# celestine

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# **CONTENTS:**

1	Boundary Conditions	1
2	Dimensional Params	3
3	Enthalpy Method	9
4	Flux	11
5	Forcing	13
6	Grids	15
7	Initial Conditions	17
8	Logging Config	19
9	Params	21
10	Phase Boundaries	23
11	Run Simulation	25
12	State	27
13	Velocities	29
14	Brine Drainage	33
15	Brine Channel Sink Terms	37
16	Template	39
17	Reduced Model Solver	41
18	Scipy Solver	43
19	Indices and tables	45
Py	Python Module Index	
Inc	lex	49

# **BOUNDARY CONDITIONS**

Module to provide functions to add boundary conditions to each quantity on the centered grid that needs to be on the ghost grid for the upwind scheme.

```
celestine.boundary_conditions.dissolved_gas_BCs(dissolved_gas_centers, cfg: Config)
Add ghost cells with BCs to center quantity

celestine.boundary_conditions.enthalpy_BCs(enthalpy_centers, cfg: Config)
Add ghost cells with BCs to center quantity

celestine.boundary_conditions.gas_BCs(gas_centers, cfg: Config)
Add ghost cells with BCs to center quantity
```

 $\verb|celestine.boundary_conditions.gas_fraction_BCs(|\textit{gas_fraction\_centers}, \textit{cfg}: \texttt{Config})|$ 

Add ghost cells with BCs to center quantity

 $\verb|celestine.boundary_conditions.liquid_fraction_BCs(|liquid_fraction_centers, cfg: Config)| \\$ 

Add ghost cells to liquid fraction such that top and bottom boundaries take the same value as the top and bottom cell center

celestine.boundary\_conditions.liquid\_salinity\_BCs(liquid\_salinity\_centers, cfg: Config)
Add ghost cells with BCs to center quantity

celestine.boundary\_conditions.pressure\_BCs(pressure\_centers, cfg: Config)
Add ghost cells to pressure so that W\_l=0 at z=0 and p=0 at z=-1

celestine.boundary\_conditions.salt\_BCs(salt\_centers, cfg: Config)

Add ghost cells with BCs to center quantity

celestine.boundary\_conditions.temperature\_BCs(temperature\_centers, time, cfg: Config)

Add ghost cells with BCs to center quantity

Note this needs the current time as well as top temperature is forced.

# **TWO**

# **DIMENSIONAL PARAMS**

Dimensional parameters required to run a simulation and convert output to dimensional variables.

The DimensionalParams class contains all the dimensional parameters needed to produce a simulation configuration.

The Scales class contains all the dimensional parameters required to convert simulation output between physical and non-dimensional variables.

class celestine.dimensional\_params.DimensionalParams(name: str, total\_time\_in\_days: float = 365,

 $savefreq\_in\_days: float = 1, lengthscale: float$ = 1, liquid density: float = 1028, gas density: float = 1,  $saturation\_concentration$ : float =1e-05, ocean salinity: float = 34, eutectic salinity: float = 270,  $eutectic\_temperature: float = -21.1$ . latent heat: float = 334000.0, specific heat capacity: float = 4184,  $phase\_average\_conductivity:\ bool = False,$  $liquid\_thermal\_conductivity: float = 0.54,$  $solid\_thermal\_conductivity: float = 2.22,$  $salt\_diffusivity: float = 0, gas\_diffusivity: float$ = 0, frame\_velocity\_dimensional: float = 0, gravity: float = 9.81, liquid\_viscosity: float = 0.00278, bubble\_radius: float = 0.001,  $pore\_radius: float = 0.001,$ pore throat scaling: float = 0.5, drag exponent: float = 6.0, bubble size distribution type: str = 'mono',  $wall\_drag\_law\_choice: str = 'power',$ bubble distribution power: float = 1.5,  $minimum\_bubble\_radius: float = 1e-06,$ maximum bubble radius: float = 0.001, porosity threshold: bool = False,porosity threshold value: float = 0.024, brine\_convection\_parameterisation: bool = False, couple\_bubble\_to\_horizontal\_flow: bool = True, couple\_bubble\_to\_vertical\_flow: bool  $= True, Rayleigh\_critical: float = 40,$  $convection\_strength: float = 0.03,$ haline\_contraction\_coefficient: float = 0.00075, reference\_permeability: float = 1e-08,  $initial\_conditions\_choice: str = 'uniform',$ far gas sat: float = 1e-05, far temp: float = -0.81,  $far\_bulk\_salinity$ : float = 34, temperature forcing choice: str = 'constant',  $constant\_top\_temperature$ : float = -30.32, Barrow top temperature data choice: str = 'air', Barrow\_initial\_bulk\_gas\_in\_ice: float = 0.2, offset: float = -1.0, amplitude: float = 0.75, period: float = 4.0, I: int = 50, timestep: float = 0.0002, regularisation: float = 1e-06, solver: str = 'SCI')

Contains all dimensional parameters needed to calculate non dimensional numbers.

To see the units each input should have look at the comment next to the default value.

#### property B

calculate the non dimensional scale for buoyant rise of gas bubbles as

$$\mathcal{B} = \frac{\rho_l g R_0^2 h}{3\mu\kappa}$$

#### property Rayleigh\_salt

Calculate the haline Rayleigh number as

$$Ra_S = \frac{\rho_l g \beta \Delta S H K_0}{\kappa \mu}$$

#### property bubble\_radius\_scaled

calculate the bubble radius divided by the pore scale

$$\Lambda = R_B/R_0$$

#### property concentration\_ratio

Calculate concentration ratio as

$$C = S_i/\Delta S$$

#### property conductivity\_ratio

Calculate the ratio of solid to liquid thermal conductivity

$$\lambda = \frac{k_s}{k_l}$$

#### property expansion\_coefficient

calculate

$$\chi = \rho_l \xi_{\rm sat} / \rho_g$$

#### property frame\_velocity

calculate the frame velocity in non dimensional units

#### get\_config()

Return a Config object for the simulation.

physical parameters and Darcy law parameters are calculated from the dimensional input. You can modify the numerical parameters and boundary conditions and forcing provided for the simulation.

#### get\_darcy\_law\_params()

return a DarcyLawParams object

#### get\_physical\_params()

return a PhysicalParams object

#### get\_scales()

return a Scales object used for converting between dimensional and non dimensional variables.

