users/jdunn/Documents/XCode/how\_much\_data/include/student\_t\_distribution.h$ 

## **Chapter 6**

## **File Documentation**

6.1 /Users/jdunn/Documents/XCode/how\_much\_data/include/exponential\_distribution.h File Reference

```
#include <boost/random.hpp>
#include <boost/math/distributions/exponential.hpp>
```

#### **Classes**

struct exponential\_distribution < RealType >
 instances of struct exponential\_distribution generate random variates for exponential distribution

6.2 /Users/jdunn/Documents/XCode/how\_much\_data/include/lognormal\_distribution.h File Reference

```
#include <iostream>
#include <iomanip>
#include <complex>
#include <string>
#include <memory>
#include <utility>
#include <boost/random.hpp>
#include <boost/math/constants/constants.hpp>
#include <boost/math/distributions/lognormal.hpp>
#include <boost/math/special_functions/erf.hpp>
#include <boost/math/tools/roots.hpp>
#include "adaptive_integration.h"
#include "lambert_w.h"
```

#### **Classes**

struct lognormal\_distribution< RealType >

a class with functions related to the lognormal distribution.

46 File Documentation

#### **Functions**

```
    template<typename RealType >
        complex< RealType > expm1 (complex< RealType > z)
    template<typename RealType >
        RealType sign (RealType x)
```

## 6.3 /Users/jdunn/Documents/XCode/how\_much\_data/include/normal\_switch\_mean.h File Reference

```
#include <iostream>
#include <iomanip>
#include <complex>
#include <string>
#include <utility>
#include <boost/random.hpp>
#include <boost/random/bernoulli_distribution.hpp>
#include <boost/random/normal_distribution.hpp>
#include <boost/math/distributions/normal.hpp>
#include <boost/math/constants/constants.hpp>
#include <boost/math/tools/roots.hpp>
```

#### Classes

class normal\_switch\_mean< RealType >

class whose instances represent a 50/50 mixture of normal distriubtions with sigma = 1 and means +d and -d

## 6.4 /Users/jdunn/Documents/XCode/how\_much\_data/include/normal\_switch\_stddev.h File Reference

```
#include <iostream>
#include <iomanip>
#include <complex>
#include <string>
#include <utility>
#include <boost/random.hpp>
#include <boost/random/bernoulli_distribution.hpp>
#include <boost/random/normal_distribution.hpp>
#include <boost/math/distributions/normal.hpp>
#include <boost/math/constants/constants.hpp>
#include <boost/math/tools/roots.hpp>
```

#### Classes

class normal\_switch\_stddev< RealType >

class whose instances represent a mixture of normal distribbtions with mu=0 and stddev of 1 w prob.

## 6.5 /Users/jdunn/Documents/XCode/how\_much\_data/include/pareto\_distribution.h File Reference

```
#include <iostream>
#include <sstream>
#include <complex>
#include <random>
#include "adaptive_integration.h"
```

#### **Classes**

- class pareto\_distribution< RealType >
  - Instantiations of class template pareto\_distribution model a Pareto Type 2 distribution.
- class pareto\_distribution< RealType >::param\_type

# 6.6 /Users/jdunn/Documents/XCode/how\_much\_data/include/student\_t\_distribution.h File Reference

```
#include <boost/random.hpp>
#include <boost/math/distributions/students_t.hpp>
#include <boost/math/special_functions/beta.hpp>
#include <boost/math/special_functions/bessel.hpp>
```

#### Classes

struct student\_t\_distribution< RealType >

Instances of class student\_t\_distribution give random variates for a student t distribution with parameter n = alpha.

#### **Functions**

template < typename T >
 int sign (T val)
 returns sign of its argument

# 6.7 /Users/jdunn/Documents/XCode/how\_much\_data/monte\_carlo\_test/monte\_carlo\_← test.cpp File Reference

```
#include <iostream>
#include <iomanip>
#include <string>
#include <sstream>
#include <fstream>
#include <random>
#include <vector>
```

