Problem Set 4

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December 20, 2020

Partial and general equilibrium models.

I The Model

I.I Utility

A representative household solve the following program,

$$
\begin{aligned}
\max_{\{c_t\}_t^T} \quad & \mathbb{E}_0 \sum_{t=0}^{T} \beta^t u(c_t) \\
\text{s.t.} \quad & c_t + a_{t+1} = w_t y_t + (1+r_t)a_t, \\
& \beta = \frac{1}{1+\rho} \in (0,1)
\end{aligned} \tag{1}
$$

to maximise future streams of consumption. Throughout the following, the discussion can be replicated for either of the following utility functions,

1. Quadratic utility

$$u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2$$

2. CRRA utility,

$$u(c_t) = \begin{cases} \frac{c_t^{1-\sigma}-1}{1-\sigma}, & \sigma > 1 \\ ln(c_t), & \sigma = 1 \end{cases}$$

   σ measures the degree of relative risk aversion. If $\sigma = 1$, using l'hôpital's rule it can be be shown that preferences are a log utility, in such a case the income and substitution effects perfectly offset each other.

At first set $r = 4\% < \rho = 6\%$ and normalise $w = 1$. Under this specification agents prefer a decreasing consumption profile. For example consider CRRA preferences, the generic Euler equation in sequential formulation is,

$$
\begin{aligned}
\frac{u'(c_t)}{u'(c_{t+1})} &= \beta[1 + f'(k_{t+1})] \\
\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} &= \beta[1 + f'(k_{t+1})] \\
\frac{c_{t+1}}{c_t} &= \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}} \\
\frac{c_{t+1}}{c_t} &= \left(\frac{1+0.04}{1+0.06}\right)^{\frac{1}{\sigma}} < 1
\end{aligned}
$$

Consumption is decreasing over time if $r < \rho$ and vice versa.

I.II Income

Assume that income is governed by a two state process,

$$Y = \{1 - \sigma_y, 1 + \sigma_y\}$$
$$\begin{bmatrix} \frac{1+\gamma}{2} & \frac{1-\gamma}{2} \\ \frac{1+\gamma}{2} & \frac{1-\gamma}{2} \end{bmatrix}$$

where the variance and persistence are governed by,

$$var(y) = \sigma_y^2$$
$$corr(y', y) = \frac{cov(y', y)}{\sqrt{var(y')}\sqrt{var(y)}} = \gamma$$

I will examine two cases, one in which agents are able to borrow but face a borrowing limit,

1. $$a_{t+1} = -\bar{A} = -\frac{1+r}{r} y_{min},$$

2. Agents are unable to borrow,

$$a_{t+1} \geq 0$$

I.III Recursive Formulation

In each state there are two state variables, assets today and the income shock, a, y. The agent faces two choice variables, c, a and therefore the solution will provide us with two policy functions, one for consumption and the other assets. Due to the constraint holding with equality on consumption we can substitute into the utility function, here represented in generic $u(c_t)$ form.

$$v(a, y) = \max_{a' \in \Gamma} \{u(wy + a - qa') + \beta \mathbb{E}[v(a', y')|y]\}$$
$$\Gamma = \{a' : a' \geq -\bar{A}\}$$

where $-\bar{A}$ is either of the previous two cases depending on agents ability to borrow. The expectation is taken with respect to the distribution of y,

$$v(a, y) = \max_{a' \in \Gamma} \{u(wy + a - qa') + \beta \sum_{y'} \pi_{y'|y} v(a', y')\}$$
$$\Gamma = \{a' : a' \geq -\bar{A}\}$$

The Euler equation is then found from the standard first order conditions and application of the envelope theorem. Attach the Lagrange multiplier, μ to the constraint,

$$\frac{\partial v(a, y)}{\partial a'} = -qu_c(wy + a - qa') + \mu_{a'} + \beta \sum_{y'} \pi_{y'|y} \frac{\partial v(a', y')}{\partial a'}$$

setting the left hand side to zero to find the optimum, we get that,

$$u_c(wy + a - qa') = \mu_{a'} + \frac{\beta}{q} \sum_{y'} \pi_{y'|y} \frac{\partial v(a', y')}{\partial a'}$$

The envelope theorem states that the derivative of the value function w.r.t a state variable is equal to the derivative of the utility function w.r.t the same state variable such that,

$$\frac{\partial v(a', y')}{\partial a'} = u_c(wy + a - qa')$$

and the Euler equation reads,

$$u_c(wy + a - qa') = \mu_{a'} + \frac{1+r}{1+\rho} \sum_{y'} \pi_{y'|y} u_{c'}(w'y' + a' - qa'')$$

The constraint therefore plays a key role and we will see this in the subsequent sections. With either of the utility specifications the Euler functions are,

1. Quadratic utility:

$$(wy + a + qa' - \bar{c}) = \frac{1+r}{1+\rho} \sum_{y'} \pi_{y'|y}(w'y' + a' + qa'' - \bar{c})$$

2. CRRA utility:

$$1 = \frac{1+r}{1+\rho} \sum_{y'} \pi_{y'|y} \left(\frac{w'y' + a' - qa''}{wy + a + qa'} \right)^{-\sigma}$$

The stationary competitive equilibrium will be a value function, $V : \mathcal{Z} \times \mathcal{M} \to \mathcal{R}$, two policy functions on assets and consumption, $a', c : \mathcal{Z} \times \mathcal{M} \to \mathcal{R}$ and two pricing functions, $w, r : \mathcal{M} \to \mathcal{R}$ such that,

1. Given prices, V solves the Bellman equation specified above along with the associated policy functions a', c.

2. Given prices, K satisfies,

$$r(\Phi) = F_K(K(\Phi)) - \delta$$
$$w(\Phi) = F_L(K(\Phi)) - \delta$$

3. Markets clear,

$$K(H(\Phi)) = \int a'(a, y; \Phi) d\Phi$$
$$\int c(a, y; \Phi) d\Phi + \int a'(a, y; \Phi) d\Phi = F_K(K(\Phi)) + (1-\delta)K(\Phi)$$

4. The aggregate law of motion H satisfies consistency

Program

I have written a program called SolveModel which can solve,

1. Discrete or continuous models

2. With a natural or zero borrowing limit

3. Range of parameter values for

    - Prudence σ

- Persistance γ
- Shocks σ_y

The following models are presented for a range of parameters. The equilibrium results, a value function and both policy functions are presented along side simulated consumption paths for 45 years for a hypothetical agent, the income shocks in each period are displayed in grey. The baseline parameters are,

Parameter	Value
σ	2
σ_y	0
γ	0

Any changes to the baseline parameters is highlighted in the captions.

I have employed both discrete and continuous methods in approximating the value and policy functions. Results from the continuous method are in the appendix as to reduce the number of graphs and results to be presented.

To approximate the value function continuously I used a 4th order spline and introduced a smoothing parameter, s. I set s equal to 6 as it provided the best fit. Setting a lower degree spline or lower smoothing factor resulted in problems in capturing the policy functions when increasing uncertainty in the model. The utilised the scipy functions, bisplrep and bisplev[1] which enable you to change such parameters.

Order of results

Since there are many sections to this problem set I have included a quick summary to show the order of what is presented as my solutions to the problem set. At all stages both CRRA and quadratic preference results are presented,

1. Discrete models
   (a) Certainty, natural borrowing limit & infinite horizon
   (b) Certainty, zero borrowing limit & infinite horizon
   (c) Uncertainty, natural borrowing limit & infinite horizon
   (d) Uncertainty, zero borrowing limit & infinite horizon
   (e) Life cycle models
   (f) Increase the prudence σ
   (g) Partial Equilibrium : Stationary distribution
   (h) General Equilibrium : Stationary distribution
   (i) Replicate Aiyagari
2. Continuous methods (Appendix)

With Certainty

When agents face certain returns, the change from no borrowing to permitting borrowing until the natural borrowing limit enables the agents to reach a slightly higher level of consumption. The overall shape of all three equilibrium results is very similar.

[1] https://docs.scipy.org/doc/scipy/reference/interpolate.html

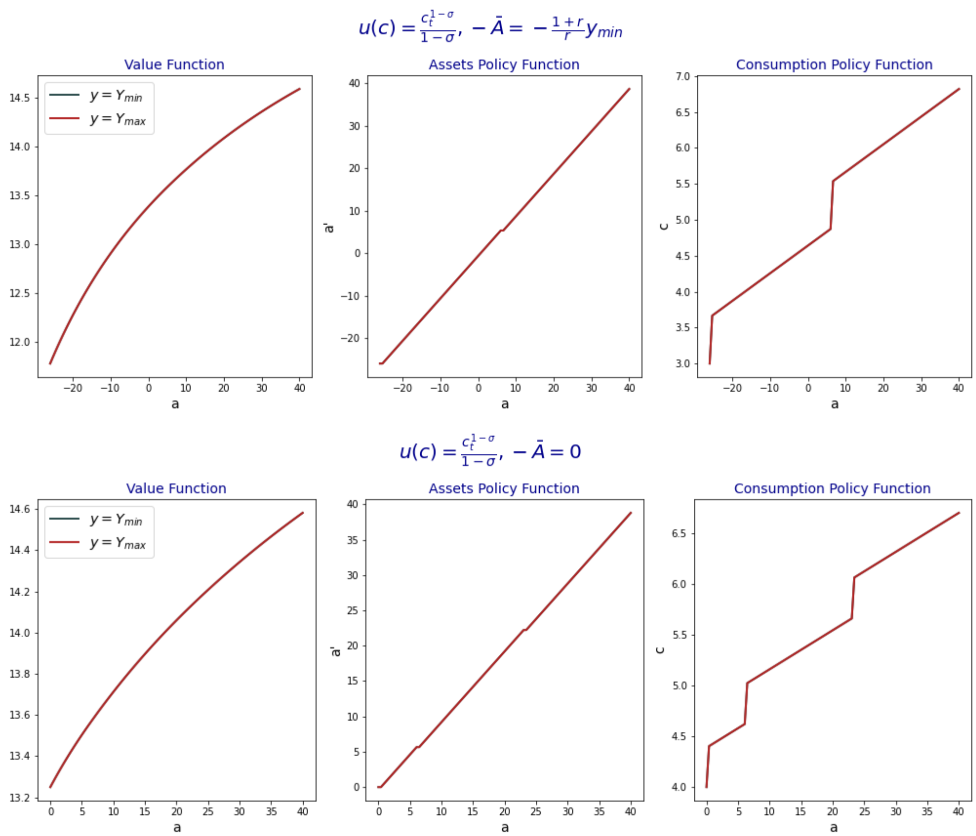
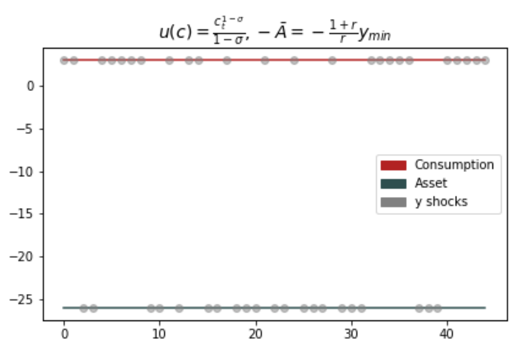
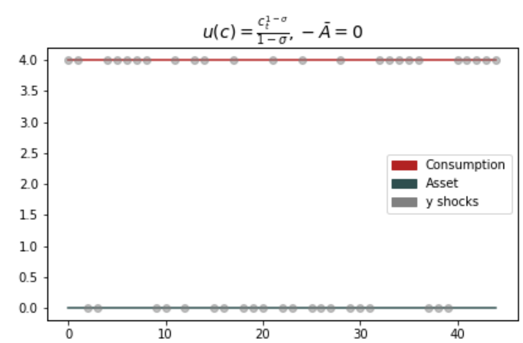


Figure 1: CRRA utility



(a) Natural borrowing limit



(b) Zero borrowing limit

Figure 2: Consumption paths for T = 45

Since agents suffer no income shocks they maintain a constant consumption path. This should increase utility overall due to the convexity of the utility function as they prefer to avoid wild changes in consumption.

For the quadratic utility case the result is very similar, since there is no variation in the economy the need for precautionary saving or certainty equivalence is removed as agents can plan consumption profiles perfectly over the infinite horizon.