#### property lewis\_gas

Calculate the lewis number for dissolved gas, return np.inf if there is no dissolved gas diffusion.

$$Le_{\xi} = \kappa/D_{\xi}$$

#### property lewis\_salt

Calculate the lewis number for salt, return np.inf if there is no salt diffusion.

$$Le_S = \kappa/D_s$$

#### classmethod load(path)

load this object from a yaml configuration file.

#### property maximum\_bubble\_radius\_scaled

calculate the bubble radius divided by the pore scale

$$\Lambda = R_B/R_0$$

#### property minimum\_bubble\_radius\_scaled

calculate the bubble radius divided by the pore scale

$$\Lambda = R_B/R_0$$

#### property ocean\_freezing\_temperature

calculate salinity dependent freezing temperature using liquidus for typical ocean salinity

$$T_i = T_L(S_i) = T_E S_i / S_E$$

### property salinity\_difference

calculate difference between eutectic salinity and typical ocean salinity

$$\Delta S = S_E - S_i$$

#### save(directory: Path)

save this object to a yaml file in the specified directory.

The name will be the name given with \_dimensional appended to distinguish it from a saved non-dimensional configuration.

#### property savefreq

calculate the save frequency in non dimensional time

#### property stefan\_number

calculate Stefan number

$$St = L/c_p \Delta T$$

#### property temperature\_difference

calculate

$$\Delta T = T_i - T_E$$

#### property thermal\_diffusivity

Return thermal diffusivity in m2/s

$$\kappa = \frac{k}{\rho_l c_p}$$

#### property total\_time

calculate the total time in non dimensional units for the simulation

```
class celestine.dimensional_params.Scales(lengthscale: float, thermal diffusivity: float, ocean salinity:
                                                 float, salinity_difference: float, ocean_freezing_temperature:
                                                 float, temperature difference: float, gas density: float,
                                                 saturation_concentration: float)
     convert_dimensional_bulk_air_to_argon_content(dimensional_bulk_gas)
          Convert kg/m3 of air to micromole of Argon per Liter of ice
     convert_from_dimensional_bulk_gas(dimensional bulk gas)
          Non dimensionalise bulk gas content in kg/m3
     convert_from_dimensional_bulk_salinity(dimensional_bulk_salinity)
          Non dimensionalise bulk salinity in g/kg
     convert_from_dimensional_dissolved_gas(dimensional dissolved gas)
          convert from dissolved gas in kg(gas)/kg(liquid) to dimensionless
     convert_from_dimensional_grid(dimensional_grid)
          Non dimensionalise domain depths in meters
     convert_from_dimensional_temperature(dimensional_temperature)
          Non dimensionalise temperature in deg C
     convert_from_dimensional_time(dimensional_time)
          Non dimensionalise time in days
     convert_to_dimensional_bulk_gas(bulk gas)
          Convert dimensionless bulk gas content to kg/m3
     convert_to_dimensional_bulk_salinity(bulk salinity)
          Convert non dimensional bulk salinity to g/kg
     convert_to_dimensional_dissolved_gas(dissolved gas)
          convert from non dimensional dissolved gas to dimensional dissolved gas in kg(gas)/kg(liquid)
     convert_to_dimensional_grid(grid)
          Get domain depths in meters from non dimensional values
     convert_to_dimensional_temperature(temperature)
          get temperature in deg C from non dimensional temperature
     convert_to_dimensional_time(time)
          Convert non dimensional time into time in days since start of simulation
celestine.dimensional_params.calculate_timescale_in_days(lengthscale, thermal diffusivity)
     calculate timescale given domain height and thermal diffusivity.
          Parameters
                • lengthscale (float) – domain height in m
                • thermal_diffusivity (float) – thermal diffusivity in m2/s
          Returns
              timescale in days
```

celestine.dimensional\_params.calculate\_velocity\_scale\_in\_m\_day(lengthscale, thermal\_diffusivity)

calculate the velocity scale given domain height and thermal diffusivity

#### **Parameters**

- lengthscale (float) domain height in m
- thermal\_diffusivity (float) thermal diffusivity in m2/s

#### Returns

velocity scale in m/day

THREE

# **ENTHALPY METHOD**

Module containing enthalpy method to calculate state variables from bulk enthalpy, bulk salinity and bulk gas.

class celestine.enthalpy\_method.EnthalpyMethod(physical\_params: PhysicalParams)

Template for an enthalpy method. To implement a new method overwrite the initializer to initialise the physical parameters and a suitable phase boundaries object. Then implement a calculate enthalpy method that takes a state and uses bulk enthalpy, salt and gas to return (temperature, liquid\_fraction, gas\_fraction, solid\_fraction, liquid\_salinity, dissolved\_gas).

class celestine.enthalpy\_method.FullEnthalpyMethod(physical\_params: PhysicalParams)

class celestine.enthalpy\_method.ReducedEnthalpyMethod(physical\_params: PhysicalParams)

celestine.enthalpy\_method.get\_enthalpy\_method(cfg)

Return the enthalpy method object required depending on solver choice

LU: Full RED: Reduced SCI: Reduced

**Parameters** 

**cfg** – configuration for simulation

# **FOUR**

# **FLUX**

Module for calculating the fluxes using upwind scheme

celestine.flux.calculate\_conductive\_heat\_flux(state\_BCs, D\_g, cfg)

Calculate conductive heat flux as

$$-\frac{\partial \theta}{\partial z}$$

or alteratively if the phase\_average\_conductivity configuration parameter is set to True then we use the conductivity ratio as follows

$$-[(\phi_l + \lambda \phi_s) \frac{\partial \theta}{\partial z}]$$

#### **Parameters**

- temperature (Numpy Array of size I+2) temperature including ghost cells
- **D\_g** (Numpy Array) difference matrix for ghost grid
- cfg (celestine.params.Config) Simulation configuration

#### Returns

conductive heat flux

celestine.flux.calculate\_diffusive\_salt\_flux(liquid\_salinity, liquid\_fraction, D\_g, cfg)

Take liquid salinity and liquid fraction on ghost grid and interpolate liquid fraction geometrically

celestine.flux.take\_forward\_euler\_step(quantity, flux, timestep, D\_e)

Advance the given quantity one forward Euler step using the given flux

The quantity is given on cell centers and the flux on cell edges.

