

Predefined-time Formation Tracking for Multi-agent Systems

Kai-Lun Huang, Ming-Feng Ge, *Member, IEEE*,

Chang-Duo Liang, Teng-Fei Ding

School of Mechanical Engineering and Electronic Information

China University of Geosciences

Wuhan 430074, China

huangkailun@cug.edu.cn (Kai-Lun Huang),

gemf@cug.edu.cn (Ming-Feng Ge),

cdliang@cug.edu.cn (Chang-Duo Liang),

dingtf@cug.edu.cn (Teng-Fei Ding)

Guang-Hui Xu

Hubei Key Laboratory for High-efficiency Utilization
of Solar Energy and Operation Control

of Energy Storage System

Hubei University of Technology

Wuhan 4300068, China

xgh@hbut.edu.cn (Guang-Hui Xu)

Abstract—A novel predefined-time formation tracking (PTFT) problem of the multi-agent systems (MASs) is investigated by employing a time base generators (TBG) in this paper. In addition, the information interaction of MASs is represented by an acyclic directed graph. To solve the aforementioned challenging problem, a novel TBG-based control algorithm for the MASs is designed. Proposed TBG-based control algorithm significantly reduces the order of magnitude for the initial control input. Besides, the desired settling time can be predefined through adjusting a TBG gain and it is achieved independently of the initial state. Eventually, simulation experiments are performed to illustrate that the proposed algorithm for the MASs is effective.

Index Terms—predefined-time formation tracking (PTFT), time base generators (TBG), multi-agent systems (MASs).

I. INTRODUCTION

Considerate attention has recently been paid to the cooperative control problem of the MASs [1] due to it has a comprehensive range of potential uses, such as target enclosing [2], large components assembly [3], and attitude synchronization of spacecraft [4]. Therefore, it proposes a series of related topics, including consensus [5], target tracking [6], bipartite consensus/tracking [7], formation tracking [8], containment control [9].

With the development of the cooperative control, there is a trend that researchers start to study the problem of formation tracking. There have been several articles on formation tracking algorithms for different types of systems. Specifically, the time-varying output formation-tracking of heterogeneous linear MASs has been studied in [10]. Contrasting with the uniparted agent, there are more obvious advantages that the MASs generally contain higher efficiency, better reliability and extra flexibility of executing control algorithms. Therefore, Ge et al. [11] solved problem of formation tracking for MASs in case of the undirected interaction topology. Furthermore, it makes a lot of sense for the scholars to investigate the formation tracking of MASs under the directed graph. Hence, the formation tracking problem of MASs via finite-time control under the directed interaction topology has been addressed in [8]. Moreover, it should be pointed out that the aforementioned

works can only achieve formation tracking in an asymptotic and finite-time manner, where convergence rate is a function related to interaction information of the topology.

It is worthwhile noting that convergence rate is an extremely important performance indicator to value the efficacy of the formation tracking control algorithm. As a consequence, in order to deal with the formation tracking problem with Prior time constraints, the concept of predefined-time convergence has been put forward. To be specific, the time at which formation tracking is implemented in predefined rules through adjusting a TBG gain and it has been completed independently of the initial state. Different from some conventional finite-time and fixed-time formation tracking algorithms developed in [8], [11], [12], the control input of the TBG-based algorithms presented in [13] have relatively smaller magnitude when there is a large initial states tracking errors, which is more preferable in practice. As far as we know, the PTFT problem of MASs is still up in the air. Hence, it is highly significant and challenging to study the PTFT problem of MASs under the directed interaction graph, which enriches the control field of MASs.

Inspired by the aforementioned discussion, this paper proposes a TBG-based control algorithm for MASs under the directed interaction graph to deal with the PTFT problem. The main contributions are listed below.

- (1) The unsolved PTFT problem of MASs is successfully addressed by using a novel TBG-based control algorithm.
- (2) The desired settling time of the formation tracking can be predefined independently of the initial state.
- (3) Proposed TBG-based controllers yield relatively smaller order of magnitude for the initial control input.

The content of this article is organized as following. The section II provides the relevant preliminaries statement, including the formulation of the MASs and the definitions of the considered control problems. The section III proposes the main results, including the TBG-based control algorithm and its stability analysis for PTFT. The simulation experiment results

are represented in section IV to validate the effectiveness of the proposed TBG-based control algorithm to deal with the aforementioned problem. Finally, section V draws conclusions.

