
celestine

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Joseph Fishlock

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CONTENTS:

1	Boundary Conditions	1
2	Dimensional Params	3
3	Enthalpy Method	9
4	Flux	11
5	Forcing	13
6	Grids	15
7	Initial Conditions	17
8	Logging Config	19
9	Params	21
10	Phase Boundaries	23
11	Run Simulation	25
12	State	27
13	Velocities	29
14	Brine Drainage	33
15	Brine Channel Sink Terms	37
16	Template	39
17	Reduced Model Solver	41
18	Scipy Solver	43
19	Indices and tables	45
	Python Module Index	47
	Index	49

BOUNDARY CONDITIONS

Module to provide functions to add boundary conditions to each quantity on the centered grid that needs to be on the ghost grid for the upwind scheme.

`celestine.boundary_conditions.dissolved_gas_BCs(dissolved_gas_centers, cfg: Config)`

Add ghost cells with BCs to center quantity

`celestine.boundary_conditions.enthalpy_BCs(enthalpy_centers, cfg: Config)`

Add ghost cells with BCs to center quantity

`celestine.boundary_conditions.gas_BCs(gas_centers, cfg: Config)`

Add ghost cells with BCs to center quantity

`celestine.boundary_conditions.gas_fraction_BCs(gas_fraction_centers, cfg: Config)`

Add ghost cells with BCs to center quantity

`celestine.boundary_conditions.liquid_fraction_BCs(liquid_fraction_centers, cfg: Config)`

Add ghost cells to liquid fraction such that top and bottom boundaries take the same value as the top and bottom cell center

`celestine.boundary_conditions.liquid_salinity_BCs(liquid_salinity_centers, cfg: Config)`

Add ghost cells with BCs to center quantity

`celestine.boundary_conditions.pressure_BCs(pressure_centers, cfg: Config)`

Add ghost cells to pressure so that $W_l=0$ at $z=0$ and $p=0$ at $z=-1$

`celestine.boundary_conditions.salt_BCs(salt_centers, cfg: Config)`

Add ghost cells with BCs to center quantity

`celestine.boundary_conditions.temperature_BCs(temperature_centers, time, cfg: Config)`

Add ghost cells with BCs to center quantity

Note this needs the current time as well as top temperature is forced.

DIMENSIONAL PARAMS

Dimensional parameters required to run a simulation and convert output to dimensional variables.

The `DimensionalParams` class contains all the dimensional parameters needed to produce a simulation configuration.

The `Scales` class contains all the dimensional parameters required to convert simulation output between physical and non-dimensional variables.

```
class celestine dimensional_params.DimensionalParams(name: str, total_time_in_days: float = 365,
                                                    savefreq_in_days: float = 1, lengthscale: float
                                                    = 1, liquid_density: float = 1028, gas_density:
                                                    float = 1, saturation_concentration: float =
                                                    1e-05, ocean_salinity: float = 34,
                                                    eutectic_salinity: float = 270,
                                                    eutectic_temperature: float = - 21.1,
                                                    latent_heat: float = 334000.0,
                                                    specific_heat_capacity: float = 4184,
                                                    phase_average_conductivity: bool = False,
                                                    liquid_thermal_conductivity: float = 0.54,
                                                    solid_thermal_conductivity: float = 2.22,
                                                    salt_diffusivity: float = 0, gas_diffusivity: float
                                                    = 0, frame_velocity_dimensional: float = 0,
                                                    gravity: float = 9.81, liquid_viscosity: float =
                                                    0.00278, bubble_radius: float = 0.001,
                                                    pore_radius: float = 0.001,
                                                    pore_throat_scaling: float = 0.5,
                                                    drag_exponent: float = 6.0,
                                                    bubble_size_distribution_type: str = 'mono',
                                                    wall_drag_law_choice: str = 'power',
                                                    bubble_distribution_power: float = 1.5,
                                                    minimum_bubble_radius: float = 1e-06,
                                                    maximum_bubble_radius: float = 0.001,
                                                    porosity_threshold: bool = False,
                                                    porosity_threshold_value: float = 0.024,
                                                    brine_convection_parameterisation: bool =
                                                    False, couple_bubble_to_horizontal_flow: bool
                                                    = True, couple_bubble_to_vertical_flow: bool
                                                    = True, Rayleigh_critical: float = 40,
                                                    convection_strength: float = 0.03,
                                                    haline_contraction_coefficient: float =
                                                    0.00075, reference_permeability: float = 1e-08,
                                                    tolerable_super_saturation_fraction: float = 1,
                                                    initial_conditions_choice: str = 'uniform',
                                                    far_gas_sat: float = 1e-05, far_temp: float = -
                                                    0.81, far_bulk_salinity: float = 34,
                                                    temperature_forcing_choice: str = 'constant',
                                                    constant_top_temperature: float = - 30.32,
                                                    Barrow_top_temperature_data_choice: str =
                                                    'air', Barrow_initial_bulk_gas_in_ice: float =
                                                    0.2, offset: float = - 1.0, amplitude: float =
                                                    0.75, period: float = 4.0, I: int = 50, timestep:
                                                    float = 0.0002, regularisation: float = 1e-06,
                                                    solver: str = 'SCI')
```

Contains all dimensional parameters needed to calculate non dimensional numbers.

To see the units each input should have look at the comment next to the default value.

property B

calculate the non dimensional scale for buoyant rise of gas bubbles as

$$B = \frac{\rho_l g R_0^2 h}{3\mu\kappa}$$

property Rayleigh_salt

Calculate the haline Rayleigh number as

$$\text{Ra}_S = \frac{\rho_l g \beta \Delta S H K_0}{\kappa \mu}$$

property bubble_radius_scaled

calculate the bubble radius divided by the pore scale

$$\Lambda = R_B / R_0$$

property concentration_ratio

Calculate concentration ratio as

$$\mathcal{C} = S_i / \Delta S$$

property conductivity_ratio

Calculate the ratio of solid to liquid thermal conductivity

$$\lambda = \frac{k_s}{k_l}$$

property expansion_coefficient

calculate

$$\chi = \rho_l \xi_{\text{sat}} / \rho_g$$

property frame_velocity

calculate the frame velocity in non dimensional units

get_config()

Return a Config object for the simulation.