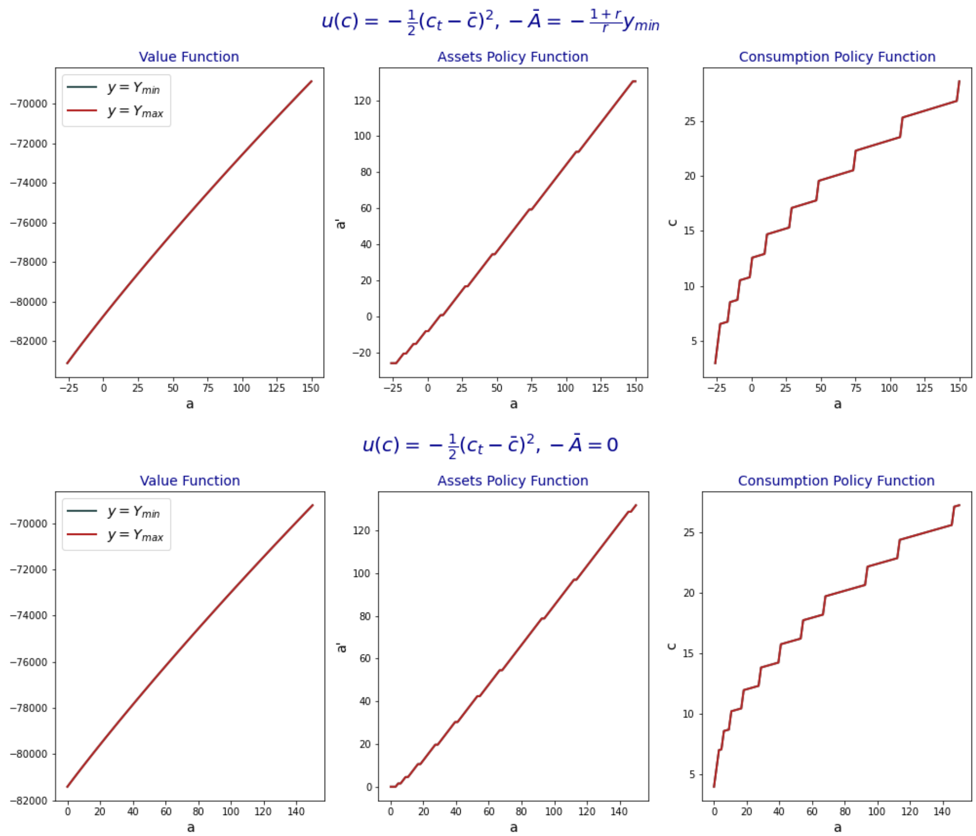
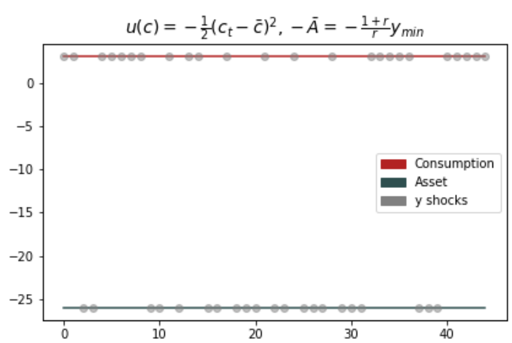
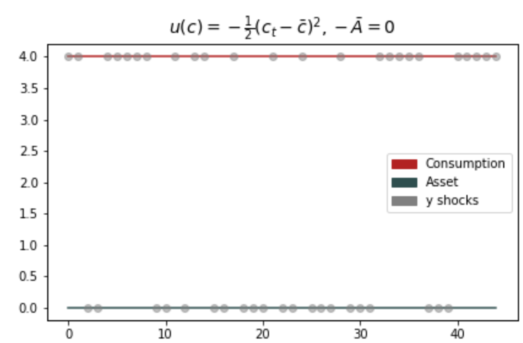


Figure 3: Quadratic utility



(a) Natural borrowing limit

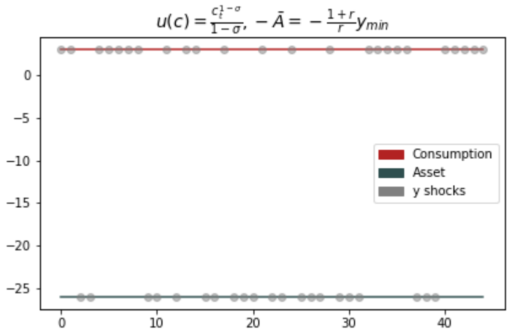


(b) Zero borrowing limit

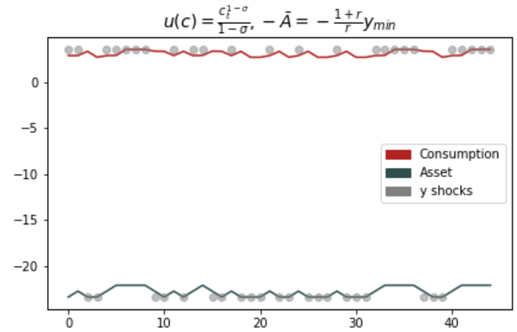
Figure 4: Consumption paths for T = 45

Uncertainty

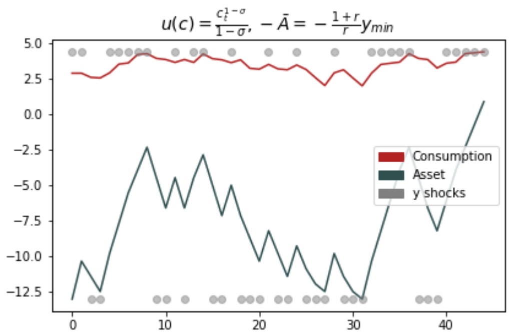
Introducing uncertainty to the economy provides much more interesting results. You can see from the savings function that given a higher income shock, agents consistently save more, consume more and enjoy higher utility returns. Changing the range of income shocks from ± 0.1 to ± 0.5 increases the difference in consumption and savings paths for those with high or low shocks as expected. In addition, it increases the curvature of the value function as agents who receive a bad shock and are around the natural borrowing limit suffer more severe utility loses from conusuming at the extreme end of the consumption profiles as mentioned before.



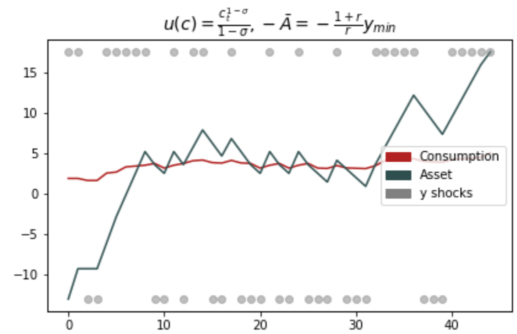
(a) $\sigma = 2$, $\sigma_y = 0$, γ=0



(b) $\sigma = 2$, $\sigma_y = 0.1$, γ=0



(c) $\sigma = 2$, σ_y=0.5, γ=0



(d) $\sigma = 2$, σ_y=0.5, γ=0.95

Figure 5: Natural borrowing limit

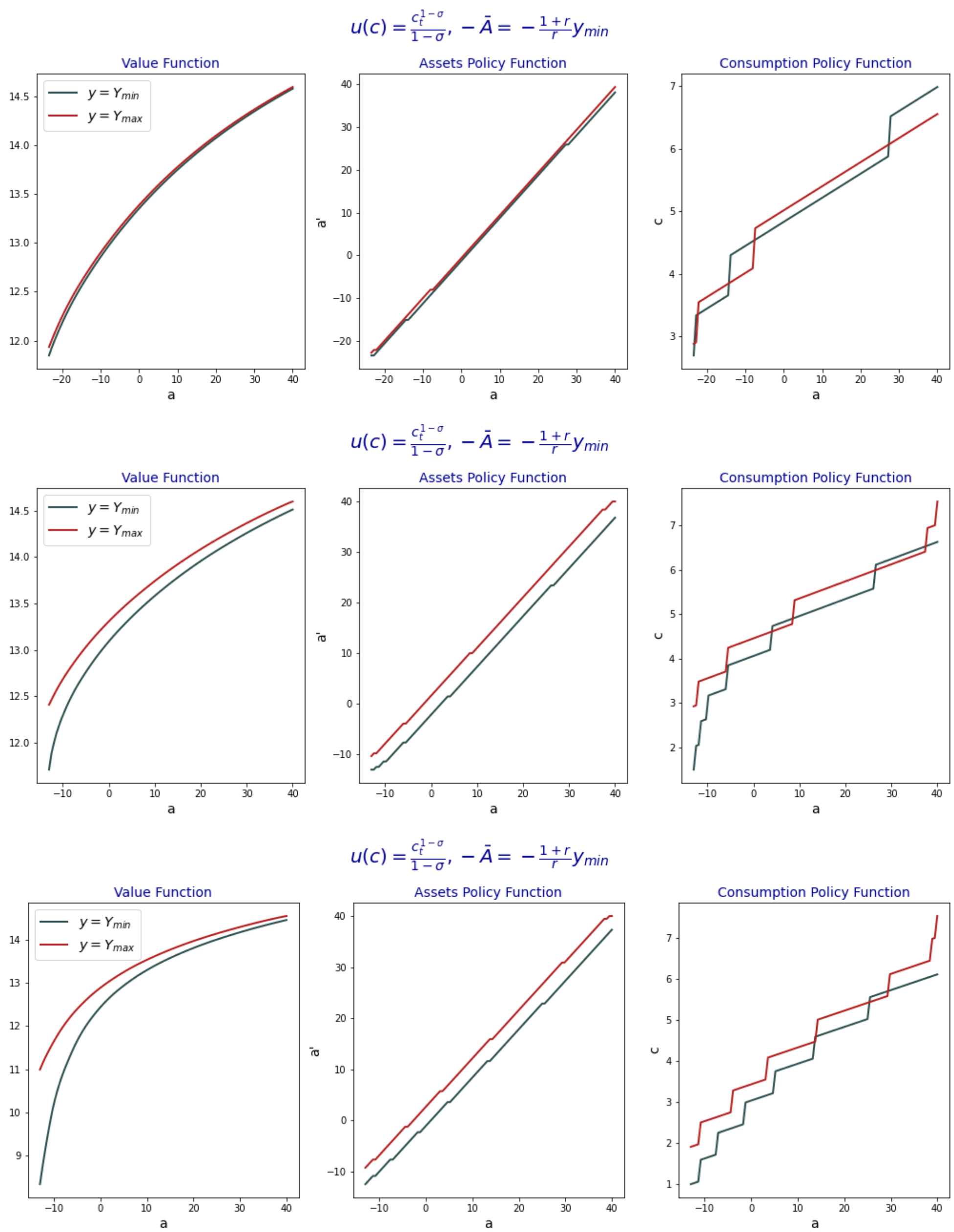


Figure 6: Top) $\sigma_y = 0.1$, Middle) $\sigma_y = 0.5$, Bottom) $\sigma_y = 0.5, \gamma = 0.95$, Natural Borrowing

For the consumption paths note that the random seed generator allows me to hold the shock distribution constant through the problem set, thus allowing me to compare across models. The standard case with no uncertainty is presented as a reference case. As we increase the idiosyncratic risk in the economy the consumption profiles become increasingly volatile. In response, as to smooth consumption patterns saving patterns also increase in volatility. Average consumption across each year in each model remains relatively constant and the biggest changes are in the saving rates, this represents consumption smoothing well and is expected under the precautionary savings motive utility specification.

The most drastic case comes from increasing the persistence of shocks. Since agents are fully rational, they know that a bad shock today is a strong predictor of a bad shock tomorrow, therefore if you compare the income shocks in grey and saving patterns we can see that they borrow heavily in periods of numerous bad shocks and save heavily in periods of good shocks. This clearly demonstrates the precautionary saving hypothesis.

For the quadratic case the findings over the infinite horizon are presented below.

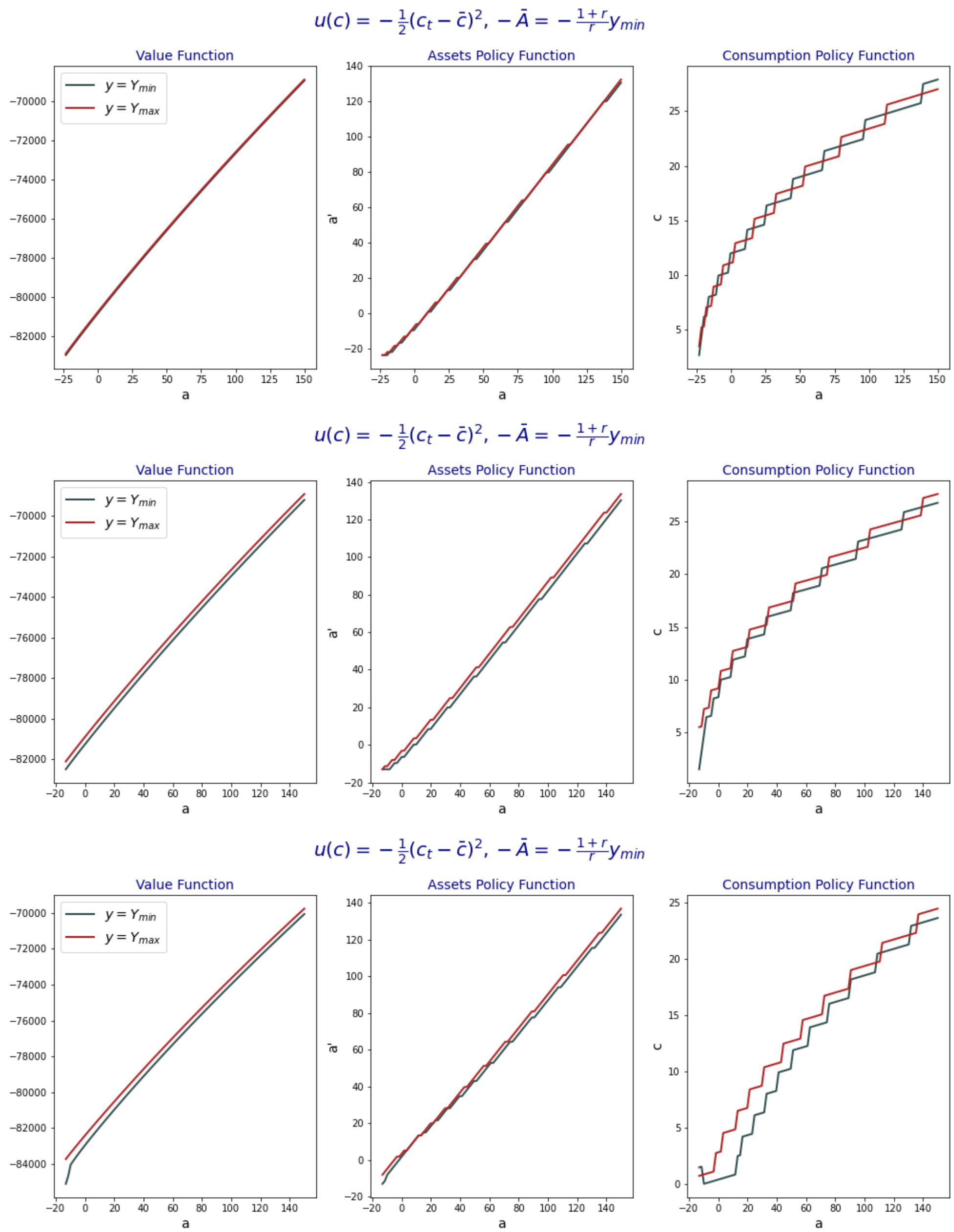
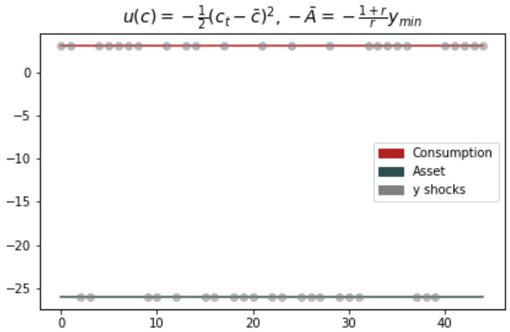
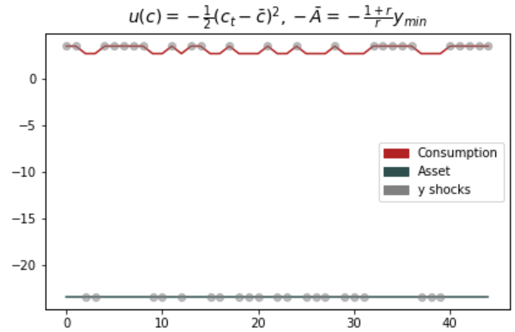