Notations: \mathbb{R}^n represents the n -dimensional Euclidean space and $\mathbb{R}^{n \times n}$ is $n \times n$ real matrix. \otimes is the Kronecker product. I_n represents n -order identity matrix. $\mathbf{1}_N$ stands for the N -dimensional column vector whose elements are 1. $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of the corresponding matrix. $\min\{\cdot\}$ and $\max\{\cdot\}$ respectively denotes the minimum and maximum value of all elements in a given set. $\|\cdot\|$ and $\|\cdot\|_p$ respectively is the Euclidean norm and the p -norm. $\text{col}(\cdot)$ denotes the column vector and $\text{sign}(\cdot)$ represents the sign function.

II. PRELIMINARIES STATEMENT

A. Algebraic Graph Theory

The considered MASs consist of N agents, the interactions communication of the MASs is described here by a directed graph $\mathcal{G} = \{V, E, A\}$, where $V = \{1, \dots, N\}$, $E \subseteq V \times V$ and $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ represents a set of vertices (i.e., agents in the MASs), a set of edges and the adjacent matrix, respectively. An edge $e_{ij} \in E$ signifies that the i -th agent can obtain the information from the j -th agent. If the interaction between the i -th agent and the j -th agent is cooperative, then $a_{ij} > 0$, otherwise $a_{ij} = 0$, i.e., $e_{ij} \notin E$. Besides, the vertex has no self-loops, i.e., $a_{ii} = 0$, $\forall i \in V$. The Laplacian matrix with respect to graph \mathcal{G} is defined as $\mathcal{L} = [l_{ij}]$, where $l_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}$ and $l_{ij} = -a_{ij}$, $\forall i \neq j \in V$, where $\mathcal{N}_i = \{j \in V \mid e_{ij} \in E\}$ represents the neighbor set of the agent i . A diagonal weighted matrix $\mathcal{B} = \text{diag}(b_1, b_1, \dots, b_N)$ is utilized to illustrate the interactions between the agents and their target. Furthermore, if the i -th agent can receive the information from their target, then $b_i > 0$, otherwise, $b_i = 0$, $\forall i \in V$.

Assumption 1: It is assumed that the directed graph \mathcal{G} is detail-balanced, which means there exist positive constants p_1, p_2, \dots, p_N such that $p_j a_{ij} = p_i a_{ji}$, $\forall i, j \in V$.

Assumption 2: The directed graph \mathcal{G} has a directed spanning tree if there exists a directed path between at least one vertex and the other arbitrary vertex in the directed graph.

Lemma 1: Based on Assumption 1 and 2, let the diagonal matrix $P = \text{diag}(p_1, p_2, \dots, p_N)$ and $\mathcal{M} = \mathcal{L} + \mathcal{B}$, then there is a positive-definite matrix \mathcal{MP} .

B. System Formulation

Considered MASs consist of N agents with single-integrator dynamics of

$$\dot{x}_i(t) = \tau_i(t), \quad i \in V, \quad (1)$$

where $x_i(t)$, $\tau_i(t) \in \mathbb{R}^n$ denote the system state and the system control input, respectively.

On the other hand, the target can be described as follows:

$$\dot{x}_0(t) = \tau_0(t), \quad (2)$$

where $x_0(t)$, $\tau_0(t) \in \mathbb{R}^n$ represent the position and the input for the target, respectively.

Due to the restraint of the motor, the velocity of the agent is bounded in practical application, therefore, there is a corresponding assumption here.

Assumption 3: The input $\tau_0(t)$ of the target has an upper-bounded, i.e., $\|\tau_0(t)\| \leq \bar{\delta}_{\tau_0}$, where $\bar{\delta}_{\tau_0}$ is a positive constant.

C. Problem Formulation

The primary purpose is to address the PTFT problem defined below. Let us define $\varepsilon_i = x_i - \Delta_i - x_o$, $\forall i \in V$, where ε_i represents the position tracking errors and Δ_i denotes the formation offset. Then, some significant definitions are given as follows.

Definition 1: The PTFT problem of the MASs is solved if there is a predefined constant T_P so as to

$$\lim_{t \rightarrow T_P} \|\varepsilon_i(t)\| \leq \bar{\delta}_\varepsilon, \quad (3)$$

where $\bar{\delta}_\varepsilon > 0$, besides, it is necessary to ensure that $\bar{\delta}_\varepsilon$ is arbitrary small by choosing appropriate control parameters, then $\|\varepsilon_i(t)\| \leq \bar{\delta}_\varepsilon$ when $t \in [T_P, \infty]$.