physical parameters and Darcy law parameters are calculated from the dimensional input. You can modify the numerical parameters and boundary conditions and forcing provided for the simulation.

get_darcy_params()

return a DarcyLawParams object

get_physical_params()

return a PhysicalParams object

get_scales()

return a Scales object used for converting between dimensional and non dimensional variables.

property lewis_gas

Calculate the lewis number for dissolved gas, return np.inf if there is no dissolved gas diffusion.

$$\text{Le}_\xi = \kappa / D_\xi$$

property lewis_salt

Calculate the lewis number for salt, return np.inf if there is no salt diffusion.

$$\text{Le}_S = \kappa / D_s$$

classmethod load(*path*)

load this object from a yaml configuration file.

property maximum_bubble_radius_scaled

calculate the bubble radius divided by the pore scale

$$\Lambda = R_B / R_0$$

property minimum_bubble_radius_scaled

calculate the bubble radius divided by the pore scale

$$\Lambda = R_B / R_0$$

property ocean_freezing_temperature

calculate salinity dependent freezing temperature using liquidus for typical ocean salinity

$$T_i = T_L(S_i) = T_E S_i / S_E$$

property salinity_difference

calculate difference between eutectic salinity and typical ocean salinity

$$\Delta S = S_E - S_i$$

save(*directory: Path*)

save this object to a yaml file in the specified directory.

The name will be the name given with `_dimensional` appended to distinguish it from a saved non-dimensional configuration.

property savefreq

calculate the save frequency in non dimensional time

property stefan_number

calculate Stefan number

$$St = L / c_p \Delta T$$

property temperature_difference

calculate

$$\Delta T = T_i - T_E$$

property thermal_diffusivity

Return thermal diffusivity in m²/s

$$\kappa = \frac{k}{\rho_l c_p}$$

property total_time

calculate the total time in non dimensional units for the simulation

```
class celestine dimensional_params.Scales(lengthscale: float, thermal_diffusivity: float, ocean_salinity:  
float, salinity_difference: float, ocean_freezing_temperature:  
float, temperature_difference: float, gas_density: float,  
saturation_concentration: float)
```

```
convert_dimensional_bulk_air_to_argon_content(dimensional_bulk_gas)
```

Convert kg/m3 of air to micromole of Argon per Liter of ice

```
convert_from_dimensional_bulk_gas(dimensional_bulk_gas)
```

Non dimensionalise bulk gas content in kg/m3

```
convert_from_dimensional_bulk_salinity(dimensional_bulk_salinity)
```

Non dimensionalise bulk salinity in g/kg

```
convert_from_dimensional_dissolved_gas(dimensional_dissolved_gas)
```

convert from dissolved gas in kg(gas)/kg(liquid) to dimensionless

```
convert_from_dimensional_grid(dimensional_grid)
```

Non dimensionalise domain depths in meters

```
convert_from_dimensional_temperature(dimensional_temperature)
```

Non dimensionalise temperature in deg C

```
convert_from_dimensional_time(dimensional_time)
```

Non dimensionalise time in days

```
convert_to_dimensional_bulk_gas(bulk_gas)
```

Convert dimensionless bulk gas content to kg/m3

```
convert_to_dimensional_bulk_salinity(bulk_salinity)
```

Convert non dimensional bulk salinity to g/kg

```
convert_to_dimensional_dissolved_gas(dissolved_gas)
```

convert from non dimensional dissolved gas to dimensional dissolved gas in kg(gas)/kg(liquid)

```
convert_to_dimensional_grid(grid)
```

Get domain depths in meters from non dimensional values

```
convert_to_dimensional_temperature(temperature)
```

get temperature in deg C from non dimensional temperature

```
convert_to_dimensional_time(time)
```

Convert non dimensional time into time in days since start of simulation

```
celestine dimensional_params.calculate_timescale_in_days(lengthscale, thermal_diffusivity)
```

calculate timescale given domain height and thermal diffusivity.

Parameters

- **lengthscale** (*float*) – domain height in m
- **thermal_diffusivity** (*float*) – thermal diffusivity in m2/s

Returns

timescale in days

```
celestine dimensional_params.calculate_velocity_scale_in_m_day(lengthscale, thermal_diffusivity)
```

calculate the velocity scale given domain height and thermal diffusivity

Parameters

- **lengthscale** (*float*) – domain height in m
- **thermal_diffusivity** (*float*) – thermal diffusivity in m²/s

Returns

velocity scale in m/day

ENTHALPY METHOD

Module containing enthalpy method to calculate state variables from bulk enthalpy, bulk salinity and bulk gas.

class celestine.enthalpy_method.**EnthalpyMethod**(*physical_params*: [PhysicalParams](#))

Template for an enthalpy method. To implement a new method overwrite the initializer to initialise the physical parameters and a suitable phase boundaries object. Then implement a calculate enthalpy method that takes a state and uses bulk enthalpy, salt and gas to return (temperature, liquid_fraction, gas_fraction, solid_fraction, liquid_salinity, dissolved_gas).

class celestine.enthalpy_method.**FullEnthalpyMethod**(*physical_params*: [PhysicalParams](#))

class celestine.enthalpy_method.**ReducedEnthalpyMethod**(*physical_params*: [PhysicalParams](#))

celestine.enthalpy_method.**get_enthalpy_method**(*cfg*)

Return the enthalpy method object required depending on solver choice

LU: Full RED: Reduced SCI: Reduced

Parameters

cfg – configuration for simulation

Module for calculating the fluxes using upwind scheme

`celestine.flux.calculate_conductive_heat_flux(state_BCs, D_g, cfg)`

Calculate conductive heat flux as

$$-\frac{\partial \theta}{\partial z}$$

or alternatively if the `phase_average_conductivity` configuration parameter is set to `True` then we use the conductivity ratio as follows

$$-[(\phi_l + \lambda \phi_s) \frac{\partial \theta}{\partial z}]$$

Parameters

- **temperature** (*Numpy Array of size I+2*) – temperature including ghost cells
- **D_g** (*Numpy Array*) – difference matrix for ghost grid
- **cfg** (`celestine.params.Config`) – Simulation configuration

Returns

conductive heat flux

`celestine.flux.calculate_diffusive_salt_flux(liquid_salinity, liquid_fraction, D_g, cfg)`

Take liquid salinity and liquid fraction on ghost grid and interpolate liquid fraction geometrically

`celestine.flux.take_forward_euler_step(quantity, flux, timestep, D_e)`

Advance the given quantity one forward Euler step using the given flux

The quantity is given on cell centers and the flux on cell edges.

