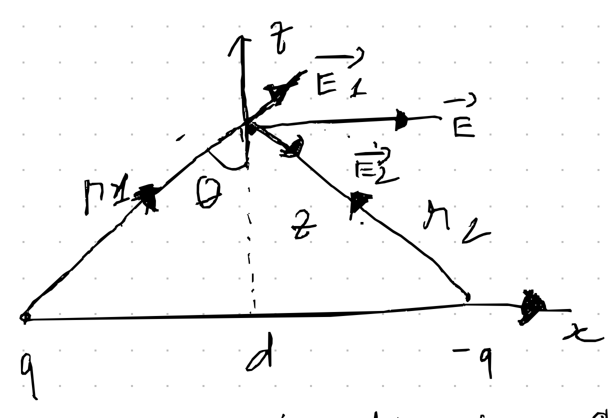
HW3:

Problem 1



find magnitude and direction of E.

$$r = \left(z^2 + \frac{d}{2}\right)^2$$

$$\cos\theta = \frac{z}{r}$$

Since they have opposite charge.

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}_1 ; \qquad \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{r^2}\hat{r}_2$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\left(\vec{r_1} - \hat{r}_2\right) \qquad \hat{r} = \frac{\vec{r}}{r} \text{ (unit vector)}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\left(\frac{\vec{r}_1}{r} - \frac{\vec{r}_2}{r}\right) = \frac{d}{4\pi r^3}\left(\vec{r_1} - \vec{r_2}\right)$$

$$\vec{r_2} = -\frac{d}{2}\hat{x} + z\hat{z}$$ (because $\vec{r_2}$ is pushing in the negative direction of $\hat{x}$)

$$\vec{r_1} = \frac{d}{2}\hat{x} + z\hat{z}$$

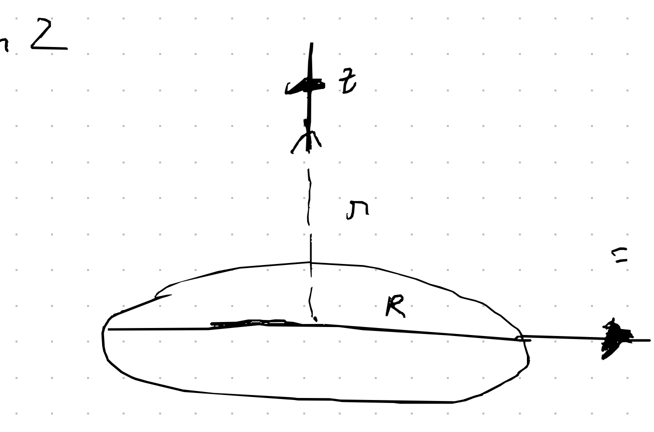
$$r_2, r_1 = \sqrt{\left(\pm\frac{d}{2}\right)^2 + z^2}$$

Substitute back to our equation.

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\left(\sqrt{\left(\pm\frac{d}{2}\right)^2 + z^2}\right)^3}\left[\left(\frac{d}{2}\hat{x} + z\hat{z}\right) - \left(-\frac{d}{2}\hat{x} + z\hat{z}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\left(\sqrt{\left(\frac{d}{2}\right)^2 + z^2}\right)^3} d\hat{x}$$

Problem 2



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(r')}{r^2}\hat{r}\,da'$$

$$= \frac{1}{4\pi\varepsilon_0}\iint\frac{\sigma(r')}{r^2}\hat{r}\,\underline{d\phi\,dr}$$

polar Coordinate

$\rightarrow$ z direction

$$\vec{r} = \vec{r} - \vec{r}' = z\hat{z} - (r\hat{r})$$

vector points to points on disk

$$|\vec{r}| = \sqrt{z^2 + r^2}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{z\hat{z} - r\hat{r}}{\sqrt{z^2 + r^2}}$$

Substitute back to equation.