Discretise the conservation equation

$$\frac{\partial Q}{\partial t} = -\frac{\partial F}{\partial z}$$

as

$$Q^{n+1} = Q^n - \Delta t(\frac{\partial F}{\partial z})$$

12 Chapter 4. Flux

### **FIVE**

# **FORCING**

Module for providing surface temperature forcing to simulation.

Note that the barrow temperature data is read in from a file if needed by the simulation configuration.

#### celestine.forcing.barrow\_ocean\_temperature\_forcing(time, cfg)

Take non dimensional time and return non dimensional ocean temperature at the Barrow site in 2009.

For this to work you must have created the configuration cfg from dimensional parameters as it must have the conversion scales object.

# ${\tt celestine.forcing.barrow\_temperature\_forcing}({\it time}, {\it cfg})$

Take non dimensional time and return non dimensional air/snow/ice temperature at the Barrow site in 2009.

For this to work you must have created the configuration cfg from dimensional parameters as it must have the conversion scales object.

### celestine.forcing.dimensional\_barrow\_ocean\_temperature\_forcing(time\_in\_days, cfg: Config)

Take time in days and linearly interp 2009 Barrow ocean temperature data to get temperature in degrees Celsius.

#### celestine.forcing.dimensional\_barrow\_temperature\_forcing(time\_in\_days, cfg: Config)

Take time in days and linearly interp 2009 Barrow air/snow/ice temperature data to get temperature in degrees Celsius.

14 Chapter 5. Forcing

# SIX

# **GRIDS**

Module providing functions to initialise the different grids and interpolate quantities between them.

celestine.grids.add\_ghost\_cells(centers, bottom, top)

Add specified bottom and top value to center grid

#### **Parameters**

- centers (Numpy array) numpy array on centered grid (size I).
- **bottom** (*float*) bottom value placed at index 0.
- top (float) top value placed at index -1.

#### **Returns**

numpy array on ghost grid (size I+2).

celestine.grids.average(ghosts)

Returns arithmetic mean pairwise of first dimension of an array

This should get values on the ghost grid and returns the arithmetic average onto the edge grid celestine.grids.geometric(ghosts)

Returns geometric mean of the first dimension of an array

16 Chapter 6. Grids

**SEVEN** 

### INITIAL CONDITIONS

Module to provide initial state of bulk enthalpy, bulk salinity and bulk gas for the simulation.

celestine.initial\_conditions.apply\_value\_in\_ice\_layer(depth\_of\_ice, ice\_value, liquid\_value, grid)

assume that top part of domain contains mushy ice of given depth and lower part of domain is liquid. This function returns output on the given grid where the ice part of the domain takes one value and the liquid a different.

This is useful for initialising the barrow simulation where we have an initial ice layer.

#### celestine.initial\_conditions.get\_barrow\_initial\_conditions(cfg: Config)

initialise domain with an initial ice layer of given temperature and bulk salinity. These values are hard coded in from Moreau paper to match barrow study. They also assume that the initial ice layer has 1/5 the saturation amount in pure liquid of dissolved gas to account for previous gas loss.

Initialise with bulk gas being (1/5) in ice and saturation in liquid. Bulk salinity is 5.92 g/kg in ice and ocean value in liquid. Enthalpy is calculated by inverting temperature relation in ice and ocean. Ice temperature is given as -8.15 degC and ocean is the far value from boundary config.

#### celestine.initial\_conditions.get\_uniform\_initial\_conditions(cfg)

Generate uniform initial solution on the ghost grid

#### Returns

initial solution arrays on ghost grid (enthalpy, salt, gas, pressure)

CHAPT	ER
EIGH	НT

# **LOGGING CONFIG**

Module to create logger for simulation

NINE

# **PARAMS**

Classes containing parameters required to run a simulation

The config class contains all the parameters needed to run a simulation as well as methods to save and load this configuration to a yaml file.

```
class celestine.params.BoundaryConditionsConfig(initial_conditions_choice: str = 'uniform', far\_gas\_sat: float = 1.0, far\_temp: float = 0.1, far\_bulk\_salinity: float = 0)
```

values for bottom (ocean) boundary

class celestine.params.Config(name: str, physical\_params: PhysicalParams =

PhysicalParams(expansion coefficient=0.029, concentration ratio=0.17, stefan\_number=4.2, lewis\_salt=inf, lewis\_gas=inf, frame\_velocity=0, phase average conductivity=False, conductivity ratio=4.11), boundary\_conditions\_config: BoundaryConditionsConfig = BoundaryConditionsConfig(initial\_conditions\_choice='uniform', far\_gas\_sat=1.0, far\_temp=0.1, far\_bulk\_salinity=0), darcy\_law\_params: DarcyLawParams = DarcyLawParams(B=100, pore\_throat\_scaling=0.5, bubble\_size\_distribution\_type='mono', wall\_drag\_law\_choice='power', drag\_exponent=6.0, bubble\_radius\_scaled=1.0, bubble\_distribution\_power=1.5, minimum\_bubble\_radius\_scaled=0.001, maximum\_bubble\_radius\_scaled=1, porosity\_threshold=False, porosity\_threshold\_value=0.024, brine\_convection\_parameterisation=False, Rayleigh\_salt=44105, Rayleigh\_critical=40, convection\_strength=0.03, couple\_bubble\_to\_horizontal\_flow=True, couple\_bubble\_to\_vertical\_flow=True), forcing\_config: ForcingConfig = ForcingConfig(temperature forcing choice='constant', constant\_top\_temperature=- 1.5, offset=- 1.0, amplitude=0.75, period=4.0, Barrow top temperature data choice='air', Barrow\_initial\_bulk\_gas\_in\_ice=0.2), numerical\_params: NumericalParams = NumericalParams(I=50, timestep=0.0002, regularisation=1e-06,solver='SCI'), scales: Optional[int] = None,  $total\_time: float = 4.0$ , savefreq: float = 0.0005)

contains all information needed to run a simulation and save output

this config object can be saved and loaded to a yaml file.