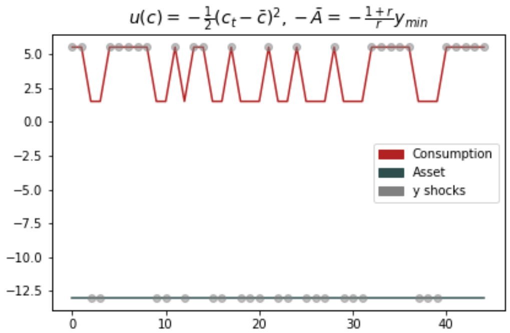
Figure 7: 1) $\sigma_y = 0.1$, 2) $\sigma_y = 0.5$, 3) $\sigma_y = 0.5, \gamma = 0.95$, Natural borrowing



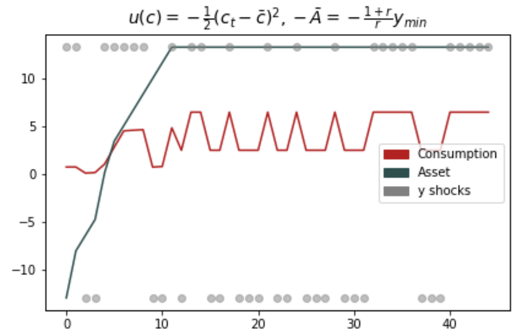
(a) σ = 2, σ_y=0, γ=0



(b) σ = 2, σ_y = 0.1, γ=0



(c) σ = 2, σ_y=0.5, γ=0



(d) σ = 2, σ_y=0.5, γ=0.95

Figure 8: Natural borrowing limit

As we increase the variance the consumption function at low asset levels tends to zero. As the uncertainty increases, the agent shows less tendency to increase assets than under the precautionary saving motive, in line with the theory and consumption decreases to a minimum level given a bad shock. Only a good shock allows the agent to increase consumption again. Interestingly, from including the persistence of shocks then the saving behaviour changes drastically as agents prepare for either long periods of good or bad times.

I now remove the natural borrowing limit and present the results with zero borrowing. Since agents are face tighter borrowing constraints their consumption and saving profiles change. Overall agents are less happy as they suffer utility losses from not being able to smooth consumption as before, this can be seen from the value functions, the consumption and asset functions are similar but the story comes out in the time profiles.

For CRRA preferences the equilibrium results are presented below. We see that given the new borrowing restrictions agents see more drastic swings in savings and greater variation in consumption under less uncertainty than given the natural borrowing limit. Increasing the range of income shocks from 0.1 to 0.5 causes a far greater level of variation than before. As uncertainty is increased further we see the hypothetical agent presented, given a run of early good shocks was able to amass a sufficiently large level savings and they use that as a buffer given future shocks. This result matches the data and observed saving and consumption patterns for wealthy individuals well.

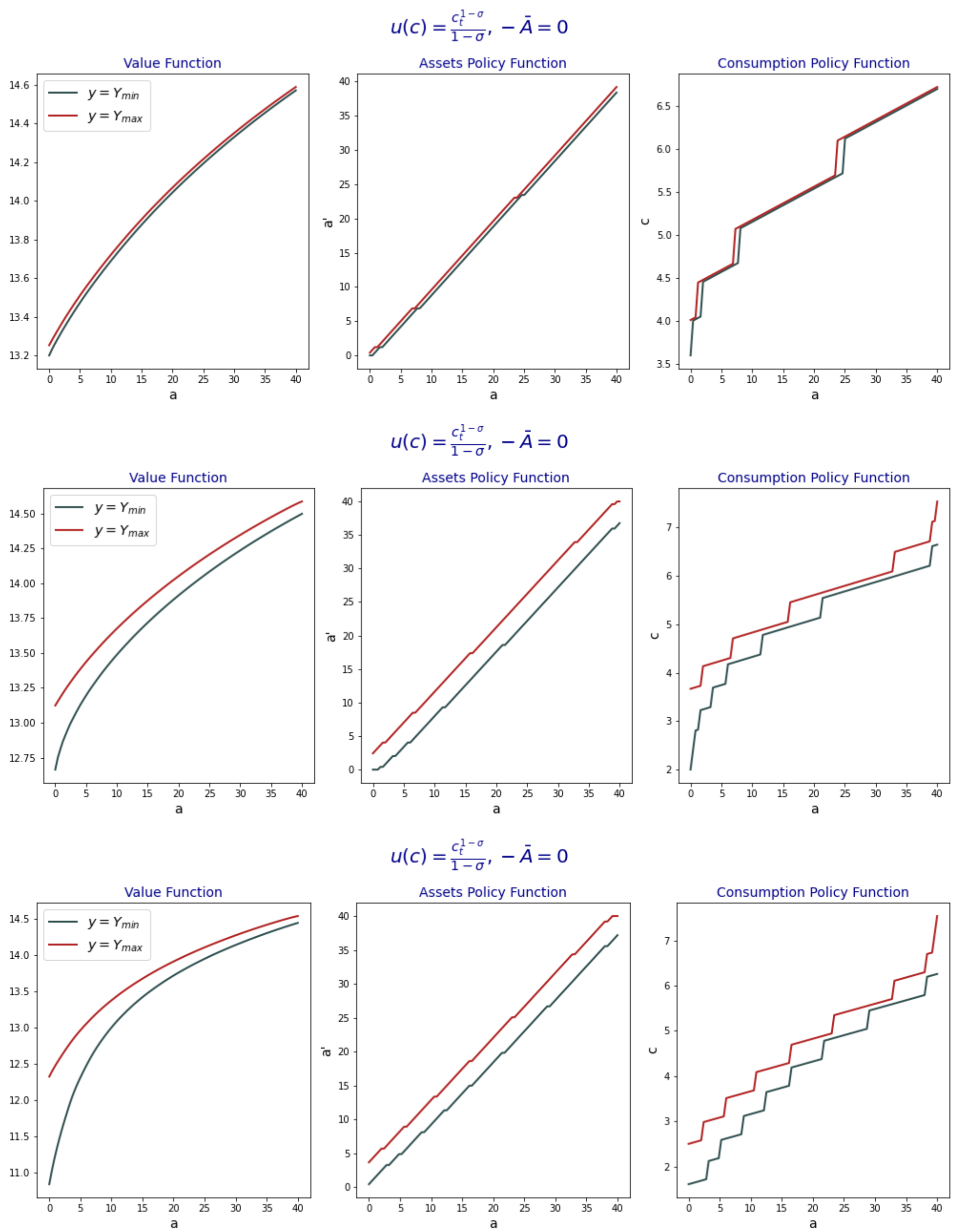
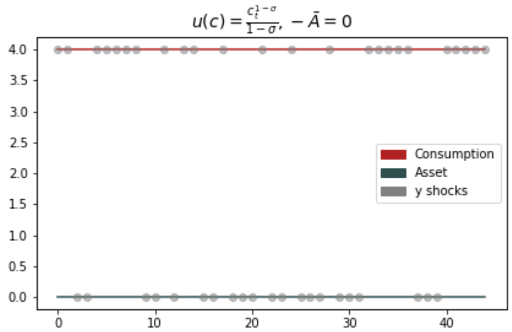
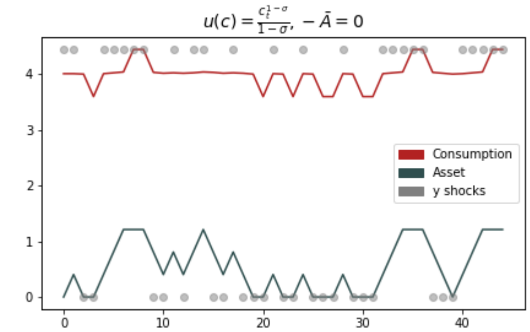


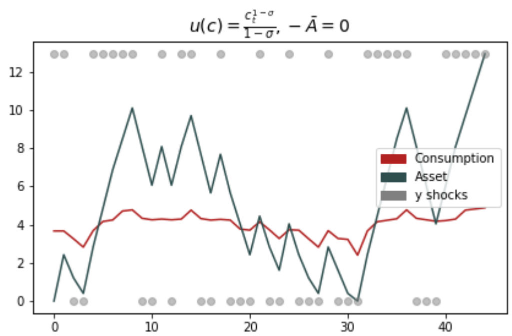
Figure 9: Top) $\sigma_y = 0.1$, Middle) $\sigma_y = 0.5$, Bottom) $\sigma_y = 0.5, \gamma = 0.95$, Zero borrowing