Definition 2: The time base generator (TBG) is described by a timing function. Besides, the timing function itself and its first-order derivative satisfy the specific constraint conditions at its initial time and its final time.

Therefore, a useful lemma is presented as follows.

Lemma 2: Based on the definition 2, for system $\dot{z}(t) = -\alpha(t)z(t)$, $z(0) = z_0$, where $z(t)$, $z_0 \in \mathbb{R}$ denote the system state and the initial state, respectively, and a TBG gain $\alpha(t)$ is defined as follows.

$$\alpha(t) = \frac{\dot{\gamma}(t)}{1 - \gamma(t) + \rho}, \quad (4)$$

where $\gamma(t)$ is the TBG, and $0 < \rho \leq 1$. The following conditions must be maintained at the initial time $t_0 = 0$ and final time T :

$$\begin{cases} \gamma(t) = 0, & t = 0, \\ \gamma(t) = 1, & t \geq T, \end{cases} \quad (5)$$

$$\begin{cases} \dot{\gamma}(t) = 0, & t = 0 \text{ and } t \geq T, \\ \dot{\gamma}(t) > 0, & t \in (0, T). \end{cases} \quad (6)$$

Eventually the system evolved to reach the state of $\frac{\rho}{1+\rho}z_0$ in the predefined-time T independently of the initial state.

Remark 1: The parameter ρ in (4) guarantees that the TBG gain $\alpha(t)$ is well-defined when $t \geq T$.

III. MAIN RESULTS

A novel TBG-based control algorithm is designed to solve PTFT problem for MASs in this section. Besides, the corresponding stability analysis is shown to demonstrate the feasibility of the designed algorithm in this paper.

A. TBG-based Control Algorithm

For PTFT problem, we propose a novel TBG-based control algorithm for MASs as follows.

$$\begin{aligned} \tau_i = & -(\eta+1) \left[\sum_{j \in \mathcal{N}_i} a_{ij} (x_i - x_j + \Delta_j - \Delta_i) + b_i (x_i - \Delta_i - x_0) \right] \\ & - \varpi \text{sign} \left[\sum_{j \in \mathcal{N}_i} a_{ij} (x_i - x_j + \Delta_j - \Delta_i) + b_i (x_i - \Delta_i - x_0) \right], \end{aligned} \quad (7)$$

where $\varpi > 0$. The TBG gains $\eta(t)$ is designed as follows.

$$\eta(t) = \frac{\bar{\delta}_p \dot{\gamma}(t)}{2\lambda_{\min}(\mathcal{M}P)(1 - \gamma(t) + \rho)}, \quad (8)$$

where

$$\bar{\delta}_p = \max \{p_1, p_2, \dots, p_N\}, \quad (9)$$

$0 < \rho \leq 1$, $\mathcal{M}P$ have been defined in Lemma 1, $\gamma(t)$ represents the TBG.

Remark 2: It is worthy noting that the TBG-based term $-\eta(\sum_{j \in \mathcal{N}_i} a_{ij} (x_i - x_j + \Delta_j - \Delta_i) + b_i (x_i - \Delta_i - x_0))$ in (7) is to guarantee that the position tracking errors are smaller than a desired bounded regions in a predefined-time. Besides, the compensation term $-(\sum_{j \in \mathcal{N}_i} a_{ij} (x_i - x_j + \Delta_j - \Delta_i) + b_i (x_i - \Delta_i - x_0))$ is to ensure that the position tracking errors can converge to origin when $t \rightarrow +\infty$.

B. Analysis for Predefined-time Formation Tracking

In this subsection, it is verified that the PTFT problem can be accomplished utilizing the designed TBG-based control algorithm.

Theorem 1: Based on Assumptions 1-3. Using the proposed TBG-based control algorithm (7), if

$$\varpi \geq \frac{\bar{\delta}_p}{\delta_p} \bar{\delta}_{\tau_0}, \quad (10)$$

where δ_p will be defined in (17), $\bar{\delta}_p$ is represented in (9) and $\bar{\delta}_{\tau_0}$ has been defined in Assumption 3, then the problem defined in Definition 1 can be successfully solved, i.e., e_i eventually converges to the bounded regions that $\|e_i(t)\| \leq \bar{\delta}_e$ in predefined-time T_P .