48 File Documentation

```
#include <array>
#include <algorithm>
#include <numeric>
#include <mutex>
#include <thread>
#include <boost/timer/timer.hpp>
#include <boost/filesystem.hpp>
#include <boost/math/special_functions/beta.hpp>
#include <boost/math/distributions/students_t.hpp>
#include "adaptive_integration.h"
#include "pareto_distribution.h"
#include "student_t_distribution.h"
#include "exponential_distribution.h"
#include "lognormal_distribution.h"
#include "normal_switch_mean.h"
#include "normal_switch_stddev.h"
#include "taleb results.h"
```

#### Classes

struct KappaResult

the struct holding the resutls of a single alpha

struct KappaResults

sturcture holding the results from all runs

### **Functions**

```
    Kronrod< double > k_big (10)
```

```
    template<typename RealType >
```

```
RealType rel_err (RealType a, RealType b)
```

return the relative error between two numbers

 $\bullet \ \ \text{template}{<} \text{typename RealType} >$ 

```
vector< RealType > quantile (const vector< RealType > &x, const vector< RealType > &probs)
```

return quantiles of the ensemble of trials

ostream & operator<< (ostream &os, const KappaResult &k)</li>

output the results from a single run

ostream & operator<< (ostream &os, KappaResults &ks)</li>

output the results for all runs

template<typename Dist >

void calc\_kappa (unsigned int thread\_id, size\_t m, size\_t m\_ci\_limit, vector< int > ns, Dist dist, double ci\_← level, KappaResult \*kp, bool verbose=false)

the per thread cacluaiton engine

• template<typename Dist >

void calculate\_kappa (size\_t m, vector < int > ns, Dist dist, double ci\_level, KappaResult \*kp, bool verbose=false)

calculate kappa for sums of iid variables at specified durations

void show\_usage (path p)

show the usage. called when the wrong # of arguments is used

• int main (int argc, const char \*argv[])

main program for convolution test with one input parameter the # of trials

## Variables

```
• int noext = 0
```

- double epsabs\_double = 0
- double **epsrel\_double** = 64 \* std::numeric\_limits<double>::epsilon()
- int **limit** = 2000
- int verbose\_integration = 0
- mutex kr\_mutex

a mutex for writing to KappaResults

mutex cout\_mutex

a mutex for writing to cout

## 6.7.1 Function Documentation

## 6.7.1.1 calc\_kappa()

### the per thread cacluaiton engine

#### **Parameters**

in	thread_id	d_id the number of the thread used as seed for urng	
in	m	the maximum # of trials for mad	
in	m_ci_limit	the maximum # of trials for ci	
in	ns	the durations to save	
in	dist	the distribution	
in	ci_level	the confidence level for kappa_ci	
out	kp	the results	
in	verbose	flag for trace infomation	

#### 6.7.1.2 calculate\_kappa()

50 File Documentation

```
vector< int > ns,
Dist dist,
double ci_level,
KappaResult * kp,
bool verbose = false )
```

calculate kappa for sums of iid variables at specified durations

#### **Parameters**

in	т	the number of scenarios
in	ns	the durations to save
in	dist	the distribution
in	ci_level	the confidence level to use
out	kp	a ptr to the results
in	verbose	a flag to generate trace

#### 6.7.1.3 operator << ()

output the results from a single run

#### **Parameters**

in,out	os	the output stream
in	k	the sturct with results

## 6.7.1.4 quantile()

```
template<typename RealType > vector<RealType> quantile ( const\ vector<\ RealType> \&\ x, \\ const\ vector<\ RealType> \&\ probs\ )
```

return quantiles of the ensemble of trials

#### **Parameters**

in	X	the result by trial
in	probs	the desired probabilities

#### 6.7.1.5 rel\_err()

return the relative error between two numbers

#### **Parameters**

in	а	the first number
in	b	the second number

# 6.8 /Users/jdunn/Documents/XCode/how\_much\_data/pinelis\_taleb\_test/pinelis\_taleb\_ test.cpp File Reference

```
#include <iostream>
#include <iomanip>
#include <string>
#include <sstream>
#include <fstream>
#include <vector>
#include <array>
#include <algorithm>
#include <mutex>
#include <thread>
#include <complex>
#include <boost/timer/timer.hpp>
#include <boost/filesystem.hpp>
#include <boost/math/constants/constants.hpp>
#include "adaptive_integration.h"
#include "pareto_distribution.h"
#include "student t distribution.h"
#include "exponential_distribution.h"
#include "lognormal_distribution.h"
#include "normal_switch_mean.h"
#include "normal switch stddev.h"
#include "taleb_results.h"
```

#### **Classes**

struct Numpunct

sturcture passed by imbue to ostreams to use commas in numbers

struct KappaResult

the struct holding the resutls of a single alpha

struct KappaResults

sturcture holding the results from all runs

class PinelisTalebIntegrand
 Dist >

Functor used in the integratoin of the Pinelis Taleb result.

