Homework 1 - Electromagnetism

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I pledge my honor that I have abided by the Stevens Honor System.

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Problem 1

Prove the identity $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

Left hand-side:

$$
\begin{aligned}
\vec{B} \times \vec{C} &= (B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) \times (C_x\hat{x} + C_y\hat{y} + C_z\hat{z}) \\
&= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \\
&= (B_yC_z - B_zC_y)\hat{x} + (B_zC_x - B_xC_z)\hat{y} + (B_xC_y - B_yC_x)\hat{z}
\end{aligned}
$$

$$
\begin{aligned}
\vec{A} \times (\vec{B} \times \vec{C}) &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ (B_yC_z - B_zC_y) & (B_zC_x - B_xC_z) & (B_xC_y - B_yC_x) \end{vmatrix} \\
&= [A_y(B_xC_y - B_yC_x) - A_z(B_zC_x - B_xC_z)]\hat{x} - \\
&\quad [A_x(B_xC_y - B_yC_x) - A_z(B_yC_z - B_zC_y)]\hat{y} + \\
&\quad [A_x(B_xC_z - B_zC_x) - A_y(B_yC_z - B_zC_y)]\hat{z}
\end{aligned}
$$

Right hand-side:

$$\vec{A} \cdot \vec{C} = A_xC_x + A_yC_y + A_zC_z$$

$$
\begin{aligned}
\vec{B}(\vec{A} \cdot \vec{C}) &= (B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) \underbrace{(A_xC_x + A_yC_y + A_zC_z)}_{\text{scalar}} \\
&= (B_xA_xC_x + B_xA_yC_y + B_xA_zC_z)\hat{x} + \\
&= (B_yA_xC_x + B_yA_yC_y + B_yA_zC_z)\hat{y} + \\
&= (B_zA_xC_x + B_zA_yC_y + B_zA_zC_z)\hat{z}
\end{aligned}
$$

$$
\begin{aligned}
\vec{C}(\vec{A}\cdot\vec{B}) &= (C_x\hat{x} + C_y\hat{y} + C_z\hat{z})(A_xB_x + A_yB_y + A_zB_z) \\
&= (C_xA_xB_x + C_xA_yB_y + C_xA_zB_z)\hat{x} + \\
&\quad (C_yA_xB_x + C_yA_yB_y + C_yA_zB_z)\hat{y} + \\
&\quad (C_zA_xB_x + C_zA_yB_y + C_zA_zB_z)\hat{z}
\end{aligned}
$$

$$
\begin{aligned}
\vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}) &= \underbrace{(B_xA_yC_y + B_xA_zC_z - C_xA_yB_y - C_xA_zB_z)}_{\text{x-component}}\hat{x} + (\dots)\hat{y} + (\dots)\hat{z} \\
\vec{A}\times(\vec{B}\times\vec{C}) &= [A_y(B_xC_y - B_yC_x) - A_z(B_zC_x - B_xC_z)]\hat{x} + (\dots)\hat{y} + (\dots)\hat{z} \\
&= \underbrace{(A_yB_xC_y - A_yB_yC_x - A_zB_zC_x + A_zB_xC_z)}_{\text{x-component}}\hat{x} + (\dots)\hat{y} + (\dots)\hat{z}
\end{aligned}
$$

Problem 2

1. Prove $\nabla\cdot(\nabla\times\vec{A}) = 0$