#### check\_thermal\_Courant\_number()

Check if courant number for thermal diffusion term is low enough for explicit method and if it isn't log a warning.

```
class celestine.params.DarcyLawParams (B: float = 100, pore\_throat\_scaling: float = 0.5, bubble\_size\_distribution\_type: str = 'mono', wall\_drag\_law\_choice: str = 'power', drag\_exponent: float = 6.0, bubble\_radius\_scaled: float = 1.0, bubble\_distribution\_power: float = 1.5, minimum\_bubble\_radius\_scaled: float = 0.001, maximum\_bubble\_radius\_scaled: float = 1, porosity\_threshold: bool = False, porosity\_threshold\_value: float = 0.024, brine\_convection\_parameterisation: bool = False, Rayleigh\_salt: float = 44105, Rayleigh\_critical: float = 40, convection\_strength: float = 0.03, couple\_bubble\_to\_horizontal\_flow: bool = True, couple\_bubble\_to\_vertical\_flow: bool = True)
```

non dimensional parameters for calculating liquid and gas darcy velocities

choice of top boundary (atmospheric) forcing and required parameters

#### load\_forcing\_data()

populate class attributes with barrow dimensional air temperature and time in days (with missing values filtered out).

Note the metadata explaining how to use the barrow temperature data is also in celestine/forcing\_data. The indices corresponding to days and air temp are hard coded in as class variables.

```
class celestine.params.NumericalParams(I: int = 50, timestep: float = 0.0002, regularisation: float = 1e-06, solver: <math>str = 'SCI')
```

parameters needed for discretisation and choice of numerical method

#### property Courant

This number must be <0.5 for stability of temperature diffusion terms

```
class celestine.params.PhysicalParams(expansion_coefficient: float = 0.029, concentration_ratio: float = 0.17, stefan_number: float = 4.2, lewis_salt: float = inf, lewis_gas: float = inf, frame_velocity: float = 0, phase_average_conductivity: bool = False, conductivity_ratio: float = 4.11)
```

non dimensional numbers for the mushy layer

```
celestine.params.filter_missing_values(air_temp, days)
```

Filter out missing values are recorded as 9999

22 Chapter 9. Params

**TEN** 

# **PHASE BOUNDARIES**

Module for calculating the phase boundaries needed for the enthalpy method.

**class** celestine.phase\_boundaries.**FullPhaseBoundaries**(*physical\_params*: PhysicalParams) calculates the phase boundaries when we include gas fraction in bulk enthalpy and bulk salinity.

**class** celestine.phase\_boundaries.**PhaseBoundaries**(*physical\_params*: PhysicalParams)
Template for phase boundary calculation.

Concrete implementations should use the state containing enthalpy, salt and gas to calculate the liquidus, enthalpy, solidus and saturation boundaries and then return masks for each possible phase of the system.

class celestine.phase\_boundaries.ReducedPhaseBoundaries(physical\_params: PhysicalParams)
 calculates the phase boundaries neglecting the gas fraction so that

$$\phi_s + \phi_l = 1$$

# **ELEVEN**

# **RUN SIMULATION**

Module to run the simulation on the given configuration with the appropriate solver.

celestine.run\_simulation.run\_batch(list\_of\_cfg, directory: Path)

Run a batch of simulations from a list of configurations.

Each simulation name is logged, as well as if it successfully runs or crashes. Output from each simulation is saved in a .npz file.

#### **Parameters**

list\_of\_cfg (List[celestine.params.Config]) - list of configurations

celestine.run\_simulation.solve(cfg: Config, directory: Path)

Solve simulation choosing appropriate solver from the choice in the config.

# **TWELVE**

# **STATE**

Classes to store solution variables

State: store variables on cell centers StateBCs: add boundary conditions in ghost cells to cell center variables Solution: store primary variables at each timestep we want to save data

```
class celestine.state.Solution(cfg: Config)
    store solution at specified times on the center grid
    add_state(state: State, index: int)
        add state to stored solution at given time index
class celestine.state.State(cfg: Config, time, enthalpy, salt, gas, pressure=None)
```

class celestine.state.State(cfg: Config, time, enthalpy, salt, gas, pressure=None

Stores information needed for solution at one timestep on cell centers

class celestine.state.StateBCs(state: State)

Stores information needed for solution at one timestep with BCs on ghost cells as well

Note must initialise once enthalpy method has already run on State.

28 Chapter 12. State

# **THIRTEEN**

### **VELOCITIES**

Module to calculate Darcy velocities.

The liquid Darcy velocity must be parameterised.

The gas Darcy velocity is calculated as gas\_fraction x interstitial bubble velocity

Interstitial bubble velocity is found by a steady state Stoke's flow calculation. We have implemented two cases mono: All bubbles nucleate and remain the same size power\_law: A power law bubble size distribution with fixed max and min.

celestine.velocities.calculate\_bubble\_size\_fraction(bubble\_radius\_scaled, liquid\_fraction, cfg:

Config)

Takes bubble radius scaled and liquid fraction on edges and calculates the bubble size fraction as

$$\lambda = \Lambda/(\phi_I^q + \text{reg})$$

Returns the bubble size fraction on the edge grid.

celestine.velocities.calculate\_gas\_interstitial\_velocity(liquid\_fraction, liquid\_darcy\_velocity, wall\_drag\_factor, lag\_factor, cfg:
Config)

Calculate Vg from liquid fraction on the ghost frid and liquid interstitial velocity

$$V_q = \mathcal{B}(\phi_l^{2q} I_1) + U_0 I_2$$

Return Vg on edge grid

celestine.velocities.calculate\_lag\_function(bubble\_size\_fraction)

Calculate lag function from bubble size fraction on edge grid as

$$G(\lambda) = 1 - \lambda/2$$

for 0 < lambda < 1. Edge cases are given by G(0) = 1 and G(1) = 0.5 for values outside this range.

celestine.velocities.calculate\_lag\_integrand(bubble\_size\_fraction: float, cfg: Config)

Scalar function to calculate lag integrand for polydispersive case.

Bubble size fraction is given as a scalar input to calculate

$$\lambda^{3-p}G(\lambda)$$

celestine.velocities.calculate\_liquid\_darcy\_velocity(liquid\_fraction, liquid\_salinity, center\_grid, edge grid, cfg: Config)

Calculate liquid Darcy velocity either using brine convection parameterisation or as stagnant

#### **Parameters**

- liquid\_fraction (Numpy Array (size I+2)) liquid fraction on ghost grid
- liquid\_salinity (Numpy Array (size I+2)) liquid salinity on ghost grid
- center\_grid (Numpy Array of shape (I,)) vertical coordinates of cell centers
- edge\_grid (Numpy Array (size I+1)) Vertical coordinates of cell edges
- cfg (celestine.params.Config) simulation configuration object

#### Returns

liquid darcy velocity on edge grid

celestine.velocities.calculate\_mono\_lag\_factor(liquid\_fraction, cfg: Config)

Take liquid fraction on the ghost grid and calculate the lag factor for a mono bubble size distribution as

$$I_2 = G(\lambda)$$

returns lag factor on the edge grid

celestine.velocities.calculate\_mono\_wall\_drag\_factor(liquid\_fraction, cfg: Config)

Take liquid fraction on the ghost grid and calculate the wall drag factor for a mono bubble size distribution as

$$I_1 = \frac{\lambda^2}{K(\lambda)}$$

returns wall drag factor on the edge grid

celestine.velocities.calculate\_power\_law\_lag\_factor(liquid\_fraction, cfg: Config)

Take liquid fraction on the ghost grid and calculate the lag factor for power law bubble size distribution.