Proof: To simplify the TBG-based control algorithm, due to $\varepsilon_i = x_i - \Delta_i - x_0$ has been defined above, $\dot{\varepsilon}_i$ can be derived as $\dot{\varepsilon}_i = \dot{x}_i - \dot{x}_0$. Thus, substituting the proposed TBG-based control algorithm (7) into (1), the following cascade closed-loop system arises,

$$\begin{aligned} \dot{\varepsilon}_i = & -(\eta+1) \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\varepsilon_i - \varepsilon_j) + b_i \varepsilon_i \right] \\ & - \varpi \text{sign} \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\varepsilon_i - \varepsilon_j) + b_i \varepsilon_i \right] - \dot{x}_0. \end{aligned} \quad (11)$$

Besides, the compact forms of ε is defined as follows.

$$\varepsilon = \text{col}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N). \quad (12)$$

Thus, the compact forms of (11) can be derived as:

$$\dot{\varepsilon} = -(\eta+1)(\mathcal{M} \otimes I_n) \varepsilon - \varpi \text{sign}[(\mathcal{M} \otimes I_n) \varepsilon] - 1_N \otimes \tau_0, \quad (13)$$

where $\mathcal{M} = \mathcal{L} + \mathcal{B}$.

Based on Assumption 1 and the positive-definite matrix $\mathcal{M}P$ defined in Lemma 1, consider the following positive-definite and continuous differentiable Lyapunov function candidate:

$$\begin{aligned} V(t) = & \frac{1}{2} [(\mathcal{M} \otimes I_n) \varepsilon]^T (\mathcal{M}P \otimes I_n)^{-1} [(\mathcal{M} \otimes I_n) \varepsilon] \\ = & \frac{1}{2} \varepsilon^T (\mathcal{M}^T P^{-1} \otimes I_n) \varepsilon. \end{aligned} \quad (14)$$

Denote $\mathcal{M}^T P^{-1} = [c_{ij}]_{N \times N}$, let

$$\begin{aligned} \delta_c = & \min \left\{ \left\{ c_{ii} + \sum_{j \neq i}^N c_{ij} \mid c_{ii} + \sum_{j \neq i}^N c_{ij} \neq 0, \forall i, j \in V \right\}, \right. \\ & \left. \min \{ |c_{ij}| \mid c_{ij} \neq 0, \forall i \neq j \in V \} \right\}. \end{aligned} \quad (15)$$

Thus, based on the conditions defined in (10) and Assumption 3 along (13), the derivative of $V(t)$ is

$$\begin{aligned} \dot{V}(t) = & \varepsilon^T (\mathcal{M}^T P^{-1} \otimes I_n) (-(\eta+1)(\mathcal{M} \otimes I_n) \varepsilon \\ & - \varpi \text{sign}[(\mathcal{M} \otimes I_n) \varepsilon] - 1_N \otimes \tau_0) \\ \leq & -\eta \bar{\delta}_p^{-1} \|(\mathcal{M} \otimes I_n) \varepsilon\|^2 \\ & - \left(\bar{\delta}_p^{-1} \vartheta - \delta_p^{-1} \bar{\delta}_{\tau_0} \right) \|(\mathcal{M} \otimes I_n) \varepsilon\|_1 \\ \leq & -\eta \bar{\delta}_p^{-1} \|(\mathcal{M} \otimes I_n) \varepsilon\|^2 \\ \leq & -\frac{2\eta \lambda_{\min}(\mathcal{M}P)}{\bar{\delta}_p} V(t) \\ = & -\frac{\dot{\gamma}}{1 - \gamma + \sigma} V(t), \end{aligned} \quad (16)$$

where

$$\delta_p = \min \{p_1, p_2, \dots, p_N\}, \quad (17)$$

$$V(t) \leq \frac{1}{2} \lambda_{\min}^{-1}(\mathcal{M}P) \|(\mathcal{M} \otimes I_n) \varepsilon\|^2 \quad (18)$$

are utilized to obtain the aforementioned inequalities, $\mathcal{M}P$ is defined in Lemma 1, η is designed in (8). For $t \in [0, T_P]$, according to Lemma 2, $V(t)$ can converge to a set $\Omega = \left\{ V(t) \mid V(t) \leq \frac{\rho}{1+\rho} V(0) \right\}$ in predefined-time T_P that can be predefined without dependence on initial state. Furthermore, it can be derived that

$$\lim_{t \rightarrow T_P} \|\varepsilon_i\| \leq N \sqrt{\frac{2\rho}{\delta_c(1+\rho)}} V(0) \leq \bar{\delta}_e, \forall i \in V. \quad (19)$$