Discretise the conservation equation

$$\frac{\partial Q}{\partial t} = -\frac{\partial F}{\partial z}$$

as

$$Q^{n+1} = Q^n - \Delta t \left(\frac{\partial F}{\partial z} \right)$$

FORCING

Module for providing surface temperature forcing to simulation.

Note that the barrow temperature data is read in from a file if needed by the simulation configuration.

`celestine.forcing.barrow_ocean_temperature_forcing(time, cfg)`

Take non dimensional time and return non dimensional ocean temperature at the Barrow site in 2009.

For this to work you must have created the configuration `cfg` from dimensional parameters as it must have the conversion scales object.

`celestine.forcing.barrow_temperature_forcing(time, cfg)`

Take non dimensional time and return non dimensional air/snow/ice temperature at the Barrow site in 2009.

For this to work you must have created the configuration `cfg` from dimensional parameters as it must have the conversion scales object.

`celestine.forcing.dimension_barrow_ocean_temperature_forcing(time_in_days, cfg: Config)`

Take time in days and linearly interp 2009 Barrow ocean temperature data to get temperature in degrees Celsius.

`celestine.forcing.dimension_barrow_temperature_forcing(time_in_days, cfg: Config)`

Take time in days and linearly interp 2009 Barrow air/snow/ice temperature data to get temperature in degrees Celsius.

GRIDS

Module providing functions to initialise the different grids and interpolate quantities between them.

`celestine.grids.add_ghost_cells(centers, bottom, top)`

Add specified bottom and top value to center grid

Parameters

- **centers** (*Numpy array*) – numpy array on centered grid (size I).
- **bottom** (*float*) – bottom value placed at index 0.
- **top** (*float*) – top value placed at index -1.

Returns

numpy array on ghost grid (size I+2).

`celestine.grids.average(ghosts)`

Returns arithmetic mean pairwise of first dimension of an array

This should get values on the ghost grid and returns the arithmetic average onto the edge grid

`celestine.grids.geometric(ghosts)`

Returns geometric mean of the first dimension of an array

INITIAL CONDITIONS

Module to provide initial state of bulk enthalpy, bulk salinity and bulk gas for the simulation.

`celestine.initial_conditions.apply_value_in_ice_layer(depth_of_ice, ice_value, liquid_value, grid)`

assume that top part of domain contains mushy ice of given depth and lower part of domain is liquid. This function returns output on the given grid where the ice part of the domain takes one value and the liquid a different.

This is useful for initialising the barrow simulation where we have an initial ice layer.

`celestine.initial_conditions.get_barrow_initial_conditions(cfg: Config)`

initialise domain with an initial ice layer of given temperature and bulk salinity. These values are hard coded in from Moreau paper to match barrow study. They also assume that the initial ice layer has 1/5 the saturation amount in pure liquid of dissolved gas to account for previous gas loss.

Initialise with bulk gas being (1/5) in ice and saturation in liquid. Bulk salinity is 5.92 g/kg in ice and ocean value in liquid. Enthalpy is calculated by inverting temperature relation in ice and ocean. Ice temperature is given as -8.15 degC and ocean is the far value from boundary config.

`celestine.initial_conditions.get_uniform_initial_conditions(cfg)`

Generate uniform initial solution on the ghost grid

Returns

initial solution arrays on ghost grid (enthalpy, salt, gas, pressure)

LOGGING CONFIG

Module to create logger for simulation

PARAMS

Classes containing parameters required to run a simulation

The config class contains all the parameters needed to run a simulation as well as methods to save and load this configuration to a yaml file.

```
class celestine.params.BoundaryConditionsConfig(initial_conditions_choice: str = 'uniform',
                                                far_gas_sat: float = 1.0, far_temp: float = 0.1,
                                                far_bulk_salinity: float = 0)
```

values for bottom (ocean) boundary

```
class celestine.params.Config(name: str, physical_params: PhysicalParams =
    PhysicalParams(expansion_coefficient=0.029, concentration_ratio=0.17,
    stefan_number=4.2, lewis_salt=inf, lewis_gas=inf, frame_velocity=0,
    phase_average_conductivity=False, conductivity_ratio=4.11,
    tolerable_super_saturation_fraction=1), boundary_conditions_config:
    BoundaryConditionsConfig =
    BoundaryConditionsConfig(initial_conditions_choice='uniform',
    far_gas_sat=1.0, far_temp=0.1, far_bulk_salinity=0), darcy_law_params:
    DarcyLawParams = DarcyLawParams(B=100, pore_throat_scaling=0.5,
    bubble_size_distribution_type='mono', wall_drag_law_choice='power',
    drag_exponent=6.0, bubble_radius_scaled=1.0,
    bubble_distribution_power=1.5, minimum_bubble_radius_scaled=0.001,
    maximum_bubble_radius_scaled=1, porosity_threshold=False,
    porosity_threshold_value=0.024, brine_convection_parameterisation=False,
    Rayleigh_salt=44105, Rayleigh_critical=40, convection_strength=0.03,
    couple_bubble_to_horizontal_flow=True,
    couple_bubble_to_vertical_flow=True), forcing_config: ForcingConfig =
    ForcingConfig(temperature_forcing_choice='constant',
    constant_top_temperature=- 1.5, offset=- 1.0, amplitude=0.75, period=4.0,
    Barrow_top_temperature_data_choice='air',
    Barrow_initial_bulk_gas_in_ice=0.2), numerical_params: NumericalParams
    = NumericalParams(I=50, timestep=0.0002, regularisation=1e-06,
    solver='SCI'), scales: Optional[int] = None, total_time: float = 4.0, savefreq:
    float = 0.0005)
```

contains all information needed to run a simulation and save output

this config object can be saved and loaded to a yaml file.

check_thermal_Courant_number()