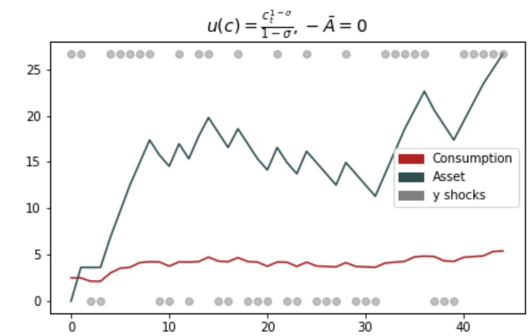
(a) $\sigma = 2$, σ_y=0, γ=0



(b) $\sigma = 2$, σ_y=0.1, γ=0



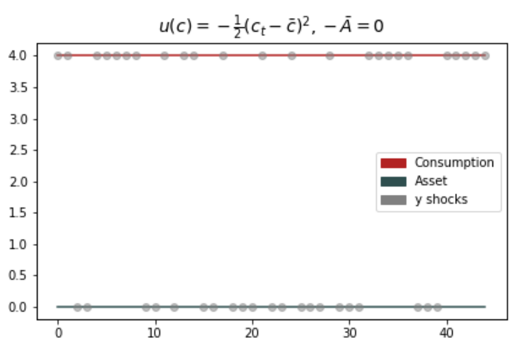
(c) $\sigma = 2$, σ_y=0.5, γ=0



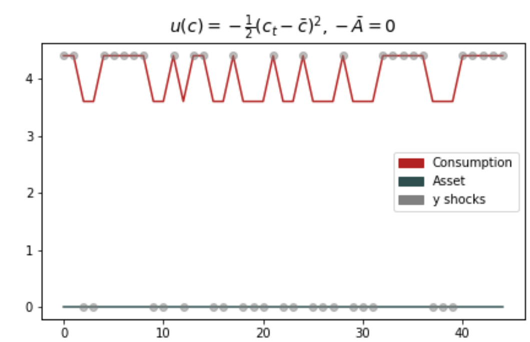
(d) $\sigma = 2$, σ_y=0.5, γ=0.95

Figure 10: Zero borrowing limit

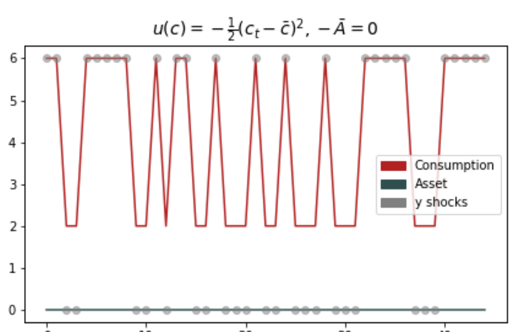
And again for the quadratic case,



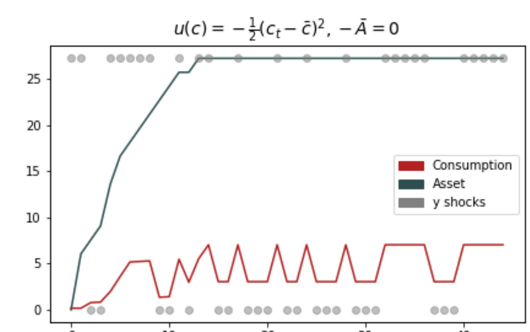
(a) $\sigma = 2$,$\sigma_y = 0$, γ=0



(b) $\sigma = 2$,σ_y=0.1, γ=0



(c) $\sigma = 2$, σ_y=05, γ=0



(d) $\sigma = 2$,σ_y=0.5, γ=09.5

Figure 11: Zero borrowing limit

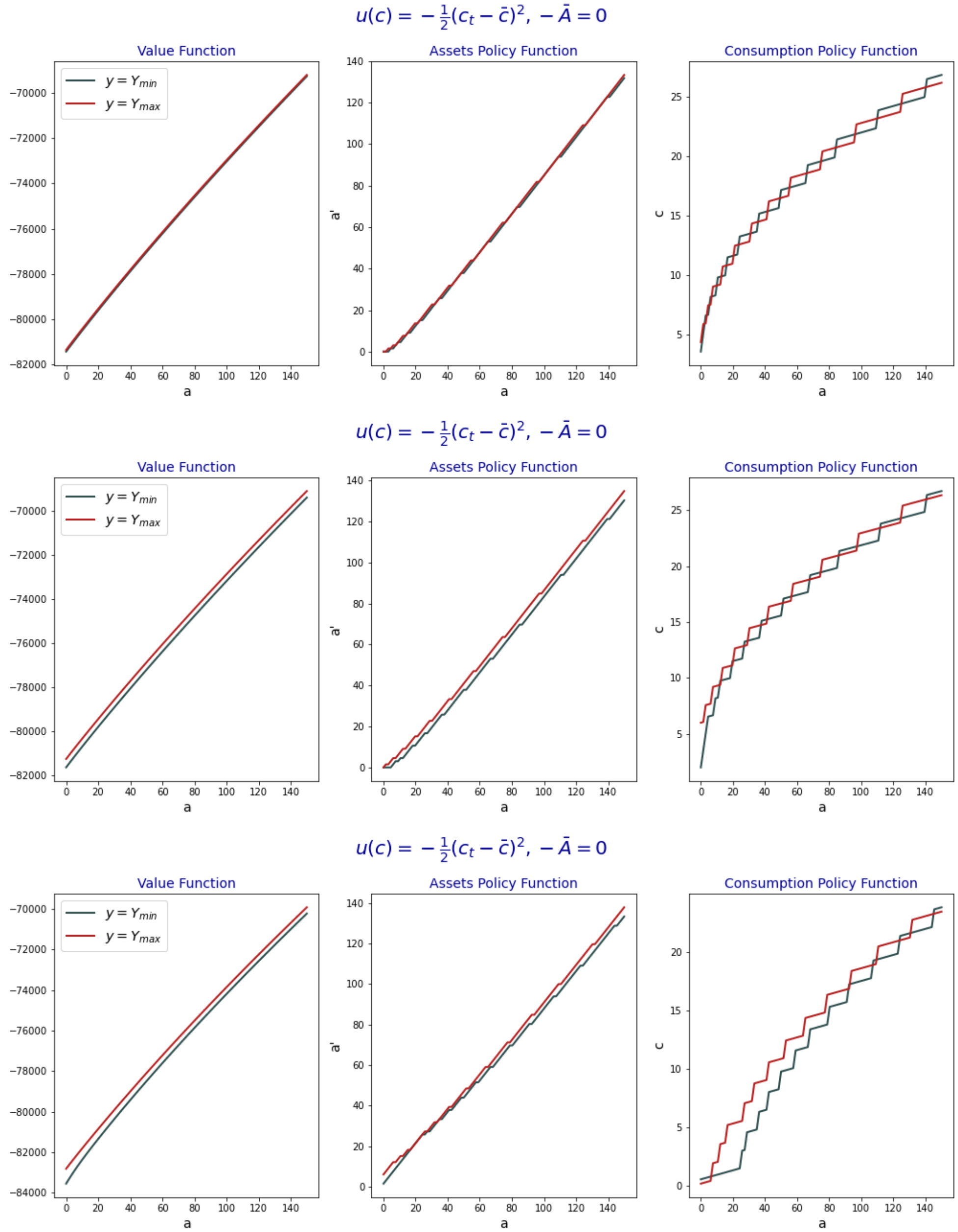


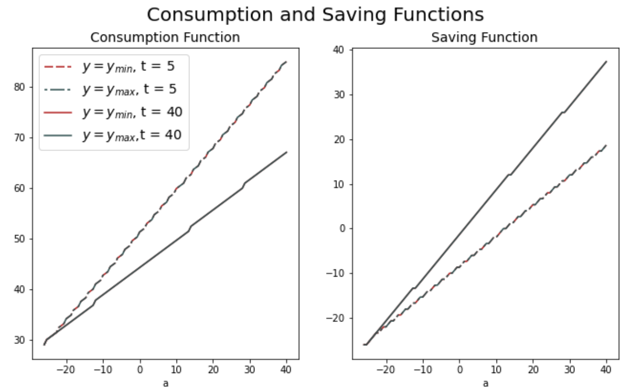
Figure 12: Top) $\sigma_y = 0.1$, Middle) $\sigma_y = 0.5$, Bottom) $\sigma_y = 0.5, \gamma = 0.95$, Zero borrowing

Here we see far larger swings in consumption given the change in the variance of income shocks than under the precautionary savings model. This change comes precisely from the lack of a the precautionary savings motive.

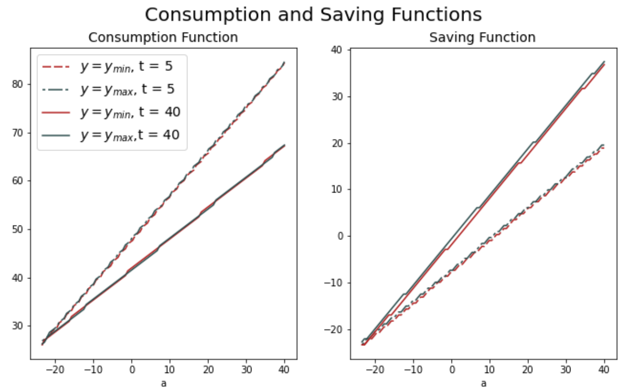
Life Cycle Models

The following were produced by iterating backwards on the value function setting $V_{45}(a, y) = 0$ as the terminal condition. For both the CRRA and quadratic preferences case I present the savings and consumption functions. The dotted line represents $t = 5$ and the block line $t = 40$. The certainty case shows us that given either income shock, young people save less and consume more than older people. This is consistent with the life cycle theory on saving patterns as people and the model's presented above. Consumption profiles are U-shaped as consumption increases at an early age to tail off again as people approach retirement. This again matches the data and observed behaviour well.

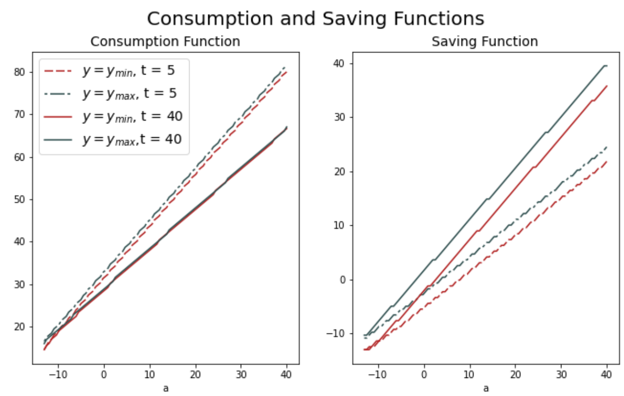
In addition, by increasing the uncertainty in the economy the gap between those who receive a good shock and a bad one widens. The figures also replicate the result from above that given a natural borrowing limit, agents will use the additional freedom to borrow to smooth consumption.



