PEP542 Homework
Spring 2025

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- There could be typos, please let me know if you find any issues.
- Do not copy solutions from Google or other online resources. It is very easy to tell whether the solution is original work or not.
- You are encouraged to discuss the problems with the other students.
- Please scan your solutions (make sure that the solution is recognizable) and upload the PDF file to Canvas.
- Late homework will not be counted in the final grade.
- Each student is granted one late-homework exemption, provided the homework is submitted within five days of the deadline. Please use it wisely.

Homework 2 (Due 2pm, Feb 14)

Question 1: Calculate the line integral of the function $\vec{v} = x^2\hat{x} + 2yz\hat{y} + y^2\hat{z}$ from the origin to the point $(1, 1, 1)$ by three different routes:

(a) $(0, 0, 0) \to (1, 0, 0) \to (1, 1, 0) \to (1, 1, 1)$.

(b) $(0, 0, 0) \to (0, 0, 1) \to (0, 1, 1) \to (1, 1, 1)$.

(c) The direct straight line.

Question 2: Calculate the volume integral of the function $T = z^2$ over the tetrahedron with corners at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. [Hint: for one of the four surfaces, $x + y + z = 1$]

Question 3: Test the divergence theorem for the function $\vec{v} = (xy)\hat{x} + (2yz)\hat{y} + (3zx)\hat{z}$. Take as your volume the cube shown in fig.1.

Question 3: Test the divergence theorem for the function $\vec{v} = (xy)\hat{x} + (2yz)\hat{y} + (3zx)\hat{z}$. Take as your volume a cube with edge length 2, with one corner at the origin and the other corners located at $(2, 0, 0)$, $(0, 2, 0)$, $(0, 0, 2)$, $(2, 2, 0)$, $(2, 0, 2)$, $(0, 2, 2)$, $(2, 2, 2)$.

Question 4: Test Stokes' theorem for the vector field $\vec{v} = (xy)\hat{x} + (2yz)\hat{y} + (3zx)\hat{z}$, using the triangular area with three corners located at $(0, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.

Question 5: Show that

$$\int_S f(\nabla \times \vec{A}) \cdot d\vec{a} = \int_S [\vec{A} \times (\nabla f)] \cdot d\vec{a} + \oint_P f\vec{A} \cdot d\vec{l} \tag{1}$$

$$\int_V \vec{B} \cdot (\nabla \times \vec{A}) d\tau = \int_V \vec{A} \cdot (\nabla \times \vec{B}) d\tau + \oint_S (\vec{A} \times \vec{B}) \cdot d\vec{a} \tag{2}$$