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_0^R\int_0^{2\pi}\frac{\sigma r}{(z^2+r^2)}\left(\frac{z\hat{z} - r\hat{r}}{\sqrt{z^2+r^2}}\right)d\phi\,dr$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\int_0^R\int_0^{2\pi}\frac{\sigma r z\hat{z}}{(z^2+r^2)^{\frac{3}{2}}}d\phi\,dr - \int_0^R\int_0^{2\pi}\frac{\sigma r^2\hat{r}}{(z^2+r^2)^{\frac{3}{2}}}d\phi\,dr\right]$$

Because this is a circular disk therefore there are alwayse equal and opposite direction of $\hat{r}$ term $\rightarrow$ they cancel out.

$$= \frac{1}{4\pi\varepsilon_0}\int_0^R\int_0^{2\pi}\frac{\sigma r z\hat{z}}{(z^2+r^2)^{\frac{3}{2}}}d\phi\,dr = \frac{\sigma z\hat{z}}{4\pi\varepsilon_0}\int_0^R\frac{2\pi r\,dr}{(z^2+r^2)^{\frac{3}{2}}}dr$$

$$= \frac{\sigma z\hat{z}}{2\varepsilon_0}\int_0^R\frac{r\,dr}{(z^2+r^2)}dr = \frac{\sigma z\hat{z}}{2\varepsilon_0}\left(\frac{1}{z} - \underbrace{\frac{1}{\sqrt{z^2-R^2}}}_{\infty}\right)\hat{z}$$

when R goes to ∞ $\quad \frac{1}{\sqrt{z^2+R^2}} \rightarrow 0$

$$= \frac{\sigma z\hat{z}}{2\varepsilon_0}\cdot\frac{1}{z} = \frac{\sigma\hat{z}}{2\varepsilon_0}$$

when $z \gg R$ or $z \rightarrow \infty$

$$E = \frac{\sigma z}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{(z^2-R^2)}}\right)$$

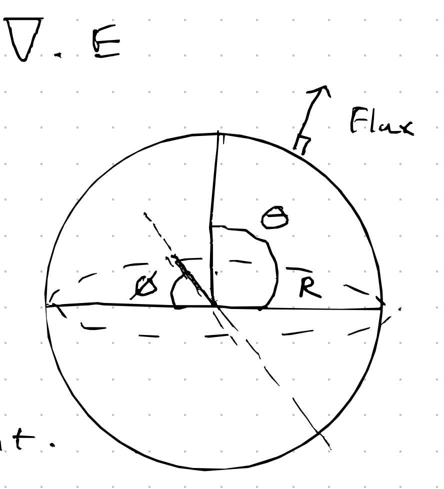
$\frac{1}{z}$ (because R is neglectable compare to z)

$$\Rightarrow \frac{\sigma z}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{z}\right) = 0\hat{z}.$$

Problem 3 $\quad E = kr^3\hat{r}. \qquad$ Coordinates (r, ϕ, θ).

(a)

$$\nabla\cdot E = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = \varepsilon_0\nabla\cdot E$$



since we only have $\hat{r}$ component.

$$\rho = \varepsilon_0\nabla\cdot E = \varepsilon_0\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\cdot E)$$

$$= \varepsilon_0\frac{1}{r^2}\cdot\frac{\partial}{\partial r}kr^5 = \varepsilon_0\frac{1}{r^2}5kr^4$$

$$= \varepsilon_0 5kr^2.$$

(b)

(1)
$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^R 5\varepsilon_0 k\underbrace{r^2\,r^2}_{r^4}\sin\theta\,dr\,d\theta\,d\phi$$

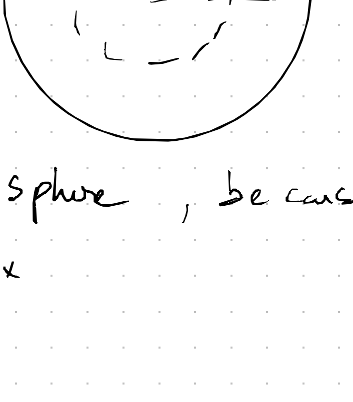
$$= 5\varepsilon_0 k\,2\pi\int_0^{\pi}\frac{R^5}{5}\sin\theta\,d\theta$$

$$= \cancel{5}\varepsilon_0 k\,2\pi\,\frac{2R^5}{\cancel{5}} = 4\varepsilon_0\pi R^5$$

(2)
$$\oint E\cdot da = kR^3\,4\pi R^2 = \frac{Q}{\varepsilon_0} \Rightarrow Q = 4kR^5\varepsilon_0\pi$$

$4\pi R^2$ (surface area).

Problem 4:



$$\underbrace{\oint\vec{E}\,da = \frac{Q_{enc}}{\varepsilon_0}}_{\text{Gauss.}}$$

for inside the sphere, because the charge is on the Surface. there is no flux

$$\Rightarrow \vec{E} = 0$$

for outside the sphere $r > R$.

we have the surface area $= 4\pi R^2$

$$\vec{E}\,4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow \vec{E} = \frac{Q}{4\pi r^2}\hat{r}.$$