For $t > T_P$, $\eta(t) = 0$, it can be derived that $\dot{V}(t) \leq -2\bar{\delta}_p^{-1} \lambda_{\min}(\mathcal{M}P) V(t)$, so that we can obtain $V(t)$ is non-increasing. Thus, it acquires not only the bound holds but also the origin is asymptotically stable, namely,

$$\|\varepsilon_i\| \leq N \sqrt{\frac{2\rho}{\delta_c(1+\rho)}} V(0) \leq \bar{\delta}_e, \forall t > T_P, \quad (20)$$

$$\lim_{t \rightarrow +\infty} \|\varepsilon_i\| = 0, \forall i \in V. \quad (21)$$

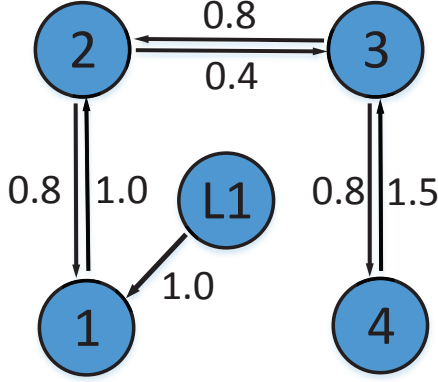


Fig. 1. The information interaction of the MASs.

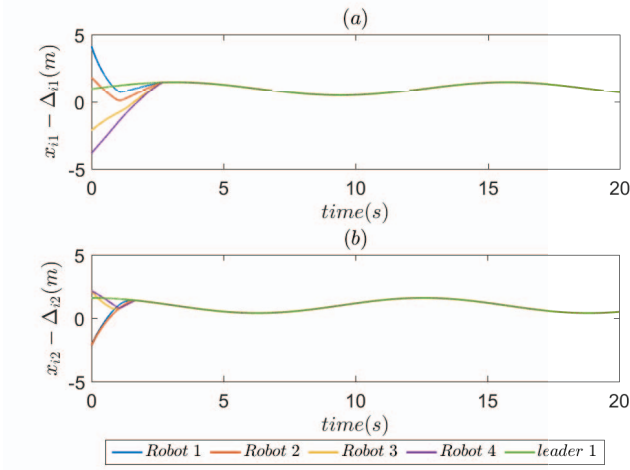


Fig. 2. The evolution of $(x_i - \Delta_i)$ for coordinates 1 and 2.

Therefore, the PTFT problem is addressed. The proof has been finished.

Remark 3: It can be derived that the boundary $N\sqrt{\frac{2\rho}{\delta_c(1+\rho)}}V(0)$ of the position tracking errors is related to the initial state, but it can be adjusted to a desired bounded regions by selecting an appropriate parameter ρ .

Remark 4: Different from some conventional finite-time, fixed-time and predefined-time research results [11], [12], [14], the proposed predefined-time control algorithm (7) uses a TBG method which avoids the large magnitude of initial control input. Because the founding value of the TBG-term is set to zero.

IV. SIMULATION EXPERIMENTS

Simulation experiments results have been presented to demonstrate the feasibility of the designed algorithm TBG-based control algorithm in this section. It is assumed that the MASs contains one leader (i.e., target) labeled by 0 and four robots (i.e., agents) labeled by 1 to 4.

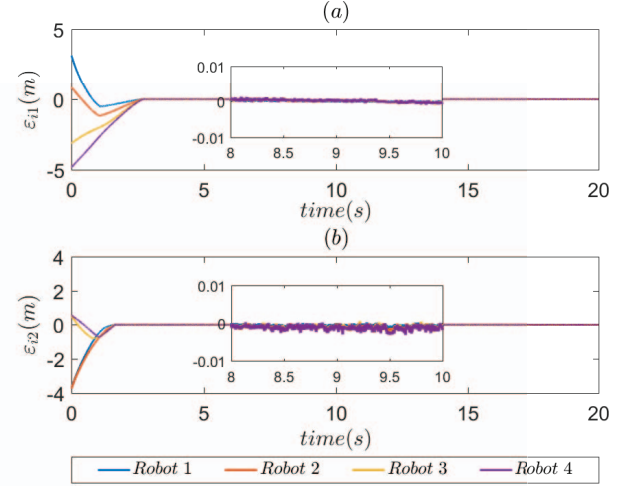


Fig. 3. The evolution of the tracking errors ϵ_i for coordinates 1 and 2.