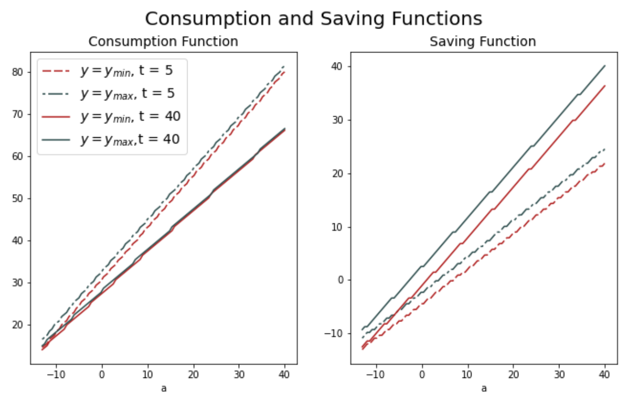
(a) σ = 2, σ_y=0, γ=0



(b) σ = 2, σ_y=0.1, γ=0

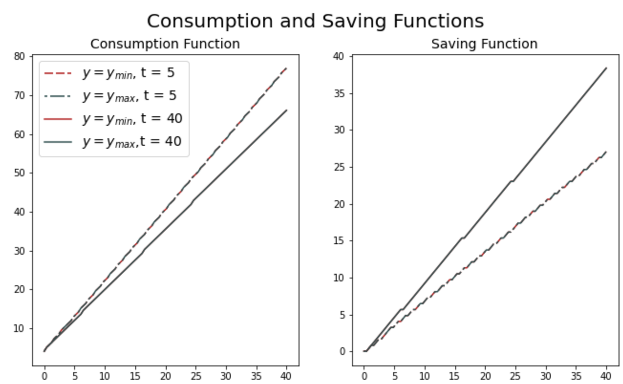


(a) σ = 2, σ_y=0.5, γ=0

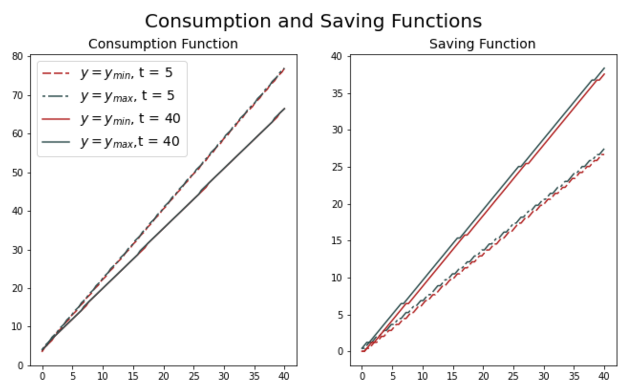


(b) σ = 2, σ_y=0.5, γ=95

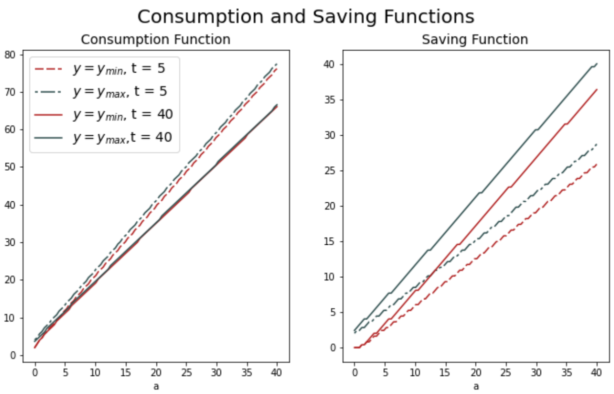
Figure 14: Natural borrowing limit



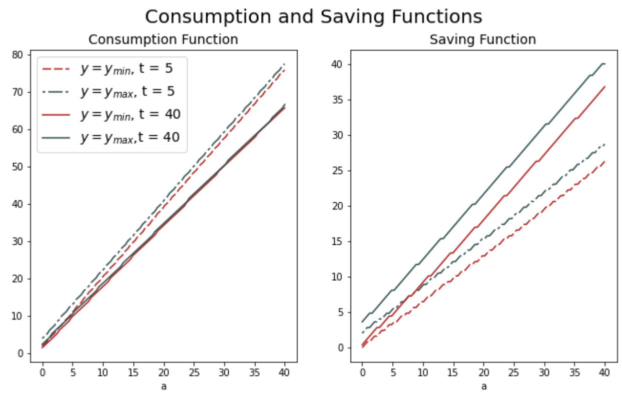
(a) σ = 2, σ_y=0, γ=0



(b) σ = 2, σ_y=0.1, γ=0



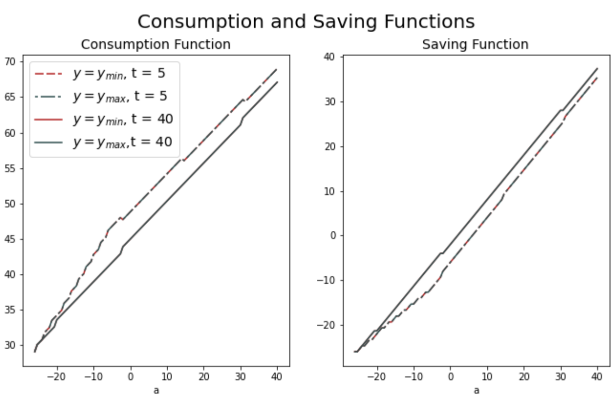
(a) $\sigma = 2$, σ_y=0.5, γ=0



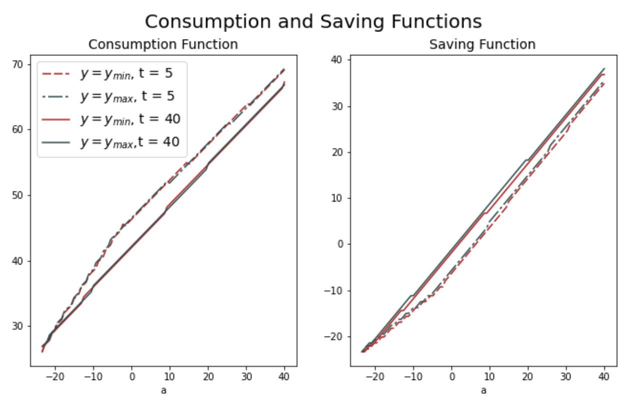
(b) $\sigma = 2$, σ_y=0.5, γ=95

Figure 16: Zero borrowing limit

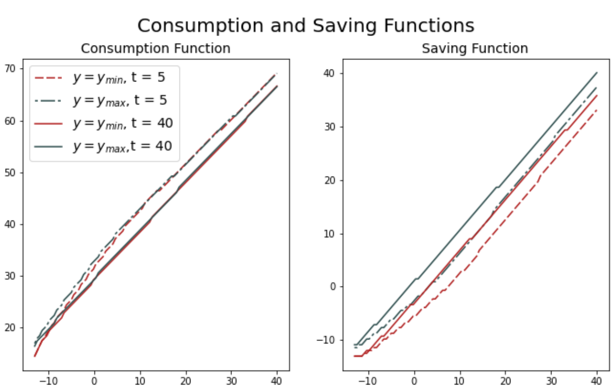
Quadratic Utility



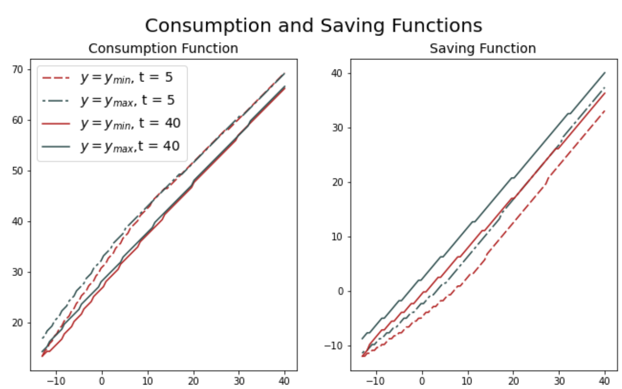
(a) $\sigma = 2$,σ_y=0, γ=0



(b) $\sigma = 2$, σ_y=0.1, γ=0

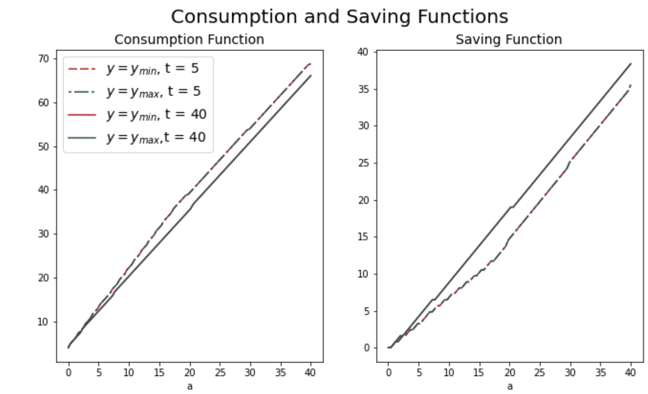


(a) $\sigma = 2$, σ_y=0.5, γ=0

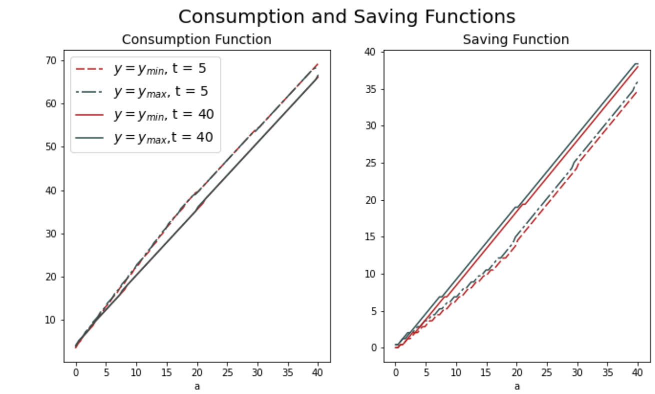


(b) $\sigma = 2$, σ_y=0.5, γ=95

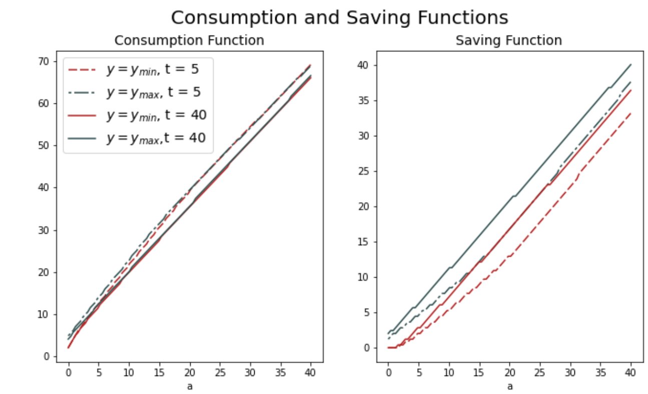
Figure 18: Natural borrowing limit



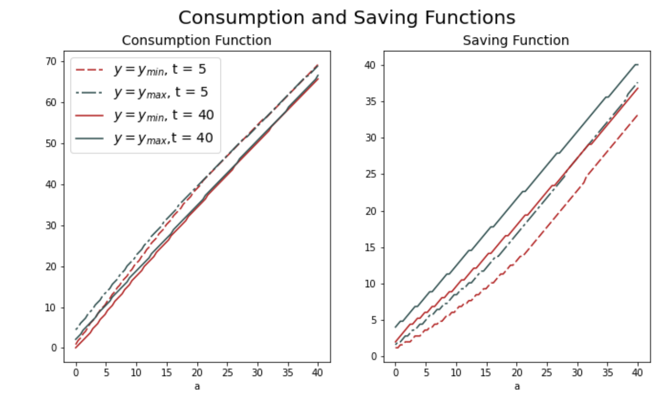
(a) $\sigma\,=\,2$, σ_y=0, γ=0



(b) $\sigma\,=\,2$, σ_y=0.1, γ=0



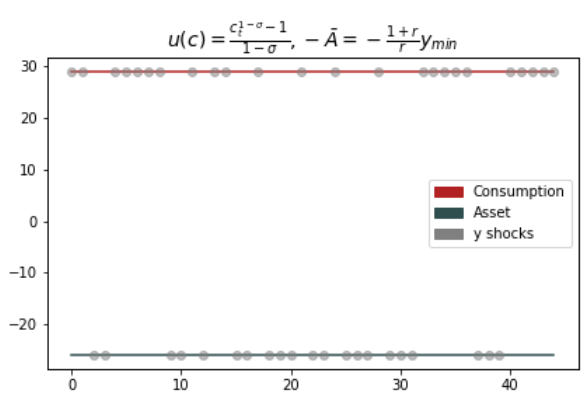
(a) $\sigma\,=\,2$, σ_y=0.5, γ=0



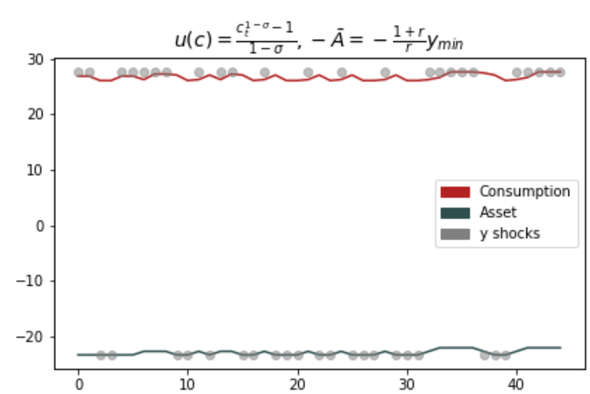
(b) $\sigma\,=\,2$, σ_y=0.5, γ=95

Figure 20: Zero borrowing limit

I have recreated a consumption and savings path for a given individual as before. To do so I loop over the value and policy functions in each time period and calculate the optimal savings and consumption paths accordingly. I first present the results under the CRRA preferences given a natural borrowing limit,

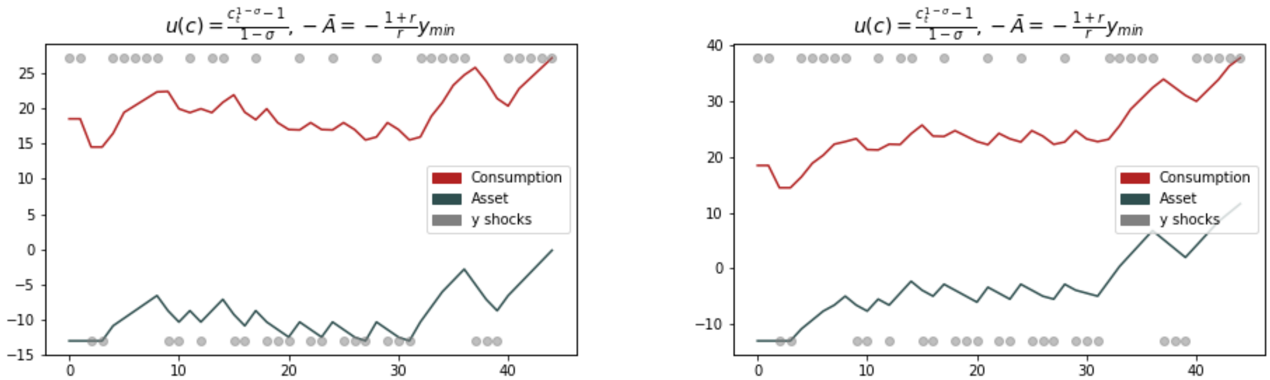


(a) $\sigma\,=\,2$,σ_y=0, γ=0



(b) $\sigma\,=\,2$, σ_y=0.1, γ=0

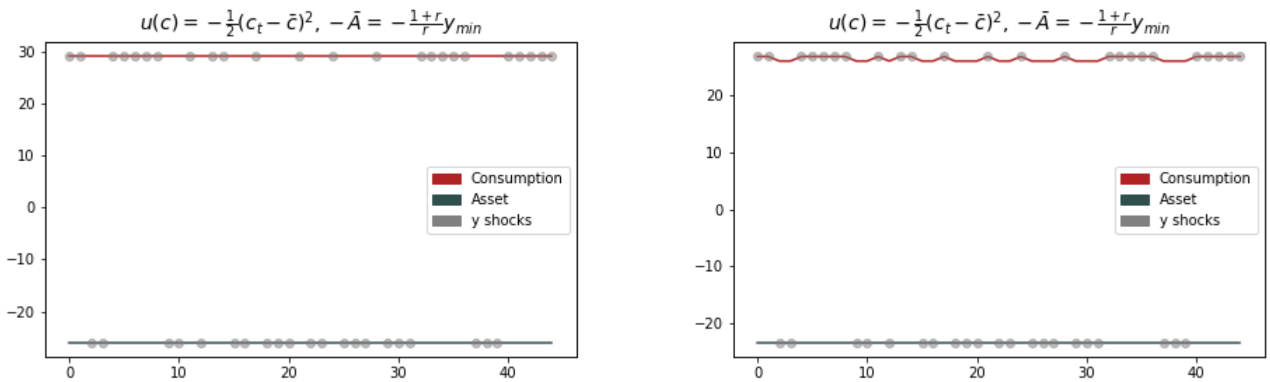
The differences between the finite and infinite models now become apparent. The agent in their later periods of life increase consumption since they are aware of the terminal condition. If the model were extend to include a bequeath motive then this would change and match behavioural observations. In the simple model presented here though it is logical since they cannot spend when they are dead.



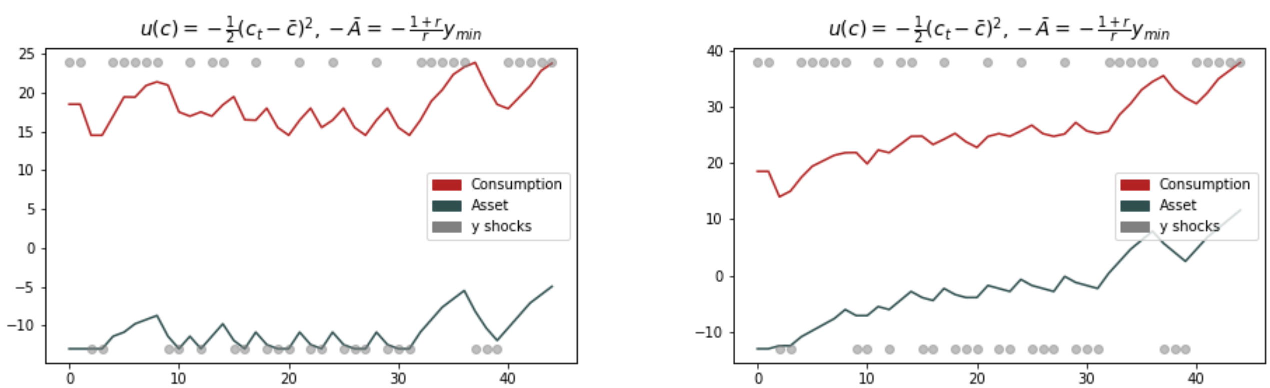
(a) σ = 2, σ_y=0.5, γ=0 (b) σ = 2, σ_y=0.5, γ=95

Figure 22: Natural borrowing limit

Over the entire period, consumption is higher than under the infinite case. Agents appear to consume and save a constant fraction of their income shocks since both curves move almost parallel to each other. Here are the equivalent results for the quadratic preferences case,



(a) σ = 2,σ_y=0, γ=0 (b) σ = 2, σ_y=0.1, γ=0



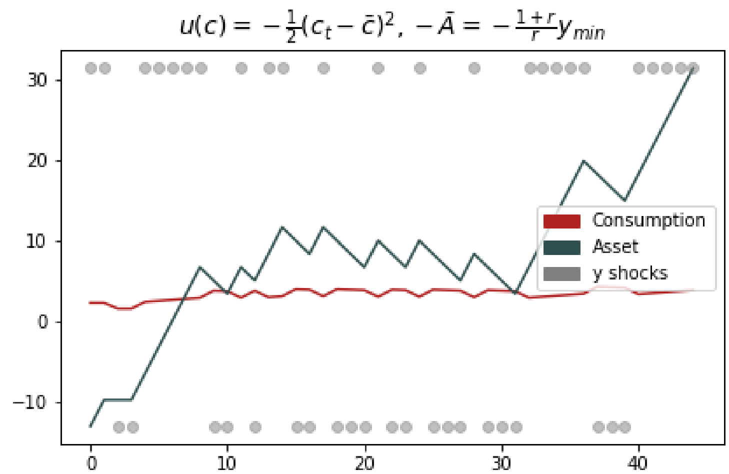
(a) σ = 2, σ_y=0.5, γ=0 (b) σ = 2, σ_y=0.5, γ=95

Figure 24: Natural borrowing limit

They follow a very similar pattern to those under the CRRA case. The results given no borrowing follow the same relationship as previously, agents suffer from not be able to smooth consumption as desired during bad shocks.