Check if courant number for thermal diffusion term is low enough for explicit method and if it isn't log a warning.

```
class celestine.params.DarcyLawParams(B: float = 100, pore_throat_scaling: float = 0.5,  
                                     bubble_size_distribution_type: str = 'mono',  
                                     wall_drag_law_choice: str = 'power', drag_exponent: float = 6.0,  
                                     bubble_radius_scaled: float = 1.0, bubble_distribution_power:  
                                     float = 1.5, minimum_bubble_radius_scaled: float = 0.001,  
                                     maximum_bubble_radius_scaled: float = 1, porosity_threshold:  
                                     bool = False, porosity_threshold_value: float = 0.024,  
                                     brine_convection_parameterisation: bool = False, Rayleigh_salt:  
                                     float = 44105, Rayleigh_critical: float = 40, convection_strength:  
                                     float = 0.03, couple_bubble_to_horizontal_flow: bool = True,  
                                     couple_bubble_to_vertical_flow: bool = True)
```

non dimensional parameters for calculating liquid and gas darcy velocities

```
class celestine.params.ForcingConfig(temperature_forcing_choice: str = 'constant',  
                                     constant_top_temperature: float = - 1.5, offset: float = - 1.0,  
                                     amplitude: float = 0.75, period: float = 4.0,  
                                     Barrow_top_temperature_data_choice: str = 'air',  
                                     Barrow_initial_bulk_gas_in_ice: float = 0.2)
```

choice of top boundary (atmospheric) forcing and required parameters

load_forcing_data()

populate class attributes with barrow dimensional air temperature and time in days (with missing values filtered out).

Note the metadata explaining how to use the barrow temperature data is also in celestine/forcing_data. The indices corresponding to days and air temp are hard coded in as class variables.

```
class celestine.params.NumericalParams(I: int = 50, timestep: float = 0.0002, regularisation: float =  
                                       1e-06, solver: str = 'SCI')
```

parameters needed for discretisation and choice of numerical method

property Courant

This number must be <0.5 for stability of temperature diffusion terms

```
class celestine.params.PhysicalParams(expansion_coefficient: float = 0.029, concentration_ratio: float =  
                                       0.17, stefan_number: float = 4.2, lewis_salt: float = inf, lewis_gas:  
                                       float = inf, frame_velocity: float = 0, phase_average_conductivity:  
                                       bool = False, conductivity_ratio: float = 4.11,  
                                       tolerable_super_saturation_fraction: float = 1)
```

non dimensional numbers for the mushy layer

```
celestine.params.filter_missing_values(air_temp, days)
```

Filter out missing values are recorded as 9999

PHASE BOUNDARIES

Module for calculating the phase boundaries needed for the enthalpy method.

class celestine.phase_boundaries.**FullPhaseBoundaries**(*physical_params*: PhysicalParams)

calculates the phase boundaries when we include gas fraction in bulk enthalpy and bulk salinity.

class celestine.phase_boundaries.**PhaseBoundaries**(*physical_params*: PhysicalParams)

Template for phase boundary calculation.

Concrete implementations should use the state containing enthalpy, salt and gas to calculate the liquidus, enthalpy, solidus and saturation boundaries and then return masks for each possible phase of the system.

class celestine.phase_boundaries.**ReducedPhaseBoundaries**(*physical_params*: PhysicalParams)

calculates the phase boundaries neglecting the gas fraction so that

$$\phi_s + \phi_l = 1$$

RUN SIMULATION

Module to run the simulation on the given configuration with the appropriate solver.

`celestine.run_simulation.run_batch(list_of_cfg, directory: Path)`

Run a batch of simulations from a list of configurations.

Each simulation name is logged, as well as if it successfully runs or crashes. Output from each simulation is saved in a .npz file.

Parameters

`list_of_cfg` (`List[celestine.params.Config]`) – list of configurations

`celestine.run_simulation.solve(cfg: Config, directory: Path)`

Solve simulation choosing appropriate solver from the choice in the config.

STATE

Classes to store solution variables

State: store variables on cell centers StateBCs: add boundary conditions in ghost cells to cell center variables Solution: store primary variables at each timestep we want to save data

```
class celestine.state.Solution(cfg: Config)
```

store solution at specified times on the center grid

```
    add_state(state: State, index: int)
```

add state to stored solution at given time index

```
class celestine.state.State(cfg: Config, time, enthalpy, salt, gas, pressure=None)
```

Stores information needed for solution at one timestep on cell centers

```
class celestine.state.StateBCs(state: State)
```

Stores information needed for solution at one timestep with BCs on ghost cells as well

Note must initialise once enthalpy method has already run on State.

VELOCITIES

Module to calculate Darcy velocities.

The liquid Darcy velocity must be parameterised.

The gas Darcy velocity is calculated as `gas_fraction` x interstitial bubble velocity

Interstitial bubble velocity is found by a steady state Stoke's flow calculation. We have implemented two cases mono: All bubbles nucleate and remain the same size `power_law`: A power law bubble size distribution with fixed max and min.

`celestine.velocities.calculate_bubble_size_fraction(bubble_radius_scaled, liquid_fraction, cfg: Config)`

Takes bubble radius scaled and liquid fraction on edges and calculates the bubble size fraction as

$$\lambda = \Lambda / (\phi_l^q + \text{reg})$$

Returns the bubble size fraction on the edge grid.

`celestine.velocities.calculate_gas_interstitial_velocity(liquid_fraction, liquid_darcy_velocity, wall_drag_factor, lag_factor, cfg: Config)`

Calculate V_g from liquid fraction on the ghost frid and liquid interstitial velocity

$$V_g = \mathcal{B}(\phi_l^{2q} I_1) + U_0 I_2$$

Return V_g on edge grid

`celestine.velocities.calculate_lag_function(bubble_size_fraction)`

Calculate lag function from bubble size fraction on edge grid as

$$G(\lambda) = 1 - \lambda/2$$

for $0 < \lambda < 1$. Edge cases are given by $G(0)=1$ and $G(1) = 0.5$ for values outside this range.

`celestine.velocities.calculate_lag_integrand(bubble_size_fraction: float, cfg: Config)`

Scalar function to calculate lag integrand for polydispersive case.