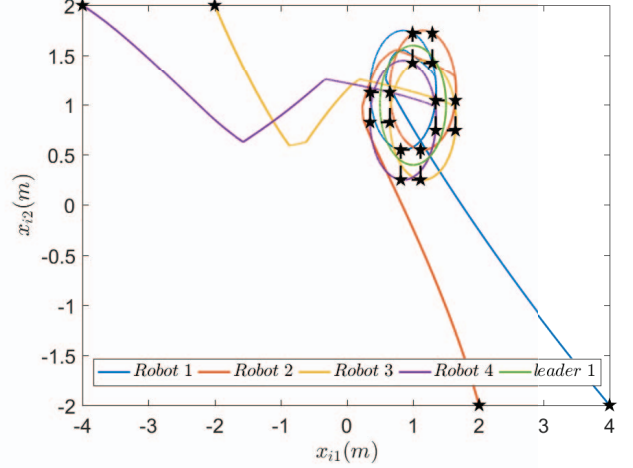


Fig. 4. The PTFT trajectories of the MASs in XY plane.

The information interaction graph of the MASs is illustrated in Fig.1, which determines the corresponding Laplacian matrix \mathcal{L} , diagonal weighted matrixes \mathcal{B} and \mathcal{P} as follows:

$$\mathcal{L} = \begin{pmatrix} 0.8 & -0.8 & 0 & 0 \\ -1 & 1.8 & -0.8 & 0 \\ 0 & -0.4 & 0.9 & -0.5 \\ 0 & 0 & -0.8 & 0.8 \end{pmatrix},$$

$$\mathcal{B} = \text{diag}(1, 0, 0, 0),$$

$$\mathcal{P} = \text{diag}(2, 2.5, 1.25, 2).$$

Consider Assumption 3, the leader labeled as L1 is chosen as

$$\begin{cases} x_o = [1 + \frac{1}{2}\sin(\frac{t}{2}), 1 + \frac{3}{5}\cos(\frac{t}{2})]^T, \\ \dot{x}_o = [\frac{1}{4}\cos(\frac{t}{2}), -\frac{3}{10}\sin(\frac{t}{2})]^T. \end{cases} \quad (22)$$

The formation is expected to form a square, which is specified by

$$\begin{cases} \Delta_1 = [-\frac{3}{20}, \frac{3}{20}], \\ \Delta_2 = [\frac{3}{20}, \frac{3}{20}], \\ \Delta_3 = [\frac{3}{20}, -\frac{3}{20}], \\ \Delta_4 = [-\frac{3}{20}, -\frac{3}{20}]. \end{cases} \quad (23)$$

The control parameters in the TBG-based control algorithm (7) are given as $\rho = 0.001$, $\varpi = 1$. A detailed of TBG function is presented and the founding moment is represented as $t_0 = 0$, namely,

$$\gamma(t) = \begin{cases} \frac{10}{46}t^6 - \frac{24}{45}t^5 + \frac{15}{44}t^4, & 0 \leq t \leq T_P, \\ 1, & t > T_P, \end{cases} \quad (24)$$

then it can be derived that $T_P = 4s$. The initial value of $x_i(0)$ and $\dot{x}_i(0)$ are randomly chosen from $[-4, 4]$.

Remark 5: Based on the TBG gains $\eta(t)$ in (8), the TBG function in (24) and the properties of the TBG defined in Lemma 2, i.e., $\gamma(0) = \dot{\gamma}(0) = 0$, $\gamma(1) = 1$, and $\dot{\gamma}(1) = 0$, it can be further concluded that (i) $T_P = 4s$, (ii) $\eta(t)$ is non-negative and increasing from zero at $t = 0$, when $0 \leq t \leq T_P$.

simulation results: The final experimental results of the PTFT problem are displayed in Figs.2-4. For a detailed description of the figures, from pictures (a) and (b) in Fig. 2, which can be clearly observed that the position $x_i - \Delta_i$ can track x_0 in predefined-time $T_P = 4s$. Fig. 3 describe the evolution of ε_i , which can be clearly observed that ε_i are bounded in $[-0.01, 0.01]$. The tracking trajectories of positions in XY plane of the MASs are shown in Fig. 4, and it shows that the MASs eventually evolve in a desired formation pattern (i.e., a square) in predefined-time $T_P = 4s$. In conclusion, the desired PTFT performance can be obtained via the proposed TBG-based control algorithm.