52 File Documentation

```
    class PinelisTalebIntegrand< pareto_distribution<>>
```

Functor used in the integration of the Pinelis Taleb result for pareto dist.

class PinelisTalebIntegrand< exponential\_distribution<>>

Functor used in the integration of the Pinelis Taleb result for exp. dist.

## **Typedefs**

• using **dcomplex** = std::complex < double >

#### **Functions**

```
    Kronrod< double > k_big (10)
```

```
\bullet \quad \text{template}{<} \text{typename RealType} >
```

```
RealType rel_err (RealType a, RealType b)
```

return the relative differnece between two numbers

ostream & operator<< (ostream &os, const KappaResult &k)</li>

output the result for one parameter value to an ostream

ostream & operator<< (ostream &os, KappaResults &ks)</li>

output the results for all parameter values to an ostream

• template<typename Dist >

```
void calculate_kappa (vector< int > ns, Dist dist, KappaResult &k, int verbose=0)
```

set up integrand and calculate the results for one parameter value

void show\_usage (path p)

show proper usage after improper command line argument

int main (int argc, const char \*argv[])

main program for pinelis-taleb run

## **Variables**

- int **noext** = 0
- double epsabs\_double = 0
- double epsrel\_double = 64 \* std::numeric\_limits<double>::epsilon()
- int limit = 2000
- int verbose\_integration = 0

#### 6.8.1 Function Documentation

#### 6.8.1.1 calculate\_kappa()

set up integrand and calculate the results for one parameter value

## **Parameters**

in	ns	the durations to calculate
in	dist	the distribution
out	k	the results
in	verbose	flag for trace

## 6.8.1.2 show\_usage()

```
void show_usage ( path \ p \ )
```

show proper usage after improper command line argument

#### **Parameters**

in <i>p</i>	the path of the executable
-------------	----------------------------

54 File Documentation

## Index

```
/Users/jdunn/Documents/XCode/how_much_data/include/exporlegtialrmal_distribution, 24
          distribution.h, 45
                                                         characteristic function
/Users/jdunn/Documents/XCode/how much data/include/lognologgalermal distribution, 25
          _distribution.h, 45
                                                         characteristic function prime
/Users/jdunn/Documents/XCode/how_much_data/include/normabgnormal_distribution, 25
          switch mean.h, 46
/Users/jdunn/Documents/XCode/how_much_data/include/rexpคลู่ential_distribution< RealType >, 11
_switch_stddev.h, 46
/Users/jdunn/Documents/XCode/how_much_data/include/pareto;—
KappaResult, 14
          _switch_stddev.h, 46
          distribution.h, 47
/Users/jdunn/Documents/XCode/how_much_data/include/studentnesult, 12
          t distribution.h, 47
                                                              calc_kappa, 14
/Users/jdunn/Documents/XCode/how much data/monte ←
                                                              initialize, 14
          carlo test/monte carlo test.cpp, 47
                                                              KappaResult, 14
/Users/jdunn/Documents/XCode/how much data/pinelis←
                                                              mad_rel_err, 15
          _taleb_test/pinelis_taleb_test.cpp, 51
                                                              ns, 15
                                                              update conf int, 14
alpha
                                                              update dev, 15
     pareto_distribution, 37
                                                         KappaResults, 16
     pareto_distribution::param_type, 33
                                                              KappaResults, 16, 17
                                                              ns, 17
calc kappa
                                                              param_label, 17
     KappaResult, 14
                                                              taleb_offset, 17
     monte carlo test.cpp, 49
calculate kappa
                                                         lognormal distribution
     monte carlo test.cpp, 49
                                                              cdf, 20
     pinelis_taleb_test.cpp, 52
                                                              cf_fourier, 21
cdf
                                                              cf_fourier_Inx, 21
     lognormal distribution, 20
                                                              cf fourier mixed, 22
     normal switch mean, 28
                                                              cf lambert w, 22
     normal_switch_stddev, 31
                                                              cf series, 22
cf fourier
                                                              cfprime fourier, 23
     lognormal distribution, 21
                                                              cfprime fourier Inx, 23
cf fourier Inx
                                                              cfprime_fourier_mixed, 24
     lognormal distribution, 21
                                                              cfprime_lambert_w, 24
cf fourier mixed
                                                              cfprime series, 24
     lognormal distribution, 22
                                                              characteristic function, 25
cf lambert w
                                                              characteristic_function_prime, 25
     lognormal_distribution, 22
                                                              lognormal_distribution, 20
cf series
                                                              mad2, 26
     lognormal distribution, 22
                                                              max, 26
cfprime_fourier
                                                              min, 26
     lognormal distribution, 23
                                                              pdf, 26
cfprime fourier Inx
                                                              quantile, 26
     lognormal distribution, 23
                                                         lognormal distribution < RealType >, 18
cfprime fourier mixed
     lognormal distribution, 24
                                                         mad2
                                                              lognormal distribution, 26
cfprime lambert w
     lognormal_distribution, 24
                                                         mad rel err
cfprime_series
                                                              KappaResult, 15
```