Bubble size fraction is given as a scalar input to calculate

$$\lambda^{3-p} G(\lambda)$$

`celestine.velocities.calculate_liquid_darcy_velocity(liquid_fraction, liquid_salinity, center_grid, edge_grid, cfg: Config)`

Calculate liquid Darcy velocity either using brine convection parameterisation or as stagnant

Parameters

- **liquid_fraction** (*Numpy Array (size I+2)*) – liquid fraction on ghost grid
- **liquid_salinity** (*Numpy Array (size I+2)*) – liquid salinity on ghost grid
- **center_grid** (*Numpy Array of shape (I,)*) – vertical coordinates of cell centers
- **edge_grid** (*Numpy Array (size I+1)*) – Vertical coordinates of cell edges
- **cfg** (`celestine.params.Config`) – simulation configuration object

Returns

liquid darcy velocity on edge grid

`celestine.velocities.calculate_mono_lag_factor(liquid_fraction, cfg: Config)`

Take liquid fraction on the ghost grid and calculate the lag factor for a mono bubble size distribution as

$$I_2 = G(\lambda)$$

returns lag factor on the edge grid

`celestine.velocities.calculate_mono_wall_drag_factor(liquid_fraction, cfg: Config)`

Take liquid fraction on the ghost grid and calculate the wall drag factor for a mono bubble size distribution as

$$I_1 = \frac{\lambda^2}{K(\lambda)}$$

returns wall drag factor on the edge grid

`celestine.velocities.calculate_power_law_lag_factor(liquid_fraction, cfg: Config)`

Take liquid fraction on the ghost grid and calculate the lag factor for power law bubble size distribution.

Return on edge grid

`celestine.velocities.calculate_power_law_wall_drag_factor(liquid_fraction, cfg: Config)`

Take liquid fraction on the ghost grid and calculate the wall drag factor for power law bubble size distribution.

Return on edge grid

`celestine.velocities.calculate_velocities(state_BCs, cfg: Config)`

Inputs on ghost grid, outputs on edge grid

`celestine.velocities.calculate_volume_integrand(bubble_size_fraction: float, cfg: Config)`

Scalar function to calculate the integrand for volume under a power law bubble size distribution given as

$$\lambda^{3-p}$$

in terms of the bubble size fraction.

`celestine.velocities.calculate_wall_drag_function(bubble_size_fraction, cfg: Config)`

Calculate wall drag function from bubble size fraction on edge grid as

$$\frac{1}{K(\lambda)} = (1 - \lambda)^r$$

in the power law case or in the Haberman case from the paper

$$\frac{1}{K(\lambda)} = \frac{1 - 1.5\lambda + 1.5\lambda^5 - \lambda^6}{1 + 1.5\lambda^5}$$

for $0 < \lambda < 1$. Edge cases are given by $K(0)=1$ and $K(1) = 0$ for values outside this range.

`celestine.velocities.calculate_wall_drag_integrand`(*bubble_size_fraction*: float, *cfg*: [Config](#))

Scalar function to calculate wall drag integrand for polydispersive case.

Bubble size fraction is given as a scalar input to calculate

$$\frac{\lambda^{5-p}}{K(\lambda)}$$

where the wall drag enhancement function K can be given by a power law fit or taken from the Haberman paper.

BRINE DRAINAGE

Module to calculate the Rees Jones and Worster 2014 parameterisation for brine convection velocity and the strenght of the sink term.

Exports the functions:

`calculate_brine_convection_liquid_velocity` To be used in velocities module when using brine convection parameterisation.

`calculate_brine_channel_sink` To be used to add sink terms to conservation equations when using brine convection parameterisation.

`celestine.brine_drainage.calculate_Rayleigh`(*cell_centers*, *edge_grid*, *liquid_salinity*, *liquid_fraction*, *cfg*: [Config](#))

Calculate the local Rayleigh number for brine convection as

$$\text{Ra}(z) = \text{Ra}_S K(z)(z + h)\Theta_l$$

Parameters

- **cell_centers** (*Numpy Array shape (I,)*) – The vertical coordinates of cell centers.
- **edge_grid** (*Numpy Array (size I+1)*) – The vertical coordinate positions of the edge grid.
- **liquid_salinity** (*Numpy Array shape (I,)*) – liquid salinity on center grid
- **liquid_fraction** (*Numpy Array (size I)*) – liquid fraction on center grid
- **cfg** ([celestine.params.Config](#)) – Configuration object for the simulation.

Returns

Array of shape (I,) of Rayleigh number at cell centers

`celestine.brine_drainage.calculate_brine_channel_sink`(*liquid_fraction*, *liquid_salinity*, *center_grid*, *edge_grid*, *cfg*: [Config](#))

Calculate the sink term due to brine channels.

$$\text{sink} = \mathcal{A}$$

in the convecting region. Zero elsewhere.

NOTE: If no ice is present or if no convecting region exists returns zero

Parameters

- **liquid_fraction** (*Numpy Array of shape (I,)*) – liquid fraction on center grid
- **liquid_salinity** (*Numpy Array of shape (I,)*) – liquid salinity on center grid

- **center_grid** (*Numpy Array of shape (I,)*) – vertical coordinate of center grid
- **edge_grid** (*Numpy Array of shape (I+1,)*) – Vertical coordinates of cell edges
- **cfg** (`celestine.params.Config`) – Configuration object for the simulation.

Returns

Strength of the sink term due to brine channels on the center grid.

`celestine.brine_drainage.calculate_brine_channel_strength(Rayleigh_number, ice_depth, convecting_region_height, cfg: Config)`

Calculate the brine channel strength in the convecting region as

$$\mathcal{A} = \frac{\alpha \text{Ra}_e}{(h + z_c)^2}$$

the effective Rayleigh number multiplied by a tuning parameter (Rees Jones and Worster 2014) over the convecting region thickness squared.

Parameters

- **Rayleigh_number** (*Numpy Array of shape (I,)*) – local Rayleigh number on center grid
- **ice_depth** (*float*) – depth of ice (positive)
- **convecting_region_height** (*float*) – position of the convecting region boundary (negative)
- **cfg** (`celestine.params.Config`) – Configuration object for the simulation.

Returns

Brine channel strength parameter

`celestine.brine_drainage.calculate_brine_convection_liquid_velocity(liquid_fraction, liquid_salinity, center_grid, edge_grid, cfg: Config)`

Calculate the vertical liquid Darcy velocity from Rees Jones and Worster 2014

$$W_l = \mathcal{A}(z_c - z)$$

in the convecting region. The velocity is stagnant above the convecting region. The velocity is constant in the liquid region and continuous at the interface.