V. CONCLUSION

In this paper, by using the TBG-based control algorithm, the PTFT problem of single-integrator MASs have been successfully solved under the directed topologies graph. It has been demonstrated that the states of all agents in MASs can converge to any small neighborhood of the target state within a predetermined time. By using the Lyapunov stability theory, it can be obtained that the effective conditions on the control parameters for the MASs to achieve stable. Specifically, simulation experiments results have been presented to described the feasibility and effectiveness of the designed TBG-based control algorithm. Future works will investigate the PTFT problem for network robotic systems and the event-triggered control.

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Conflict of interest The authors declare that they have no conflict of interest.

REFERENCES

- [1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215-233, 2007.
- [2] H. Duan, "A Binary Tree Based Coordination Scheme for Target Enclosing with Micro Aerial Vehicles," *IEEE/ASME Transactions on Mechatronics*, 2020.
- [3] S. Djezairi, B. Z. Razika, I. Akli, B. Bouzouia, and P. K. Abdellah, "Robust adaptive control of coordinated multiple mobile manipulators," *International Conference on Applied Automation and Industrial Diagnostics*, vol. 1, pp. 1-6, 2019.
- [4] B. Wu, C. Xu, and Y. Zhang, "Decentralized adaptive control for attitude synchronization of multiple spacecraft via quantized information exchange," *Acta Astronautica*, 2020.
- [5] X. Li, Y. Tang, and H. R. Karimi, "Consensus of multi-agent systems via fully distributed event-triggered control," *Automatica*, vol. 116, pp. 108898, 2020.
- [6] M. F. Ge, Z. H. Guan, B. Hu, D. X. He, and R. Q. Liao, "Distributed controller-estimator for target tracking of networked robotic systems under sampled interaction," *Automatica*, vol. 69, pp. 410-417, 2016.
- [7] J. Wu, Q. Deng, T. Han, and H. C. Yan, "Distributed bipartite tracking consensus of nonlinear multi-agent systems with quantized communication," *Neurocomputing*, 2020.
- [8] L. Tian, Y. Hua, X. Dong, Q. Li, and Z. Ren, "Distributed adaptive finite-time time-varying group formation tracking for high-order multi-agent systems with directed topologies," *Chinese Control Conference*, pp. 4627-4632, 2020.
- [9] X. Dong, Z. Shi, G. Lu, and Y. Zhong, "Formation containment analysis and design for high-order linear time-invariant swarm systems," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 17, pp. 3439-3456, 2015.
- [10] J. Duan, H. Zhang, Y. Cai, and K. Zhang, "Finite-time time-varying output formation-tracking of heterogeneous linear multi-agent systems," *Journal of the Franklin Institute*, vol. 357, no. 2, pp. 926-941, 2020.
- [11] M. F. Ge, Z. H. Guan, C. Yang, T. Li, and Y.-W. Wang, "Time-varying formation tracking of multiple manipulators via distributed finite-time control," *Neurocomputing*, vol. 202, pp. 20-26, 2016.
- [12] X. Chai, J. Liu, Y. Yu, J. Xi, and C. Sun, "Practical Fixed-Time Event-Triggered Time-Varying Formation Tracking Control for Disturbed Multi-Agent Systems with Continuous Communication Free," *Unmanned Systems*, pp. 1-12, 2020.
- [13] V. Parra-Vega, "Second order sliding mode control for robot arms with time base generators for finite-time tracking," *Dynamics and Control*, vol. 11, no. 2, pp. 175-186, 2001.
- [14] R. Aldana-López, D. Gómez-Gutiérrez, E. Jiménez-Rodríguez, J. D. Sánchez-Torres, and A. G. Loukianov, "On predefined-time consensus protocols for dynamic networks," *Journal of The Franklin Institute-engineering and Applied Mathematics*, 2019.