56 INDEX

max	pareto_distribution< RealType >, 35
lognormal_distribution, 26	pareto_distribution< RealType >::param_type, 32
pareto_distribution, 37	pareto_distribution::param_type
min	alpha, 33
lognormal_distribution, 26	mu, 33
pareto_distribution, 37	operator!=, 34
monte_carlo_test.cpp	operator<<, 34
calc_kappa, 49	operator>>, 34
calculate_kappa, 49	operator==, 34
operator<<, 50	param_type, 33
quantile, 50	sigma, 33
•	
rel_err, 50	pdf
mu	lognormal_distribution, 26
pareto_distribution, 38	normal_switch_mean, 28
pareto_distribution::param_type, 33	normal_switch_stddev, 31
1 9 1	pinelis_taleb_test.cpp
normal_switch_mean	calculate_kappa, 52
cdf, 28	show_usage, 53
normal_switch_mean, 28	PinelisTalebIntegrand
pdf, 28	PinelisTalebIntegrand, 39
quantile, 29	PinelisTalebIntegrand < exponential_distribution <>
normal_switch_mean< RealType >, 27	>, 40
normal_switch_stddev	PinelisTalebIntegrand< pareto_distribution<> >,
cdf, 31	41
normal_switch_stddev, 30	PinelisTalebIntegrand < Dist >, 39
pdf, 31	PinelisTalebIntegrand< exponential_distribution<> >,
quantile, 31	40
normal_switch_stddev< RealType >, 29	PinelisTalebIntegrand, 40
ns	PinelisTalebIntegrand< pareto_distribution<>>, 41
KappaResult, 15	PinelisTalebIntegrand, 41
KappaResults, 17	Tillolis falosificgrafia, 41
Numpunct, 32	quantile
Trampanot, oz	lognormal distribution, 26
operator!=	monte_carlo_test.cpp, 50
pareto_distribution::param_type, 34	normal_switch_mean, 29
operator<<	normal_switch_stddev, 31
monte_carlo_test.cpp, 50	normal_switch_studev, or
pareto_distribution::param_type, 34	rel_err
	monte_carlo_test.cpp, 50
operator>>	тото_оато_состорр, оо
pareto_distribution, 38	show_usage
pareto_distribution::param_type, 34	pinelis_taleb_test.cpp, 53
operator==	sigma
pareto_distribution::param_type, 34	pareto_distribution, 38
	pareto_distribution::param_type, 33
param	student_t_distribution< RealType >, 42
pareto_distribution, 38	student_t_distribution< riearrype >, 42
param_label	taleb_offset
KappaResults, 17	KappaResults, 17
param_type	rappartesuito, 17
pareto_distribution::param_type, 33	update_conf_int
pareto_distribution	KappaResult, 14
alpha, 37	update_dev
max, 37	KappaResult, 15
min, 37	παρμαπισσαπ, το
mu, 38	
operator>>, 38	
param, 38	
pareto_distribution, 37	
sigma, 38	