NOTE: If no ice is present or if no convecting region exists returns zero velocity

Parameters

- **liquid_fraction** (*Numpy Array of shape (I,)*) – liquid fraction on center grid
- **liquid_salinity** (*Numpy Array of shape (I,)*) – liquid salinity on center grid
- **center_grid** (*Numpy Array of shape (I,)*) – vertical coordinate of center grid
- **edge_grid** (*Numpy Array of shape (I+1,)*) – Vertical coordinates of cell edges
- **cfg** (`celestine.params.Config`) – Configuration object for the simulation.

Returns

Liquid darcy velocity on the edge grid.

`celestine.brine_drainage.calculate_ice_ocean_boundary_depth(liquid_fraction, edge_grid)`

Calculate the depth of the ice ocean boundary as the edge position of the first cell from the bottom to be not completely liquid. I.e the first time the liquid fraction goes below 1.

If the ice has made it to the bottom of the domain raise an error.

If the domain is completely liquid set h=0.

NOTE: depth is a positive quantity and our grid coordinate increases from -1 at the bottom of the domain to 0 at the top.

Parameters

- **liquid_fraction** (*Numpy Array (size I)*) – liquid fraction on center grid
- **edge_grid** (*Numpy Array (size I+1)*) – The vertical coordinate positions of the edge grid.

Returns

positive depth value of ice ocean interface

`celestine.brine_drainage.calculate_integrated_mean_permeability(z, liquid_fraction, ice_depth, cell_centers, cfg: Config)`

Calculate the harmonic mean permeability from the base of the ice up to the cell containing the specified z value using the expression of ReesJones2014.

$$K(z) = \left(\frac{1}{h+z} \int_{-h}^z \frac{1}{\Pi(\phi_l(z'))} dz' \right)^{-1}$$

Parameters

- **z** (*float*) – height to integrate permeability up to
- **liquid_fraction** (*Numpy Array shape (I,)*) – liquid fraction on the center grid
- **ice_depth** (*float*) – positive depth position of ice ocean interface
- **cell_centers** (*Numpy Array of shape (I,)*) – cell center positions
- **cfg** (`celestine.params.Config`) – Configuration object for the simulation.

Returns

permeability averaged from base of the ice up to given z value

`celestine.brine_drainage.calculate_permeability(liquid_fraction, cfg: Config)`

Calculate the absolute permeability as a function of liquid fraction

$$\Pi(\phi_l) = \phi_l^3$$

Alternatively if the porosity threshold flag is true

$$\Pi(\phi_l) = \phi_l^2(\phi_l - \phi_c)$$

Parameters

- **liquid_fraction** (*Numpy Array*) – liquid fraction
- **cfg** (`celestine.params.Config`) – Configuration object for the simulation.

Returns

permeability on the same grid as liquid fraction

`celestine.brine_drainage.get_convecting_region_height(Rayleigh_number, edge_grid, cfg: Config)`

Calculate the height of the convecting region as the top edge of the highest cell in the domain for which the quantity

$$Ra(z) - Ra_c$$

is greater than or equal to zero.

NOTE: if no convecting region exists return `np.NaN`

Parameters

- **Rayleigh_number** (*Numpy Array of shape (I,)*) – local rayleigh number on center grid
- **edge_grid** (*Numpy Array (size I+1)*) – The vertical coordinate positions of the edge grid.
- **cfg** (`celestine.params.Config`) – Configuration object for the simulation.

Returns

Edge grid value at convecting boundary.

`celestine.brine_drainage.get_effective_Rayleigh_number(Rayleigh_number, cfg: Config)`

Calculate the effective Rayleigh Number as the maximum of

$$Ra(z) - Ra_c$$

in the convecting region.

NOTE: if no convecting region exists returns 0.

Parameters

- **Rayleigh_number** (*Numpy Array of shape (I,)*) – local rayleigh number on center grid
- **cfg** (`celestine.params.Config`) – Configuration object for the simulation.

Returns

Effective Rayleigh number.

BRINE CHANNEL SINK TERMS

Module to calculate the sink terms for conservation equations when using the Rees Jones and Worster 2014 brine drainage parameterisation.

These terms represent loss through the brine channels and need to be added in the convecting region when using this parameterisation

TEMPLATE

Template for a solver concrete solvers should inherit and overwrite required methods

```
class celestine.solvers.template.SolverTemplate(cfg: Config)
```

```
    generate_initial_solution()
```

Generate initial solution on the ghost grid

Returns

initial solution arrays on ghost grid (enthalpy, salt, gas, pressure)

```
    pre_solve_checks()
```

Optionally implement this method if you want to check anything before running the solver.

For example to check the timestep and grid step satisfy some constraint.

```
    abstract take_timestep(state: State) → State
```

advance enthalpy, salt, gas and pressure to the next timestep.

Note as of 2023-05-17 removed ability to have adaptive timestepping for simplicity.

Parameters

state (celestine.solvers.template.State) – object containing current time, enthalpy, salt, gas, pressure and surface temperature.

Returns

state of system (new enthalpy, salt, gas and pressure) after one timestep.

REDUCED MODEL SOLVER

class celestine.solvers.reduced_solver.ReducedSolver(*cfg*: Config)

Take timestep using forward Euler upwind scheme using reduced model.

pre_solve_checks()

Optionally implement this method if you want to check anything before running the solver.

For example to check the timestep and grid step satisfy some constraint.

take_timestep(*state*: State)

advance enthalpy, salt, gas and pressure to the next timestep.

Note as of 2023-05-17 removed ability to have adaptive timestepping for simplicity.

Parameters

state (celestine.solvers.template.State) – object containing current time, enthalpy, salt, gas, pressure and surface temperature.

Returns

state of system (new enthalpy, salt, gas and pressure) after one timestep.

celestine.solvers.reduced_solver.prevent_gas_rise_into_saturated_cell(*Vg*, *state_BCs*: StateBCs)

Modify the gas interstitial velocity to prevent bubble rise into a cell which is already theoretically saturated with gas.

From the state with boundary conditions calculate the gas and solid fraction in the cells (except at lower ghost cell). If any of these are such that there is more gas fraction than pore space available then set gas interstitial velocity to zero on the edge below. Make sure the very top boundary velocity is not changed as we want to always allow flux to the atmosphere regardless of the boundary conditions imposed.