Return on edge grid

celestine.velocities.calculate\_power\_law\_wall\_drag\_factor(liquid fraction, cfg: Config)

Take liquid fraction on the ghost grid and calculate the wall drag factor for power law bubble size distribution.

Return on edge grid

celestine.velocities.calculate\_velocities(state\_BCs, cfg: Config)

Inputs on ghost grid, outputs on edge grid

celestine.velocities.calculate\_volume\_integrand(bubble\_size\_fraction: float, cfg: Config)

Scalar function to calculate the integrand for volume under a power law bubble size distribution given as

$$\lambda^{3-p}$$

in terms of the bubble size fraction.

celestine.velocities.calculate\_wall\_drag\_function(bubble\_size\_fraction, cfg: Config)

Calculate wall drag function from bubble size fraction on edge grid as

$$\frac{1}{K(\lambda)} = (1 - \lambda)^r$$

in the power law case or in the Haberman case from the paper

$$\frac{1}{K(\lambda)} = \frac{1 - 1.5\lambda + 1.5\lambda^5 - \lambda^6}{1 + 1.5\lambda^5}$$

for 0 < lambda < 1. Edge cases are given by K(0)=1 and K(1)=0 for values outside this range.

celestine.velocities.calculate\_wall\_drag\_integrand(bubble\_size\_fraction: float, cfg: Config)

Scalar function to calculate wall drag integrand for polydispersive case.

Bubble size fraction is given as a scalar input to calculate

$$\frac{\lambda^{5-p}}{K(\lambda)}$$

where the wall drag enhancement function K can be given by a power law fit or taken from the Haberman paper.

## **FOURTEEN**

## **BRINE DRAINAGE**

Module to calculate the Rees Jones and Worster 2014 parameterisation for brine convection velocity and the strenght of the sink term.

Exports the functions:

calculate\_brine\_convection\_liquid\_velocity To be used in velocities module when using brine convection parameterisation.

calculate\_brine\_channel\_sink To be used to add sink terms to conservation equations when using brine convection parameterisation.

celestine.brine\_drainage.calculate\_Rayleigh(cell\_centers, edge\_grid, liquid\_salinity, liquid\_fraction, cfg: Config)

Calculate the local Rayleigh number for brine convection as

$$Ra(z) = Ra_S K(z)(z+h)\Theta_l$$

#### **Parameters**

- **cell\_centers** (Numpy Array shape (I,)) The vertical coordinates of cell centers.
- edge\_grid (Numpy Array (size I+1)) The vertical coordinate positions of the edge grid.
- liquid\_salinity (Numpy Array shape (I,)) liquid salinity on center grid
- liquid\_fraction (Numpy Array (size I)) liquid fraction on center grid
- **cfg** (celestine.params.Config) Configuration object for the simulation.

#### Returns

Array of shape (I,) of Rayleigh number at cell centers

celestine.brine\_drainage.calculate\_brine\_channel\_sink(liquid\_fraction, liquid\_salinity, center\_grid, edge\_grid, cfg: Config)

Calculate the sink term due to brine channels.

$$sink = A$$

in the convecting region. Zero elsewhere.

NOTE: If no ice is present or if no convecting region exists returns zero

#### **Parameters**

- $\bullet \ \ \textbf{liquid\_fraction} \ (\textit{Numpy Array of shape} \ \ (\textit{I,})) liquid \ \text{fraction on center grid}$
- liquid\_salinity (Numpy Array of shape (I,)) liquid salinity on center grid

- center\_grid (Numpy Array of shape (I,)) vertical coordinate of center grid
- edge\_grid (Numpy Array of shape (I+1,)) Vertical coordinates of cell edges
- cfg (celestine.params.Config) Configuration object for the simulation.

#### Returns

Strength of the sink term due to brine channels on the center grid.

celestine.brine\_drainage.calculate\_brine\_channel\_strength(Rayleigh\_number, ice\_depth, convecting region height, cfg: Config)

Calculate the brine channel strength in the convecting region as

$$\mathcal{A} = \frac{\alpha \mathbf{R} \mathbf{a}_e}{(h + z_c)^2}$$

the effective Rayleigh number multiplied by a tuning parameter (Rees Jones and Worster 2014) over the convecting region thickness squared.

#### **Parameters**

- Rayleigh\_number (Numpy Array of shape (I,)) local Rayleigh number on center grid
- ice\_depth (float) depth of ice (positive)
- convecting\_region\_height (float) position of the convecting region boundary (negative)
- **cfg** (celestine.params.Config) Configuration object for the simulation.

#### Returns

Brine channel strength parameter

celestine.brine\_drainage.calculate\_brine\_convection\_liquid\_velocity(liquid\_fraction,

liquid\_salinity, center\_grid,
edge\_grid, cfg: Config)

Calculate the vertical liquid Darcy velocity from Rees Jones and Worster 2014

$$W_l = \mathcal{A}(z_c - z)$$

in the convecting region. The velocity is stagnant above the convecting region. The velocity is constant in the liquid region and continuous at the interface.

NOTE: If no ice is present or if no convecting region exists returns zero velocity

#### **Parameters**

- liquid\_fraction (Numpy Array of shape (I,)) liquid fraction on center grid
- liquid\_salinity (Numpy Array of shape (I,)) liquid salinity on center grid
- center\_grid (Numpy Array of shape (I,)) vertical coordinate of center grid
- edge\_grid (Numpy Array of shape (I+1,)) Vertical coordinates of cell edges
- cfg (celestine.params.Config) Configuration object for the simulation.

