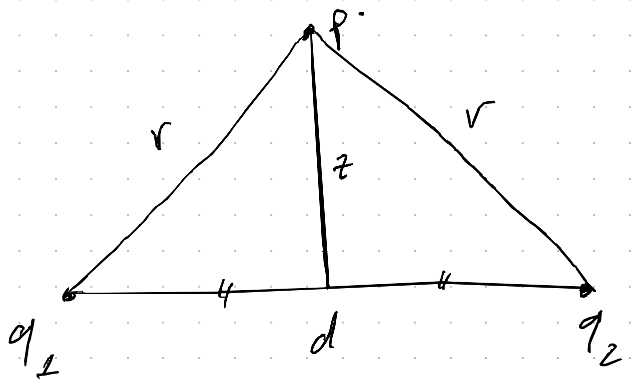
Homework 5.

(4)



$\vec{E} = -\nabla V$.

a) Calculate the electrostatic potential at point A.

potential of point charge q.

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad (2.26). \qquad r = \sqrt{z^2 + \left(\frac{d}{2}\right)^2}$$

$$V_1 = V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2 + \left(\frac{d}{2}\right)^2}}$$

$$\Rightarrow V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\sqrt{z^2 + \left(\frac{d}{2}\right)^2}}$$

b) $\vec{E} = -\nabla V$.

$$= -\Big(\underbrace{\frac{\partial V}{\partial x}}_{0}\hat{x} + \underbrace{\frac{\partial V}{\partial y}}_{0}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\Big)$$

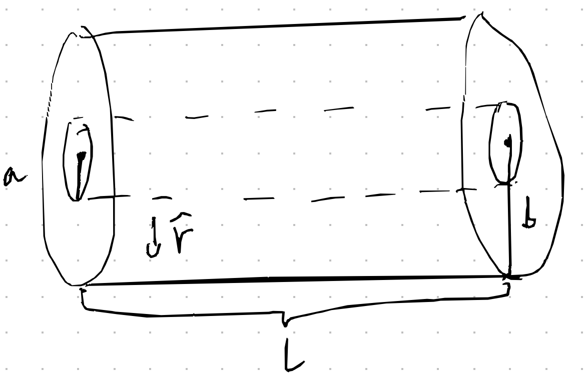
(no $\hat{x}$ and $\hat{y}$).

$$\Rightarrow \vec{E} = -\frac{\partial V}{\partial z}\hat{z} = \frac{-2q}{4\pi\varepsilon_0}\frac{d}{dz}\frac{1}{\sqrt{z^2 - \left(\frac{d}{2}\right)^2}}\hat{z}.$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2qz}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}}}\hat{z}$$

$$C = \frac{Q}{V}$$

(2)



Gauss Law.

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = E\cdot\underbrace{2\pi r\cdot L}_{\text{Gaussian surface}} = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \vec{E} = \frac{Q}{2\pi r L\varepsilon_0}\hat{r}$$

Electric potential difference between 2 cylinders.

$$V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$= -\int_a^b \frac{Q}{2\pi r L\varepsilon_0}\,dr =$$

$$= -\frac{Q}{2\pi L\varepsilon_0}\int_a^b \frac{1}{r}\,dr$$

$$= -\frac{Q}{2\pi L\varepsilon_0}\ln\left(\frac{b}{a}\right).$$

$$\Rightarrow C = \frac{Q}{V} = \frac{Q}{-\frac{Q}{2\pi L\varepsilon_0}\ln\left(\frac{b}{a}\right)} = \frac{-2\pi L\varepsilon_0}{\ln\left(\frac{b}{a}\right)}.$$