Increase the prudence

I have also run the model increasing the prudence parameter σ for the CRRA preferences specification. As an example here is a the consumption path for a given individual when $\sigma = 20$,

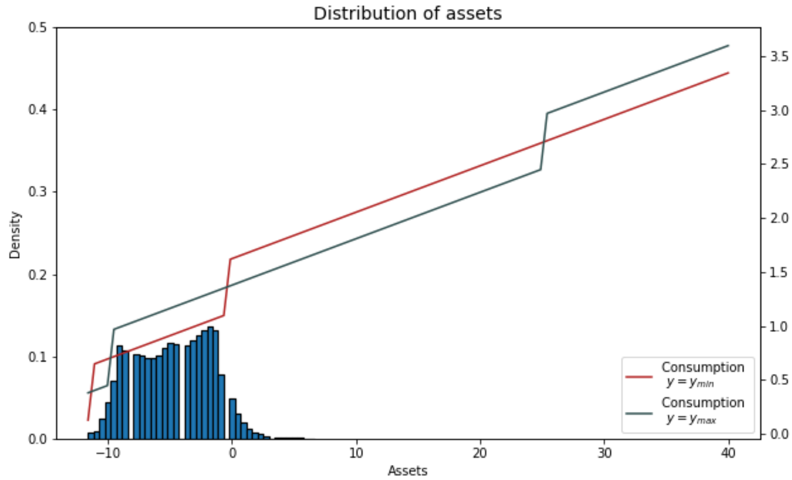


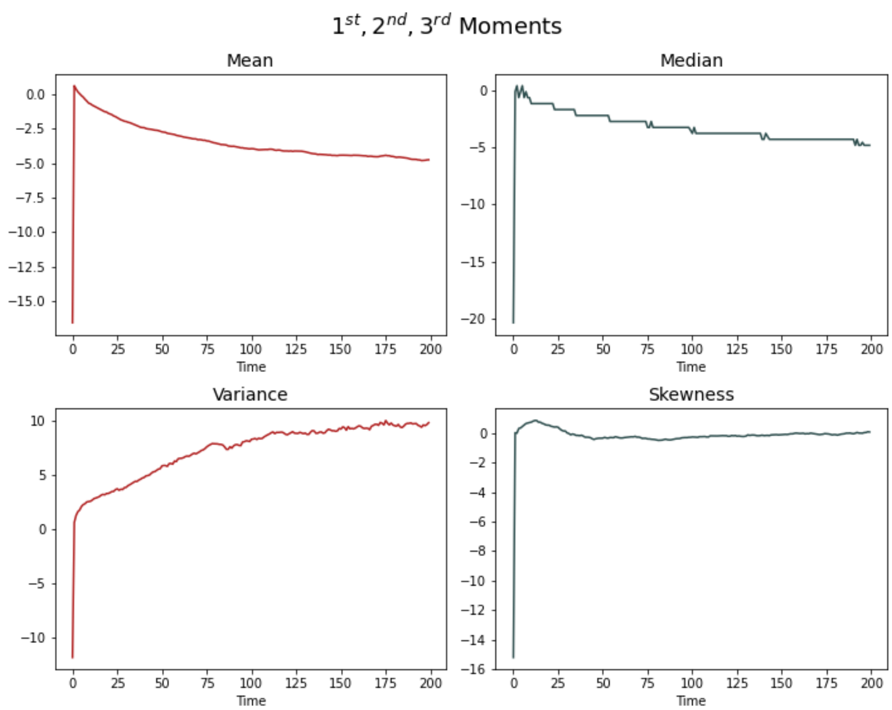
The consumption path is smoother than with a lower prudence level and the agent increases the difference between saving in a good period and therefore spending in the bad as they have increased their aversion to fluctuations in consumption patterns.

I.IV Stationary Distribution

Given the partial equilibrium result, we can then compute the long run stationary equilibrium. Using the saving and consumption policy functions I have calculated the long run equilibrium using Monte Carlo simulation. By taking 200 individuals over 1000 time periods, by repeatedly sampling from the income shock process and simulating consumption paths we converge on the invariant distribution.

The first 3 moments, the mean, variance and skewness are presented along side the median to justify that we have reached a long run equilibrium. All four moments flatten off, thus indicating the invariant distribution has been found.





This can be compare to the results found in Krueger, Mitman, and Perri 2016 who present the following baseline economy distribution,

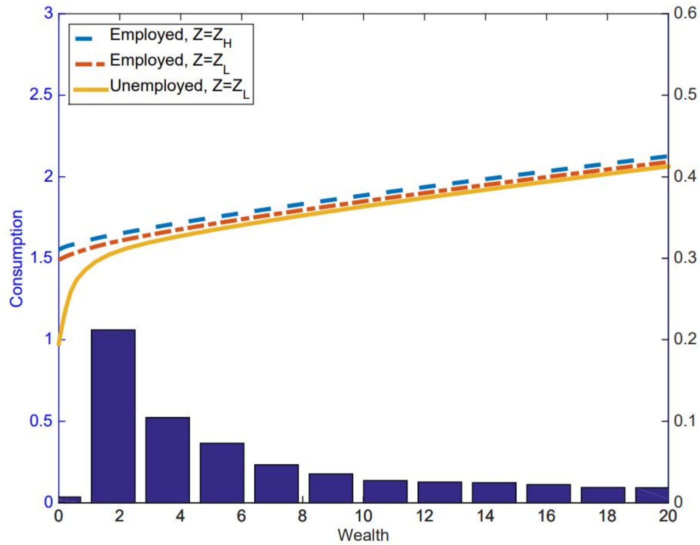


Figure 25: Caption

The summary statistics of this partial equilibrium are,

	Value
% share by	
Q1	3.83
Q2	11.88
Q3	19.93
Q4	27.73
Q5	36.63
90-95	9.18
95-99	8.29
Top 1%	2.52
Gini	0.341
Sample Size	25000

This partial equilibrium result under represents the inequality in the economy when compared to Krueger, Mitman, and Perri 2016. I will come back to this in more detail under the general equilibrium specification.

I.V General Equilibrium

The above results were all calculated in partial equilibrium, for a given interest rate and wage. This only constitutes half the answer since agents make savings decisions as a function of an endogenous interest rate which fluctuates with variation in the level of capital supplied and demanded.

I have calculated the general equilibrium in principle, following Aiyagari 1994. He utilises a bisection method to update the interest rate however I found that using discrete methods in my model was an unstable approach. To overcome this problem I defined,

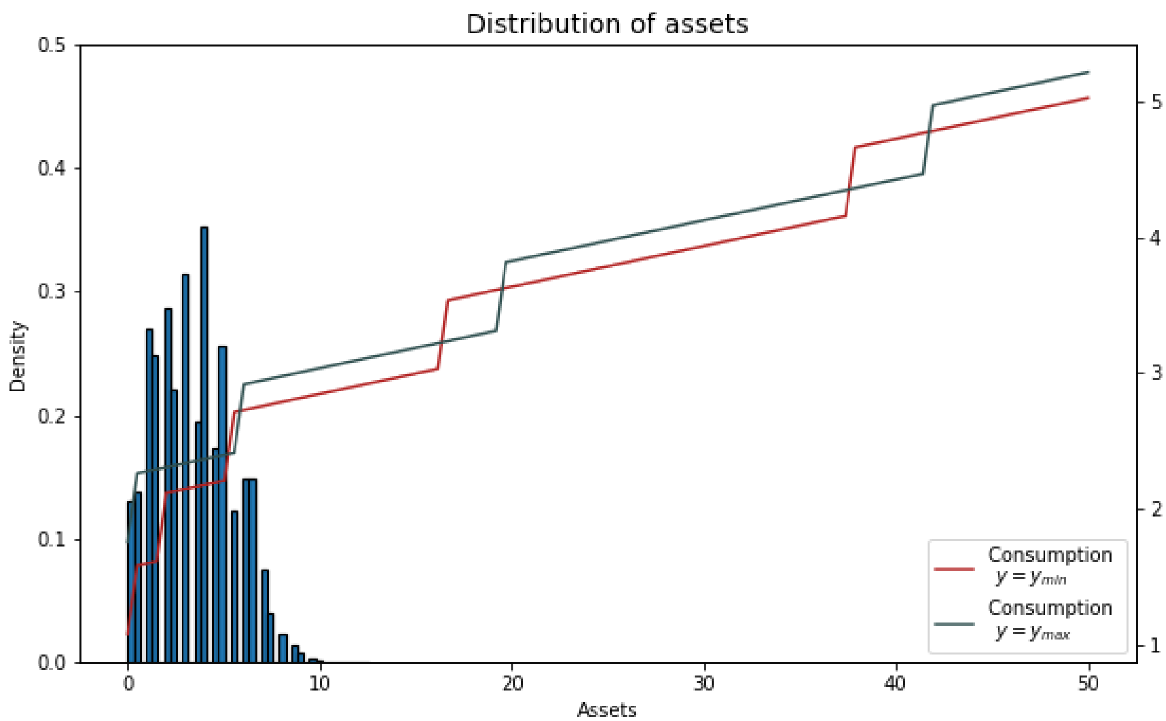
$$d(r) = K(r) - A(r)$$

and used a sympy routine solver to find the root of this function over r. This was my new interest rate.

The general equilibrium results are,

Equilibrium	Value
Interest rate	0.031
Capital supply	33.393
Capital demand	33.393
Wage rate	2.132

The equilibrium distribution is,



The summary statistics are,

	Value
% share by	
Q1	4.51
Q2	12.03
Q3	19.46
Q4	26.40
Q5	37.60
90-95	9.63
95-99	8.64
Top 1%	2.61
Gini	0.338
Sample Size	25000

and the corresponding Lorenz curve,

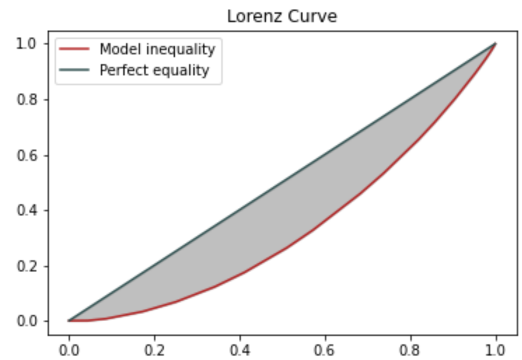


Figure 26: Gini coefficient : 0.338

The shape of the general equilibrium wealth distribution resembles the data to a reasonable extent, however when comparing the summary statistics to table 1 in Krueger, Mitman, and Perri 2016, the distribution produced under my model here does not demonstrate sufficiently high top level wealth accumulation. Particularly the top 1% own far too small a share of the total wealth.

The gini coefficient on wealth is also too low and we can see from the Lorenz curve that we are too close to the 45 line, which represents a perfectly equal economy. From the partial equilibrium, the interest rate has decreased from 3.6% to 3.1% and the economy has become more equal as the Gini has fallen slightly. Higher interest rates benefit more the rich who can earn a higher return on their capital in the market, the poor, unable to amass significant wealth levels are excluded from these gains.

Aiyagari

Aiyagari 1994 presented in his seminal paper the outline for the general approach taken in this problem set to solving such problems. In his model he has a larger range of parameter values and I have recreated selected results from table 2 in his paper.

