

# Homework 1 - Electromagnetism

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I pledge my honor that I have abided by the Stevens Honor System.

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## Problem 1

Prove the identity  $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

Left hand-side:

$$\begin{aligned}\vec{B} \times \vec{C} &= (B_x \hat{x} + B_y \hat{y} + B_z \hat{z}) \times (C_x \hat{x} + C_y \hat{y} + C_z \hat{z}) \\ &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \\ &= (B_y C_z - B_z C_y) \hat{x} + (B_x C_z - B_z C_x) \hat{y} + (B_x C_y - B_y C_x) \hat{z} \\ \vec{A} \times (\vec{B} \times \vec{C}) &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ (B_y C_z - B_z C_y) & (B_x C_z - B_z C_x) & (B_x C_y - B_y C_x) \end{vmatrix} \\ &= [A_y(B_x C_y - B_y C_x) - A_z(B_x C_z - B_z C_x)] \hat{x} + \\ &\quad [A_x(B_x C_y - B_y C_x) - A_z(B_y C_z - B_z C_y)] \hat{y} + \\ &\quad [A_x(B_x C_z - B_z C_x) - A_y(B_y C_z - B_z C_y)] \hat{z}\end{aligned}$$

Right hand-side:

$$\begin{aligned}\vec{A} \cdot \vec{C} &= A_x C_x + A_y C_y + A_z C_z \\ \vec{B}(\vec{A} \cdot \vec{C}) &= (B_x \hat{x} + B_y \hat{y} + B_z \hat{z}) \underbrace{(A_x C_x + A_y C_y + A_z C_z)}_{\text{scalar}} \\ &= (B_x A_x C_x + B_x A_y C_y + B_x A_z C_z) \hat{x} + \\ &= (B_y A_x C_x + B_y A_y C_y + B_y A_z C_z) \hat{y} + \\ &= (B_z A_x C_x + B_z A_y C_y + B_z A_z C_z) \hat{z}\end{aligned}$$

$$\begin{aligned}
\vec{C}(\vec{A} \cdot \vec{B}) &= (C_x \hat{x} + C_y \hat{y} + C_z \hat{z})(A_x B_x + A_y B_y + A_z B_z) \\
&= (C_x A_x B_x + C_x A_y B_y + C_x A_z B_z) \hat{x} + \\
&\quad (C_y A_x B_x + C_y A_y B_y + C_y A_z B_z) \hat{y} + \\
&\quad (C_z A_x B_x + C_z A_y B_y + C_z A_z B_z) \hat{z}
\end{aligned}$$

$$\vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) = \underbrace{(B_x A_y C_y + B_x A_z C_z - C_x A_y B_y - C_x A_z B_z)}_{\text{x-component}} \hat{x} + (\dots) \hat{y} + (\dots) \hat{z}$$

$$\begin{aligned}
\vec{A} \times (\vec{B} \times \vec{C}) &= [A_y (B_x C_y - B_y C_x) - A_z (B_x C_z - B_z C_x)] \hat{x} + (\dots) \hat{y} + (\dots) \hat{z} \\
&= \underbrace{(A_y B_x C_y - A_y B_y C_x - A_z B_x C_z + A_z B_z C_x)}_{\text{x-component}} \hat{x} + (\dots) \hat{y} + (\dots) \hat{z}
\end{aligned}$$