Parameters

- **Vg** (Numpy array (size $I+1$)) – gas interstitial velocity on cell edges
- **state_BCs** (celestine.state.StateBCs) – state of system with boundary conditions

Returns

filtered gas interstitial velocities on edges to prevent gas rise into a fully gas saturated cell

SCIPY SOLVER

class `celestine.solvers.scipy.ScipySolver`(*cfg*: [Config](#))

Solve reduced model using `scipy solve_ivp` using RK23 solver. This is the “SCI” solver option.

Impose a maximum timestep constraint using `courant` number for thermal diffusion as this is an explicit method.

This solver uses adaptive timestepping which makes it a good choice for running simulations with large buoyancy driven gas bubble velocities and we save the output at intervals given by the `savefreq` parameter in configuration.

The interface of this class is a little different as we overwrite the `solve` method from the template and must provide a function to calculate the ode forcing for `solve_ivp`. However the `solve` function still saves the data in the same format using the `celestine.state.Solution` class.

take_timestep(*state*: [State](#))

advance enthalpy, salt, gas and pressure to the next timestep.

Note as of 2023-05-17 removed ability to have adaptive timestepping for simplicity.

Parameters

state (`celestine.solvers.template.State`) – object containing current time, enthalpy, salt, gas, pressure and surface temperature.

Returns

state of system (new enthalpy, salt, gas and pressure) after one timestep.

INDICES AND TABLES

- `genindex`
- `modindex`
- `search`

PYTHON MODULE INDEX

C

- `celestine.boundary_conditions`, 1
- `celestine.brine_channel_sink_terms`, 37
- `celestine.brine_drainage`, 33
- `celestine.dimensional_params`, 3
- `celestine.enthalpy_method`, 9
- `celestine.flux`, 11
- `celestine.forcing`, 13
- `celestine.grids`, 15
- `celestine.initial_conditions`, 17
- `celestine.logging_config`, 19
- `celestine.params`, 21
- `celestine.phase_boundaries`, 23
- `celestine.run_simulation`, 25
- `celestine.solvers.reduced_solver`, 41
- `celestine.solvers.scipy`, 43
- `celestine.solvers.template`, 39
- `celestine.state`, 27
- `celestine.velocities`, 29

A

add_ghost_cells() (in module *celestine.grids*), 15
 add_state() (*celestine.state.Solution* method), 27
 apply_value_in_ice_layer() (in module *celestine.initial_conditions*), 17
 average() (in module *celestine.grids*), 15

B

B (*celestine.dimensionals_params.DimensionalsParams* property), 4
 barrow_ocean_temperature_forcing() (in module *celestine.forcing*), 13
 barrow_temperature_forcing() (in module *celestine.forcing*), 13
 BoundaryConditionsConfig (class in *celestine.params*), 21
 bubble_radius_scaled (*celestine.dimensionals_params.DimensionalsParams* property), 5

C

calculate_brine_channel_sink() (in module *celestine.brine_drainage*), 33
 calculate_brine_channel_strength() (in module *celestine.brine_drainage*), 34
 calculate_brine_convection_liquid_velocity() (in module *celestine.brine_drainage*), 34
 calculate_bubble_size_fraction() (in module *celestine.velocities*), 29
 calculate_conductive_heat_flux() (in module *celestine.flux*), 11
 calculate_diffusive_salt_flux() (in module *celestine.flux*), 11
 calculate_gas_interstitial_velocity() (in module *celestine.velocities*), 29
 calculate_ice_ocean_boundary_depth() (in module *celestine.brine_drainage*), 34
 calculate_integrated_mean_permeability() (in module *celestine.brine_drainage*), 35
 calculate_lag_function() (in module *celestine.velocities*), 29

calculate_lag_integrand() (in module *celestine.velocities*), 29
 calculate_liquid_darcy_velocity() (in module *celestine.velocities*), 29
 calculate_mono_lag_factor() (in module *celestine.velocities*), 30
 calculate_mono_wall_drag_factor() (in module *celestine.velocities*), 30
 calculate_permeability() (in module *celestine.brine_drainage*), 35
 calculate_power_law_lag_factor() (in module *celestine.velocities*), 30
 calculate_power_law_wall_drag_factor() (in module *celestine.velocities*), 30
 calculate_Rayleigh() (in module *celestine.brine_drainage*), 33
 calculate_timescale_in_days() (in module *celestine.dimensionals_params*), 7
 calculate_velocities() (in module *celestine.velocities*), 30
 calculate_velocity_scale_in_m_day() (in module *celestine.dimensionals_params*), 7
 calculate_volume_integrand() (in module *celestine.velocities*), 30
 calculate_wall_drag_function() (in module *celestine.velocities*), 30
 calculate_wall_drag_integrand() (in module *celestine.velocities*), 31
 celestine.boundary_conditions module, 1
 celestine.brine_channel_sink_terms module, 37
 celestine.brine_drainage module, 33
 celestine.dimensionals_params module, 3
 celestine.enthalpy_method module, 9
 celestine.flux module, 11
 celestine.forcing module, 13

celestine.grids
 module, 15
 celestine.initial_conditions
 module, 17
 celestine.logging_config
 module, 19
 celestine.params
 module, 21
 celestine.phase_boundaries
 module, 23
 celestine.run_simulation
 module, 25
 celestine.solvers.reduced_solver
 module, 41
 celestine.solvers.scipy
 module, 43
 celestine.solvers.template
 module, 39
 celestine.state
 module, 27
 celestine.velocities
 module, 29
 check_thermal_Courant_number() (celestine.params.Config method), 21
 concentration_ratio (celestine dimensional_params.DimensionalParams property), 5
 conductivity_ratio (celestine dimensional_params.DimensionalParams property), 5
 Config (class in celestine.params), 21
 convert_dimensional_bulk_air_to_argon_content() (celestine dimensional_params.Scales method), 7
 convert_from_dimensional_bulk_gas() (celestine dimensional_params.Scales method), 7
 convert_from_dimensional_bulk_salinity() (celestine dimensional_params.Scales method), 7
 convert_from_dimensional_dissolved_gas() (celestine dimensional_params.Scales method), 7
 convert_from_dimensional_grid() (celestine dimensional_params.Scales method), 7
 convert_from_dimensional_temperature() (celestine dimensional_params.Scales method), 7
 convert_from_dimensional_time() (celestine dimensional_params.Scales method), 7
 convert_to_dimensional_bulk_gas() (celestine dimensional_params.Scales method), 7
 convert_to_dimensional_bulk_salinity() (celestine dimensional_params.Scales method), 7
 convert_to_dimensional_dissolved_gas() (celestine dimensional_params.Scales method), 7
 convert_to_dimensional_grid() (celestine dimensional_params.Scales method), 7
 convert_to_dimensional_temperature() (celestine dimensional_params.Scales method), 7
 convert_to_dimensional_time() (celestine dimensional_params.Scales method), 7
 Courant (celestine.params.NumericalParams property), 22

