MA331 HW4

To get full credit you need to:

- Clearly state the correct answer.
- Justify and explain each step in your solution and the final answer.
- Make very clear that you understand the logic behind the steps.
- Using a symbolic algebra system to perform computations does not exonerate you to explain and motivate every step as mentioned above.

Very important: Honor code applies fully. You are required to sign the pledge. You must submit your own work only.

Problem 1

Prove the following:

(a) If $\hat{\Theta}_1$ is an unbiased estimator for θ, and W is a zero mean random variable, then

$$\hat{\Theta}_2 = \hat{\Theta}_1 + W$$

is also an unbiased estimator for θ.

(b) If $\hat{\Theta}_1$ is an estimator for θ such that $\mathbb{E}\left[\hat{\Theta}_1\right] = a\theta + b$, where $a \neq 0$, show that

$$\hat{\Theta}_2 = \frac{\hat{\Theta}_1 - b}{a}$$

is an unbiased estimator for θ.

Problem 2

Let $X_1, X_2, X_3, ..., X_n$ be a random sample with unknown mean $\mathbb{E}[X_i] = \mu$, and unknown variance $\mathrm{Var}[X_i] = \sigma^2$. Suppose that we would like to estimate $\theta = \mu^2$. We define the estimator $\hat{\Theta}$ as

$$\hat{\Theta} = (\bar{X})^2 = \left[\frac{1}{n}\sum_{k=1}^{n} X_k\right]^2$$

to estimate θ. Is $\hat{\Theta}$ an unbiased estimator for θ? Why?

Problem 3

Let $X_1, X_2, X_3, ..., X_n$ be a random sample from the following distribution

$$f_X(x) = \begin{cases} \theta(x - \frac{1}{2}) + 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta \in [-2, 2]$ is a unknown parameter. We define the estimator $\hat{\Theta}_n$ as

$$\hat{\Theta}_n = 12\bar{X} - 6$$

to estimate θ.

(a) is $\hat{\Theta}_n$ an unbiased estimator of θ?

(b) Is $\hat{\Theta}_n$ a consistent estimator of θ?

(c) Find the mean squared error (MSE) of $\hat{\Theta}_n$.

Problem 4

Let $X_1, ..., X_n$ be a random sample from a Poisson(λ) distribution. Find the log likelihood function and use that the obtain the ML estimator for λ, $\hat{\lambda}_{ML}$.