## Returns

Liquid darcy velocity on the edge grid.

celestine.brine\_drainage.calculate\_ice\_ocean\_boundary\_depth(liquid\_fraction, edge\_grid)

Calculate the depth of the ice ocean boundary as the edge position of the first cell from the bottom to be not completely liquid. I.e the first time the liquid fraction goes below 1.

If the ice has made it to the bottom of the domain raise an error.

If the domain is completely liquid set h=0.

NOTE: depth is a positive quantity and our grid coordinate increases from -1 at the bottom of the domain to 0 at the top.

#### **Parameters**

- liquid\_fraction (Numpy Array (size I)) liquid fraction on center grid
- edge\_grid (Numpy Array (size I+1)) The vertical coordinate positions of the edge grid.

#### Returns

positive depth value of ice ocean interface

celestine.brine\_drainage.calculate\_integrated\_mean\_permeability(z, liquid\_fraction, ice\_depth, cell\_centers, cfg: Config)

Calculate the harmonic mean permeability from the base of the ice up to the cell containing the specified z value using the expression of ReesJones2014.

$$K(z) = \left(\frac{1}{h+z} \int_{-h}^{z} \frac{1}{\Pi(\phi_{l}(z'))} dz'\right)^{-1}$$

#### **Parameters**

- **z** (*float*) height to integrate permeability up to
- liquid\_fraction (Numpy Array shape (I,)) liquid fraction on the center grid
- **ice\_depth** (*float*) positive depth position of ice ocean interface
- cell\_centers (Numpy Array of shape (I,)) cell center positions
- cfg (celestine.params.Config) Configuration object for the simulation.

#### Returns

permeability averaged from base of the ice up to given z value

celestine.brine\_drainage.calculate\_permeability(liquid\_fraction, cfg: Config)

Calculate the absolute permeability as a function of liquid fraction

$$\Pi(\phi_l) = \phi_l^3$$

Alternatively if the porosity threshold flag is true

$$\Pi(\phi_l) = \phi_l^2(\phi_l - \phi_c)$$

#### **Parameters**

- liquid\_fraction (Numpy Array) liquid fraction
- **cfg** (celestine.params.Config) Configuration object for the simulation.

#### Returns

permeability on the same grid as liquid fraction

celestine.brine\_drainage.get\_convecting\_region\_height(Rayleigh\_number, edge\_grid, cfg: Config)

Calculate the height of the convecting region as the top edge of the highest cell in the domain for which the quantity

$$Ra(z) - Ra_c$$

is greater than or equal to zero.

NOTE: if no convecting region exists return np.NaN

#### **Parameters**

- Rayleigh\_number (Numpy Array of shape (I,)) local rayleigh number on center grid
- edge\_grid (Numpy Array (size I+1)) The vertical coordinate positions of the edge grid.
- **cfg** (celestine.params.Config) Configuration object for the simulation.

#### Returns

Edge grid value at convecting boundary.

celestine.brine\_drainage.get\_effective\_Rayleigh\_number(Rayleigh\_number, cfg: Config)

Calculate the effective Rayleigh Number as the maximum of

$$Ra(z) - Ra_c$$

in the convecting region.

NOTE: if no convecting region exists returns 0.

#### **Parameters**

- Rayleigh\_number (Numpy Array of shape (I,)) local rayleigh number on center grid
- **cfg** (celestine.params.Config) Configuration object for the simulation.

#### Returns

Effective Rayleigh number.

# **FIFTEEN**

# **BRINE CHANNEL SINK TERMS**

Module to calculate the sink terms for conservation equations when using the Rees Jones and Worster 2014 brine drainage parameterisation.

These terms represent loss through the brine channels and need to be added in the convecting region when using this parameterisation

## **SIXTEEN**

## **TEMPLATE**

Template for a solver concrete solvers should inherit and overwrite required methods

class celestine.solvers.template.SolverTemplate(cfg: Config)

#### generate\_initial\_solution()

Generate initial solution on the ghost grid

#### Returns

initial solution arrays on ghost grid (enthalpy, salt, gas, pressure)

#### pre\_solve\_checks()

Optionally implement this method if you want to check anything before running the solver.

For example to check the timestep and grid step satisfy some constraint.

#### abstract take\_timestep(state: State) $\rightarrow State$

advance enthalpy, salt, gas and pressure to the next timestep.

Note as of 2023-05-17 removed ability to have adaptive timestepping for simplicity.

#### **Parameters**

**state** (celestine.solvers.template.State) — object containing current time, enthalpy, salt, gas, pressure and surface temperature.

#### Returns

state of system (new enthalpy, salt, gas and pressure) after one timestep.

## **SEVENTEEN**

## REDUCED MODEL SOLVER

class celestine.solvers.reduced\_solver.ReducedSolver(cfg: Config)

Take timestep using forward Euler upwind scheme using reduced model.

#### pre\_solve\_checks()

Optionally implement this method if you want to check anything before running the solver.

For example to check the timestep and grid step satisfy some constraint.

#### take\_timestep(state: State)

advance enthalpy, salt, gas and pressure to the next timestep.

Note as of 2023-05-17 removed ability to have adaptive timestepping for simplicity.

#### **Parameters**

 $\begin{tabular}{ll} \textbf{state} & (celestine.solvers.template.State) - object containing current time, enthalpy, salt, gas, pressure and surface temperature. \\ \end{tabular}$ 

#### Returns

state of system (new enthalpy, salt, gas and pressure) after one timestep.

Modify the gas interstitial velocity to prevent bubble rise into a cell which is already theoretically saturated with gas.

From the state with boundary conditions calculate the gas and solid fraction in the cells (except at lower ghost cell). If any of these are such that there is more gas fraction than pore space available then set gas insterstitial velocity to zero on the edge below. Make sure the very top boundary velocity is not changed as we want to always alow flux to the atmosphere regardless of the boundary conditions imposed.

#### **Parameters**

- **Vg** (Numpy array (size I+1)) gas insterstitial velocity on cell edges
- state\_BCs (celestine.state.StateBCs) state of system with boundary conditions

#### Returns

filtered gas interstitial velocities on edges to prevent gas rise into a fully gas saturated cell

## **EIGHTEEN**

## **SCIPY SOLVER**

## class celestine.solvers.scipy.ScipySolver(cfg: Config)

Solve reduced model using scipy solve\_ivp using RK23 solver. This is the "SCI" solver option.

Impose a maximum timestep constraint using courant number for thermal diffusion as this is an explicit method.