The state space is now four dimensional not two. I have used four since I had trouble producing a system which converged using seven as Aiyagari does in his paper however the underlying principle is the same. I have discretised the underlying auto regressive process which determines the transition probabilities for each state. The transition follows the following process,

$$log(y_t) = \rho log(y_{t-1}) + \hat{\sigma}(1-\rho^2)^{\frac{1}{2}}\epsilon_t, \epsilon_t \sim \mathcal{N}(0,1)$$

where,

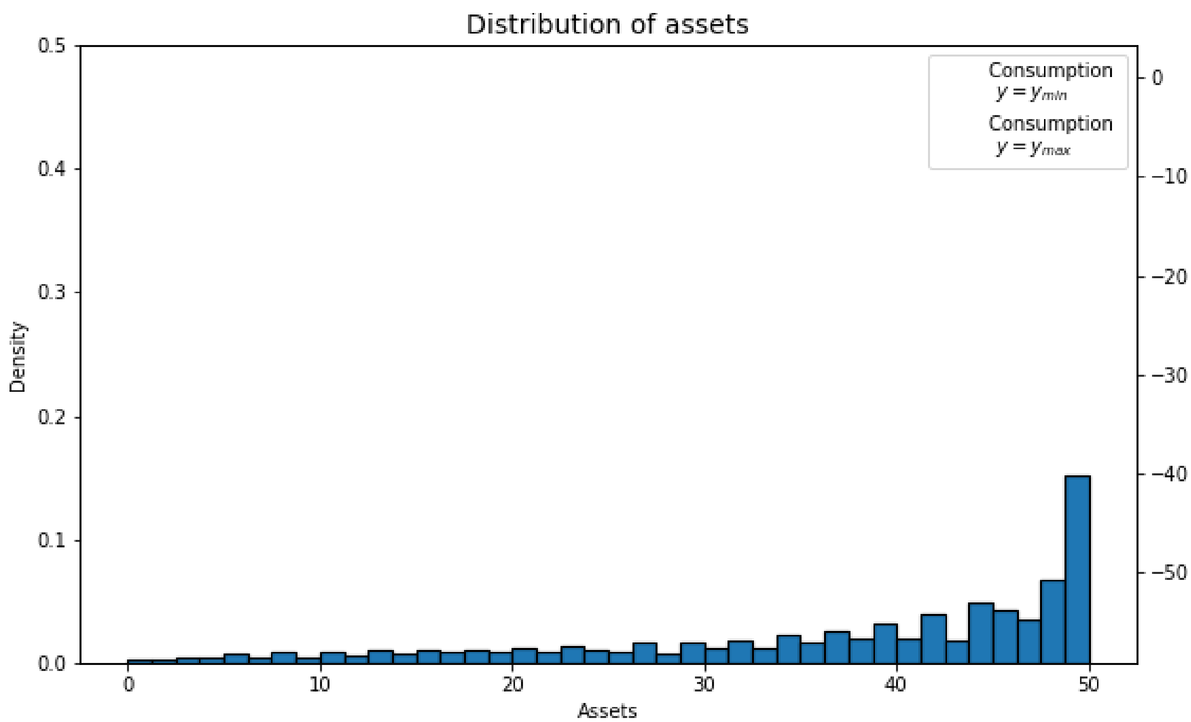
$$\hat{\sigma} = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}$$
$$\rho = \begin{bmatrix} 0 & 0.3 \end{bmatrix}$$

The results are presented in the table below where each entry is ,

$$\frac{\text{interest rate}}{\text{saving rate}}$$

$\sigma = 3$		
ρ / $\hat{\sigma}$	0.2	0.4
0	$\frac{0.0079}{0.3277}$	$\frac{0.0084}{0.3259}$
0.3	$\frac{0.0081}{0.3269}$	$\frac{0.0082}{0.3264}$

As you can see from the table, my model is not very accurate. The interest rate is far too low and the saving rate is too high which would in most cases be a contradiction. I would like to try this again and rewrite the model to attempt to get a better fit.



As you can the graph confirms I am not approximating the data well using the Aiyagari specifications. The consumption functions and moments are not well defined or stationary using the same Monte Carlo process as under only a two-chain Markov process. The increased complexity in the economy clearly requires revising my model.

Appendix

I will now present some results using a continuous representation of the value function.

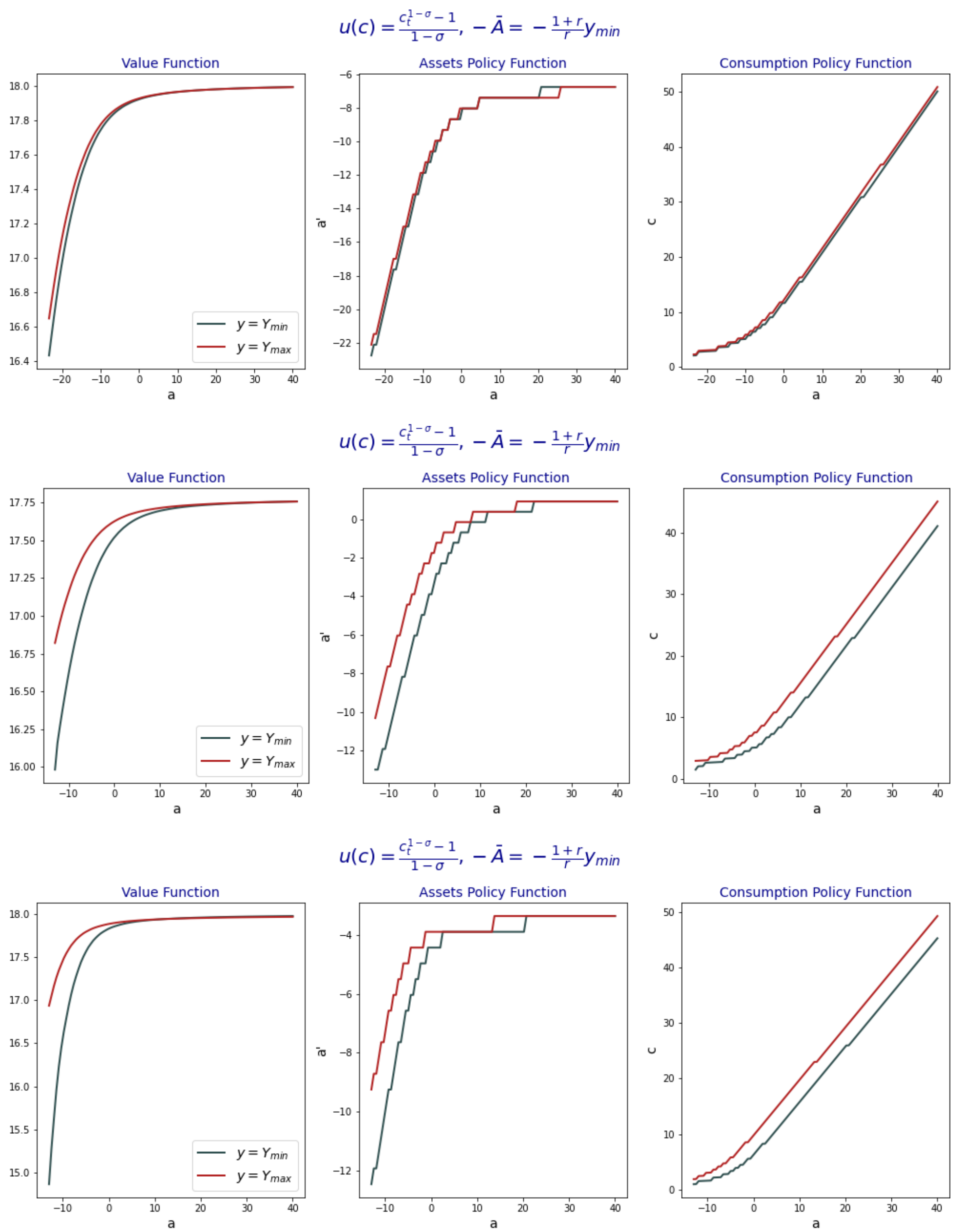
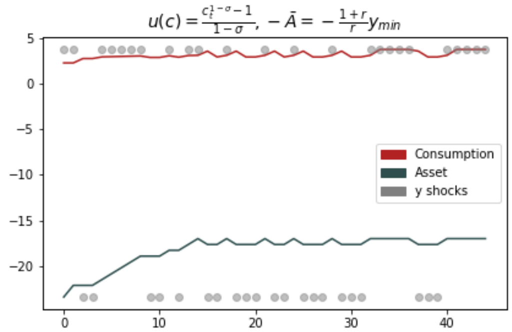
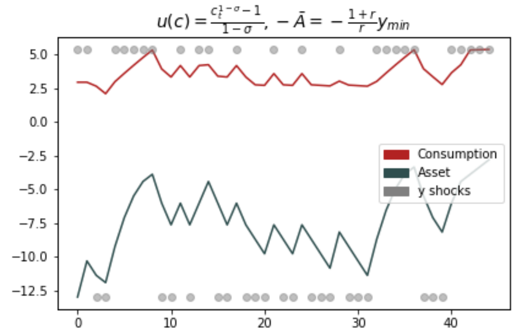


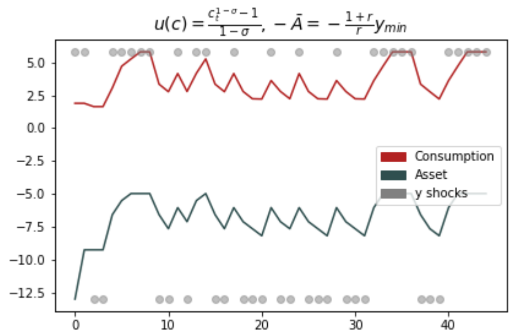
Figure 27: 1) $\sigma_y = 0.1$, 2) $\sigma_y = 0.5$, 3) $\sigma_y = 0.5, \gamma = 0.95$, Natural Borrowing



(a) sigma = 2, $sigma_y = 0$, gamma=0



(b) sigma = 2, $sigma_y = 01$, gamma=0



(c) sigma = 2, sigmay=05, gamma=95

Figure 28: Natural borrowing limit

Again repeated for the quadratic utility case

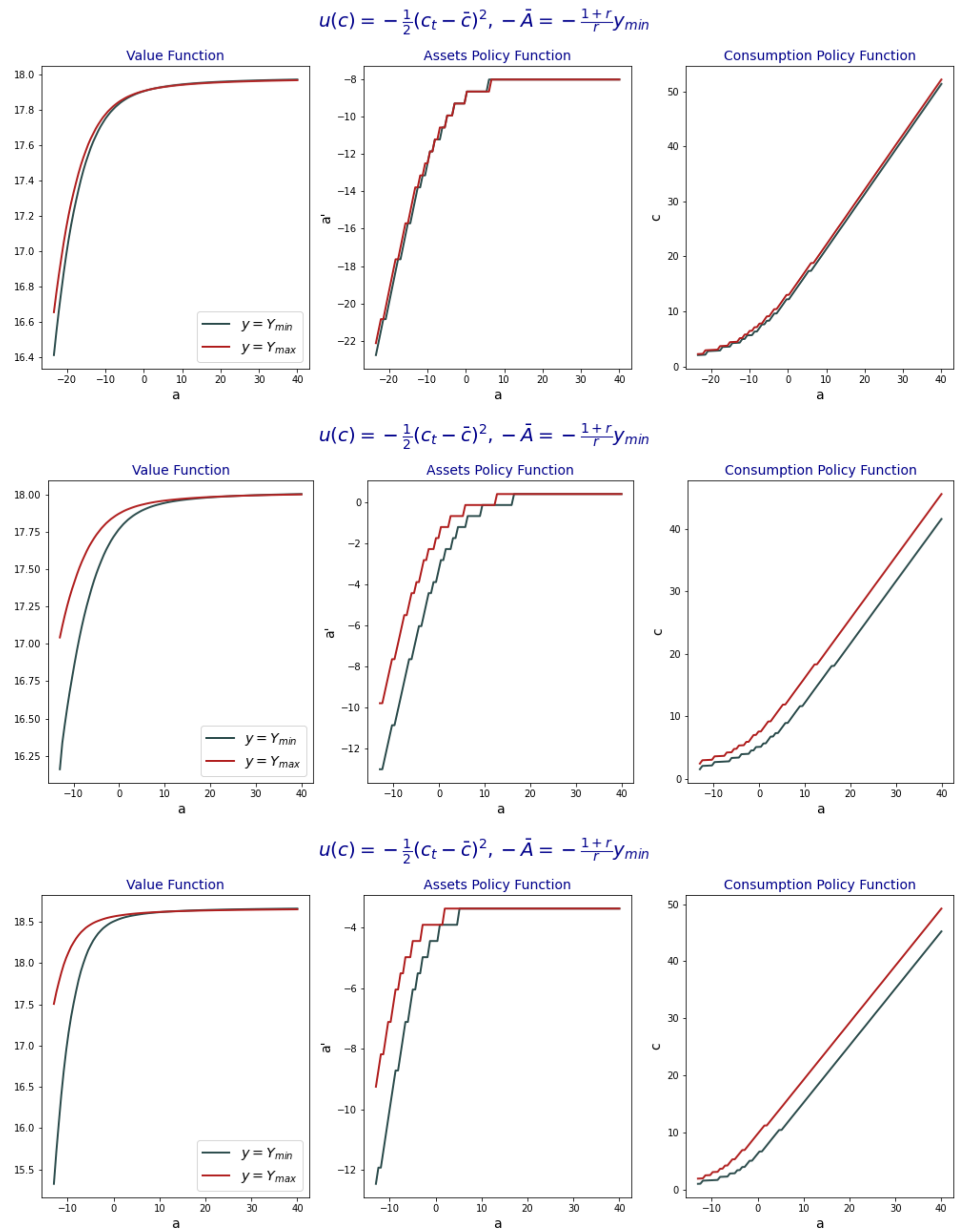
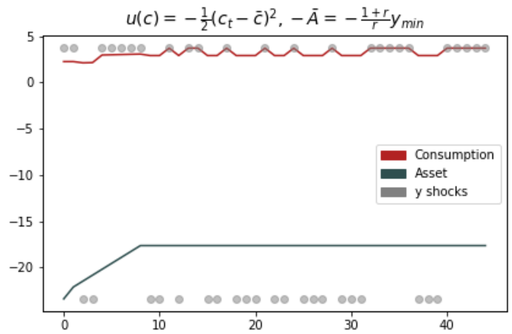
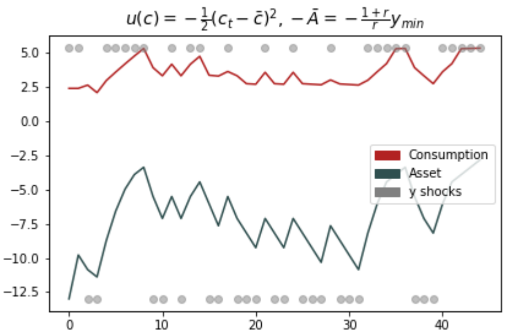


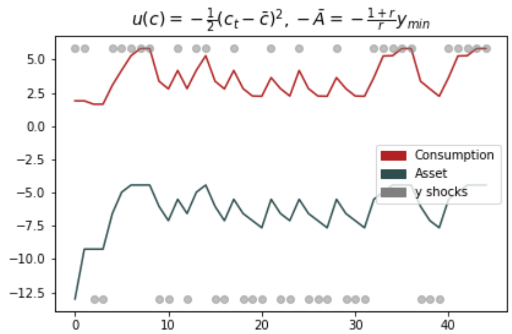
Figure 29: 1) $\sigma_y = 0.1$, 2) $\sigma_y = 0.5$, 3) $\sigma_y = 0.5, \gamma = 0.95$, Natural Borrowing