D
 DarcyLawParams (class in celestine.params), 21
 dimensional_barrow_ocean_temperature_forcing() (in module celestine.forcing), 13
 dimensional_barrow_temperature_forcing() (in module celestine.forcing), 13
 DimensionalParams (class in celestine dimensional_params), 3
 dissolved_gas_BCs() (in module celestine.boundary_conditions), 1

E
 enthalpy_BCs() (in module celestine.boundary_conditions), 1
 EnthalpyMethod (class in celestine.enthalpy_method), 9
 expansion_coefficient (celestine dimensional_params.DimensionalParams property), 5

F
 filter_missing_values() (in module celestine.params), 22
 ForcingConfig (class in celestine.params), 22
 frame_velocity (celestine dimensional_params.DimensionalParams property), 5
 FullEnthalpyMethod (class in celestine.enthalpy_method), 9
 FullPhaseBoundaries (class in celestine.phase_boundaries), 23

G
 gas_BCs() (in module celestine.boundary_conditions), 1
 gas_fraction_BCs() (in module celestine.boundary_conditions), 1
 generate_initial_solution() (celestine.solvers.template.SolverTemplate method), 39
 geometric() (in module celestine.grids), 15

get_barrow_initial_conditions() (in module celestine.initial_conditions), 17
 get_config() (celestine dimensional_params.DimensionalParams method), 5
 get_convecting_region_height() (in module celestine.brine_drainage), 35
 get_darcy_law_params() (celestine dimensional_params.DimensionalParams method), 5
 get_effective_Rayleigh_number() (in module celestine.brine_drainage), 36
 get_enthalpy_method() (in module celestine.enthalpy_method), 9
 get_physical_params() (celestine dimensional_params.DimensionalParams method), 5
 get_scales() (celestine dimensional_params.DimensionalParams method), 5
 get_uniform_initial_conditions() (in module celestine.initial_conditions), 17

L

lewis_gas (celestine dimensional_params.DimensionalParams property), 5
 lewis_salt (celestine dimensional_params.DimensionalParams property), 5
 liquid_fraction_BCs() (in module celestine.boundary_conditions), 1
 liquid_salinity_BCs() (in module celestine.boundary_conditions), 1
 load() (celestine dimensional_params.DimensionalParams class method), 5
 load_forcing_data() (celestine.params.ForcingConfig method), 22

M

maximum_bubble_radius_scaled (celestine dimensional_params.DimensionalParams property), 6
 minimum_bubble_radius_scaled (celestine dimensional_params.DimensionalParams property), 6
 module

- celestine.boundary_conditions, 1
- celestine.brine_channel_sink_terms, 37
- celestine.brine_drainage, 33
- celestine dimensional_params, 3
- celestine.enthalpy_method, 9
- celestine.flux, 11
- celestine.forcing, 13
- celestine.grids, 15
- celestine.initial_conditions, 17
- celestine.logging_config, 19
- celestine.params, 21
- celestine.phase_boundaries, 23
- celestine.run_simulation, 25
- celestine solvers.reduced_solver, 41
- celestine solvers.scipy, 43
- celestine solvers.template, 39
- celestine.state, 27
- celestine.velocities, 29

N

NumericalParams (class in celestine.params), 22

O

ocean_freezing_temperature (celestine dimensional_params.DimensionalParams property), 6

P

PhaseBoundaries (class in celestine.phase_boundaries), 23
 PhysicalParams (class in celestine.params), 22
 pre_solve_checks() (celestine solvers.reduced_solver.ReducedSolver method), 41
 pre_solve_checks() (celestine solvers.template.SolverTemplate method), 39
 pressure_BCs() (in module celestine.boundary_conditions), 1
 prevent_gas_rise_into_saturated_cell() (in module celestine solvers.reduced_solver), 41

R

Rayleigh_salt (celestine dimensional_params.DimensionalParams property), 4
 ReducedEnthalpyMethod (class in celestine.enthalpy_method), 9
 ReducedPhaseBoundaries (class in celestine.phase_boundaries), 23
 ReducedSolver (class in celestine solvers.reduced_solver), 41
 run_batch() (in module celestine.run_simulation), 25

S

salinity_difference (celestine dimensional_params.DimensionalParams property), 6
 salt_BCs() (in module celestine.boundary_conditions), 1
 save() (celestine dimensional_params.DimensionalParams method), 6
 savefreq (celestine dimensional_params.DimensionalParams property), 6

Scales (*class in celestine dimensional_params*), 6
 ScipySolver (*class in celestine solvers.scipy*), 43
 Solution (*class in celestine.state*), 27
 solve() (*in module celestine.run_simulation*), 25
 SolverTemplate (*class in celestine solvers.template*), 39
 State (*class in celestine.state*), 27
 StateBCs (*class in celestine.state*), 27
 stefan_number (*celestine dimensional_params.DimensionarParams property*), 6

T

take_forward_euler_step() (*in module celestine.flux*), 11
 take_timestep() (*celestine solvers.reduced_solver.ReducedSolver method*), 41
 take_timestep() (*celestine solvers.scipy.ScipySolver method*), 43
 take_timestep() (*celestine solvers.template.SolverTemplate method*), 39
 temperature_BCs() (*in module celestine.boundary_conditions*), 1
 temperature_difference (*celestine dimensional_params.DimensionarParams property*), 6
 thermal_diffusivity (*celestine dimensional_params.DimensionarParams property*), 6
 total_time (*celestine dimensional_params.DimensionarParams property*), 6