This solver uses adaptive timestepping which makes it a good choice for running simulations with large buoyancy driven gas bubble velocities and we save the output at intervals given by the savefreq parameter in configuration.

The interface of this class is a little different as we overwrite the solve method from the template and must provide a function to calculate the ode forcing for solve\_ivp. However the solve function still saves the data in the same format using the *celestine.state.Solution* class.

#### take\_timestep(state: State)

advance enthalpy, salt, gas and pressure to the next timestep.

Note as of 2023-05-17 removed ability to have adaptive timestepping for simplicity.

#### **Parameters**

**state** (celestine.solvers.template.State) — object containing current time, enthalpy, salt, gas, pressure and surface temperature.

#### Returns

state of system (new enthalpy, salt, gas and pressure) after one timestep.

# **NINETEEN**

# **INDICES AND TABLES**

- genindex
- modindex
- search

## **PYTHON MODULE INDEX**

#### С

```
{\tt celestine.boundary\_conditions}, 1
celestine.brine_channel_sink_terms, 37
celestine.brine_drainage, 33
celestine.dimensional_params, 3
celestine.enthalpy_method,9
celestine.flux, 11
celestine.forcing, 13
celestine.grids, 15
celestine.initial_conditions, 17
celestine.logging_config, 19
celestine.params, 21
celestine.phase_boundaries, 23
celestine.run_simulation, 25
celestine.solvers.reduced_solver,41
celestine.solvers.scipy, 43
celestine.solvers.template, 39
celestine.state, 27
celestine.velocities, 29
```

48 Python Module Index

# **INDEX**

A	calculate_lag_integrand() (in module celes-
add_ghost_cells() (in module celestine.grids), 15 add_state() (celestine.state.Solution method), 27	<pre>tine.velocities), 29 calculate_liquid_darcy_velocity() (in module</pre>
<pre>apply_value_in_ice_layer() (in module celes- tine.initial_conditions), 17 average() (in module celestine.grids), 15</pre>	<pre>calculate_mono_lag_factor() (in module celes- tine.velocities), 30</pre>
В	calculate_mono_wall_drag_factor() (in module celestine.velocities), 30
B (celestine.dimensional_params.DimensionalParams property), 4	<pre>calculate_permeability() (in module celes- tine.brine_drainage), 35 calculate_power_law_lag_factor() (in module ce-</pre>
<pre>barrow_ocean_temperature_forcing() (in module</pre>	lestine.velocities), 30 calculate_power_law_wall_drag_factor() (in
barrow_temperature_forcing() (in module celestine.forcing), 13	module celestine.velocities), 30 calculate_Rayleigh() (in module celes-
BoundaryConditionsConfig (class in celes- tine.params), 21 bubble_radius_scaled (celes-	<pre>tine.brine_drainage), 33 calculate_timescale_in_days() (in module celes-</pre>
tine.dimensional_params.DimensionalParams property), 5	<pre>tine.dimensional_params), 7 calculate_velocities() (in module celes- tine.velocities), 30</pre>
С	calculate_velocity_scale_in_m_day() (in module celestine.dimensional_params), 7
<pre>calculate_brine_channel_sink() (in module celes- tine.brine_drainage), 33</pre>	calculate_volume_integrand() (in module celes- tine.velocities), 30
<pre>calculate_brine_channel_strength() (in module</pre>	<pre>calculate_wall_drag_function() (in module celes- tine.velocities), 30</pre>
calculate_brine_convection_liquid_velocity() (in module celestine.brine_drainage), 34	<pre>calculate_wall_drag_integrand() (in module ce- lestine.velocities), 31</pre>
calculate_bubble_size_fraction() (in module celestine.velocities), 29	<pre>celestine.boundary_conditions   module, 1</pre>
<pre>calculate_conductive_heat_flux() (in module ce- lestine.flux), 11</pre>	<pre>celestine.brine_channel_sink_terms   module, 37</pre>
calculate_diffusive_salt_flux() (in module celestine.flux), 11	celestine.brine_drainage module, 33
calculate_gas_interstitial_velocity() (in mod- ule celestine.velocities), 29	<pre>celestine.dimensional_params   module, 3</pre>
<pre>calculate_ice_ocean_boundary_depth() (in mod-</pre>	celestine.enthalpy_method module, 9
<pre>calculate_integrated_mean_permeability() (in</pre>	celestine.flux module, 11
calculate_lag_function() (in module celestine.velocities), 29	celestine.forcing module, 13

celestine.grids module,15	<pre>convert_to_dimensional_dissolved_gas() (celes- tine.dimensional_params.Scales method), 7</pre>
celestine.initial_conditions module,17	convert_to_dimensional_grid() (celes- tine.dimensional_params.Scales method),
celestine.logging_config	7
module, 19	<pre>convert_to_dimensional_temperature() (celes-</pre>
celestine.params	tine.dimensional_params.Scales method),
module, 21	7
celestine.phase_boundaries	<pre>convert_to_dimensional_time() (celes-</pre>
module, 23	tine.dimensional_params.Scales method),
celestine.run_simulation	7
module, 25	Courant (celestine.params.NumericalParams property),
celestine.solvers.reduced_solver	22
module, 41	<b>D</b>
celestine.solvers.scipy	D
module, 43	DarcyLawParams (class in celestine.params), 21
celestine.solvers.template	<pre>dimensional_barrow_ocean_temperature_forcing()</pre>
module, 39	(in module celestine.forcing), 13
celestine.state	<pre>dimensional_barrow_temperature_forcing() (in</pre>
module, 27	module celestine.forcing), 13
celestine.velocities	DimensionalParams (class in celes-
module, 29	tine.dimensional_params), 3
check_thermal_Courant_number() (celes-	dissolved_gas_BCs() (in module celes-
tine.params.Config method), 21	$tine.boundary\_conditions), 1$
concentration_ratio (celes-	_
$tine. dimensional\_params. Dimensional Params$	E
property), 5	enthalpy_BCs() (in module celes-
conductivity_ratio (celes-	tine.boundary_conditions), 1
$tine. dimensional\_params. Dimensional Params$	EnthalpyMethod (class in celestine.enthalpy_method), 9
property), 5	expansion_coefficient (celes-
Config (class in celestine.params), 21	tine.dimensional_params.DimensionalParams
convert_dimensional_bulk_air_to_argon_content	O property), 5
$(celestine.dimensional\_params.Scales\ method),$	
7	F
convert_from_dimensional_bulk_gas() (celes- tine.dimensional_params.Scales method),	filter_missing_values() (in module celestine.params), 22
7	ForcingConfig (class in celestine.params), 22
<pre>convert_from_dimensional_bulk_salinity() (ce- lestine.dimensional_params.Scales method), 7</pre>	frame_velocity (celes-
convert_from_dimensional_dissolved_gas() (celestine.dimensional_params.Scales method), 7	tine.dimensional_params.DimensionalParams property), 5
convert_from_dimensional_grid() (celes-	FullEnthalpyMethod (class in celestine.enthalpy_method), 9
tine.dimensional_params.Scales method), 7	FullPhaseBoundaries (class in celes-
<pre>convert_from_dimensional_temperature() (celes-</pre>	tine.phase_boundaries), 23
tine.dimensional_params.Scales method), 7	G
convert_from_dimensional_time() (celes-	
tine.dimensional_params.Scales method),	gas_BCs() (in module celestine.boundary_conditions), 1
7	<pre>gas_fraction_BCs() (in module celes-</pre>
convert_to_dimensional_bulk_gas() (celes-	$tine.boundary\_conditions), 1$
tine.dimensional_params.Scales method),	generate_initial_solution() (celes-
7	tine.solvers.template.SolverTemplate method),
<pre>convert_to_dimensional_bulk_salinity() (celes-</pre>	39
$tine.dimensional\_params.Scales\ method), 7$	<pre>geometric() (in module celestine.grids), 15</pre>