(a) sigma = 2, $sigma_y = 0$, gamma=0



(b) sigma = 2, $sigma_y = 01$, gamma=0



(c) sigma = 2, sigmay=05, gamma=95

Figure 30: Natural borrowing limit

For the Zero borrowing limit case

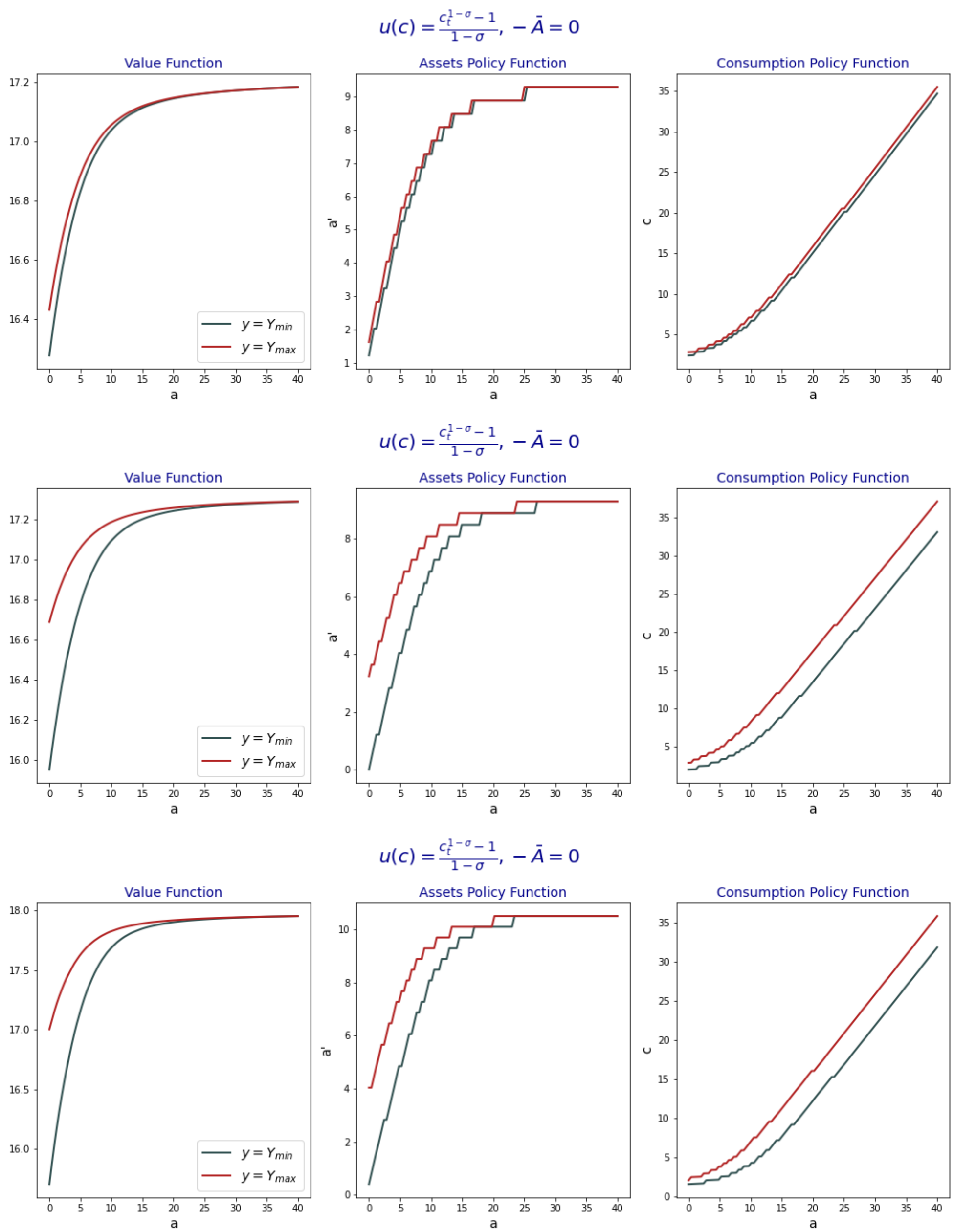
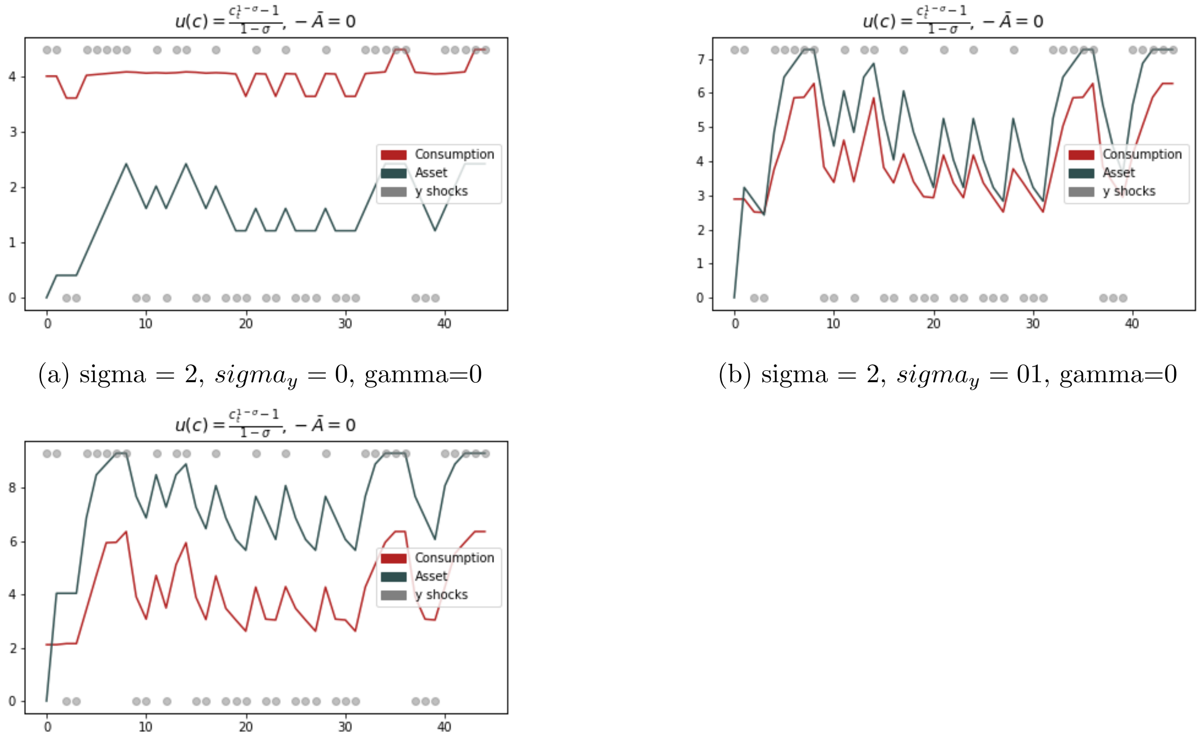


Figure 31: 1) $\sigma_y = 0.1$, 2) $\sigma_y = 0.5$, 3) $\sigma_y = 0.5, \gamma = 0.95$, Zero Borrowing



(a) sigma = 2, $sigma_y = 0$, gamma=0

(b) sigma = 2, $sigma_y = 01$, gamma=0

(c) sigma = 2, sigmay=05, gamma=95

Figure 32: Zero borrowing limit

Again repeated for the quadratic utility case

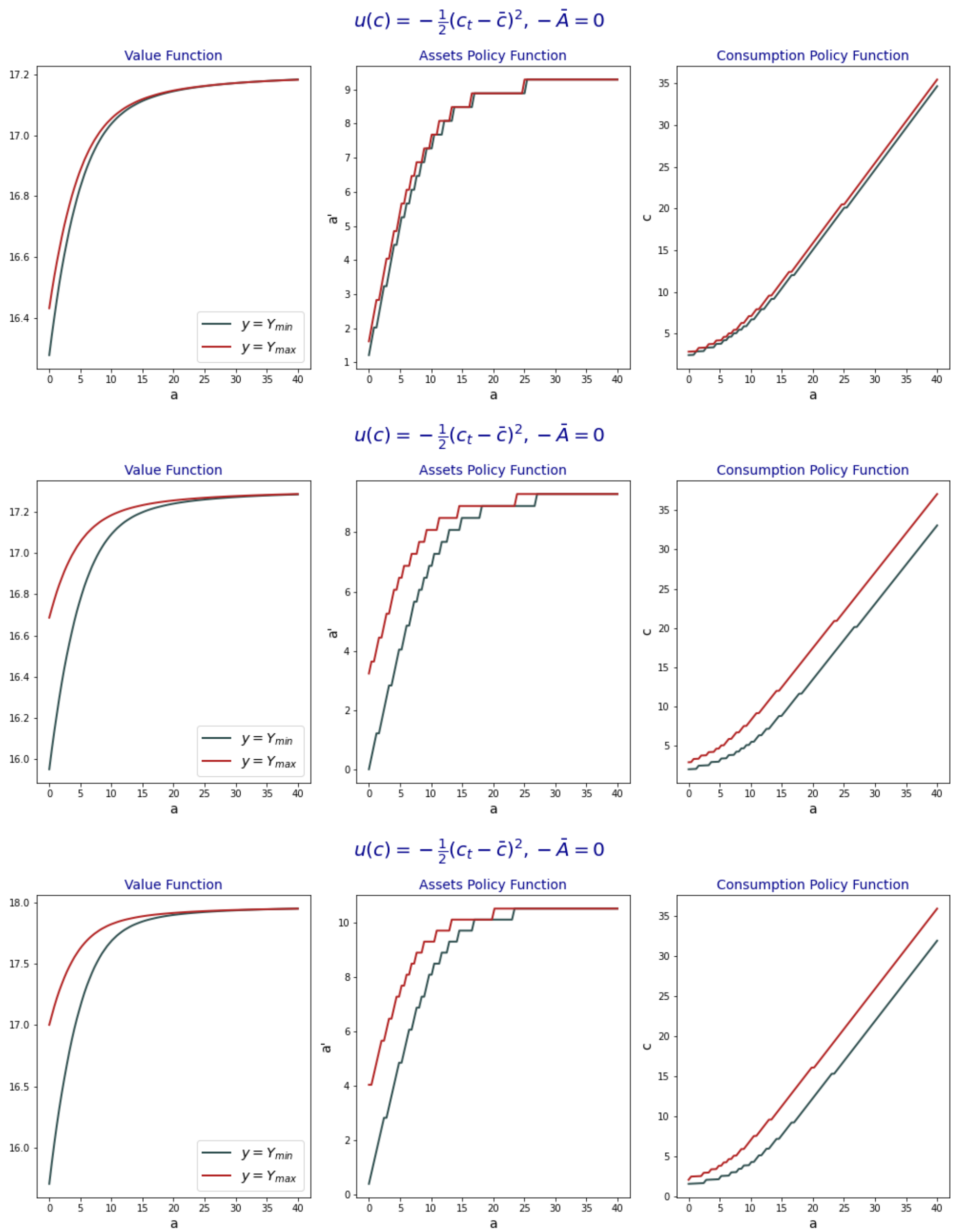
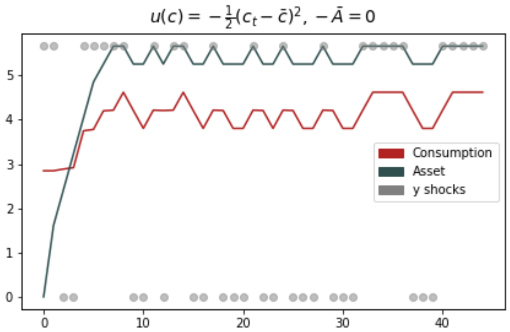
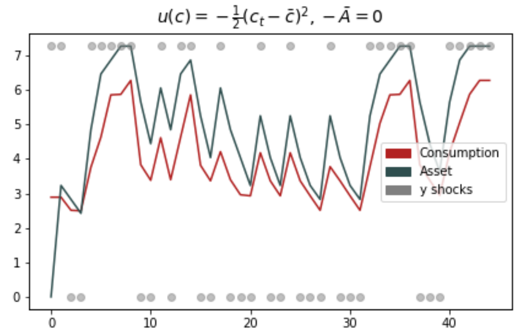


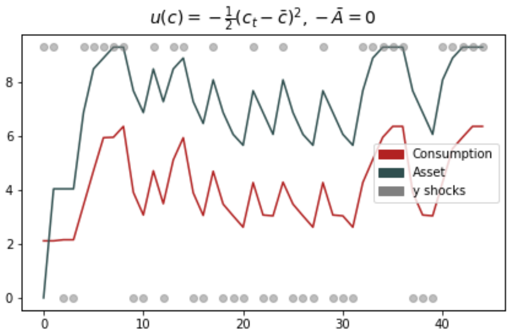
Figure 33: 1) $\sigma_y = 0.1$, 2) $\sigma_y = 0.5$, 3) $\sigma_y = 0.5, \gamma = 0.95$, Zero Borrowing



(a) sigma = 2, $sigma_y = 0$, gamma=0



(b) sigma = 2, $sigma_y = 01$, gamma=0



(c) sigma = 2, sigmay=05, gamma=95

Figure 34: Zero borrowing limit

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Aiyagari, S. Rao (1994). "Uninsured Idiosyncratic Risk and Aggregate Saving". In: *The Quarterly Journal of Economics* 109.3, pp. 659–684. ISSN: 00335533, 15314650. URL: http://www.jstor.org/stable/2118417.

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