50 Index

<pre>get_barrow_initial_conditions() (in module ce- lestine.initial_conditions), 17 get_config() (celestine.dimensional_params.Dimensional_method), 5</pre>	celestine.phase_boundaries, 23 celestine.run_simulation, 25 alParamedestine.solvers.reduced_solver, 41 celestine.solvers.scipy, 43
get_convecting_region_height() (in module celes-	celestine.solvers.template, 39
tine.brine_drainage), 35	celestine.state, 27
get_darcy_law_params() (celes-	celestine.velocities, 29
tine.dimensional_params.DimensionalParams method), 5	N
<pre>get_effective_Rayleigh_number() (in module ce- lestine.brine_drainage), 36</pre>	NumericalParams (class in celestine.params), 22
<pre>get_enthalpy_method() (in module celes-</pre>	0
tine.enthalpy_method), 9	ocean_freezing_temperature (celes-
<pre>get_physical_params() (celes- tine.dimensional_params.DimensionalParams method), 5</pre>	tine.dimensional_params.DimensionalParams property), 6
<pre>get_scales() (celestine.dimensional_params.Dimensional method), 5</pre>	a <b>P</b> arams
get_uniform_initial_conditions() (in module celestine.initial_conditions), 17	PhaseBoundaries (class in celestine.phase_boundaries), 23
	PhysicalParams (class in celestine.params), 22
L	pre_solve_checks() (celes-
<pre>lewis_gas (celestine.dimensional_params.DimensionalParams.property), 5</pre>	meinoa), 41
property), 5 lewis_salt (celestine.dimensional_params.DimensionalF	pre_solve_checks() (celes-
property), 5	tine.solvers.template.SolverTemplate method),
liquid_fraction_BCs() (in module celes-	39
tine.boundary_conditions), 1	pressure_BCs() (in module celes-
liquid_salinity_BCs() (in module celes-	$tine.boundary\_conditions), 1$
tine.boundary_conditions), 1	<pre>prevent_gas_rise_into_saturated_cell() (in</pre>
load() (celestine.dimensional_params.DimensionalParam	module celestine.solvers.reduced_solver), 41
class method), 5	_
load_forcing_data() (celes-	R
tine.params.ForcingConfig method), 22	Rayleigh_salt (celes-
interparament or entry congress mentous), 22	$tine. dimensional\_params. Dimensional Params$
M	property), 4
maximum_bubble_radius_scaled (celes-	ReducedEnthalpyMethod (class in celes-
tine.dimensional_params.DimensionalParams	tine.enthalpy_method), 9
	ReducedPhaseBoundaries (class in celes-
<pre>property), 6 minimum_bubble_radius_scaled (celes-</pre>	tine.phase_boundaries), 23
tine.dimensional_params.DimensionalParams	ReducedSolver (class in celes-
property), 6	tine.solvers.reduced_solver), 41
module	run_batch() (in module celestine.run_simulation), 25
celestine.boundary_conditions, 1	0
celestine.boundary_conditions, 1 celestine.brine_channel_sink_terms, 37	S
celestine.brine_drainage, 33	salinity_difference (celes-
celestine.dimensional_params, 3	$tine. dimensional\_params. Dimensional Params$
celestine.enthalpy_method, 9	property), 6
celestine.flux, 11	<pre>salt_BCs() (in module celestine.boundary_conditions),</pre>
celestine.forcing, 13	1
celestine.grids, 15	<pre>save() (celestine.dimensional_params.DimensionalParams</pre>
celestine.initial_conditions, 17	method), 6
celestine.logging_config, 19	${\tt savefreq} \ (celestine. dimensional\_params. Dimensional Params$
celestine.params, 21	property), 6

Index 51

```
Scales (class in celestine.dimensional_params), 6
ScipySolver (class in celestine.solvers.scipy), 43
Solution (class in celestine.state), 27
solve() (in module celestine.run_simulation), 25
SolverTemplate (class in celestine.solvers.template), 39
State (class in celestine.state), 27
StateBCs (class in celestine.state), 27
stefan_number
                                               (celes-
         tine.dimensional\_params.DimensionalParams
         property), 6
Т
take_forward_euler_step() (in module celes-
         tine.flux), 11
take_timestep()
                                               (celes-
         tine.solvers.reduced\_solver.ReducedSolver
         method), 41
take_timestep() (celestine.solvers.scipy.ScipySolver
         method), 43
take_timestep()
                                               (celes-
         tine.solvers.template.SolverTemplate method),
temperature_BCs()
                          (in
                                   module
                                               celes-
         tine.boundary_conditions), 1
temperature_difference
                                               (celes-
         tine.dimensional\_params.DimensionalParams
         property), 6
thermal_diffusivity
                                               (celes-
         tine.dimensional_params.DimensionalParams
         property), 6
total\_time(celestine.dimensional\_params.DimensionalParams
         property), 6
```

52 Index